1. (4 pts)

Which probability involving rolling 2 fair, six-sided dice is estimated by the following simulation?

```
def simulate(num_samples=10000):
    die = np.array([1,2,3,4,5,6])
    roll1 = np.random.choice(die, size=num_samples)
    roll2 = np.random.choice(die, size=num_samples)
    data1 = (roll1 + roll2) == 4
    data2 = np.logical_or(roll1==3, roll2==3)
    both = np.logical_and(data1, data2)
    return np.sum(both)/np.sum(data1)
```

$P(A)$

$P(B)$

$= \frac{P(B \cap A)}{P(A)} = P(B|A)$

- ○ $P(\text{Dice sum to 4})$
- ○ $P(\text{Dice sum to 4} \mid \text{Roll at least one 3})$
- ○ $P(\text{Roll at least one 4} \mid \text{Roll at least one 3})$
- ○ $P(\text{Dice sum to 4} \cap \text{Roll at least one 3})$
- ○ $P(\text{Roll no 3}s \mid \text{Dice sum to 4})$
- ✓ $P(\text{Roll at least one 3} \mid \text{Dice sum to 4})$
- ○ $P(\text{Roll at least one 3})$
- ○ None of these

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2. (7 pts)

Suppose the probability of heavy snow tomorrow is 50%, the probability of strong winds tomorrow is 40%, and the probability of heavy snow **or** strong winds tomorrow is 70%.

Based on these probabilities, do you have enough information to determine if the events "heavy snow tomorrow" (S), and "strong winds tomorrow" (W) are independent or dependent? If so, determine if they are independent or dependent and justify your answer mathematically. If not, explain what additional information you would need.

$P(S) = 0.50$, $P(W) = 0.40$, $P(S \cup W) = 0.70$

$P(S \cup W) = P(S) + P(W) - P(S \cap W) \Rightarrow P(S \cap W) = P(S) + P(W) - P(S \cup W)$

$= 0.50 + 0.40 - 0.70 = 0.20$

$P(S \cap W) = P(S) \cdot P(W) = 0.50 \cdot 0.40 = 0.20$

$\Rightarrow (P(S \cap W) = 0.20) = (P(S) \cdot P(W) = 0.20)$ ✓ $\therefore$ Independent

3. (9 pts)

The probability that a quiz question is about the Kansas City Chiefs is $\frac{4}{5}$ if Professor Oscamou is tired and $\frac{2}{5}$ if she's not tired. Professor Oscamou is tired with probability $\frac{3}{4}$. Given that this problem is about the Kansas City Chiefs, what is the probability that Professor Oscamou is tired? Show all steps justifying your work. You can leave your answer unsimplified.

$\boxed{P(T|Q) = \frac{30}{35} \approx 0.857}$

$P(Q|T) = \frac{4}{5}$, $P(Q|T') = \frac{2}{5}$, $P(T) = \frac{3}{4}$

	P(T)	P(T')	
P(Q)	3/5	1/10	7/10
P(Q')	3/20	3/20	3/10
	3/4	1/4	1

$P(Q|T') = \frac{P(Q \cap T')}{P(T')}$

$\frac{10}{40} = \frac{4}{40} + x$

$\Rightarrow x = \frac{6}{40}$

$P(Q \cap T') = P(T')P(Q|T') = \frac{1}{4} \cdot \frac{2}{5} = \frac{1}{10}$

$P(Q|T) = \frac{P(Q \cap T)}{P(T)}$

$\frac{15}{20} = \frac{12}{20} + x$

$\Rightarrow x = \frac{3}{20}$

$P(Q \cap T) = P(T)P(Q|T) = \frac{3}{4} \cdot \frac{4}{5} = \frac{3}{5}$

$P(T|Q) = \frac{P(Q|T) \cdot P(T)}{P(Q)}$

$= \frac{\frac{4}{5} \cdot \frac{3}{4}}{\frac{7}{10}} = \frac{\frac{3}{5}}{\frac{7}{10}} = \frac{30}{35} \approx 0.857 : 85.7\%$