

I). Independent vs Dependent Events



Warm-Up: Suppose you roll a fair die two times.

Let A be the event "the sum of the throws equals 4" and

Let B be the event "at least one of the throws is a 3"

a). What is $P(A)$?

b). What is $P(B)$?

c). What is $P(A, B)$?

d). Are events A and B independent or dependent?

Independence: Events A and B are independent if and only if any one of the following holds

$$P(A | B) = P(A)$$

$$P(B | A) = P(B)$$

$$P(A \cap B) = P(A)P(B)$$

Note: The three statements above are equivalent. They will either all be true or all will be false.

Similarly, events A_1, A_2, \dots, A_m are **independent** if

$$P(A_1 \cap A_2 \cap \dots \cap A_m) = P(A_1)P(A_2) \dots P(A_m)$$

Ex 1).

Suppose you flip a fair coin twice.

Let A = "Heads on flip 1",

B = "Heads on flip 2", and

C = "Same outcome on both flips".

a). Are events A and B independent or dependent? Justify your answer using the mathematical definition.

b). Are events A and C independent or dependent? Justify your answer using the mathematical definition.

c). Are events A , B and C independent or dependent? Justify your answer using the mathematical definition.

Careful with terminology:

Independence is not the same as mutually exclusive/disjoint

A & B are DISJOINT if and only if:

A & B are INDEPENDENT if and only if:

II) Simulating Probabilities: See Lec12 notebook