

# 7.1 Binary trees

## Binary tree basics

In a list, each node has up to one successor. In a **binary tree**, each node has up to two children, known as a *left child* and a *right child*. "Binary" means two, referring to the two children. Some more definitions related to a binary tree:

- **Leaf**: A tree node with no children.
- **Internal node**: A node with at least one child.
- **Parent**: A node with a child is said to be that child's parent. A node's **ancestors** include the node's parent, the parent's parent, etc., up to the tree's root.
- **Root**: The one tree node with no parent (the "top" node).

Another section discusses binary tree usefulness; this section introduces definitions.

Below, each node is represented by just the node's key, as in B, although the node may have other data.

PARTICIPATION  
ACTIVITY

7.1.1: Binary tree basics.



### Animation captions:

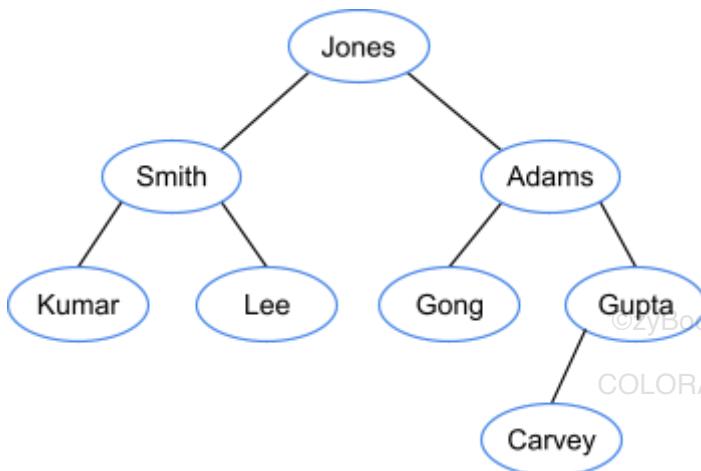
1. In a list, each node has up to one successor.
2. In a binary tree, each node has up to two children.
3. A tree is normally drawn vertically. Edge arrows are optional.
4. A node can have a left child and right child. A node with a child is called the child's parent.
5. First node: Root node. Node without child: Leaf node. Node with child: Internal nodes.

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7.1.2: Binary tree basics.



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- 1) Root node: \_\_\_\_\_.

 //**Check****Show answer**

- 2) Smith's left child: \_\_\_\_\_.

 //**Check****Show answer**

- 3) The tree has four leaf nodes:

Kumar, Lee, Gong, and \_\_\_\_\_.

 //**Check****Show answer**

- 4) How many internal nodes does the tree have?

 //**Check****Show answer**

- 5) The tree has an internal node, Gupta, with only one child rather than two. Is the tree still a binary tree? Type yes or no.

 //

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**Check****Show answer**

## Depth, level, and height

A few additional terms:

- The link from a node to a child is called an **edge**.
- A node's **depth** is the number of edges on the path from the root to the node. The root node thus has depth 0.
- All nodes with the same depth form a tree **level**.
- A tree's **height** is the largest depth of any node. A tree with just one node has height 0.

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7.1.3: Binary tree terminology: height, depth, and level.

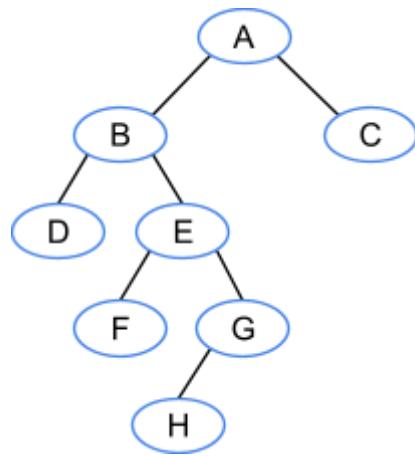


### Animation captions:

- The binary tree has edges A to B, A to C, and C to D.
- A node's depth is the number of edges from the root to the node.
- Nodes with the same depth form a level.
- A tree's height is the largest depth of any node.

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7.1.4: Binary tree height, depth, and level.



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1) A's depth is 1.

- True  
 False

2) E's depth is 2.

- True



False

3) B and C form level 1.

 True False

4) D, F, and H form level 2.

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 True False

5) The tree's height is 4.

 True False

6) A tree with just one node has height 0.

 True False

## Special types of binary trees

Certain binary tree structures can affect the speed of operations on the tree. The following describe special types of binary trees:

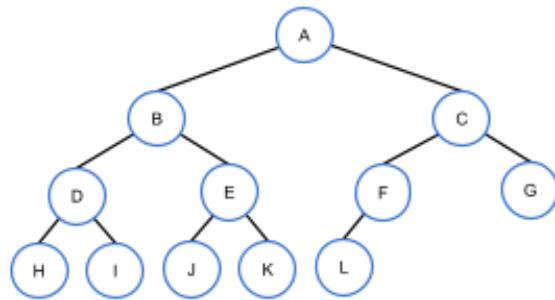
- A binary tree is **full** if every node contains 0 or 2 children.
- A binary tree is **complete** if all levels, except possibly the last level, contain all possible nodes and all nodes in the last level are as far left as possible.
- A binary tree is **perfect**, if all internal nodes have 2 children and all leaf nodes are at the same level.

Figure 7.1.1: Special types of binary trees: full, complete, perfect.

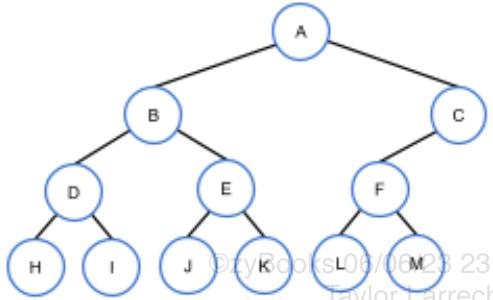
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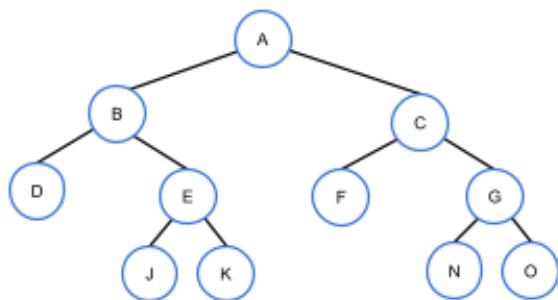
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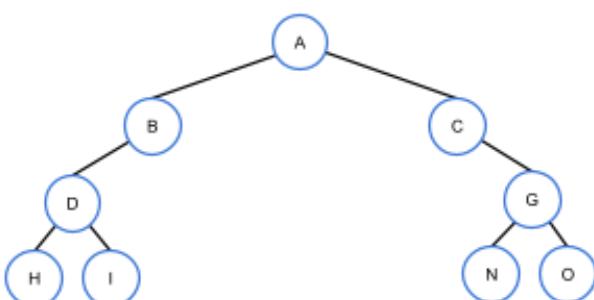
Not full, complete, not perfect



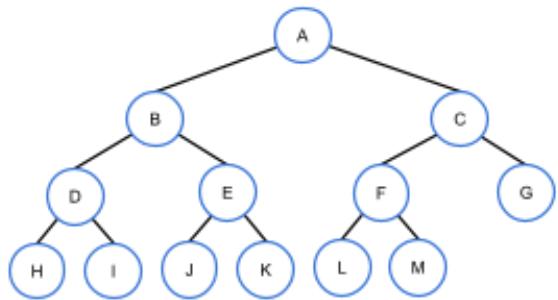
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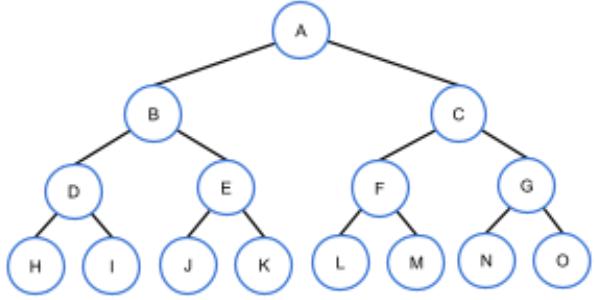
Full, not complete, not perfect



Not full, not complete, not perfect



Full, complete, not perfect



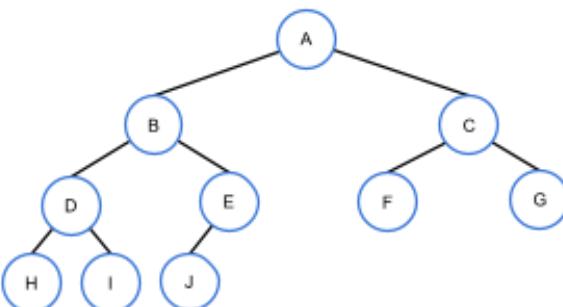
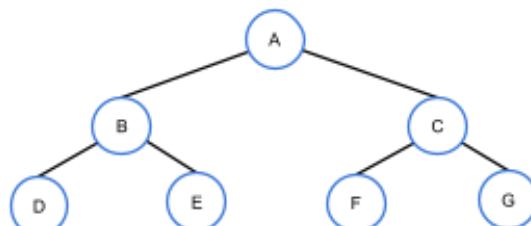
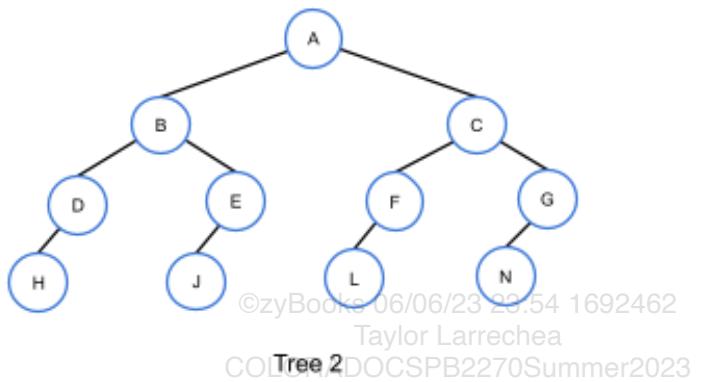
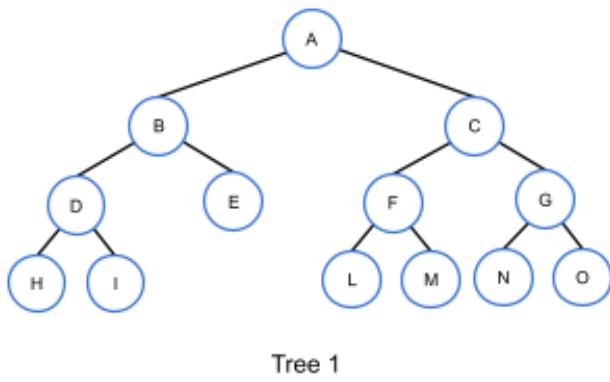
Full, complete, perfect

PARTICIPATION ACTIVITY

7.1.5: Identifying special types of binary trees.



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If unable to drag and drop, refresh the page.

**Not full, not complete, not perfect**

**Full, complete, perfect**

**Full, not complete, not perfect**

**Not full, complete, not perfect**

Tree 1

Tree 2

Tree 3

Tree 4

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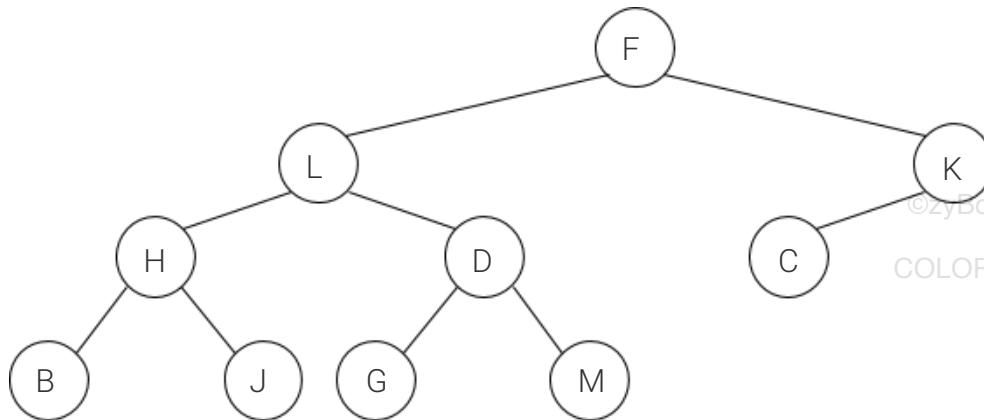
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**CHALLENGE ACTIVITY**

7.1.1: Binary trees.



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**Start**

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What is the root node?

Ex: A

What is the parent of node J?

Ex: A

What are the ancestors of node J?

Ex: A, B, C

What are the leaf nodes?

Ex: A, B, C

How many internal nodes does the tree have?

Ex: 9

1

2

3

**Check****Next**

## 7.2 Applications of trees

### File systems

Trees are commonly used to represent hierarchical data. A tree can represent files and directories in a file system, since a file system is a hierarchy.

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**PARTICIPATION ACTIVITY**

7.2.1: A file system is a hierarchy that can be represented by a tree.



### Animation content:

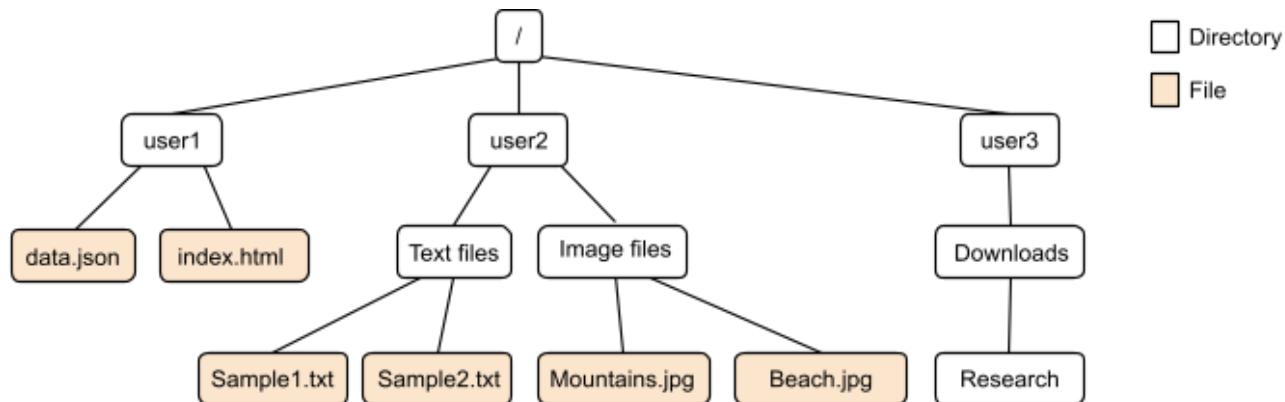
undefined

## Animation captions:

1. A tree representing a file system has the filesystem's root directory ("/"), represented by the root node.
  2. The root contains 2 configuration text files and 2 additional directories: user1 and user2.
  3. Directories contain additional entries. Only empty directories will be leaf nodes. All files are leaf nodes.
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### 7.2.2: Analyzing a file system tree.



1) What is the depth of the "Mountains.jpg" file node?



- 3
- 4

2) What is the tree's height?



- 3
- 4
- 14

3) What is the parent of the "Text files" node?



- The "user2" directory node
- The "Image files" directory node
- The "Sample1.txt" file node.

4) Which operation would increase the height of the tree?



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- Adding a new file into the user1 directory
- Adding a new directory into the "Image files" directory
- Adding a new directory into the "Research" directory

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## 7.2.3: File system trees.



- 1) A file in a file system tree is always a leaf node.
  - True
  - False
- 2) A directory in a file system tree is always an internal node.
  - True
  - False
- 3) Using a tree data structure to implement a file system requires that each directory node support a variable number of children.
  - True
  - False



## Binary space partitioning

**Binary space partitioning (BSP)** is a technique of repeatedly separating a region of space into 2 parts and cataloging objects contained within the regions. A **BSP tree** is a binary tree used to store information for binary space partitioning. Each node in a BSP tree contains information about a region of space and which objects are contained in the region.

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In graphics applications, a BSP tree can be used to store all objects in a multidimensional world. The BSP tree can then be used to efficiently determine which objects must be rendered on screen. The viewer's position in space is used to perform a lookup within the BSP tree. The lookup quickly eliminates a large number of objects that are not visible and therefore should not be rendered.

**PARTICIPATION ACTIVITY**

## 7.2.4: A BSP tree is used to quickly determine which objects do not need to be rendered.



## Animation content:

undefined

## Animation captions:

1. Data for a large, open 2-D world contains many objects. Only a few objects are visible on screen at any given moment.
2. Avoiding rendering off-screen objects is crucial for realtime graphics. But checking the intersection of all objects with the screen's rectangle is too time consuming.
3. A BSP tree represents partitioned space. The root represents the entire world and stores a list of all objects in the world, as well as the world's geometric boundary.
4. The root's left child represents the world's left half. The node stores information about the left half's geometric boundary, and a list of all objects contained within.
5. The root's right child contains similar information for the right half.
6. Using the screen's position within the world as a lookup into the BSP tree quickly yields the right node's list of objects. A large number of objects are quickly eliminated from the list of potential objects on screen.
7. Further partitioning makes the tree even more useful.

### PARTICIPATION ACTIVITY

#### 7.2.5: Binary space partitioning.



- 1) When traversing down a BSP tree, half the objects are eliminated each level.

- True
- False



- 2) A BSP implementation could choose to split regions in arbitrary locations, instead of right down the middle.

- True
- False



- 3) In the animation, if the parts of the screen were in 2 different regions, then all objects from the 2 regions would have to be analyzed when rendering.

- True
- False

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- 4) BSP can be used in 3-D graphics as well as 2-D.

- True
- False

## Using trees to store collections

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Most of the tree data structures discussed in this book serve to store a collection of values. Numerous tree types exist to store data collections in a structured way that allows for fast searching, inserting, and removing of values.

## 7.3 Binary search



This section has been set as optional by your instructor.

### Linear search vs. binary search

Linear search may require searching all list elements, which can lead to long runtimes. For example, searching for a contact on a smartphone one-by-one from first to last can be time consuming. Because a contact list is sorted, a faster search, known as binary search, checks the middle contact first. If the desired contact comes alphabetically before the middle contact, binary search will then search the first half and otherwise the last half. Each step reduces the contacts that need to be searched by half.

#### PARTICIPATION ACTIVITY

7.3.1: Using binary search to search contacts on your phone.



#### Animation captions:

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1. A contact list stores contacts sorted by name. Searching for Pooja using a binary search starts by checking the middle contact.
2. The middle contact is Muhammad. Pooja is alphabetically after Muhammad, so the binary search only searches the contacts after Muhammad. Only half the contacts now need to be searched.

3. Binary search continues by checking the middle element between Muhammad and the last contact. Pooja is before Sharod, so the search continues with only those contacts between Muhammad and Sharod, which is one fourth of the contacts.
4. The middle element between Muhammad and Sharod is Pooja. Each step reduces the number of contacts to search by half.

**PARTICIPATION ACTIVITY**

7.3.2: Using binary search to search a contact list.

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A contact list is searched for Bob.

Assume the following contact list: ( Amy, Bob, Chris, Holly, Ray, Sarah, Zoe )

- 1) What is the first contact searched?

**Check****Show answer**

- 2) What is the second contact searched?

**Check****Show answer**

## Binary search algorithm

**Binary search** is a faster algorithm for searching a list if the list's elements are sorted and directly accessible (such as an array). Binary search first checks the middle element of the list. If the search key is found, the algorithm returns the matching location. If the search key is not found, the algorithm repeats the search on the remaining left sublist (if the search key was less than the middle element) or the remaining right sublist (if the search key was greater than the middle element).

**PARTICIPATION ACTIVITY**

7.3.3: Binary search efficiently searches sorted list by reducing the search space by half each iteration.

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## Animation captions:

1. Elements with indices between low and high remain to be searched.
2. Search starts by checking the middle element.
3. If search key is greater than element, then only elements in right sublist need to be searched.
4. Each iteration reduces search space by half. Search continues until key found or search space is empty.

## Figure 7.3.1: Binary search algorithm.

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```
BinarySearch(numbers, numbersSize, key) {  
    mid = 0  
    low = 0  
    high = numbersSize - 1  
  
    while (high >= low) {  
        mid = (high + low) / 2  
        if (numbers[mid] < key) {  
            low = mid + 1  
        }  
        else if (numbers[mid] > key) {  
            high = mid - 1  
        }  
        else {  
            return mid  
        }  
    }  
  
    return -1 // not found  
}  
  
main() {  
    numbers = { 2, 4, 7, 10, 11, 32, 45, 87 }  
    NUMBERS_SIZE = 8  
    i = 0  
    key = 0  
    keyIndex = 0  
  
    print("NUMBERS: ")  
    for (i = 0; i < NUMBERS_SIZE; ++i) {  
        print(numbers[i] + " ")  
    }  
    printLine()  
  
    print("Enter a value: ")  
    key = getIntFromUser()  
  
    keyIndex = BinarySearch(numbers, NUMBERS_SIZE, key)  
  
    if (keyIndex == -1) {  
        printLine(key + " was not found.")  
    }  
    else {  
        printLine("Found " + key + " at index " + keyIndex + ".")  
    }  
}
```

```
NUMBERS: 2 4 7 10 11 32 45 87  
Enter a value: 10  
Found 10 at index 3.  
...  
NUMBERS: 2 4 7 10 11 32 45 87  
Enter a value: 17  
17 was not found.
```

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**PARTICIPATION ACTIVITY**

7.3.4: Binary search algorithm execution.



Given list: ( 4, 11, 17, 18, 25, 45, 63, 77, 89, 114 ).

- 1) How many list elements will be checked to find the value 77 using binary search?

**Check****Show answer**

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- 2) How many list elements will be checked to find the value 17 using binary search?

**Check****Show answer**

- 3) Given an array with 32 elements, how many list elements will be checked if the key is less than all elements in the list, using binary search?

**Check****Show answer**

## Binary search efficiency

Binary search is incredibly efficient in finding an element within a sorted list. During each iteration or step of the algorithm, binary search reduces the search space (i.e., the remaining elements to search within) by half. The search terminates when the element is found or the search space is empty (element not found). For a 32 element list, if the search key is not found, the search space is halved to have 16 elements, then 8, 4, 2, 1, and finally none, requiring only 6 steps. For an  $N$  element list, the maximum number of steps required to reduce the search space to an empty sublist is

. Ex:

**PARTICIPATION ACTIVITY**

7.3.5: Speed of linear search versus binary search to find a number within a sorted list.



## Animation captions:

1. A binary search begins with the middle element of the list. Each subsequent search reduces the search space by half. Using binary search, a match was found with only 3 comparisons.
2. Using linear search, a match was found after 6 comparisons. Compared to a linear search, binary search is incredibly efficient in finding an element within a sorted list.

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If each comparison takes 1  $\mu$ s (1 microsecond), a binary search algorithm's runtime is at most 20  $\mu$ s to search a list with 1,000,000 elements, 21  $\mu$ s to search 2,000,000 elements, 22  $\mu$ s to search 4,000,000 elements, and so on. Ex: Searching Amazon's online store, which has more than 200 million items, requires less than 28  $\mu$ s; up to 7,000,000 times faster than linear search.

### PARTICIPATION ACTIVITY

#### 7.3.6: Linear and binary search efficiency.



- 1) Suppose a list of 1024 elements is searched with linear search. How many distinct list elements are compared against a search key that is less than all elements in the list?

elements

**Check**

[Show answer](#)



- 2) Suppose a sorted list of 1024 elements is searched with binary search. How many distinct list elements are compared against a search key that is less than all elements in the list?

elements

**Check**

[Show answer](#)



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### CHALLENGE ACTIVITY

#### 7.3.1: Binary search.



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**Start**

A last names list is searched for Boyd using binary search.

Last names list: ( Boyd, Diaz, Ford, Hall, Hill, King, Lee, Long, Page, Webb )

What is the first last name searched?

Ex: Webb

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What is the second last name searched?

1

2

3

4

5

Check

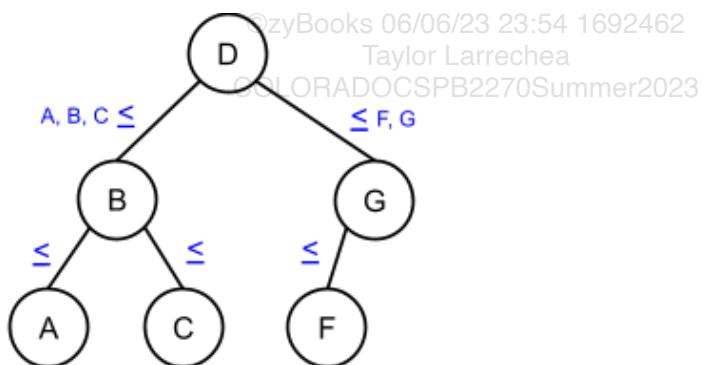
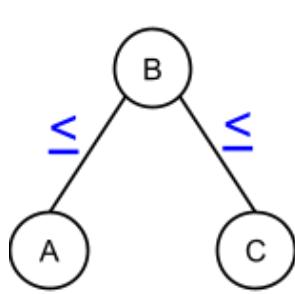
Next

## 7.4 Binary search trees

### Binary search trees

An especially useful form of binary tree is a **binary search tree** (BST), which has an ordering property that any node's left subtree keys  $\leq$  the node's key, and the right subtree's keys  $\geq$  the node's key. That property enables fast searching for an item, as will be shown later.

Figure 7.4.1: BST ordering property: For three nodes, left child is less-than-or-equal-to parent, parent is less-than-or-equal-to right child. For more nodes, all keys in subtrees must satisfy the property, for every node.



**PARTICIPATION ACTIVITY**

7.4.1: BST ordering properties.

**Animation content:**

undefined

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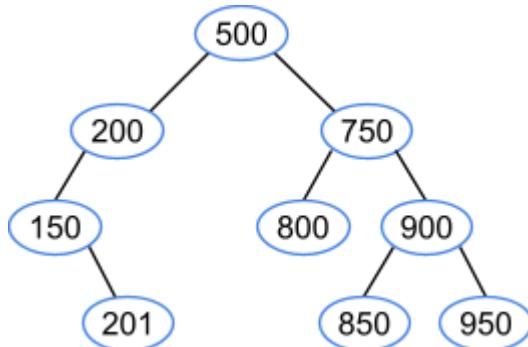
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**Animation captions:**

1. BST ordering property: Left subtree's keys  $\leq$  node's key, right subtree's keys  $\geq$  node's key.
2. All keys in subtree must obey the ordering property. Not a BST.
3. All keys in subtree must obey the ordering property. Not a BST.
4. All keys in subtree must obey the ordering property. Valid BST.

**PARTICIPATION ACTIVITY**

7.4.2: Binary search tree: Basic ordering property.



1) Does node 900 and the node's subtrees obey the BST ordering property?

- Yes  
 No

2) Does node 750 and the node's subtrees obey the BST ordering property?

- Yes  
 No

3) Does node 150 and the node's subtrees obey the BST ordering property?

- Yes  
 No

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- 4) Does node 200 and the node's subtrees obey the BST ordering property?

Yes  
 No

- 5) Is the tree a binary search tree?

Yes  
 No

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- 6) Is the tree a binary tree?

Yes  
 No



- 7) Would inserting 300 as the right child of 200 obey the BST ordering property (considering only nodes 300, 200, and 500)?

Yes  
 No



## Searching

To **search** nodes means to find a node with a desired key, if such a node exists. A BST may yield faster searches than a list. Searching a BST starts by visiting the root node (which is the first currentNode below):

Figure 7.4.2: Searching a BST.

```
if (currentNode->key == desiredKey) {  
    return currentNode; // The desired node was  
    found  
}  
else if (desiredKey < currentNode->key) {  
    // Visit left child, repeat  
}  
else if (desiredKey > currentNode->key) {  
    // Visit right child, repeat  
}
```

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If a child to be visited doesn't exist, the desired node does not exist. With this approach, only a small fraction of nodes need be compared.

**PARTICIPATION ACTIVITY**

7.4.3: A BST may yield faster searches than a list.


**Animation captions:**

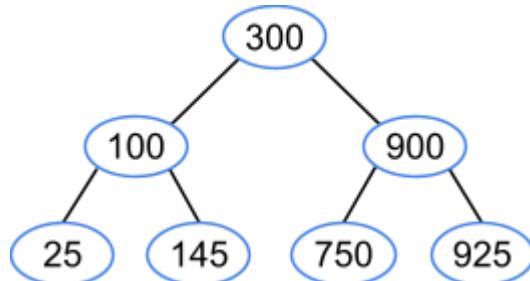
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1. Searching a 7-node list may require up to 7 comparisons. COLORADOCSPB2270Summer2023
2. In a BST, if desired key equals current node's key, return found. If less, descend to left child. If greater, descend to right child.
3. Searching a BST may require fewer comparisons, in this case 3 vs. 7.

**PARTICIPATION ACTIVITY**

7.4.4: Searching a BST.



- 1) In searching for 145, what node is visited first?



**Check**

[Show answer](#)

- 2) In searching for 145, what node is visited second?



**Check**

[Show answer](#)

- 3) In searching for 145, what node is visited third?



**Check**

[Show answer](#)

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- 4) Which nodes would be visited when searching for 900? Write nodes in order visited, as: 5, 10

 //**Check****Show answer**

- 5) Which nodes would be visited when searching for 800? Write nodes in order visited, as: 5, 10, 15

 //**Check****Show answer**

- 6) What is the worst case (largest) number of nodes visited when searching for a key?

 //**Check****Show answer**

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## BST search runtime

Searching a BST in the worst case requires  $H + 1$  comparisons, meaning  $O(H)$  comparisons, where  $H$  is the tree height. Ex: A tree with a root node and one child has height 1; the worst case visits the root and the child:  $1 + 1 = 2$ . A major BST benefit is that an  $N$ -node binary tree's height may be as small as  $O(\sqrt{N})$ , yielding extremely fast searches. Ex: A 10,000 node list may require 10,000 comparisons, but a 10,000 node BST may require only 14 comparisons.

A binary tree's height can be minimized by keeping all levels full, except possibly the last level. Such an "all-but-last-level-full" binary tree's height is .

Table 7.4.1: Minimum binary tree heights for  $N$  nodes are equivalent to

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Nodes $N$	Height $H$			Nodes per level
1	0	0	0	1

2	1	1	1	1/1
3	1	1.6	1	1/2
4	2	2	2	1/2/1
5	2	2.3	2	1/2/2
6	2	2.6	2	1/2/3
7	2	2.8	2	1/2/4
8	3	3	3	1/2/4/1
9	3	3.2	3	1/2/4/2
...				
15	3	3.9	3	1/2/4/8
16	4	4	4	1/2/4/8/1

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7.4.5: Searching a perfect BST with N nodes requires only O(      ) comparisons.

**Animation captions:**

1. A perfect binary tree has height .
2. A perfect binary tree search is O(   ), so O(   ).
3. Searching a BST may be faster than searching a list.

**PARTICIPATION ACTIVITY**

7.4.6: Searching BSTs with N nodes.



What is the worst case (largest) number of comparisons given a BST with N nodes?

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- 1) Perfect BST with N = 7

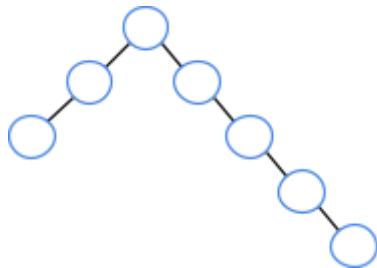
- 
- 
- 

- 2) Perfect BST with N = 31



- 31
- 4
- 5

3) Given the following tree.



- 3
- 5

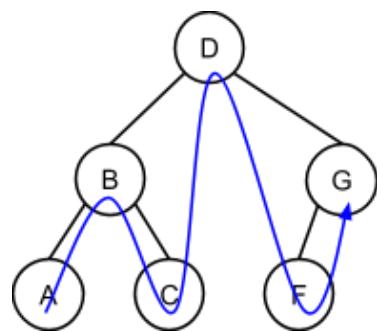
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## Successors and predecessors

A BST defines an ordering among nodes, from smallest to largest. A BST node's **successor** is the node that comes after in the BST ordering, so in A B C, A's successor is B, and B's successor is C. A BST node's **predecessor** is the node that comes before in the BST ordering.

If a node has a right subtree, the node's successor is that right subtree's leftmost child: Starting from the right subtree's root, follow left children until reaching a node with no left child (may be that subtree's root itself). If a node doesn't have a right subtree, the node's successor is the first ancestor having this node in a left subtree. Another section provides an algorithm for printing a BST's nodes in order.

Figure 7.4.3: A BST defines an ordering among nodes.



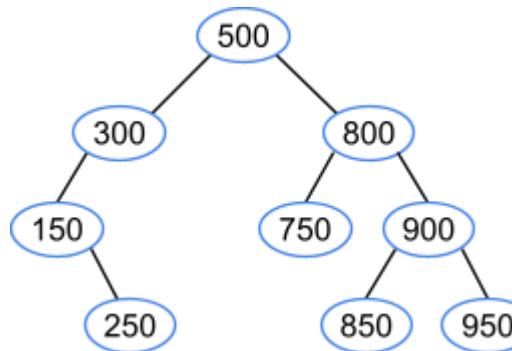
BST ordering:  
A B C D F G

Successor follows in ordering.  
Ex: D's successor is F.

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### PARTICIPATION ACTIVITY

7.4.7: Binary search tree: Defined ordering.



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- 1) The first node in the BST ordering is 150.

True  
 False

- 2) 150's successor is 250.

True  
 False

- 3) 250's successor is 300.

True  
 False

- 4) 500's successor is 850.

True  
 False

- 5) 950's successor is 150.

True  
 False

- 6) 950's predecessor is 900.

True  
 False

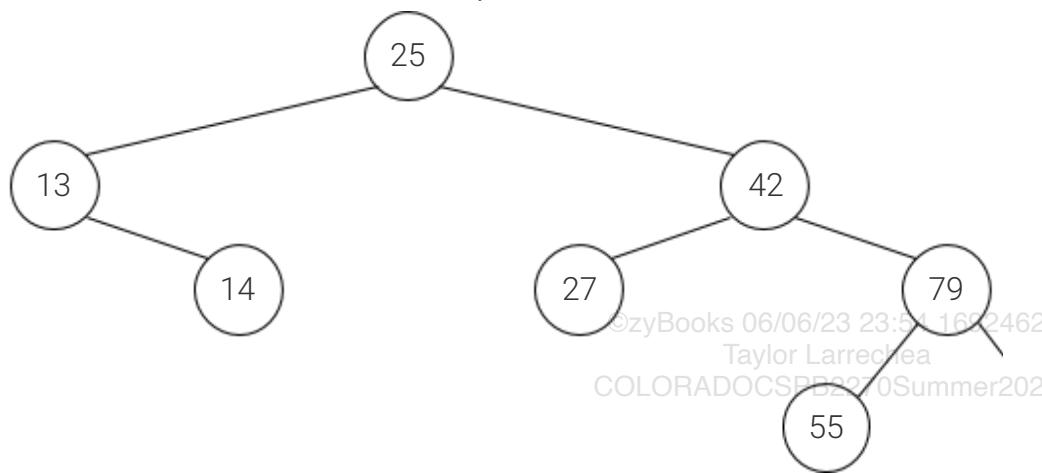
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**CHALLENGE ACTIVITY**

7.4.1: Binary search trees.

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Start



Does node 13 and the node's subtrees obey the BST ordering property? ▼

Does node 42 and the node's subtrees obey the BST ordering property? ▼

Is the tree a BST? ▼

1

2

3

4

5

Check

Next

## 7.5 BST search algorithm

Given a key, a **search** algorithm returns the first node found matching that key, or returns null if a matching node is not found. A simple BST search algorithm checks the current node (initially the tree's root), returning that node as a match, else assigning the current node with the left (if key is less) or right (if key is greater) child and repeating. If such a child is null, the algorithm returns null (matching node not found).

PARTICIPATION ACTIVITY

7.5.1: BST search algorithm.



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### Animation content:

undefined

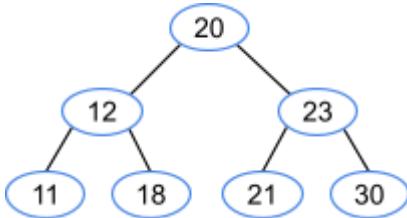
### Animation captions:

1. BST search algorithm checks current node, returning a match if found. Otherwise, assigns current node with left (if key is less) or right (if key is greater) child and continues search.

2. If the child to be visited does not exist, the algorithm returns null indicating no match found.

**PARTICIPATION ACTIVITY****7.5.2: BST search algorithm.**

Consider the following tree.



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1) When searching for key 21, what node is visited first?

- 20
- 21



2) When searching for key 21, what node is visited second?

- 12
- 23



3) When searching for key 21, what node is visited third?

- 21
- 30



4) If the current node matches the key, when does the algorithm return the node?

- Immediately
- Upon exiting the loop



5) If the child to be visited is null, when does the algorithm return null?

- Immediately
- Upon exiting the loop



6) What is the maximum loop iterations for a perfect binary tree with 7 nodes, if a node matches?



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3 7

- 7) What is the maximum loop iterations for a perfect binary tree with 7 nodes, if no node matches?

 3 7

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- 8) What is the maximum loop iterations for a perfect binary tree with 255 nodes?

 8 255

- 9) Suppose node 23 was instead 21, meaning two 21 nodes exist (which is allowed in a BST). When searching for 21, which node will be returned?

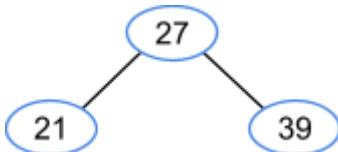
 Leaf Internal
**PARTICIPATION ACTIVITY**

7.5.3: BST search algorithm decisions.



Determine cur's next assignment given the key and current node.

- 1) key = 40, cur = 27

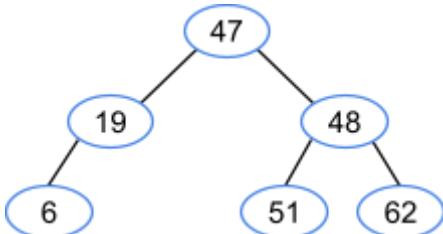
 27 21 39

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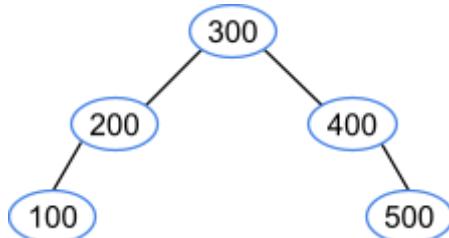
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- 2) key = 6, cur = 47



- 6
- 19
- 48

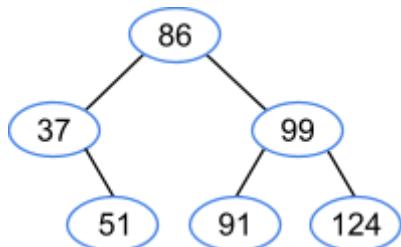
3) key 350, cur = 400



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- Search terminates and returns null.
- 400
- 500

4) key 91, cur = 99



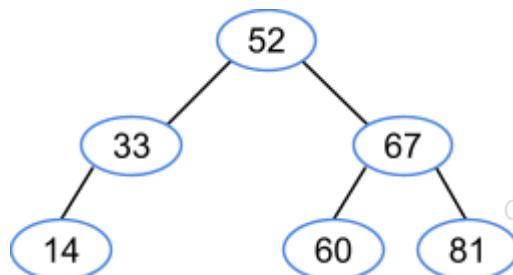
- 86
- 91
- 124

PARTICIPATION ACTIVITY

7.5.4: Tracing a BST search.



Consider the following tree. If node does not exist, enter null.



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1) When searching for key 45, what node is visited first?



**Check****Show answer**

- 2) When searching for key 45, what node is visited second?

**Check****Show answer**

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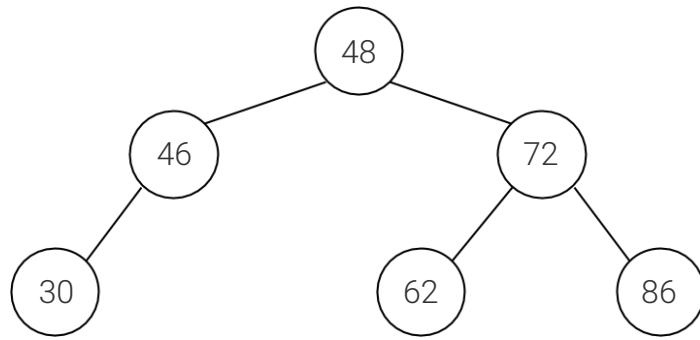
- 3) When searching for key 45, what node is visited third?

**Check****Show answer****CHALLENGE ACTIVITY**

7.5.1: BST search algorithm.



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**Start**

What does BSTSearch(tree, 50) return?



1

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?

**Check****Next**

## 7.6 BST insert algorithm

Given a new node, a BST **insert** operation inserts the new node in a proper location obeying the BST ordering property. A simple BST insert algorithm compares the new node with the current node (initially the root).

- *Insert as left child:* If the new node's key is less than the current node, and the current node's left child is null, the algorithm assigns that node's left child with the new node.
- *Insert as right child:* If the new node's key is greater than or equal to the current node, and the current node's right child is null, the algorithm assigns the node's right child with the new node.
- *Search for insert location:* If the left (or right) child is not null, the algorithm assigns the current node with that child and continues searching for a proper insert location.

**PARTICIPATION ACTIVITY**

7.6.1: Binary search tree insertions.

**Animation content:**

undefined

**Animation captions:**

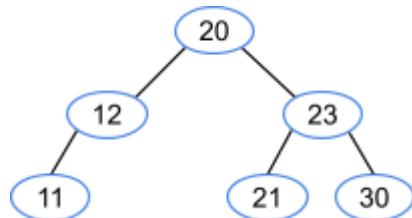
1. A node inserted into an empty tree will become the tree's root.
2. The BST is searched to find a suitable location to insert the new node as a leaf node.

**PARTICIPATION ACTIVITY**

7.6.2: BST insert algorithm.



Consider the following tree.



- 1) Where will a new node 18 be inserted?

- 12's right child
- 11's right child

- 2) Where will a new node 11 be inserted?

(So two nodes of 11 will exist).

- 11's left child

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11's right child

- 3) Assume a perfect 7-node BST. How many algorithm loop iterations will occur for an insert?

3

7

- 4) Assume a perfect 255-node BST. How many algorithm loop iterations will occur for an insert?

8

255

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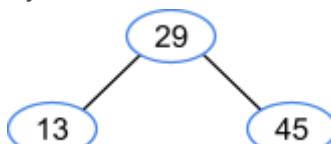
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**PARTICIPATION ACTIVITY**

7.6.3: BST insert algorithm decisions.

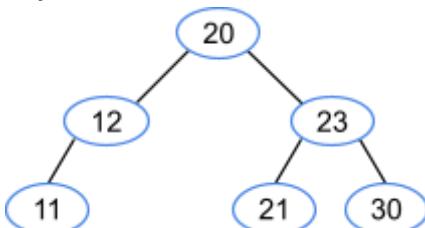
Determine the insertion algorithm's next step given the new node's key and the current node.

- 1) key = 7, currentNode = 29



- currentNode->left = node
- currentNode =  
currentNode->right
- currentNode = currentNode->left

- 2) key = 18, currentNode = 12



- currentNode->left = node
- currentNode->right = node
- currentNode =  
currentNode->right

- 3) key = 87, currentNode = null, tree->root  
= null (empty tree)

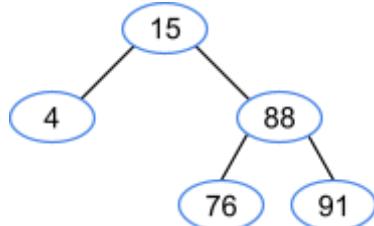
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- tree $\rightarrow$ root = node
- currentNode $\rightarrow$ right = node
- currentNode $\rightarrow$ left = node

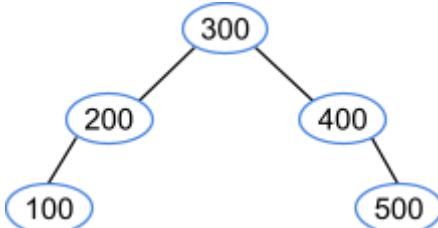
4) key = 53, currentNode = 76



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- currentNode $\rightarrow$ left = node
- currentNode $\rightarrow$ right = node
- tree $\rightarrow$ root = node

5) key = 600, currentNode = 400

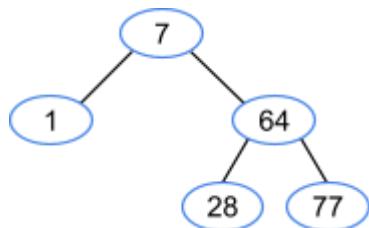


- currentNode $\rightarrow$ left = node
- currentNode =  
currentNode $\rightarrow$ right
- currentNode $\rightarrow$ right = node

**PARTICIPATION  
ACTIVITY**

7.6.4: Tracing BST insertions.

Consider the following tree.



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1) When inserting a new node with key 35, what node is visited first?

**Check**

**Show answer**



- 2) When inserting a new node with key 35, what node is visited second?

 //**Check****Show answer**

- 3) When inserting a new node with key 35, what node is visited third?

 //**Check****Show answer**

- 4) Where is the new node inserted?

Type: left or right

 //**Check****Show answer**

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## BST insert algorithm complexity

The BST insert algorithm traverses the tree from the root to a leaf node to find the insertion location. One node is visited per level. A BST with  $N$  nodes has at least

levels and at most levels. Therefore, the runtime complexity of insertion is best case and worst case .

The space complexity of insertion is because only a single pointer is used to traverse the tree to find the insertion location.

### CHALLENGE ACTIVITY

#### 7.6.1: BST insert algorithm.

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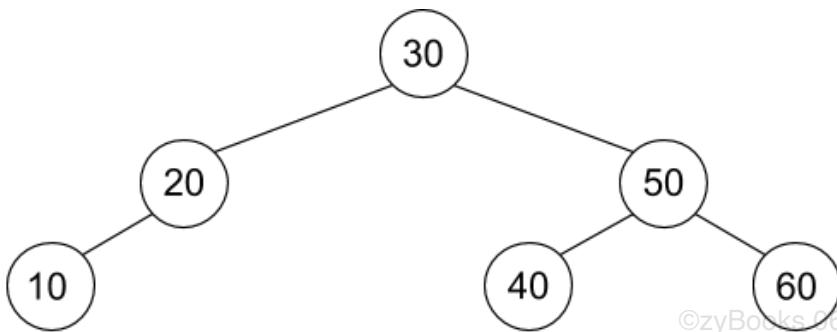
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**Start**



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Where will a new node 25 be inserted?

 child of node

1	2	3
---	---	---

[Check](#) [Next](#)

Exploring further:

- [Binary search tree visualization](#)

## 7.7 BST remove algorithm

Given a key, a BST **remove** operation removes the first-found matching node, restructuring the tree to preserve the BST ordering property. The algorithm first searches for a matching node just like the search algorithm. If found (call this node X), the algorithm performs one of the following sub-algorithms:

- *Remove a leaf node*: If X has a parent (so X is not the root), the parent's left or right child (whichever points to X) is assigned with null. Else, if X was the root, the root pointer is assigned with null, and the BST is now empty.
- *Remove an internal node with single child*: If X has a parent (so X is not the root), the parent's left or right child (whichever points to X) is assigned with X's single child. Else, if X was the root, the root pointer is assigned with X's single child.
- *Remove an internal node with two children*: This case is the hardest. First, the algorithm locates X's successor (the leftmost child of X's right subtree), and copies the successor to X. Then, the algorithm recursively removes the successor from the right subtree.

**PARTICIPATION ACTIVITY**

7.7.1: BST remove: Removing a leaf, or an internal node with a single child.

**Animation captions:**

1. Removing a leaf node: The parent's right child is assigned with null.
2. Remove an internal node with a single child: The parent's right child is assigned with node's single child.

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7.7.2: BST remove: Removing internal node with two children.

**Animation captions:**

1. Find successor: Leftmost child in node 25's right subtree is node 27.
2. Copy successor to current node.
3. Remove successor from right subtree.

Figure 7.7.1: BST remove algorithm.

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```
BSTRemove(tree, key) {
    par = null
    cur = tree->root
    while (cur is not null) { // Search for node
        if (cur->key == key) { // Node found
            if (cur->left is null && cur->right is null) { // Remove leaf
                if (par is null) // Node is root
                    tree->root = null
                else if (par->left == cur)
                    par->left = null
                else
                    par->right = null
            }
            else if (cur->right is null) { // Remove node with
                only left child
                    if (par is null) // Node is root
                        tree->root = cur->left
                    else if (par->left == cur)
                        par->left = cur->left
                    else
                        par->right = cur->left
                }
            else if (cur->left is null) { // Remove node with
                only right child
                    if (par is null) // Node is root
                        tree->root = cur->right
                    else if (par->left == cur)
                        par->left = cur->right
                    else
                        par->right = cur->right
                }
            else { // Remove node with two
                children
                    // Find successor (leftmost child of right subtree)
                    suc = cur->right
                    while (suc->left is not null)
                        suc = suc->left
                    successorData = Create copy of suc's data
                    BSTRemove(tree, suc->key) // Remove successor
                    Assign cur's data with successorData
                }
            return // Node found and removed
        }
        else if (cur->key < key) { // Search right
            par = cur
            cur = cur->right
        }
        else { // Search left
            par = cur
            cur = cur->left
        }
    }
    return // Node not found
}
```

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## BST remove algorithm complexity

The BST remove algorithm traverses the tree from the root to find the node to remove. When the node being removed has two children, the node's successor is found and a recursive call is made. One node is visited per level, and in the worst case scenario the tree is traversed twice from the root to a leaf. A BST with ~~nodes has at least~~ ©zyBooks 06/06/23 23:54 1692462 Taylor Larrechea COLORADOCSPB2270Summer2023 nodes has at least ~~levels and at most~~ ICSPB2270Sformer2023 levels and at most ~~levels~~ for a tree with levels. So removal's worst case time complexity is ~~for a tree with~~ for a tree with levels.

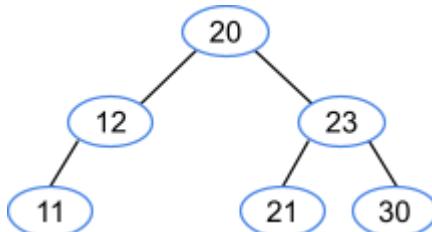
Two pointers are used to traverse the tree during removal. When the node being removed has two children, a third pointer and a copy of one node's data are also used, and one recursive call is made. So removal's space complexity is always ~~.~~ .

### PARTICIPATION ACTIVITY

#### 7.7.3: BST remove algorithm.



Consider the following tree. Each question starts from the original tree. Use this text notation for the tree: (20 (12 (11, -), 23 (21, 30))). The - means the child does not exist.



1) What is the tree after removing 21?



- (20 (12 (11, -), 23 (-, 30)))
- (20 (12 (11, -), 23))

2) What is the tree after removing 12?



- (20 (- (11, -), 23 (21, 30)))
- (20 (11, 23 (21, 30)))

3) What is the tree after removing 20?



- (21 (12 (11, -), 23 (-, 30)))
- (23 (12 (11, -), 30 (21, -)))

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- 4) Removing a node from an N-node nearly-full  
BST has what computational complexity?

- O( $\dots$ )
- O( $n$ )

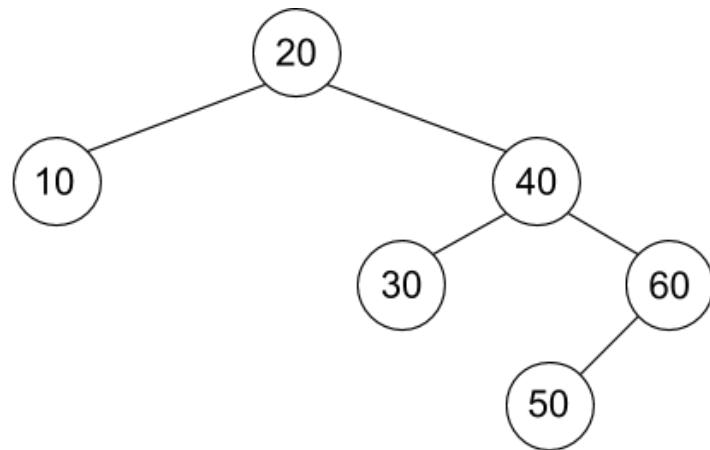
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**CHALLENGE ACTIVITY****7.7.1: BST remove algorithm.**

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**Start**

BSTRemove(tree, 30) executes.

What is the left child of 40? What is the right child of 40? 

1

2

3

**Check****Next**

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## 7.8 BST inorder traversal

A **tree traversal** algorithm visits all nodes in the tree once and performs an operation on each node. An **inorder traversal** visits all nodes in a BST from smallest to largest, which is useful for example to print the tree's nodes in sorted order. Starting from the root, the algorithm recursively prints the left subtree, the current node, and the right subtree.

Figure 7.8.1: BST inorder traversal algorithm.

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```
BSTPrintInorder(node) {
    if (node is null)
        return

    BSTPrintInorder(node->left)
    Print node
    BSTPrintInorder(node->right)
}
```

#### PARTICIPATION ACTIVITY

7.8.1: BST inorder print algorithm.



#### Animation captions:

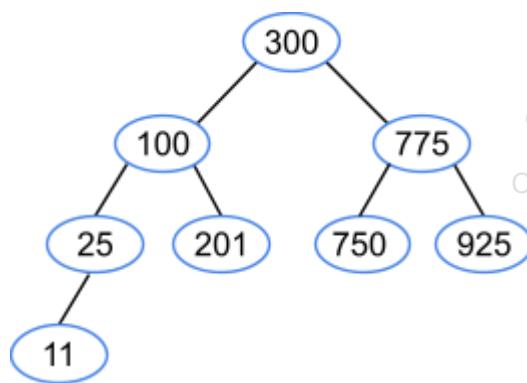
1. An inorder traversal starts at the root. Recursive call descends into left subtree.
2. When left done, current is printed, then recursively descend into right subtree.
3. Return from recursive call causes ascending back up the tree; left is done, so do current and right.
4. Continues similarly.

#### PARTICIPATION ACTIVITY

7.8.2: Inorder traversal of a BST.



Consider the following tree.



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- 1) What node is printed first?



[Show answer](#)

- 2) Complete the tree traversal after node 300's left subtree has been printed.

11 25 100 201

  
[Show answer](#)

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- 3) How many nodes are visited?

  
[Check](#)[Show answer](#)

- 4) Using left, current, and right, what ordering will print the BST from largest to smallest? Ex: An inorder traversal uses left current right.

  
[Check](#)[Show answer](#)

## 7.9 BST height and insertion order

### BST height and insertion order

Recall that a tree's **height** is the maximum edges from the root to any leaf. (Thus, a one-node tree has height 0.)

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The *minimum* N-node binary tree height is \_\_\_\_\_, achieved when each level is full except possibly the last. The *maximum* N-node binary tree height is  $N - 1$  (the  $- 1$  is because the root is at height 0).

Searching a BST is fast if the tree's height is near the minimum. Inserting items in random order naturally keeps a BST's height near the minimum. In contrast, inserting items in nearly-sorted order leads to a nearly-maximum tree height.

**PARTICIPATION  
ACTIVITY**

7.9.1: Inserting in random order keeps tree height near the minimum.  
Inserting in sorted order yields the maximum.

**Animation captions:**

1. Inserting in random order naturally keeps tree height near the minimum, in this case 3  
(minimum: 2)
2. Inserting in sorted order yields the maximum height, in this case 6.
3. If nodes are given beforehand, randomizing the ordering before inserting keeps tree height near minimum.

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**PARTICIPATION  
ACTIVITY**

7.9.2: BST height.



Draw a BST by hand, inserting nodes one at a time, to determine a BST's height.

- 1) A new BST is built by inserting nodes in this order:

6 2 8



What is the tree height?

(Remember, the root is at height 0)

**Check****Show answer**

- 2) A new BST is built by inserting nodes in this order:

20 12 23 18 30



What is the tree height?

**Check****Show answer**

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- 3) A new BST is built by inserting nodes in this order:

30 23 21 20 18



What is the tree height?

**Check****Show answer**

- 4) A new BST is built by inserting nodes in this order:

30 11 23 21 20

What is the tree height?

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**Check****Show answer**

- 5) A new BST is built by inserting 255 nodes in sorted order. What is the tree height?

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**Check****Show answer**

- 6) A new BST is built by inserting 255 nodes in random order. What is the minimum possible tree height?

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**Check****Show answer**

## BSTGetHeight algorithm

Given a node representing a BST subtree, the height can be computed as follows:

- If the node is null, return -1.
- Otherwise recursively compute the left and right child subtree heights, and return 1 plus the greater of the 2 child subtrees' heights.

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PARTICIPATION  
ACTIVITY

7.9.3: BSTGetHeight algorithm.



### Animation content:

undefined

## Animation captions:

1. BSTGetHeight(tree $\rightarrow$ root) is called to get the height of the tree. The height of the root's left child is determined first using a recursive call.
2. BSTGetHeight for node 18 makes a recursive call on node 12. BSTGetHeight on node 12 makes a recursive call on the null left child, which returns -1. ©zyBooks 06/06/23 23:54 1692462 Taylor Larrechea
3. Returning to the BSTGetHeight(node 12) call, a recursive call is now made on the right child. Node 14 is a leaf, so both recursive calls return -1.
4. BSTGetHeight(node 14) returns  $1 + \max(-1, -1) = 1 + -1 = 0$ .
5. BSTGetHeight(node 12) has completed 2 recursive calls and returns  $1 + \max(-1, 0) = 1$ . BSTGetHeight(node 18) makes the recursive call on the null right child, which returns -1.
6. A recursive call is made for each node in the tree. BSTGetHeight(tree $\rightarrow$ root) returns  $1 + \max(2, 1) = 3$ , which is the tree's height.

**PARTICIPATION ACTIVITY**

### 7.9.4: BSTGetHeight algorithm.



- 1) BSTGetHeight returns 0 for a tree with a single node.

- True  
 False



- 2) The base case for BSTGetHeight is when the node argument is null.

- True  
 False



- 3) The worst-case time complexity for BSTGetHeight is  $O(\log N)$ , where N is the number of nodes in the tree.

- True  
 False



- 4) BSTGetHeight would also work if the recursive call on the right child was made before the recursive call on the left child.

- True  
 False

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## 7.10 BST parent node pointers

A BST implementation often includes a parent pointer inside each node. A balanced BST, such as an AVL tree or red-black tree, may utilize the parent pointer to traverse up the tree from a particular node to find a node's parent, grandparent, or siblings. The BST insertion and removal algorithms below insert or remove nodes in a BST with nodes containing parent pointers.

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Figure 7.10.1: BSTInsert algorithm for BSTs with nodes containing parent pointers.

```
BSTInsert(tree, node) {
    if (tree->root == null) {
        tree->root = node
        node->parent = null
        return
    }

    cur = tree->root
    while (cur != null) {
        if (node->key < cur->key) {
            if (cur->left == null)
            {
                cur->left = node
                node->parent = cur
                cur = null
            }
            else
                cur = cur->left
        }
        else {
            if (cur->right == null)
            {
                cur->right = node
                node->parent = cur
                cur = null
            }
            else
                cur = cur->right
        }
    }
}
```

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Figure 7.10.2: BSTReplaceChild algorithm.

```
BSTReplaceChild(parent, currentChild,
newChild) {
    if (parent->left != currentChild &&
        parent->right != currentChild)
        return false

    if (parent->left == currentChild)
        parent->left = newChild
    else
        parent->right = newChild

    if (newChild != null)
        newChild->parent = parent
    return true
}
```

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Figure 7.10.3: BSTRemoveKey and BSTRemoveNode algorithms for BSTs with nodes containing parent pointers.

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```
BSTRemoveKey(tree, key) {
    node = BSTSearch(tree, key)
    BSTRemoveNode(tree, node)
}

BSTRemoveNode(tree, node) {
    if (node == null)
        return

    // Case 1: Internal node with 2 children
    if (node->left != null && node->right != null) {
        // Find successor
        succNode = node->right
        while (succNode->left)
            succNode = succNode->left

        // Copy value/data from succNode to node
        node = Copy succNode

        // Recursively remove succNode
        BSTRemoveNode(tree, succNode)
    }

    // Case 2: Root node (with 1 or 0 children)
    else if (node == tree->root) {
        if (node->left != null)
            tree->root = node->left
        else
            tree->root = node->right

        // Make sure the new root, if non-null, has a null
parent
        if (tree->root != null)
            tree->root->parent = null
    }

    // Case 3: Internal with left child only
    else if (node->left != null)
        BSTReplaceChild(node->parent, node, node->left)

    // Case 4: Internal with right child only OR leaf
    else
        BSTReplaceChild(node->parent, node, node->right)
}
```

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**PARTICIPATION ACTIVITY**

7.10.1: BST parent node pointers.

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- 1) **BSTInsert** will not work if the tree's root is null.

 True

False

- 2) `BSTReplaceChild` will not work if the parent pointer is null.

 True False

- 3) `BSTRemoveKey` will not work if the key is not in the tree.

 True False

- 4) `BSTRemoveNode` will not work to remove the last node in a tree.

 True False

- 5) `BSTRemoveKey` uses `BSTRemoveNode`.

 True False

- 6) `BSTRemoveNode` uses `BSTRemoveKey`.

 True False

- 7) `BSTRemoveNode` may use recursion.

 True False

- 8) `BSTRemoveKey` will not properly update parent pointers when a non-root node is being removed.

 True False

- 9) All calls to `BSTRemoveNode` to remove a non-root node will result in a call to `BSTReplaceChild`.

 True

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False

## 7.11 BST: Recursion

### BST recursive search algorithm

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BST search can be implemented using recursion. A single node and search key are passed as arguments to the recursive search function. Two base cases exist. The first base case is when the node is null, in which case null is returned. If the node is non-null, then the search key is compared to the node's key. The second base case is when the search key equals the node's key, in which case the node is returned. If the search key is less than the node's key, a recursive call is made on the node's left child. If the search key is greater than the node's key, a recursive call is made on the node's right child.

**PARTICIPATION ACTIVITY**

7.11.1: BST recursive search algorithm.



### Animation content:

undefined

### Animation captions:

1. A call to BSTSearch(tree, 40) calls the BSTSearchRecursive function with the tree's root as the node argument.
2. The search key 40 is less than 64, so a recursive call is made on the root node's left child.
3. An additional recursive call searches node 32's right child. The key 40 is found and node 40 is returned.
4. Each function returns the result of a recursive call, so BSTSearch(tree, 40) returns node 40.

**PARTICIPATION ACTIVITY**

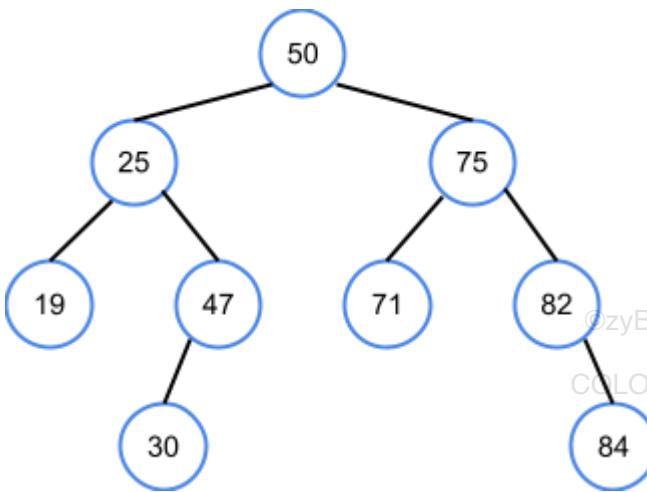
7.11.2: BST recursive search algorithm.



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- 1) How many calls to BSTSearchRecursive are made by calling BSTSearch(tree, 71)?

- 2
- 3
- 4

- 2) How many calls to BSTSearchRecursive are made by calling BSTSearch(tree, 49)?

- 3
- 4
- 5

- 3) What is the maximum possible number of calls to BSTSearchRecursive when searching the tree?

- 4
- 5

## BST get parent algorithm

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A recursive BST get-parent algorithm searches for a parent in a way similar to the normal BST search algorithm. But instead of comparing the search key with a candidate node's key, the search key is compared with the keys of the candidate node's children.

Figure 7.11.1: BST get parent algorithm.

```
BSTGetParent(tree, node) {
    return BSTGetParentRecursive(tree->root, node)
}

BSTGetParentRecursive(subtreeRoot, node) {
    if (subtreeRoot is null)
        return null

    if (subtreeRoot->left == node or
        subtreeRoot->right == node) {
        return subtreeRoot
    }

    if (node->key < subtreeRoot->key) {
        return BSTGetParentRecursive(subtreeRoot->left,
node)
    }
    return BSTGetParentRecursive(subtreeRoot->right,
node)
}
```

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**PARTICIPATION ACTIVITY****7.11.3: BST get parent algorithm.**

- 1) BSTGetParent() returns null when the node parameter is the tree's root.

- True
- False

- 2) BSTGetParent() returns a leaf node when the node parameter is null.

- True
- False

- 3) The base case for BSTGetParentRecursive() is when subtreeRoot is null or is node's parent.

- True
- False

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## Recursive BST insertion and removal

BST insertion and removal can also be implemented using recursion. The insertion algorithm uses recursion to traverse down the tree until the insertion location is found. The removal algorithm uses

the recursive search functions to find the node and the node's parent, then removes the node from the tree. If the node to remove is an internal node with 2 children, the node's successor is recursively removed.

Figure 7.11.2: Recursive BST insertion and removal.

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```
BSTInsert(tree, node) {
    if (tree->root is null)
        tree->root = node
    else
        BSTInsertRecursive(tree->root, node)
}

BSTInsertRecursive(parent, nodeToInsert) {
    if (nodeToInsert->key < parent->key) { ©zyBooks 06/06/23 23:54 1692462
        if (parent->left is null)
            parent->left = nodeToInsert
        else
            BSTInsertRecursive(parent->left,
nodeToInsert)
    }
    else { Taylor Larrechea
        if (parent->right is null)
            parent->right = nodeToInsert
        else
            BSTInsertRecursive(parent->right,
nodeToInsert)
    }
}

BSTRemove(tree, key) {
    node = BSTSearch(tree, key)
    parent = BSTGetParent(tree, node)
    BSTRemoveNode(tree, parent, node)
}

BSTRemoveNode(tree, parent, node) {
    if (node == null)
        return false

    // Case 1: Internal node with 2 children
    if (node->left != null && node->right != null) { COLORADO CSPB2270 Summer 2023
        // Find successor and successor's parent
        succNode = node->right
        successorParent = node
        while (succNode->left != null) {
            successorParent = succNode
            succNode = succNode->left
        }

        // Copy the value from the successor node
        node = Copy succNode

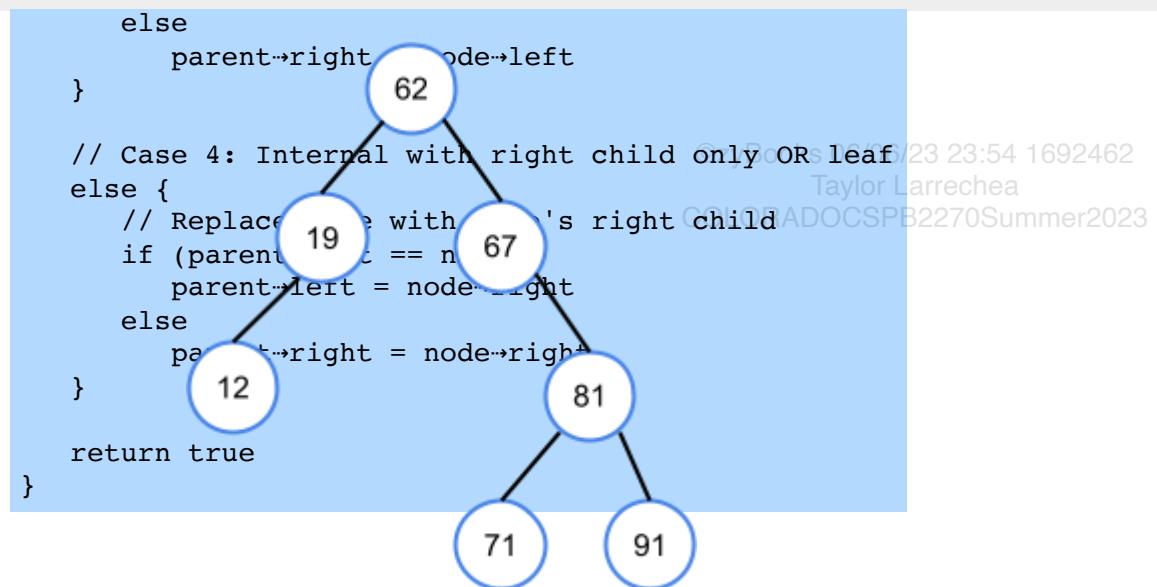
        // Recursively remove successor
        BSTRemoveNode(tree, successorParent, succNode) ©zyBooks 06/06/23 23:54 1692462
    }
}

// Case 2: Root node (with 1 or 0 children)
else if (node == tree->root) {
    if (node->left != null)
        tree->root = node->left
    else
        tree->root = node->right
}
```

```
// Case 3: Internal with left child only
else if (node->left != null) {
```

**PARTICIPATION ACTIVITY**

7.11.4: Recursive BST insertion and removal.



The following operations are executed on the above tree:

BSTInsert(tree, node 70)

BSTInsert(tree, node 56)

BSTRemove(tree, 67)

1) Where is node 70 inserted?

- Node 67's left child
- Node 71's left child
- Node 71's right child

2) How many times is BSTInsertRecursive called when inserting node 56?

- 2
- 3
- 5

3) How many times is BSTRemoveNode called when removing node 67?

- 1
- 2
- 3

4) What is the maximum number of calls to BSTRemoveNode when removing one of the tree's nodes?

- 2
- 4
- 5

## 7.12 Tries

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### Overview

A **trie** (or **prefix tree**) is a tree representing a set of strings. Each non-root node represents a single character. Each node has at most one child per distinct alphabet character. A **terminal node** is a node that represents a terminating character, which is the end of a string in the trie.

Tries provide efficient storage and quick search for strings, and are often used to implement auto-complete and predictive text input.

PARTICIPATION  
ACTIVITY

7.12.1: Trie representing the set of strings: bat, cat, and cats.



### Animation content:

undefined

### Animation captions:

1. The following trie represents a set of two strings. Each string can be determined by traversing the path from the root to a leaf.
2. Suppose "cats" is added to the trie. Adding another child for 'c' would violate the trie requirements.
3. So the existing child for 'c' is reused.
4. Similarly, nodes for 'a' and 't' are reused. Node 't' has a new child added for 's'.
5. Exactly one terminal node exists for each string. Other nodes may be shared by multiple strings.

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PARTICIPATION  
ACTIVITY

7.12.2: Trie representing the set of strings: bat, cat, and cats.



Refer to the trie above.

- 1) The terminal nodes can be removed, and instead the last character of a



string can be a leaf.

- True
- False

2) Adding the string "balance" would create a new branch off the root node.

- True
- False

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3) Inserting a string that doesn't already exist in the trie requires allocation of at least 1 new node.

- True
- False



## Trie insert algorithm

Given a string, a **trie insert** operation creates a path from the root to a terminal node that visits all the string's characters in sequence.

A current node pointer initially points to the root. A loop then iterates through the string's characters. For each character C:

1. A new child node is added only if the current node does not have a child for C.
2. The current node pointer is assigned with the current node's child for C.

After all characters are processed, a terminal node is added and returned.

### PARTICIPATION ACTIVITY

7.12.3: Trie insert algorithm.



## Animation content:

undefined

## Animation captions:

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1. The trie starts with an empty root node. Adding "APPLE" first adds a new child for 'A' to the root node.
2. New nodes are built for each remaining character in "APPLE".
3. The terminal node is added, completing insertion of "APPLE".
4. When adding "APPLY", the first 4 character nodes are reused.
5. Node L has no child for 'Y', so a new node is added. The terminal node is also added.

6. When adding "APP", only 1 new node is needed, the terminal node.

**PARTICIPATION ACTIVITY****7.12.4: Trie insert algorithm.**

Assume a trie is built by executing the following code.

```
trieRoot = new TrieNode()
TrieInsert(trieRoot, "cat")
TrieInsert(trieRoot, "cow")
TrieInsert(trieRoot, "crow")
```

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- 1) When inserting "cat", \_\_\_\_ new nodes are created.

- 1
- 3
- 4



- 2) When inserting "crow", \_\_\_\_ new nodes are created.

- 1
- 4
- 5



- 3) If `TrieInsert(trieRoot, "cow")` is called a second time, \_\_\_\_ new nodes are created.

- 0
- 1
- 4



## Trie search algorithm

Given a string, a **trie search** operation returns the terminal node corresponding to that string, or null if the string is not in the trie.

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**PARTICIPATION ACTIVITY****7.12.5: Trie search algorithm.**

### Animation content:

undefined

## Animation captions:

1. The search for "PAPAYA" starts at the root and iterates through one node per character. The terminal node is returned, indicating that the string was found.
2. The search for "GRAPE" ends quickly, since the root has no child for 'G'.
3. The search for "PEA" gets to the node for 'A'. No terminal child node exists after 'A', so null is returned.

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**PARTICIPATION ACTIVITY**

7.12.6: Trie search algorithm.



Refer to the trie above. Assume that to "visit" a node means accessing the node's children in TrieSearch.

- 1) `TrieSearch(root, "PINEAPPLE")` returns \_\_\_\_.

- the trie's root node
- the terminal node for "APPLE"
- null



- 2) When searching for "PLUM", \_\_\_\_ visited.

- only the root node is
- the root and the root's 'P' child node are
- all nodes in the root's 'P' child subtree are



- 3) `TrieSearch` will visit at most \_\_\_\_ nodes in a single search.

- 7
- 9
- 41



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## Trie remove algorithm

Given a string, a **trie remove** operation removes the string's corresponding terminal node and all non-root ancestors with 0 children.

**PARTICIPATION ACTIVITY**

7.12.7: Trie remove algorithm.



## Animation content:

undefined

## Animation captions:

1. TrieRemove is called to remove "BANANA". TrieRemove then calls TrieRemoveRecursive, passing the trie's root, the string "BANANA", and a character index of 0.  
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2. The root has a child for 'B'. A recursive removal call is made for the child and the next character index.  
#other2023
3. Recursive calls continue until the terminal node's parent is reached.
4. The terminal node is removed from the node's children and true is returned.
5. After returning from each recursive call, child nodes with 0 children are also removed.
6. When removing "APPLE", 4 nodes are removed.
7. Removal operations only remove nodes that are exclusive to the string being removed.

### PARTICIPATION ACTIVITY

7.12.8: Trie remove algorithm.



Refer to the trie above. Match each operation to the statement that is true when the operation executes. Assume that "BANANA" and "APPLE" have already been removed.

If unable to drag and drop, refresh the page.

`TrieRemove(root, "AVOCADO")  
TrieRemove(root, "APRICOT")`

`TrieRemove(root, "PAPAYA")`

`TrieRemove(root, "PEAR")`

`TrieRemove(root, "CHERRY")  
TrieRemove(root, "PLUM")`

Remove's the root's child node for 'A'.

charIndex is 6 at the moment a terminal node is removed.

Has no effect.

Removes a total of 2 nodes from the trie.

**Reset**

## Trie time complexities

Implementations commonly use a lookup table for a trie node's children, allowing retrieval of a child node from a character in  $O(1)$  time. Therefore, to insert, remove, or search for a string of length  $M$  in a trie takes  $O(M)$  time. The trie's current size does not affect each operation's time complexity.

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### CHALLENGE ACTIVITY

#### 7.12.1: Tries.



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Start

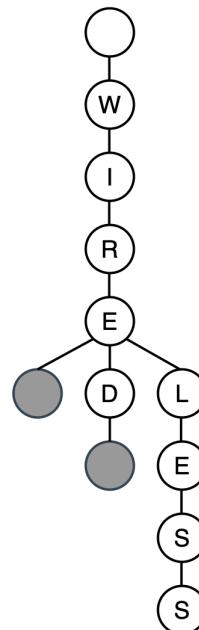
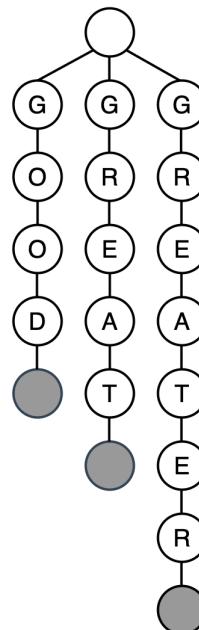
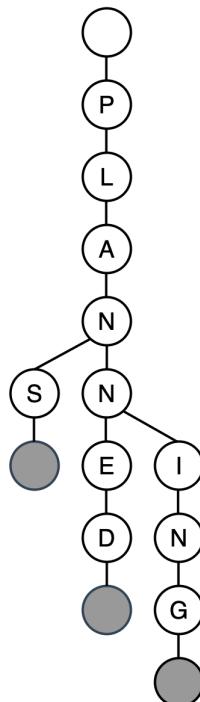
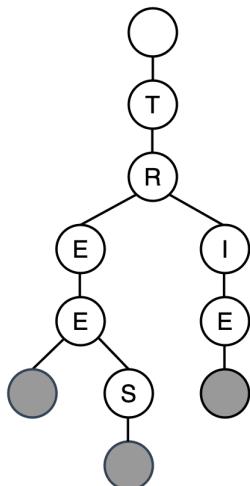
Select all valid tries.

A

B

C

D



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1

2

3

4

Check

Next