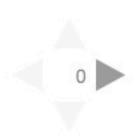


Department of Computer Science CSCI 2824: Discrete Structures Chris Ketelsen

Algorithms and Complexity

Sequences



Sequences

A sequence is a discrete structure used to represent an ordered list. It's one of the most fundamental structures in computer science.

Examples:

- 0, 1, 1, 2, 3, 5, 8, 13, ...
- 2, 4, 8, 16, 32, 64, ...
- 3, 7, 11, 15, 19, 23, ...

Def: A **sequence** is a function that maps from a subset of the integers (usually $\{0, 1, 2, ...\}$ or $\{1, 2, 3, ...\}$) to a set S.

We use the notation $\{a_n\}$ to represent a sequence where a_n represents the n^{th} individual term in the sequence



Sequences

Example: Consider the sequence $\{a_n\}$ where $a_n = \frac{1}{n}$

In this case the sequence starts with n=1

The first three terms several terms in the sequence are:

Geometric Sequences

Some sequential patterns are so common we have names for them

Example: A geometric progression is a sequence of the form

$$a, ar, ar^2, ar^3, \ldots, ar^n, \ldots$$

where the *initial term* a and *common ratio* r are real numbers

Example: What are a and r in the geometric sequence

Arithmetic Sequences

Example: Another common sequence is the arithmetic progression, which has the form

$$a, a + d, a + 2d, a + 3d, \dots, a + nd, \dots$$

which has initial term a and common difference d

Example: Find a and d in the arithmetic sequence



Sequences from Recurrences

Sometimes we don't give explicit formulas for a_n .

Instead define sequence by a **recurrance** specifying the first few terms, and then giving a formula to find additional terms

Example: Let $a_0 = 1$, $a_1 = 3$, and $a_n = 2a_{n-1} - a_{n-2}$

Sequences

Example: Show that $a_n = 4^n$ is a solution to $a_n = 8a_{n-1} - 16a_{n-2}$

Strategy: Plug in and show that both sides are equal

The most famous sequence ever is the following:

This is the Fibonacci Sequence

It's defined by the recurrence: $F_0=0$, $F_1=1$, $F_n=F_{n-1}+F_{n-2}$

Lots of amazing things hidden in Fibonacci Sequence:

- Hidden in Pascal's Triangle
- Ratio of terms approaches **Golden Ratio** $\frac{1+\sqrt{5}}{2}$
- Pattern found all over the place in nature

The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55

Notice any interesting patterns?

Parity Pattern: even, odd, odd, even, odd, odd, ...

Could we find the sequence of just the even Fibonacci numbers?

Example: Determine a recurrence for the even Fibonacci numbers

Strategy: Want a recurrence of the form $F_n = aF_{n-3} + bF_{n-6}$

Example: Determine a recurrence for the even Fibonacci numbers

Strategy: Want a recurrence of the form $F_n = aF_{n-3} + bF_{n-6}$