Search

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Note: Some slides were adapted from the materials from Prof. Pieter Abbeel and Prof. Dan Klein at UC Berkeley, and Russell & Norvig's AI book.

Constraint Satisfaction Problems

Review Search

- Assumptions: a single agent, deterministic actions, fully observed state, discrete state space

- Path planning
  - The path to the goal is the important thing
  - Paths have various costs, depths
  - Heuristics give problem-specific guidance

Different Kinds of Search

Planning

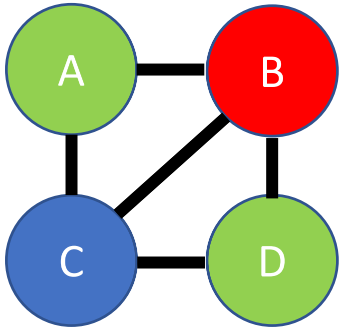
- Sequences of actions
- The path to the goal is the important thing
- Paths have various costs, depths
- Heuristics give problem-specific guidance
- Shortest path finding problem

Identification

- Assignments to variables
- The goal itself is important, not the path
- All paths at the same depth
- Uses general-purpose guideline
- Constraint satisfaction problem

What is Constraint Satisfaction Problems?

- A special subset of search problems
- State is defined by variables $\{X\}$ with values from a domain $\{D\}$



X = {A,B,C,D}

D = {red, green, blue}

- Goal test is a set of constraints specifying allowable combinations of values for subsets of variables
- Allows useful general-purpose algorithms with more power than standard search algorithms

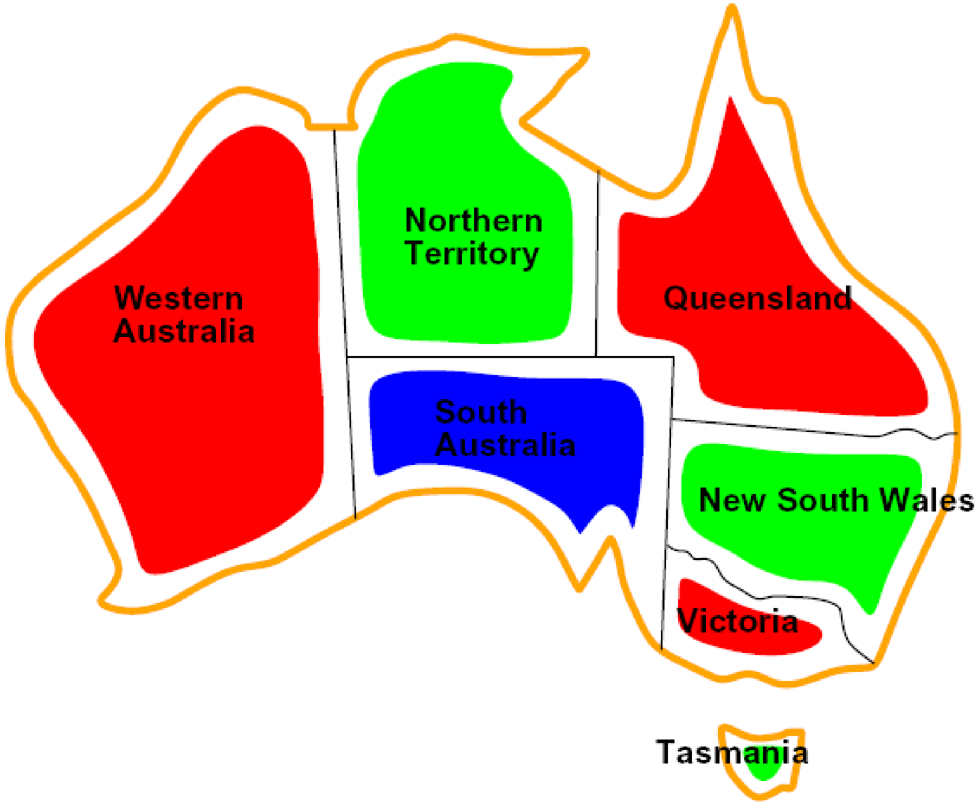
Example: Map Coloring

- Variables: WA, NT, Q, NSW, V, SA, T
- Domains: $D = \{\text{red}, \text{green}, \text{blue}\}$
- Constraints: adjacent regions must have different colors

  Implicit: $WA \neq NT$

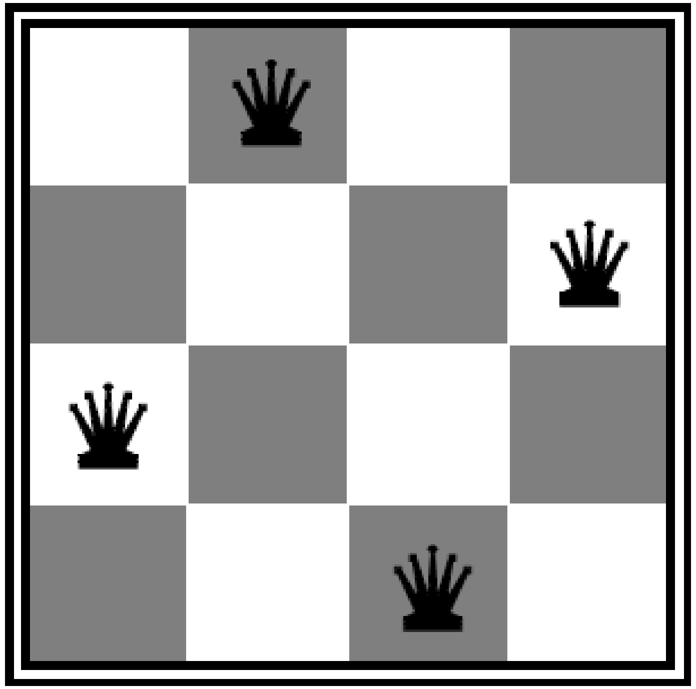
  Explicit: $(WA, NT) \in \{(\text{red}, \text{green}), (\text{red}, \text{blue}), \ldots\}$
- Solutions are assignments satisfying all constraints, e.g.:

  {WA=red, NT=green, Q=red, NSW=green, V=red, SA=blue, T=green}



Example: N-Queens

- Formulation 1
  - Variables: X_{ij}
  - Domains: $\{0, 1\}$
  - Constraints



$$\forall i, j, k \;\; (X_{ij}, X_{ik}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{kj}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{i+k,j+k}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{i+k,j-k}) \in \{(0,0), (0,1), (1,0)\}$$

$$\sum_{i,j} X_{ij} = N$$

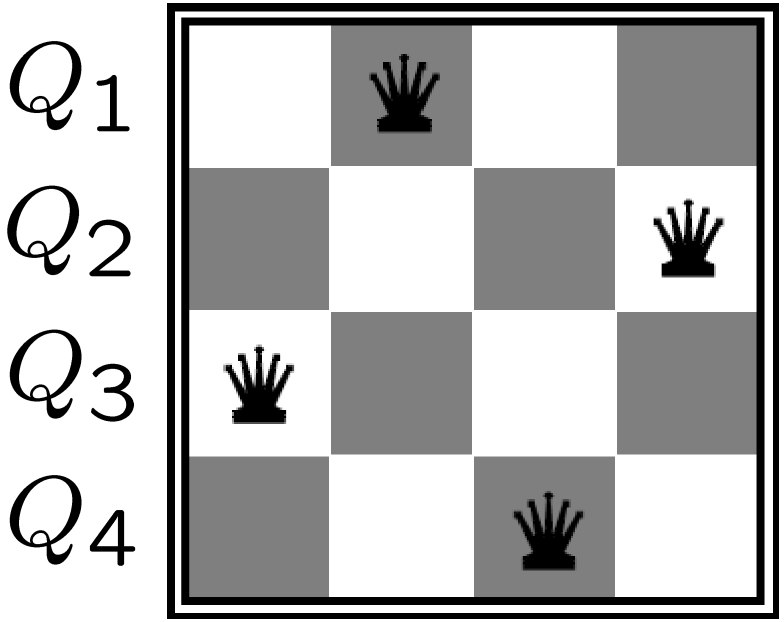
Example: N-Queens

- Formulation 2
  - Variables: Q_k
  - Domains: $\{1, 2, 3, \ldots N\}$
  - Constraints:

Implicit: $\forall i, j$ non-threatening(Q_i, Q_j)

Explicit: $(Q_1, Q_2) \in \{(1,3), (1,4), \ldots\}$

$\cdots$



Example: Sudoku

					8			4
	8	4		1	6			
			5			1		
1		3	8			9		
6		8				4		3
		2			9	5		1
		7			2			
			7	8		2	6	
2			3					

- Variables:

  Each (open) square
- Domains:

  {1,2,…,9}
- Constraints:

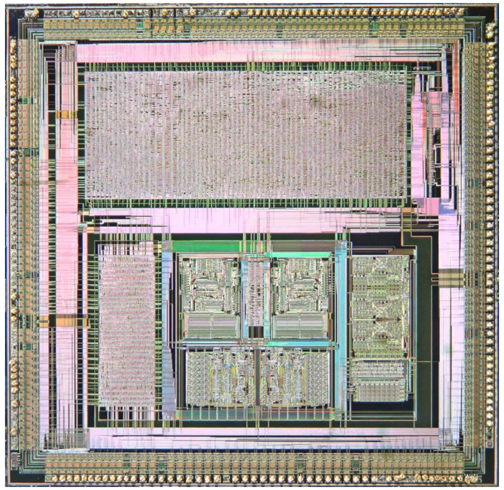
  1-9 each column

  1-9 each row

  1-9 each region

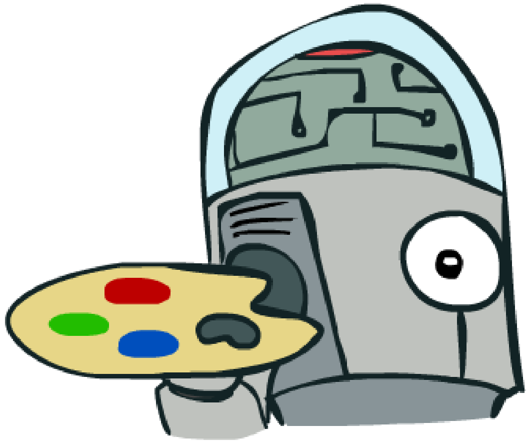
Real-World CSPs

- Scheduling problems: e.g., when can we all meet?
- Timetabling problems: e.g., which class is offered when and where?
- Assignment problems: e.g., who teaches what class
- Transportation scheduling
- Factory scheduling
- Hardware configuration
- Circuit layout
- Fault diagnosis
- … lots more!



Varieties of CSPs

- Discrete Variables
  - Finite domains
    - Size d means $O(d^n)$ complete assignments
    - E.g., Boolean CSPs, including Boolean satisfiability (NP-complete)
  - Infinite domains (integers, strings, etc.)
    - E.g., job scheduling, variables are start/end times for each job
    - Linear constraints solvable, nonlinear undecidable



- Continuous variables
  - E.g., start/end times for Hubble Telescope observations
  - Linear constraints solvable in polynomial time by LP methods (see cs170 for a bit of this theory)

Varieties of Constraints

- Varieties of Constraints
  - Unary constraints $\mathrm{SA} \neq \mathrm{green}$
  - Binary constraints $\mathrm{SA} \neq \mathrm{WA}$
  - Higher-order constraints
- Preferences (soft constraints):
  - E.g., red is better than green
  - Often representable by a cost for each variable assignment
  - Gives constrained optimization problems