7.3. Incidence matrix

Incidence matrix of a graph. We create the incidence matrix of the network shown in Figure 7.3 in VMLS.

In [ ]:
```
A = np.array([[-1,-1,0,1,0], [1,0,-1,0,0], [0,0,1,-1,-1],
↪  [0,1,0,0,1]])
xcirc = np.array([1,-1,1,0,1]) #A circulation
A @ xcirc
```

Out[ ]:
```
array([0, 0, 0, 0])
```

In [ ]:
```
s = np.array([1,0,-1,0,]) # A source vector
x = np.array([0.6,0.3,0.6,-0.1,-0.3]) #A flow vector
A @ x + s #Total incoming flow at each node
```

Out[ ]:
```
array([1.11022302e-16, 0.00000000e+00, 0.00000000e+00,
↪  0.00000000e+00])
```

Dirichlet energy. On page 135 of VMLS we compute the Dirichlet energy of two potential vectors associated with the graph of Figure 7.2 in VMLS.

In [ ]:
```
A = np.array([[-1,-1,0,1,0], [1,0,-1,0,0], [0,0,1,-1,-1],
↪  [0,1,0,0,1]])
vsmooth = np.array([1,2,2,1])
np.linalg.norm(A.T @ vsmooth)**2 #Dirichlet energy of vsmooth
```

Out[ ]:
```
2.9999999999999996
```

In [ ]:
```
vrough = np.array([1,-1, 2, -1])
np.linalg.norm(A.T @ vrough)**2 # Dirichlet energy of vrough
```

Out[ ]:
```
27.0
```

7.4. Convolution

The numpy function `np.convolve()` can be used to compute the convolution of the vectors `a` and `b`. Let's use this to find the coefficients of the polynomial

$$p(x) = (1+x)(2-x+x^2)(1+x-2x^2) = 2+3x-3x^2-x^3+x^4-2x^5$$