College of Engineering & Applied Sciences

CSPB 2824

Discrete Structures

Induction Mastery Workbook

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Sections

Mastery Workbook 9

Counting And Binomials Workbook

I have neither given nor received unauthorized assistance.

Taylor James Larrechea

Problem 1

Problem Statement

Problem 1 from the mastery workbook quiz can be found on the following page.

Solution

Question 1

Correct

Mark 10.00 out of 10.00

The n = 6 row of Pascal's triangle reads

1, 6, 15, 20, 15, 6, 1

What is the next row of the triangle?

Select one:

- a. 1, 7, 21, 35, 35, 21, 7, 1 ✔
- b. 1, 7, 21, 35, 54, 88, 1
- c. 1, 8, 21, 35, 70, 35, 21, 8, 1
- d. 1, 2, 3, 4, 5, 6, 7
- e. 1, 7, 21, 35, 70, 28, 8, 1

Your answer is correct.

Correct

Marks for this submission: 10.00/10.00.

Problem 1 Summary

Procedure

- Read off the $n = 7$ line of Pascals triangle and select the corresponding answer.

Key Concepts

- This problem showcases Pascals triangle and what the rows of his triangle are.

Variations

- We could be asked to select a different row from Pascals triangle.
  - In that case we would just look at Pascals triangle and select the correct row.

Problem 2

Problem Statement

Problem 2 from the mastery workbook quiz can be found on the following page.

Solution

Correct

Mark 10.00 out of 10.00

Try the Pascal's triangle activities in the Pascal's Triangle supplement.

Which of the following are patterns found in Pascal's Triangle?

Select one or more:

- [x] a. The sequence of squares. ✔
- [ ] b. The Sequence of Derivatives
- [x] c. The sequence of integers (1,2,3,4...) ✔
- [x] d. The Fibonacci Sequence ✔
- [x] e. The coefficients of the binomial expansion. ✔

Your answer is correct.

Correct

Marks for this submission: 10.00/10.00.

Problem 2 Summary

Procedure

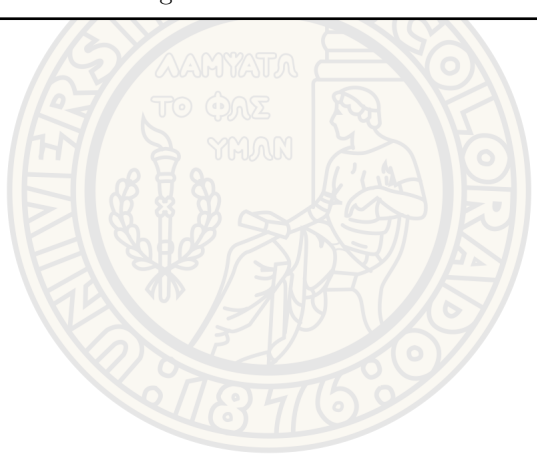
- Observe Pascals triangle and look at the patterns inside of Pascals triangle.
- Select the correct options from the given choices.

Key Concepts

- Pascals triangle shows a sequence of squares.
- Pascals triangle shows a sequence of integers.
- Pascals triangle shows the Fibonacci sequence.
- Pascals triangle shows the coefficients of the binomial expansion.

Variations

- We could be asked to examine different patterns inside of Pascals triangle.
  - We would then have to sift through the choices to see which ones are the correct answers.



Problem 3

Problem Statement

Problem 3 from the mastery workbook quiz can be found on the following page.

Solution

Question 3

Correct

Mark 10.00 out of 10.00

Read thru the example in Combinatorics supplement.

What is the probability of having 8 children, exactly 5 girls and 3 boys?

(Answer in percent, as a decimal - so 65.7% would be written as 65.7)

Answer: 21.9 ✔

Correct

Marks for this submission: 10.00/10.00.

Problem 3 Summary

Procedure

- Calculate the overall probability from having 8 children, namely

$$\left(\frac{1}{2}\right)^8.$$

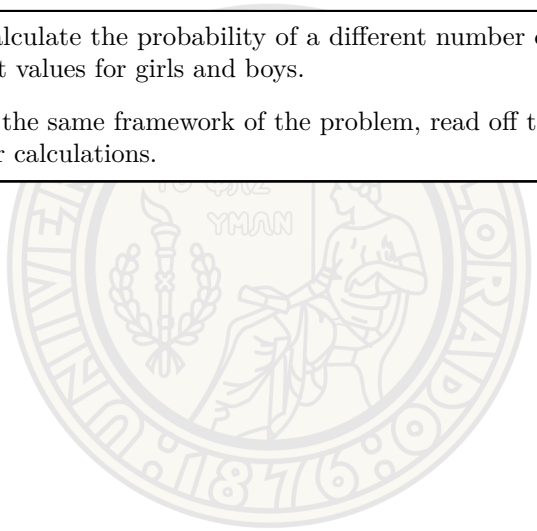
- Read off the column value from Pascals triangle for there to be 5 out of 8 binary possibilities.
- Multiply this value (56) by $(\frac{1}{2})^8$ to get the overall probability.

Key Concepts

- This problem showcases how we can use Pascals triangle to calculate binary choice probabilities.

Variations

- We could be asked to calculate the probability of a different number of children (or binary choices) being born with different values for girls and boys.
  - We would then use the same framework of the problem, read off the value from Pascals triangle, and perform similar calculations.



Problem 4

Problem Statement

Problem 4 from the mastery workbook quiz can be found on the following page.

Solution

Question **4**

Correct

Mark 10.00 out of 10.00

Calculate the value of the following summation involving binomial coefficients:

$$\binom{6}{0}\frac{1}{8}^6 + \binom{6}{1}\frac{1}{8}^5\frac{7}{8} + \binom{6}{2}\frac{1}{8}^4\frac{7}{8}^2 + \binom{6}{3}\frac{1}{8}^3\frac{7}{8}^3 + \binom{6}{4}\frac{1}{8}^2\frac{7}{8}^4 + \binom{6}{5}\frac{1}{8}\frac{7}{8}^5 + \binom{6}{6}\frac{7}{8}^6$$

(hint:there is a tedious way and a no- so-tedious-way to solve)

(hint:hint: See Binomial Theorem p. 416 Rosen)

Answer: 1 ✔

Correct

Marks for this submission: 10.00/10.00.

Problem 4 Summary

Procedure

- Use the Binomial Theorem, namely

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

  where the choose formula is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

  where $x = \frac{1}{8}$, $y = \frac{7}{8}$, and $n = 6$.

- Round the final answer from the Binomial Theorem result.

Key Concepts

- The Binomial Theorem can be used to calculate a complex sum in this context.
- In this context, the value for x and y represent the the coefficients in front of two variables that are being summed and raised to a power.

Variations

- We could be given a different expression to evaluate with the Binomial Theorem.
  - We would then use the Binomial theorem to determine what this new sum evaluates to.

Problem 5

Problem Statement

Problem 5 from the mastery workbook quiz can be found on the following page.

Solution

Question **5**

Correct

Mark 5.00 out of 5.00

The coefficients of (x +y)^6 expand in expanded form are:

1, 6, 15, 20, 15, 6, 1

$(x^6 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + y^6)$

What are the coefficients of (x+y)^7 in expanded form?

Select one:

- a. 1, 7, 21, 35, 35, 21, 7, 1 ✔
- b. 0, 1, 7, 21, 35, 35, 21, 7, 1, 0
- c. 1, 2, 3, 4, 5, 6, 7
- d. 1, 7, 21, 35, 35, 28, 8, 1
- e. 1, 7, 21, 35, 21, 7, 1

Your answer is correct.

Correct

Marks for this submission: 5.00/5.00.

Problem 5 Summary

Procedure

- Determine the value of the exponent that is being used in the Binomial Theorem.
- Go to this line of Pascals triangle and choose the correct choice.

Key Concepts

- This problem encapsulates determining what row of Pascals triangle is present in the expansion.

Variations

- We could be given a different expansion to decide which row of Pascals triangle is being shown.
  - We would then just read off of Pascals triangle what row is being represented.

Problem 6

Problem Statement

Problem 6 from the mastery workbook quiz can be found on the following page.

Solution

Question 6

Correct

Mark 5.00 out of 5.00

What are the coefficients (in expanded form) of $(2x + 3y)^2$? (This can be done by hand.)

What is the coefficient of x^2?

4

What is the coefficient of the xy?

12

What is the coefficient of the y^2?

9

Your answer is correct.

Correct

Marks for this submission: 5.00/5.00.

Problem 6 Summary

Procedure

- Expand the expression.
- Read off the coefficients from the expansion and submit the correct choices.

Key Concepts

- This problem encapsulates how the Binomial Theorem can be used to find the coefficients in an expansion.

Variations

- We could be given a different expression to calculate the coefficients for.
  - We would then use the Binomial Theorem to calculate the coefficients for the expansion. Or just do it by hand if it is simple enough.

Problem 7

Problem Statement

Problem 7 from the mastery workbook quiz can be found on the following page.

Solution

Question **7**

Correct

Mark 10.00 out of 10.00

What is the coefficient of the x^2y^3 in $(2x + 3y)^5$

Use the binomial theorem - remember you can substitution 2x for x.

(It may be helpful to rewrite the theorem with a,b's to make this clearer.)

Answer: 1080 ✔

Correct

Marks for this submission: 10.00/10.00.

Problem 7 Summary

Problem Statement

- Use the Binomial Theorem where $x = 2$, $y = 3$, and $n = 5$.
- Read off the coefficient from the x^2y^3 term.

Key Concepts

- This problem incorporates how to use the Binomial Theorem for an expansion.

Variations

- We could be given a different expression to find the coefficients for.
  - We would then use the Binomial Theorem for this new expression and read off the correct coefficients.

Problem 8

Problem Statement

Problem 8 from the mastery workbook quiz can be found on the following page.

Solution

Question 8

Correct

Mark 10.00 out of 10.00

Write some code (on your own) to calculate all the coefficients of $(2x + 3y)^5$ using the binomial theorem.

What is the coefficient of x^5 ?

32

What is the coefficient of $x^4 y$?

240

What is the coefficient of $x^3 y^2$?

720

What is the coefficient of $x^2 y^3$?

1080

What is the coefficient of xy^4 ?

810

What is the coefficient of y^5 ?

243

Your answer is correct.

Correct

Marks for this submission: 10.00/10.00.

Problem 8 Summary

Procedure

- Write code in Python that will return the coefficients in a Binomial expansion that are stored in an array.

Key Concepts

- This problem encapsulates how to use the Binomial Theorem in the context of code in Python.

Variations

- We could be asked to do something different with the coefficients.
  - We would then use the same general algorithm and then make the modifications that are needed to achieve the final result.

Problem 9

Problem Statement

Problem 9 from the mastery workbook quiz can be found on the following page.

Solution

Question **9**

Correct

Mark 5.00 out of 5.00

Calculate the product of the coefficients in the binomial expansion of $(2x + 3y)^5$

The product is 1175462461440000.

Select one:

- True ✔
- False

Correct

Marks for this submission: 5.00/5.00.

Problem 9 Summary

Procedure

- Modify the code from Problem 8 to multiply the coefficients of an expansion.
- Select the correct choice that corresponds with the result.

Key Concepts

- This problem incorporates how to utilizes code for a Binomial Theorem algorithm so that it returns a different result.

Variations

- We could be asked to calculate something different like the sum of coefficients.
  - We would then slightly modify the algorithm so that it returns the result that is desired.

Problem 10

Problem Statement

Problem 10 from the mastery workbook quiz can be found on the following page.

Solution

Question **10**

Correct

Mark 25.00 out of 25.00

Write a Python function **binom_product(a,b,n)** that takes three integer arguments a and b and positive integer argument n and returns the product of the coefficients in the expansion of $(ax + by)^n$.

Example: Let $a = 2$, $b = -1$, and $n = 3$. Then

$(2x - y)^3 = 8x^3 - 12x^2y + 6xy^2 - y^3$

The product of the expansion coefficients is $8 \times -12 \times 6 \times -1 = 576$

Notes: There are two visible test cases and three hidden test cases. Integers a and b will satisfy $-10 \leq a, b \leq 10$ and integer n will satisfy $1 \leq n \leq 10$.

For example:

Test	Result
`print(round(binom_product(2,-1,3)))`	576

Answer: (penalty regime: 0 %)

Reset answer

```
from math import pow, factorial
def binom_product(a,b,n):
    coefficients = []
    for j in range(n + 1):
        result = (factorial(n) / (factorial(j) * factorial(n - j))) * (pow(a,(n - j))) * (pow(b,j)
        coefficients.append(result)
    result = 1
    for i in range(len(coefficients)):
        result *= coefficients[i]
    return result
```

	Test	Expected	Got	
✔	`print(round(binom_product(2,-1,3)))`	576	576	✔
✔	`print(round(binom_product(1,3,4)))`	5668704	5668704	✔

Passed all tests! ✔

Correct

Marks for this submission: 25.00/25.00.

Problem 10 Summary

Procedure

- Write code in Python that achieves the result that we are seeking to produce.

Key Concepts

- This problem incorporates the Binomial Theorem in the context of code in Python.

Variations

- We could be asked to calculate something different with the coefficients of the expansion.
  - We would then slightly modify the algorithm so that it achieves the result that we are seeking to have.