CSPB 2824 - Stade - Discrete Structures

Question **1**

Correct

Marked out of 1.00

Recall that the *absolute value* of a real number x is simply given by removing the negative sign if present, and is denoted by $|x|$. For instance,
$|-2| = 2, \; |3.54| = 3.54, \; |0| = 0, \; |-\pi| = \pi.$

Antonio was asked to proved the following claim:

For any real number $x, \; |x| \geq x$.

Antonio decided to prove the claim by considering two different cases. If Antonio assumes $x > 0$ in Case 1, what assumption should he use in Case 2?

Select one:

a. $x \geq 0$

b. $x < 0$

c. $x < -1$

d. $x \leq 0$ Correct.

Clear my choice

Check

Your answer is correct.

Question **2**

Correct

Marked out of 1.00

Sophie is asked to prove the following claim:

For any integer n, if n^3 is odd, then n is odd.

Sophie decides to prove its contrapositive statement. So which one of the following statements is the contrapositive of the given claim?

Select one:

- a. There exists some integer n such that n^3 is odd but n is even.
- b. For any integer n, if n^3 is not odd, then n is not odd.
- c. For any integer n, if n is odd, then n^3 is odd.
- d. For any integer n, if n is not odd, then n^3 is not odd. Correct.
- e. There exists some integer n such that if n is even, then n^3 is even too.

Clear my choice

Check

Your answer is correct.

Question **3**

Correct

Marked out of 1.00

Sophie is asked to prove the following claim:

For any positive integer n, if n^3 is odd, then n is odd.

Sophie decides to prove by contradiction. So which one of the following statements is the way to start her proof after saying "We prove by contradiction."

Select one:

- a. For all n, n^3 is odd and n is even.
- b. Consider n = 3, suppose n^3 is even. n^3 is 27 so by contradiction this can't be true.
- c. If n^3 is odd, then n is even.
- d. There exists an n such that n^3 is odd and n is not odd. .

This is the negation and how we begin a proof by contradictionnote how this relates to our formal logic definition

P(n) = n^3 is odd.
Q(n) = n is odd.

∀(P(n) -> Q(n)) = ∀(!P(n) V Q(n))
Negation is ∃(P(n) ∧ !Q(n)).
It translates to "There exists a n such that n^3 is odd and n is even."

- e. none of these

Clear my choice

Check

Your answer is correct.

Question **4**

Correct

Marked out of 1.00

Joe learned these new functions the other day:

> For any real numbers x and y, $\max(x, y)$ picks the largest number among x and y, and $\min(x, y)$ picks the smallest number among x and y.

For example,

- $\max(3,\ 4.5) = 4.5, \min(3,\ 4.5) = 3;$
- $\max(-2,\ -5) = -2, \min(-2,\ -5) = -5;$
- $\max(4,\ 4) = 4, \min(4,\ 4) = 4.$

After writing down a few examples, Joe conjectured that

> for any real numbers x and y, $\max(x,\ y) + \min(x,\ y) = x + y$.

Joe tried to prove his conjecture by considering two different cases. His proof has three statements as follows:

1. If $x > y$, then $\max(x,\ y) = x$ and $\min(x,\ y) = y$, so $\max(x,\ y) + \min(x,\ y) = x + y$.
2. If $x < y$, then $\max(x,\ y) = y$ and $\min(x,\ y) = x$, so $\max(x,\ y) + \min(x,\ y) = x + y$.
3. Since the statement holds for both cases considered, the equality holds for all $x, y \in \mathbb{R}$. QED!

Is the proof correct? Check all the true statements below.

- [x] a. The statement in Step (1) is a valid argument.

  Correct.

- [x] b. The statement in Step (2) is a valid argument.

  Correct

- [ ] c. The statement in Step (3) is a valid argument.

Check

Your answer is correct.

You have correctly selected two options.

Question **5**

Correct

Marked out of 1.00

In each of the following choices, is the third statement a valid conclusion drawn from the first and second statements? Check all the choices that give a valid conclusion.

- [ ] a. Sergio likes all kinds of Japanese manga.
  Sergio likes Naruto.
  Therefore, Naruto is a manga.

- [x] b. If I play soccer, then I am sore the next day.
  I use the jacuzzi whenever I am sore.
  I didn't use jacuzzi today so I didn't play soccer.

  Correct.

- [x] c. A sports car is fun to drive.
  Bradley's car is not fun to drive.
  Therefore, Bradley's car is not a sports car.

  The conclusion follows by taking the contrapositive of the first statement.

- [x] d. All students in CS work very hard.
  Jonathan is a student in CS.
  So Jonathan works very hard.

  Correct. It follows from the first implication: if <x> is a CS student, then <x> works very hard.

Check

Your answer is correct.

You have correctly selected three options.

Question **6**

Correct

Marked out of 2.00

Now Antonio is asked to prove the following:

For any real numbers x and y, $|x + y| \leq |x| + |y|$.

If Antonio decides to use the proof by cases technique again, which of the following case-splits can he use? Choose the valid case-splits where all the real numbers x and y are covered by at least one of the cases.

(For example,

- Case 1: $x \geq 0$ and $y \in \mathbb{R}$
- Case 2: $x < 0$ and $y \in \mathbb{R}$

is a valid case-split because *any* real-number pair $\hat{x}$ and $\hat{y}$ falls in one of the two categories, whereas

- Case 1: $x > 0$ and $y > 0$
- Case 2: $x \leq 0$ and $y \in \mathbb{R}$

is not a valid case-split because the real-number pair $x = 1, y = -1$ is not considered by either of the cases.)

☐ a.
Case 1: $x \in \mathbb{R}$ and $y > 0$
Case 2: $x \in \mathbb{R}$ and $y < 0$

☐ b.
Case 1: $x > 0$ and $y > 0$
Case 2: $x > 0$ and $y < 0$
Case 3: $x < 0$ and $y > 0$
Case 4: $x < 0$ and $y < 0$
Case 5: $x = 0$ and $y = 0$

☑ c.
Case 1: $x \geq 0$ **or** $y \geq 0$ [NOTE to reader: we are using or here]
Case 2: $x < 0$ and $y < 0$

Correct. The assumption in Case 2 is the negation of that in Case 1.

☑ d.
Case 1: $x \geq 0$ and $y \geq 0$
Case 2: $x < 0$ and $y \geq 0$
Case 3: $x > 0$ and $y < 0$
Case 4: $x \leq 0$ and $y < 0$

Correct. These 4 cases cover all the possibilities for any arbitrary real numbers x and y. In particular, Cases 1 and 2 covers the scenario that $y \geq 0$ and Cases 3 and 4 cover the alternative scenario that $y < 0$.

☐ e.
Case 1: $x > 0$ and $y < 0$
Case 2: $x \leq 0$ and $y < 0$

Check

Your answer is correct.

You have correctly selected two options.

Question **7**

Correct

Marked out of 3.00

We wish to prove the statement about prime numbers using proof by contradiction

Theorem: For all natural numbers $n \geq 5$, $n! \geq 3^{n-1}$.

Which of the following statements is a valid starting assumption for a proof by contradiction?

Note: proofs by contradiction will need to assume the negation of the original statements.

Select one:

a. There exists a number \(n\) such that \(n < 5\) and \(n! < 3^{n-1}\).

b. There exists a number \(n\) such that \(n < 5\) and \(n! \geq 3^{n-1}\).

c. Assume that for all natural numbers \(n \geq 5\), \(n! \geq 3^{n-1}\).

d. There exists a number \(n\) such that \(n \geq 5\) and \(n! < 3^{n-1}\). Correct

e. There exists a number \(n\) such that if \(n \geq 5\) then \(n! \geq 3^{n-1}\).

Clear my choice

Check

Your answer is correct.

Question **8**

Correct

Marked out of 3.00

Consider the following statement.

Theorem: Every prime number is odd.

Proof (Steps are numbered for your convenience)

1. Let us assume for the sake of contradiction that there is an even prime number \(p\)
2. Since \(p\) is even, we have \(p = 2k\) for some natural number \(k\).
3. Therefore, \(p\) is divisible by \(2\).
4. Therefore, \(p\) cannot be prime.
5. However, this contradicts our assumption that \(p\) is a prime number (see step 1).

Recall a prime number \(p\) is a number that is divisible only by the numbers \(1\) and \(p\) (itself).

We know that \(2\) is prime and even and thus the theorem is wrong.

Where is the flaw in the proof above? Select the appropriate answer.

Select one:

- a. Step 2 is flawed: We cannot write \(p= 2k\).
- b. Step 3 is flawed. Not all even numbers are divisible by \(2\). For example, \(2\) is not.
- c. Step 4 is wrong: just because \(p\) is divisible by \(2\), we cannot conclude that \(p\) is not prime.

  Bingo: Prime numbers are only divisible by \(1\) and themselves. Therefore, when we said that \(p\) is divisible by \(2\), it does not mean that \(p\) is not prime. For example \(2\) is divisible by \(2\) but also a prime number.
- d. Step 1 is flawed: it should have assumed that there are prime numbers that are odd.
- e. Proof by contradiction cannot be used to prove properties of prime numbers.

Clear my choice

Check

Your answer is correct.

Question **9**

Correct

Marked out of 1.00

Prove there exists n such that 2^n -2, 2^n-1, and 2^n+1 are prime.

Select one:

- a. Consider the contrapositive, for all n, 2n - 1, 2n -2 and 2n +1 are odd and n is even. proof by nonsense.
- b. n = 1, proof by example.
- c. This is not true. These are 3 consecutive numbers so one must be even.
- d. Consider n = 2, proof by example. (selected)
- e. n = 5 is a counter example.

Clear my choice

Check

Your answer is correct.

Question **10**

Correct

Marked out of 1.00

Which are valid methods of a proof for

For all integers n, \(n^2 >= n\)

Select one or more:

- [ ] a. Proof by example
- [ ] b. Proof by contrapositive
- [x] c. Proof by contradiction
- [ ] d. Proof by counter example
- [ ] e. Proof by exhaustion.

Check

Your answer is correct.

You have correctly selected 1.