College of Engineering & Applied Sciences

CSPB 3202

Introduction To Artifical Intelligence

Assignment 3 - Constraint Satisfaction Problems

University Of Colorado

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I have neither given nor received unauthorized assistance.

Taylor Larrechea

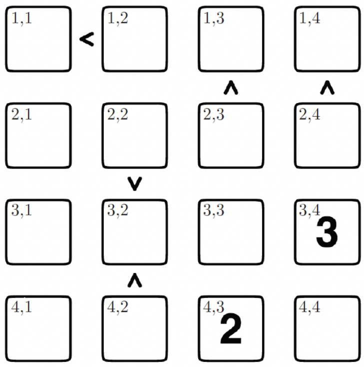
The original assignment can be found here, here, and here.

Problem 1 - CSP Futoshiki

Problem Statement

Futoshiki is a Japanese logic puzzle that is very simple, but can be quite challenging. You are given an $n \times n$ grid, and must place the numbers $1, \ldots n$ in the grid such that every row and column has exactly one of each. Additionally, the assignment must satisfy the inequalities placed between some adjacent squares.

Below is an instance of this problem, for size $n = 4$. Some of the squares have known values, such that the puzzle has a unique solution. (The letters mean nothing to the puzzle, and will be used only as labels with which to refer to certain squares). Note also that inequalities apply only to the two adjacent squares, and do not directly constrain other squares in the row or column.



Let's formulate this puzzle as a CSP. We will use 4^2 variables, one for each cell, with X_{ij} as the variable for the cell in the ith row and jth column (each cell contains its i, j label in the top left corner). The only unary constraints will be those assigning the known initial values to their respective squares (e.g. $X_{34} = 3$).

(a) Complete the formulation of the CSP using only binary constraints (in addition to the unary constraints specificed above). In particular, describe the domains of the variables, and all binary constraints you think are necessary. You do not need to enumerate them all, just describe them using concise mathematical notation. You are not permitted to use n-ary constraints where $n \geq 3$.

(b) After enforcing unary constraints, consider the binary constraints involving X_{14} and X_{24}. Enforce arc consistency on just these constraints and state the resulting domains for the two variables.

(c) Suppose we enforced unary constraints and ran arc consistency on this CSP, pruning the domains of all variables as much as possible. After this, what is the maximum possible domain size for any variable? [*Hint*: consider the least constrained variable(s); you should *not* have to run every step of arc consistency.]

(d) Suppose we enforced unary constraints and ran arc consistency on the initial CSP in the figure above. What is the maximum possible domain size for a variable adjacent to an inequality?

(e) By inspection of column 2, we find it is necessary that $X_{32} = 1$, despite not having found an assignment to any of the other cells in that column. Would running arc consistency find this requirement? Explain why or why not.

Solution - Part A

To start, the domain of these cells follows the form of

$$D(X_{ij}) = \{1, 2, \ldots, n\} \text{ for all cells with known values.}$$

The unary constraints for this problem are

$$X_{3,4} = 3 \text{ and } X_{4,3} = 2.$$

The binary constraints for the rows are such that all cells in the row must be unique, for example,

$$\forall_i \forall_{j_1} \neq j_2, X_{ij_1} \neq X_{ij_2}.$$

The binary constraint for the columns are such that all cells in a column must be unique, for example,

$$\forall_j \forall_{i_1} \neq i_2, X_{i_1 j} \neq X_{i_2 j}.$$

The inequality constraint is such that if there exists an inequality between cells, then for example,

$$X_{i_1 j_1} < X_{i_2 j_2}.$$

Solution - Part B

If we take for example the first two rows and the final two columns, we first check that $X_{14} \neq X_{24}$. We then prune the domain of X_{14} to remove values that are not possible for X_{24}.

At this point we know that 2,4 and 1,4 cannot be 3. With the constraint in place, this constitutes that 2,4 be greater than 1,4. This means that 2,4 can either be 4 or 2. If 2,4 is 4 then 1,4 can either be 2 or 1, but if 2,4 is 2, this means that 1,4 can only be 1.

Solution - Part C

If we take into account the least constrained variables, the maximum domain size should be

$$n - 1.$$

Solution - Part D

For a variable that is adjacent to an inequality, the maximum domain size will be reduced, more than likely down to

$$n - 2.$$

Solution - Part E

Arc consistency is not directly going to deduce that $X_{32} = 1$ because arc consistency only prunes inconsistent values and does not specifically deduce assignments. It will specifically deduce assignments if other values have been pruned.

Problem 2 - Course Scheduling

Problem Statement

You are in charge of scheduling for computer science classes that meet Mondays, Wednesdays and Fridays. There are 5 classes that meet on these days and 3 professors who will be teaching these classes. You are constrained by the fact that each professor can only teach one class at a time.

The classes are:

1. Class 1 - Intro to Programming: meets from 8:00-9:00am
2. Class 2 - Intro to Artificial Intelligence: meets from 8:30-9:30am
3. Class 3 - Natural Language Processing: meets from 9:00-10:00am
4. Class 4 - Computer Vision: meets from 9:00-10:00am
5. Class 5 - Machine Learning: meets from 10:30-11:30am

The professors are:

1. Professor A, who is qualified to teach Classes 1, 2, and 5.
2. Professor B, who is qualified to teach Classes 3, 4, and 5.
3. Professor C, who is qualified to teach Classes 1, 3, and 4.

(a) Formulate this problem as a CSP problem in which there is one variable per class, stating the domains (after enforcing unary constraints), and binary constraints. Constraints should be specified formally and precisely, but may be implicit rather than explicit.

(b) Draw the constraint graph associated with your CSP.

Solution - Part A

The variables in this context are professors A,B, and C and classes C_i for each class i from 1 to 5. The classes that can be taught by each professor (the domain) are then

- $D(C_1) = \{A, C\}$
- $D(C_2) = \{A\}$
- $D(C_3) = \{B, C\}$
- $D(C_4) = \{B, C\}$
- $D(C_5) = \{A, B\}$

The binary constraint for this problem is then

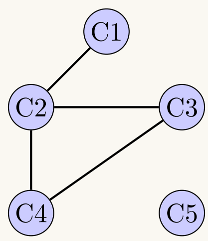
No professor can teach two classes that overlap in time.

The unary constraints for this problem are

- Class 1 cannot be taught at the same time as class 2 by professor A or C.
- Class 2 cannot be taught at the same time as class 1, 3, or 4 by professor A.
- Class 3 cannot be taught at the same time as class 2 or 4 by professor B or C.
- Class 4 cannot be taught at the same time as class 2 or 3 by professor B or C.
- Class 5 does not have any specific constraints of when it can be taught by professor A or B.

Solution - Part B

The constraint graph for this problem can be found below.



The nodes in the above image represent classes and the edges represent constraints between the classes. This means if an edge exists between the two classes, these two classes cannot be taught at the same time as one another.

In the above image, class 1 cannot be taught at the same time as class 2, class 2 cannot be taught at the same time as class 1, 3, or 4, class 3 cannot be taught at the same time as class 2 or 4, and class 4 cannot be taught at the same time as class 2 or 3. Class 5 does not have any conflicts with the other classes.

Problem 3 - CSP: Air Traffic Control

Problem Statement

We have five planes: A, B, C, D, and E and two runways: international and domestic. We would like to schedule a time slot and runway for each aircraft to **either** land or take off. We have four time slots: $\{1, 2, 3, 4\}$ for each runway, during which we can schedule a landing or take off of a plane. We must find an assignment that meets the following constraints:

- Plane B has lost an engine and must land in time slot 1.
- Plane D can only arrive at the airport to land during or after time slot 3.
- Plane A is running low on fuel but can last until at most time slot 2.
- Plane D must land before plane C takes off, because some passengers must transfer from D to C.
- No two aircrafts can reserve the same time slot for the same runway.

(a) Complete the formulation of this problem as a CSP in terms of variables, domains, and constraints (both unary and binary). Constraints should be expressed implicitly using mathematical or logical notation rather than with words.

(b) For the following subparts, we add the following two constraints:

- Planes A, B, and C cater to international flights and can only use the international runway.
- Planes D and E cater to domestic flights and can only use the domestic runway.

(i) With the addition of the two constraints above, we completely reformulate the CSP and draw the constraint graph.

(ii) What are the domains of the variables after enforcing arc-consistency? Begin by enforcing unary constraints. (Cross out values that are no longer in the domain.)

A	1	2	3	4
B	1	2	3	4
C	1	2	3	4
D	1	2	3	4
E	1	2	3	4

(iii) Arc-consistency can be rather expensive to enforce, and we believe that we can obtain faster solutions using only forward-checking on our variable assignments. Using the Minimum Remaining Values heuristic, perform backtracking search on the graph, breaking ties by picking lower values and characters first. List the (variable; assignment) pairs in the order they occur (including the assignments that are reverted upon reaching a dead end). Enforce unary constraints before starting the search.

A	1	2	3	4
B	1	2	3	4
C	1	2	3	4
D	1	2	3	4
E	1	2	3	4

Solution - Part A

The variables for each plane i have a runway assignment X_i. The domains correlate to the time slots $\{1, 2, 3, 4\}$ and the constraints are

Binary: No two planes can use the runway at the same time.

For the Unary constraints, these are when the planes are allowed to land:

- $X_A \leq 2$
- $X_B = 1$
- $X_D \geq 3$
- $X_D < X_C$

Or formally

$$\forall I, J \in \{A, B, C, D, E\}, I \neq J, X_I \neq X_J$$

Solution - Part B

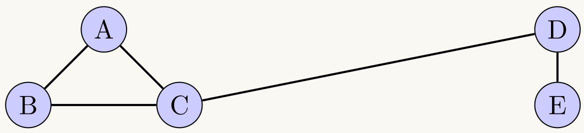
Part (i)

The domain and binary constraints do not change in this context. The only thing that changes from part (A) are the unary constraints. The unary constraints are then

- $X_A \leq 2$
- $X_B = 1$
- $X_D \geq 3$
- $X_D < X_C$
- Planes A,B, and C must use international runway.
- Planes D and E must use domestic runway.

The constraint graph would then look like the following.

International Runway **Domestic Runway**



Nodes in this graph represent planes and edges represent constraints between the planes.

Part (ii)

After enforcing arc consistency with the aforementioned constraints, the domains of each plane are then:

$$\begin{array}{c|cccc}
A & \cancel{1} & 2 & \cancel{3} & \cancel{4} \\
B & 1 & \cancel{2} & \cancel{3} & \cancel{4} \\
C & \cancel{1} & \cancel{2} & \cancel{3} & 4 \\
D & \cancel{1} & \cancel{2} & 3 & \cancel{4} \\
E & 1 & 2 & \cancel{3} & 4
\end{array}$$

Part (iii)

To address this we first start with the most constrained variable, plane B:

- $X_B = 1$ (Plane B must land first)
- We then perform forward checking and remove this time from all the planes landing on the international runway
  - $D(X_A) = \{2\}$
  - $D(X_C) = \{2, 3, 4\}$
- Next, we enforce the time in which plane A must land
  - $X_A = 2$
- We then remove this time from the same international runway planes
  - $D(X_C) = \{3, 4\}$
- The next most constrained variable is plane D since it must land before plane C takes off. Since the only possible values at this point for plane C are 3 and 4 this means that plane D must land at time 3
  - $X_D = 3$
- We then remove this time slot from plane C and we have
  - $X_C = 4$
- The next most constrained variable is then plane E and it must land at a different time than plane D leaving the domain for plane E as
  - $D(X_E) = \{1, 2, 4\}$

The order in which the planes are assigned times is then

$$(B, 1) \to (A, 2) \to (D, 3) \to (C, 4) \to (E, \{1, 2, 4\}).$$

Problem 4 - 2A Question 1

Problem Statement

Modify your code for uniform-cost search from Homework 1 so that it provides optionally as output the number of nodes expanded in completing the search.

Include a new optional logical (True/False) argument `return_nexp`, so your function calls to the new uniform cost search will look like: `uniform_cost(start, goal, state_graph, return_cost, return_nexp)`.

- If `return_nexp` is True, then the last output in the output tuple should be the number of nodes expanded.
- If `return_nexp` is False, then the code should behave exactly as it did in Homework 1.

Then, verify that your revised codes are working by checking Neal's optimal route from New York to Chicago. Include the number of nodes expanded and the path cost (using `map_distances`).

Solution

```python
import numpy as np
import heapq
import unittest

def path(previous, s):
    '''
    `previous` is a dictionary chaining together the predecessor state that led to each state
    `s` will be None for the initial state
    otherwise, start from the last state `s` and recursively trace `previous` back to the initial state,
    constructing a list of states visited as we go
    '''
    if s is None:
        return []
    else:
        return path(previous, previous[s])+[s]

def pathcost(path, step_costs):
    '''
    add up the step costs along a path, which is assumed to be a list output from the `path` function above
    '''
    cost = 0
    for s in range(len(path)-1):
        cost += step_costs[path[s]][path[s+1]]
    return cost

map_distances = dict(
    chi=dict(det=283, cle=345, ind=182),
    cle=dict(chi=345, det=169, col=144, pit=134, buf=189),
    ind=dict(chi=182, col=176),
    col=dict(ind=176, cle=144, pit=185),
    det=dict(chi=283, cle=169, buf=256),
    buf=dict(det=256, cle=189, pit=215, syr=150),
    pit=dict(col=185, cle=134, buf=215, phi=305, bal=247),
    syr=dict(buf=150, phi=253, new=254, bos=312),
    bal=dict(phi=101, pit=247),
    phi=dict(pit=305, bal=101, syr=253, new=97),
    new=dict(syr=254, phi=97, bos=215, pro=181),
    pro=dict(bos=50, new=181),
    bos=dict(pro=50, new=215, syr=312, por=107),
    por=dict(bos=107))

sld_providence = dict(
    chi=833,
    cle=531,
    ind=782,
    col=618,
    det=596,
    buf=385,
    pit=458,
    syr=253,
    bal=325,
    phi=236,
    new=157,
    pro=0,
    bos=38,
```

```
    por=136)

# Solution:

""" Frontier_PQ - Implements a priority queue ordered by path cost for uniform cost search
    Methods:
        __init__ - Initializes an empty priority queue
        is_empty - Checks if the priority queue is empty
        put - Adds an item with a specified priority to the priority queue
        get - Removes and returns the item with the lowest priority from the priority queue
    Algorithm:
        * __init__ initializes an empty list to represent the priority queue
        * is_empty returns True if the list is empty, otherwise False
        * put uses heapq.heappush to add an item to the priority queue with the given priority
        * get uses heapq.heappop to remove and return the item with the lowest priority
    Output:
        * is_empty returns a boolean indicating whether the priority queue is empty
        * put does not return a value
        * get returns the item with the lowest priority
"""
class Frontier_PQ:
    ''' frontier class for uniform search, ordered by path cost '''
    # add your code here
    def __init__(self):
        self.elements = []
    def is_empty(self):
        return len(self.elements) == 0
    def put(self, item, priority):
        heapq.heappush(self.elements, (priority, item))
    def get(self):
        return heapq.heappop(self.elements)

""" uniform_cost - Performs a Uniform Cost Search (UCS) on the state graph for the path between a
 start and goal.
    Input:
        start - Node that represents the start of the path.
        goal - Node that represents the desired end point of the path.
        state_graph - Dictionary representing the graph being searched, with costs for each edge.
        return_cost - Boolean value that indicates whether to return the cost of the path.
        return_nexp - Boolean value that indicates whether to return the number of nodes expanded.
    Algorithm:
        * Initialize a Frontier_PQ instance and add the start node with a priority of 0.
        * Initialize a dictionary of previous nodes with the start node set to None.
        * Initialize a dictionary to keep track of the cost to reach each node with the start node
 set to 0.
        * Initialize a counter to keep track of the number of nodes expanded (nodes_expanded) and set
  it to 0.
        * While the priority queue is not empty:
            * Get the node with the lowest cost from the priority queue.
            * If the current node is the goal:
                * Update the path to the goal using the previous nodes and the goal.
                * Calculate the cost if return_cost is True.
                * If return_cost is True:
                    * If return_nexp is True, return the path to the goal, the cost, and the number
 of nodes expanded.
                    * Otherwise, return the path to the goal and the cost.
                * If return_cost is False:
                    * If return_nexp is True, return the path to the goal and the number of nodes
 expanded.
                    * Otherwise, just return the path to the goal.
            * Increment the nodes_expanded counter.
            * Iterate over the neighbors of the current node:
                * Calculate the new cost to reach each neighbor.
                * If the neighbor has not been visited or the new cost is lower than the recorded
 cost:
                    * Update the cost to reach the neighbor.
                    * Add the neighbor to the priority queue with the new cost as priority.
                    * Update the previous nodes with the current node.
        * If the goal is not reachable:
            * If return_cost is True:
                * If return_nexp is True, return (None, 0, nodes_expanded).
                * Otherwise, return (None, 0).
            * If return_cost is False:
                * If return_nexp is True, return (None, nodes_expanded).
                * Otherwise, return None.
    Output:
        Returns the path to the goal.
        If return_cost is True, also returns the cost of the path.
        If return_nexp is True, also returns the number of nodes expanded.
"""
def uniform_cost(start, goal, state_graph, return_cost=False, return_nexp=True):
    frontier = Frontier_PQ()
    frontier.put(start, 0)
    previous = {start: None}
    cost_so_far = {start: 0}
    nodes_expanded = 0
    while not frontier.is_empty():
        current_priority, current = frontier.get()
        if current == goal:
            path_to_goal = path(previous, goal)
            if return_cost:
                cost = pathcost(path_to_goal, state_graph)
```

```python
                if return_nexp:
                    return path_to_goal, cost, nodes_expanded
                else:
                    return path_to_goal, cost
            else:
                if return_nexp:
                    return path_to_goal, nodes_expanded
                else:
                    return path_to_goal
        nodes_expanded += 1
        for neighbor in state_graph[current]:
            new_cost = cost_so_far[current] + state_graph[current][neighbor]
            if neighbor not in cost_so_far or new_cost < cost_so_far[neighbor]:
                cost_so_far[neighbor] = new_cost
                priority = new_cost
                frontier.put(neighbor, priority)
                previous[neighbor] = current
    if return_cost:
        if return_nexp:
            return None, 0, nodes_expanded
        else:
            return None, 0
    else:
        if return_nexp:
            return None, nodes_expanded
        else:
            return None

```

Problem 5 - 2A Question 2

Problem Statement

Define a function to take as an argument the state that Neal is in (city on our graphs), and return as output the value of the straight-line distance heuristic, between Neal's state and Providence.

Note that your function should be quite short, and amounts to looking up the proper value from the `sld_providence` dictionary defined in the helper functions. Call this function `heuristic_sld_providence`.

Solution

```python
sld_providence = dict(
    chi=833,
    cle=531,
    ind=782,
    col=618,
    det=596,
    buf=385,
    pit=458,
    syr=253,
    bal=325,
    phi=236,
    new=157,
    pro=0,
    bos=38,
    por=136)

""" heuristic_sld_providence - Returns the straight-line distance heuristic between the given state (
city) and Providence.
    Input:
        state - The current city/state as a string.
    Output:
        The straight-line distance from the given state to Providence as an integer.
"""
def heuristic_sld_providence(state):
    return sld_providence[state]

```

Problem 6 - 2A Question 3

Problem Statement

We are finally ready to help Neal use his knowledge of straight-line distances from various cities to Providence to inform his family's route to move from Chicago to Providence!

Modify your uniform-cost search codes from 1.1 even further so that they now perform A* search, using as the heuristic function the straight-line distance to Providence.

Provide heuristic as an additional argument, which should just be the function to call within the A* code. So your call to the A routine should look like: `astar_search(start, goal, state_graph, heuristic, return_cost, return_nexp)`. (This kind of modular programming will make it much easier to swap in alternative heuristic functions later, and also helps to facilitate debugging if something goes wrong.)

Solution

```
import numpy as np
import heapq
import unittest

def path(previous, s):
    '''
    `previous` is a dictionary chaining together the predecessor state that led to each state
    `s` will be None for the initial state
    otherwise, start from the last state `s` and recursively trace `previous` back to the initial
 state,
    constructing a list of states visited as we go
    '''
    if s is None:
        return []
    else:
        return path(previous, previous[s])+[s]

def pathcost(path, step_costs):
    '''
    add up the step costs along a path, which is assumed to be a list output from the `path` function
  above
    '''
    cost = 0
    for s in range(len(path)-1):
        cost += step_costs[path[s]][path[s+1]]
    return cost

map_distances = dict(
    chi=dict(det=283, cle=345, ind=182),
    cle=dict(chi=345, det=169, col=144, pit=134, buf=189),
    ind=dict(chi=182, col=176),
    col=dict(ind=176, cle=144, pit=185),
    det=dict(chi=283, cle=169, buf=256),
    buf=dict(det=256, cle=189, pit=215, syr=150),
    pit=dict(col=185, cle=134, buf=215, phi=305, bal=247),
    syr=dict(buf=150, phi=253, new=254, bos=312),
    bal=dict(phi=101, pit=247),
    phi=dict(pit=305, bal=101, syr=253, new=97),
    new=dict(syr=254, phi=97, bos=215, pro=181),
    pro=dict(bos=50, new=181),
    bos=dict(pro=50, new=215, syr=312, por=107),
    por=dict(bos=107))

sld_providence = dict(
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    buf=385,
    pit=458,
    syr=253,
    bal=325,
    phi=236,
    new=157,
    pro=0,
    bos=38,
    por=136)

# Solution:

""" heuristic_sld_providence - Returns the straight-line distance heuristic between the given state (
```

```
city) and Providence.
    Input:
        state - The current city/state as a string.
    Output:
        The straight-line distance from the given state to Providence as an integer.
"""
def heuristic_sld_providence(state):
    return sld_providence[state]

""" Frontier_PQ - Implements a priority queue ordered by path cost for uniform cost search
    Methods:
        __init__ - Initializes an empty priority queue
        is_empty - Checks if the priority queue is empty
        put - Adds an item with a specified priority to the priority queue
        get - Removes and returns the item with the lowest priority from the priority queue
    Algorithm:
        * __init__ initializes an empty list to represent the priority queue
        * is_empty returns True if the list is empty, otherwise False
        * put uses heapq.heappush to add an item to the priority queue with the given priority
        * get uses heapq.heappop to remove and return the item with the lowest priority
    Output:
        * is_empty returns a boolean indicating whether the priority queue is empty
        * put does not return a value
        * get returns the item with the lowest priority
"""
class Frontier_PQ:
    ''' frontier class for uniform search, ordered by path cost '''
    # add your code here
    def __init__(self):
        self.elements = []
    def is_empty(self):
        return len(self.elements) == 0
    def put(self, item, priority):
        heapq.heappush(self.elements, (priority, item))
    def get(self):
        return heapq.heappop(self.elements)

# Solution:
""" astar_search - Performs an A* Search on the state graph for the path between a start and goal.
    Input:
        start - Node that represents the start of the path.
        goal - Node that represents the desired end point of the path.
        state_graph - Dictionary representing the graph being searched, with costs for each edge.
        heuristic - Function to estimate the cost from a state to the goal.
        return_cost - Boolean value that indicates whether to return the cost of the path.
        return_nexp - Boolean value that indicates whether to return the number of nodes expanded.
    Algorithm:
        * Initialize a Frontier_PQ instance and add the start node with a priority of 0.
        * Initialize a dictionary of previous nodes with the start node set to None.
        * Initialize a dictionary to keep track of the cost to reach each node with the start node set to 0.
        * Initialize a counter to keep track of the number of nodes expanded (nodes_expanded) and set it to 0.
        * While the priority queue is not empty:
            * Get the node with the lowest cost from the priority queue.
            * Increment the nodes_expanded counter.
            * If the current node is the goal:
                * Update the path to the goal using the previous nodes and the goal.
                * Calculate the cost if return_cost is True.
                * If return_cost is True:
                    * If return_nexp is True, return the path to the goal, the cost, and the number of nodes expanded.
                    * Otherwise, return the path to the goal and the cost.
                * If return_cost is False:
                    * If return_nexp is True, return the path to the goal and the number of nodes expanded.
                    * Otherwise, just return the path to the goal.
            * Iterate over the neighbors of the current node:
                * Calculate the new cost to reach each neighbor.
                * If the neighbor has not been visited or the new cost is lower than the recorded cost:
                    * Update the cost to reach the neighbor.
                    * Calculate the priority by adding the heuristic value to the new cost.
                    * Add the neighbor to the priority queue with the new priority.
                    * Update the previous nodes with the current node.
        * If the goal is not reachable:
            * If return_cost is True:
                * If return_nexp is True, return (None, 0, nodes_expanded).
                * Otherwise, return (None, 0).
            * If return_cost is False:
                * If return_nexp is True, return (None, nodes_expanded).
                * Otherwise, return None.
    Output:
        Returns the path to the goal.
        If return_cost is True, also returns the cost of the path.
        If return_nexp is True, also returns the number of nodes expanded.
"""
def astar_search(start, goal, state_graph, heuristic, return_cost=False, return_nexp=False):
    frontier = Frontier_PQ()
    frontier.put(start, 0)
    previous = {start: None}
    cost_so_far = {start: 0}
```

```python
    nodes_expanded = 0
    while not frontier.is_empty():
        current_priority, current = frontier.get()
        nodes_expanded += 1
        if current == goal:
            path_to_goal = path(previous, goal)
            if return_cost:
                cost = pathcost(path_to_goal, state_graph)
                if return_nexp:
                    return path_to_goal, cost, nodes_expanded
                else:
                    return path_to_goal, cost
            else:
                if return_nexp:
                    return path_to_goal, nodes_expanded
                else:
                    return path_to_goal
        for neighbor in state_graph[current]:
            new_cost = cost_so_far[current] + state_graph[current][neighbor]
            if neighbor not in cost_so_far or new_cost < cost_so_far[neighbor]:
                cost_so_far[neighbor] = new_cost
                priority = new_cost + heuristic(neighbor)
                frontier.put(neighbor, priority)
                previous[neighbor] = current
    if return_cost:
        if return_nexp:
            return None, 0, nodes_expanded
        else:
            return None, 0
    else:
        if return_nexp:
            return None, nodes_expanded
        else:
            return None

```

Problem 7 - 2A Question 4

Problem Statement

Print the the following using your code:

1. The optimal path.
2. The optimal path cost (miles traveled).
3. The number of states expanded during the A* search.

Additionally, print how many states must be expanded to find the optimal path from Buffalo to Providence using the regular old uniform-cost search algorithm from 1.1.

Solution

```
import numpy as np
import heapq
import unittest

def path(previous, s):
    '''
    `previous` is a dictionary chaining together the predecessor state that led to each state
    `s` will be None for the initial state
    otherwise, start from the last state `s` and recursively trace `previous` back to the initial
 state,
    constructing a list of states visited as we go
    '''
    if s is None:
        return []
    else:
        return path(previous, previous[s])+[s]

def pathcost(path, step_costs):
    '''
    add up the step costs along a path, which is assumed to be a list output from the `path` function
  above
    '''
    cost = 0
    for s in range(len(path)-1):
        cost += step_costs[path[s]][path[s+1]]
    return cost

map_distances = dict(
    chi=dict(det=283, cle=345, ind=182),
    cle=dict(chi=345, det=169, col=144, pit=134, buf=189),
    ind=dict(chi=182, col=176),
    col=dict(ind=176, cle=144, pit=185),
    det=dict(chi=283, cle=169, buf=256),
    buf=dict(det=256, cle=189, pit=215, syr=150),
    pit=dict(col=185, cle=134, buf=215, phi=305, bal=247),
    syr=dict(buf=150, phi=253, new=254, bos=312),
    bal=dict(phi=101, pit=247),
    phi=dict(pit=305, bal=101, syr=253, new=97),
    new=dict(syr=254, phi=97, bos=215, pro=181),
    pro=dict(bos=50, new=181),
    bos=dict(pro=50, new=215, syr=312, por=107),
    por=dict(bos=107))

sld_providence = dict(
    chi=833,
    cle=531,
    ind=782,
    col=618,
    det=596,
    buf=385,
    pit=458,
    syr=253,
    bal=325,
    phi=236,
    new=157,
    pro=0,
    bos=38,
    por=136)

# Solution:
```

```

""" heuristic_sld_providence - Returns the straight-line distance heuristic between the given state (
 city) and Providence.
    Input:
        state - The current city/state as a string.
    Output:
        The straight-line distance from the given state to Providence as an integer.
"""
def heuristic_sld_providence(state):
    return sld_providence[state]

""" Frontier_PQ - Implements a priority queue ordered by path cost for uniform cost search
    Methods:
        __init__ - Initializes an empty priority queue
        is_empty - Checks if the priority queue is empty
        put - Adds an item with a specified priority to the priority queue
        get - Removes and returns the item with the lowest priority from the priority queue
    Algorithm:
        * __init__ initializes an empty list to represent the priority queue
        * is_empty returns True if the list is empty, otherwise False
        * put uses heapq.heappush to add an item to the priority queue with the given priority
        * get uses heapq.heappop to remove and return the item with the lowest priority
    Output:
        * is_empty returns a boolean indicating whether the priority queue is empty
        * put does not return a value
        * get returns the item with the lowest priority
"""
class Frontier_PQ:
    ''' frontier class for uniform search, ordered by path cost '''
    # add your code here
    def __init__(self):
        self.elements = []
    def is_empty(self):
        return len(self.elements) == 0
    def put(self, item, priority):
        heapq.heappush(self.elements, (priority, item))
    def get(self):
        return heapq.heappop(self.elements)

# Solution:

""" astar_search - Performs an A* Search on the state graph for the path between a start and goal.
    Input:
        start - Node that represents the start of the path.
        goal - Node that represents the desired end point of the path.
        state_graph - Dictionary representing the graph being searched, with costs for each edge.
        heuristic - Function to estimate the cost from a state to the goal.
        return_cost - Boolean value that indicates whether to return the cost of the path.
        return_nexp - Boolean value that indicates whether to return the number of nodes expanded.
    Algorithm:
        * Initialize a Frontier_PQ instance and add the start node with a priority of 0.
        * Initialize a dictionary of previous nodes with the start node set to None.
        * Initialize a dictionary to keep track of the cost to reach each node with the start node
 set to 0.
        * Initialize a counter to keep track of the number of nodes expanded (nodes_expanded) and set
  it to 0.
        * While the priority queue is not empty:
            * Get the node with the lowest cost from the priority queue.
            * Increment the nodes_expanded counter.
            * If the current node is the goal:
                * Update the path to the goal using the previous nodes and the goal.
                * Calculate the cost if return_cost is True.
                * If return_cost is True:
                    * If return_nexp is True, return the path to the goal, the cost, and the number
 of nodes expanded.
                    * Otherwise, return the path to the goal and the cost.
                * If return_cost is False:
                    * If return_nexp is True, return the path to the goal and the number of nodes
 expanded.
                    * Otherwise, just return the path to the goal.
            * Iterate over the neighbors of the current node:
                * Calculate the new cost to reach each neighbor.
                * If the neighbor has not been visited or the new cost is lower than the recorded
 cost:
                    * Update the cost to reach the neighbor.
                    * Calculate the priority by adding the heuristic value to the new cost.
                    * Add the neighbor to the priority queue with the new priority.
                    * Update the previous nodes with the current node.
        * If the goal is not reachable:
            * If return_cost is True:
                * If return_nexp is True, return (None, 0, nodes_expanded).
                * Otherwise, return (None, 0).
            * If return_cost is False:
                * If return_nexp is True, return (None, nodes_expanded).
                * Otherwise, return None.
    Output:
        Returns the path to the goal.
        If return_cost is True, also returns the cost of the path.
        If return_nexp is True, also returns the number of nodes expanded.
"""
def astar_search(start, goal, state_graph, heuristic, return_cost=False, return_nexp=False):
    frontier = Frontier_PQ()
```

```
    frontier.put(start, 0)
    previous = {start: None}
    cost_so_far = {start: 0}
    nodes_expanded = 0
    while not frontier.is_empty():
        current_priority, current = frontier.get()
        nodes_expanded += 1
        if current == goal:
            path_to_goal = path(previous, goal)
            if return_cost:
                cost = pathcost(path_to_goal, state_graph)
                if return_nexp:
                    return path_to_goal, cost, nodes_expanded
                else:
                    return path_to_goal, cost
            else:
                if return_nexp:
                    return path_to_goal, nodes_expanded
                else:
                    return path_to_goal
        for neighbor in state_graph[current]:
            new_cost = cost_so_far[current] + state_graph[current][neighbor]
            if neighbor not in cost_so_far or new_cost < cost_so_far[neighbor]:
                cost_so_far[neighbor] = new_cost
                priority = new_cost + heuristic(neighbor)
                frontier.put(neighbor, priority)
                previous[neighbor] = current
    if return_cost:
        if return_nexp:
            return None, 0, nodes_expanded
        else:
            return None, 0
    else:
        if return_nexp:
            return None, nodes_expanded
        else:
            return None

""" uniform_cost - Performs a Uniform Cost Search (UCS) on the state graph for the path between a start and goal.
    Input:
        start - Node that represents the start of the path.
        goal - Node that represents the desired end point of the path.
        state_graph - Dictionary representing the graph being searched, with costs for each edge.
        return_cost - Boolean value that indicates whether to return the cost of the path.
        return_nexp - Boolean value that indicates whether to return the number of nodes expanded.
    Algorithm:
        * Initialize a Frontier_PQ instance and add the start node with a priority of 0.
        * Initialize a dictionary of previous nodes with the start node set to None.
        * Initialize a dictionary to keep track of the cost to reach each node with the start node set to 0.
        * Initialize a counter to keep track of the number of nodes expanded (nodes_expanded) and set it to 0.
        * While the priority queue is not empty:
            * Get the node with the lowest cost from the priority queue.
            * If the current node is the goal:
                * Update the path to the goal using the previous nodes and the goal.
                * Calculate the cost if return_cost is True.
                * If return_cost is True:
                    * If return_nexp is True, return the path to the goal, the cost, and the number of nodes expanded.
                    * Otherwise, return the path to the goal and the cost.
                * If return_cost is False:
                    * If return_nexp is True, return the path to the goal and the number of nodes expanded.
                    * Otherwise, just return the path to the goal.
            * Increment the nodes_expanded counter.
            * Iterate over the neighbors of the current node:
                * Calculate the new cost to reach each neighbor.
                * If the neighbor has not been visited or the new cost is lower than the recorded cost:
                    * Update the cost to reach the neighbor.
                    * Add the neighbor to the priority queue with the new cost as priority.
                    * Update the previous nodes with the current node.
        * If the goal is not reachable:
            * If return_cost is True:
                * If return_nexp is True, return (None, 0, nodes_expanded).
                * Otherwise, return (None, 0).
            * If return_cost is False:
                * If return_nexp is True, return (None, nodes_expanded).
                * Otherwise, return None.
    Output:
        Returns the path to the goal.
        If return_cost is True, also returns the cost of the path.
        If return_nexp is True, also returns the number of nodes expanded.
"""
def uniform_cost(start, goal, state_graph, return_cost=False, return_nexp=True):
    frontier = Frontier_PQ()
    frontier.put(start, 0)
    previous = {start: None}
    cost_so_far = {start: 0}
    nodes_expanded = 0
```

```python
    while not frontier.is_empty():
        current_priority, current = frontier.get()
        nodes_expanded += 1
        if current == goal:
            path_to_goal = path(previous, goal)
            if return_cost:
                cost = pathcost(path_to_goal, state_graph)
                if return_nexp:
                    return path_to_goal, cost, nodes_expanded
                else:
                    return path_to_goal, cost
            else:
                if return_nexp:
                    return path_to_goal, nodes_expanded
                else:
                    return path_to_goal
        for neighbor in state_graph[current]:
            new_cost = cost_so_far[current] + state_graph[current][neighbor]
            if neighbor not in cost_so_far or new_cost < cost_so_far[neighbor]:
                cost_so_far[neighbor] = new_cost
                priority = new_cost
                frontier.put(neighbor, priority)
                previous[neighbor] = current
    if return_cost:
        if return_nexp:
            return None, 0, nodes_expanded
        else:
            return None, 0
    else:
        if return_nexp:
            return None, nodes_expanded
        else:
            return None

```

Problem 8 - 2A Question 5

Problem Statement

Comment on the difference in states that must be explored by each algorithm. Sanity check: No matter what your start and goal states are, how should the output from `astar_search` and `uniform_cost` search compare?

Solution

The output from the astar_search and uniform_cost search should have the same paths as one another. The difference between the two should lie in how many nodes are expanded for each.

In the astar search, there should be less nodes that are expanded (and this is true in the previous example) and for uniform cost search there should be more (and there is in the previous example).

Problem 9 - 2A Question 6

Problem Statement

How many states are expanded by each of A*search and uniform cost search to find the optimal path from Philadelphia to Providence?

Solution

```python
import numpy as np
import heapq
import unittest

def path(previous, s):
    '''
    'previous' is a dictionary chaining together the predecessor state that led to each state
    's' will be None for the initial state
    otherwise, start from the last state 's' and recursively trace 'previous' back to the initial state,
    constructing a list of states visited as we go
    '''
    if s is None:
        return []
    else:
        return path(previous, previous[s])+[s]

def pathcost(path, step_costs):
    '''
    add up the step costs along a path, which is assumed to be a list output from the 'path' function above
    '''
    cost = 0
    for s in range(len(path)-1):
        cost += step_costs[path[s]][path[s+1]]
    return cost

map_distances = dict(
    chi=dict(det=283, cle=345, ind=182),
    cle=dict(chi=345, det=169, col=144, pit=134, buf=189),
    ind=dict(chi=182, col=176),
    col=dict(ind=176, cle=144, pit=185),
    det=dict(chi=283, cle=169, buf=256),
    buf=dict(det=256, cle=189, pit=215, syr=150),
    pit=dict(col=185, cle=134, buf=215, phi=305, bal=247),
    syr=dict(buf=150, phi=253, new=254, bos=312),
    bal=dict(phi=101, pit=247),
    phi=dict(pit=305, bal=101, syr=253, new=97),
    new=dict(syr=254, phi=97, bos=215, pro=181),
    pro=dict(bos=50, new=181),
    bos=dict(pro=50, new=215, syr=312, por=107),
    por=dict(bos=107))

sld_providence = dict(
    chi=833,
    cle=531,
    ind=782,
    col=618,
    det=596,
    buf=385,
    pit=458,
    syr=253,
    bal=325,
    phi=236,
    new=157,
    pro=0,
    bos=38,
    por=136)

# Solution:

""" heuristic_sld_providence - Returns the straight-line distance heuristic between the given state (city) and Providence.
    Input:
        state - The current city/state as a string.
    Output:
        The straight-line distance from the given state to Providence as an integer.
"""
def heuristic_sld_providence(state):
    return sld_providence[state]

```

```
""" Frontier_PQ - Implements a priority queue ordered by path cost for uniform cost search
    Methods:
        __init__ - Initializes an empty priority queue
        is_empty - Checks if the priority queue is empty
        put - Adds an item with a specified priority to the priority queue
        get - Removes and returns the item with the lowest priority from the priority queue
    Algorithm:
        * __init__ initializes an empty list to represent the priority queue
        * is_empty returns True if the list is empty, otherwise False
        * put uses heapq.heappush to add an item to the priority queue with the given priority
        * get uses heapq.heappop to remove and return the item with the lowest priority
    Output:
        * is_empty returns a boolean indicating whether the priority queue is empty
        * put does not return a value
        * get returns the item with the lowest priority
"""
class Frontier_PQ:
    ''' frontier class for uniform search, ordered by path cost '''
    # add your code here
    def __init__(self):
        self.elements = []
    def is_empty(self):
        return len(self.elements) == 0
    def put(self, item, priority):
        heapq.heappush(self.elements, (priority, item))
    def get(self):
        return heapq.heappop(self.elements)

# Solution:

""" astar_search - Performs an A* Search on the state graph for the path between a start and goal.
    Input:
        start - Node that represents the start of the path.
        goal - Node that represents the desired end point of the path.
        state_graph - Dictionary representing the graph being searched, with costs for each edge.
        heuristic - Function to estimate the cost from a state to the goal.
        return_cost - Boolean value that indicates whether to return the cost of the path.
        return_nexp - Boolean value that indicates whether to return the number of nodes expanded.
    Algorithm:
        * Initialize a Frontier_PQ instance and add the start node with a priority of 0.
        * Initialize a dictionary of previous nodes with the start node set to None.
        * Initialize a dictionary to keep track of the cost to reach each node with the start node
 set to 0.
        * Initialize a counter to keep track of the number of nodes expanded (nodes_expanded) and set
  it to 0.
        * While the priority queue is not empty:
            * Get the node with the lowest cost from the priority queue.
            * Increment the nodes_expanded counter.
            * If the current node is the goal:
                * Update the path to the goal using the previous nodes and the goal.
                * Calculate the cost if return_cost is True.
                * If return_cost is True:
                    * If return_nexp is True, return the path to the goal, the cost, and the number
 of nodes expanded.
                    * Otherwise, return the path to the goal and the cost.
                * If return_cost is False:
                    * If return_nexp is True, return the path to the goal and the number of nodes
 expanded.
                    * Otherwise, just return the path to the goal.
            * Iterate over the neighbors of the current node:
                * Calculate the new cost to reach each neighbor.
                * If the neighbor has not been visited or the new cost is lower than the recorded
 cost:
                    * Update the cost to reach the neighbor.
                    * Calculate the priority by adding the heuristic value to the new cost.
                    * Add the neighbor to the priority queue with the new priority.
                    * Update the previous nodes with the current node.
        * If the goal is not reachable:
            * If return_cost is True:
                * If return_nexp is True, return (None, 0, nodes_expanded).
                * Otherwise, return (None, 0).
            * If return_cost is False:
                * If return_nexp is True, return (None, nodes_expanded).
                * Otherwise, return None.
    Output:
        Returns the path to the goal.
        If return_cost is True, also returns the cost of the path.
        If return_nexp is True, also returns the number of nodes expanded.
"""
def astar_search(start, goal, state_graph, heuristic, return_cost=False, return_nexp=False):
    frontier = Frontier_PQ()
    frontier.put(start, 0)
    previous = {start: None}
    cost_so_far = {start: 0}
    nodes_expanded = 0
    while not frontier.is_empty():
        current_priority, current = frontier.get()
        nodes_expanded += 1
        if current == goal:
            path_to_goal = path(previous, goal)
            if return_cost:
                cost = pathcost(path_to_goal, state_graph)
```

```
                if return_nexp:
                    return path_to_goal, cost, nodes_expanded
                else:
                    return path_to_goal, cost
            else:
                if return_nexp:
                    return path_to_goal, nodes_expanded
                else:
                    return path_to_goal
        for neighbor in state_graph[current]:
            new_cost = cost_so_far[current] + state_graph[current][neighbor]
            if neighbor not in cost_so_far or new_cost < cost_so_far[neighbor]:
                cost_so_far[neighbor] = new_cost
                priority = new_cost + heuristic(neighbor)
                frontier.put(neighbor, priority)
                previous[neighbor] = current
    if return_cost:
        if return_nexp:
            return None, 0, nodes_expanded
        else:
            return None, 0
    else:
        if return_nexp:
            return None, nodes_expanded
        else:
            return None

""" uniform_cost - Performs a Uniform Cost Search (UCS) on the state graph for the path between a start and goal.
    Input:
        start - Node that represents the start of the path.
        goal - Node that represents the desired end point of the path.
        state_graph - Dictionary representing the graph being searched, with costs for each edge.
        return_cost - Boolean value that indicates whether to return the cost of the path.
        return_nexp - Boolean value that indicates whether to return the number of nodes expanded.
    Algorithm:
        * Initialize a Frontier_PQ instance and add the start node with a priority of 0.
        * Initialize a dictionary of previous nodes with the start node set to None.
        * Initialize a dictionary to keep track of the cost to reach each node with the start node set to 0.
        * Initialize a counter to keep track of the number of nodes expanded (nodes_expanded) and set it to 0.
        * While the priority queue is not empty:
            * Get the node with the lowest cost from the priority queue.
            * If the current node is the goal:
                * Update the path to the goal using the previous nodes and the goal.
                * Calculate the cost if return_cost is True.
                * If return_cost is True:
                    * If return_nexp is True, return the path to the goal, the cost, and the number of nodes expanded.
                    * Otherwise, return the path to the goal and the cost.
                * If return_cost is False:
                    * If return_nexp is True, return the path to the goal and the number of nodes expanded.
                    * Otherwise, just return the path to the goal.
            * Increment the nodes_expanded counter.
            * Iterate over the neighbors of the current node:
                * Calculate the new cost to reach each neighbor.
                * If the neighbor has not been visited or the new cost is lower than the recorded cost:
                    * Update the cost to reach the neighbor.
                    * Add the neighbor to the priority queue with the new cost as priority.
                    * Update the previous nodes with the current node.
        * If the goal is not reachable:
            * If return_cost is True:
                * If return_nexp is True, return (None, 0, nodes_expanded).
                * Otherwise, return (None, 0).
            * If return_cost is False:
                * If return_nexp is True, return (None, nodes_expanded).
                * Otherwise, return None.
    Output:
        Returns the path to the goal.
        If return_cost is True, also returns the cost of the path.
        If return_nexp is True, also returns the number of nodes expanded.
"""
def uniform_cost(start, goal, state_graph, return_cost=False, return_nexp=True):
    frontier = Frontier_PQ()
    frontier.put(start, 0)
    previous = {start: None}
    cost_so_far = {start: 0}
    nodes_expanded = 0
    while not frontier.is_empty():
        current_priority, current = frontier.get()
        nodes_expanded += 1
        if current == goal:
            path_to_goal = path(previous, goal)
            if return_cost:
                cost = pathcost(path_to_goal, state_graph)
                if return_nexp:
                    return path_to_goal, cost, nodes_expanded
                else:
                    return path_to_goal, cost
```

```python
            else:
                if return_nexp:
                    return path_to_goal, nodes_expanded
                else:
                    return path_to_goal
        for neighbor in state_graph[current]:
            new_cost = cost_so_far[current] + state_graph[current][neighbor]
            if neighbor not in cost_so_far or new_cost < cost_so_far[neighbor]:
                cost_so_far[neighbor] = new_cost
                priority = new_cost
                frontier.put(neighbor, priority)
                previous[neighbor] = current
    if return_cost:
        if return_nexp:
            return None, 0, nodes_expanded
        else:
            return None, 0
    else:
        if return_nexp:
            return None, nodes_expanded
        else:
            return None
```

Problem 10 - 2A Question 7

Problem Statement

Moodle Quiz Problem 7. Pass the unit tests for the CSP class.

Solution

```python
from collections import OrderedDict

canada = OrderedDict(
    [("AB"  , ["BC","NT","SK"]),
    ("BC" , ["AB","NT","YT"]),
    ("LB" , ["NF", "NS", "PE","QC"]),
    ("MB" , ["ON","NV","SK"]),
    ("NB" , ["NS","QC"]),
    ("NF" , ["LB","QC"]),
    ("NS" , ["LB","NB","PE"]),
    ("NT" , ["AB","BC","NV","SK","YT"]),
    ("NV" , ["MB","NT"]),
    ("ON" , ["MB","QC"]),
    ("PE" , ["LB","NS","QC"]),
    ("QC" , ["LB","NB","NF","ON","PE"]),
    ("SK" , ["AB","MB","NT"]),
    ("YT" , ["BC","NT"])])

states = ["AB", "BC", "LB", "MB", "NB", "NF", "NS", "NT", "NV", "ON", "PE", "QC", "SK", "YT"]
colors = ["blue", "green", "red"]

class CSP:
    # your code here#
    """ Constructor - Creates an instance of the CSP class
        Input:
            variables - These are the variables of the CSP
            neighbors - These are the neighbors of a given variable
            domain - The domain of the variables in the CSP
        Algorithm:
            * Create instances for variables, neighbors, and domain from the constructor
        Output:
            This function does not return a value
    """
    def __init__(self, variables, neighbors, domain):
        self.variables = variables
        self.neighbors = neighbors
        self.domain = {var: domain for var in variables}

cspObj = CSP(states,canada,colors)

```