

Search



Constraint Satisfaction Problems

Review Search

- Assumptions: a single agent, deterministic actions, fully observed state, discrete state space
- Path planning
 - The path to the goal is the important thing
 - Paths have various costs, depths
 - Heuristics give problem-specific guidance

Different Kinds of Search

Planning

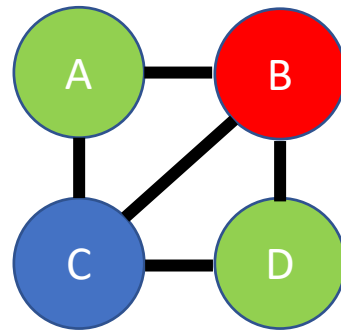
- Sequences of actions
- The path to the goal is the important thing
- Paths have various costs, depths
- Heuristics give problem-specific guidance
- Shortest path finding problem

Identification

- Assignments to variables
- The goal itself is important, not the path
- All paths at the same depth
- Uses general-purpose guideline
- Constraint satisfaction problem

What is Constraint Satisfaction Problems?

- A special subset of search problems
- State is defined by **variables $\{X\}$** with values from a **domain $\{D\}$**



$X = \{A, B, C, D\}$

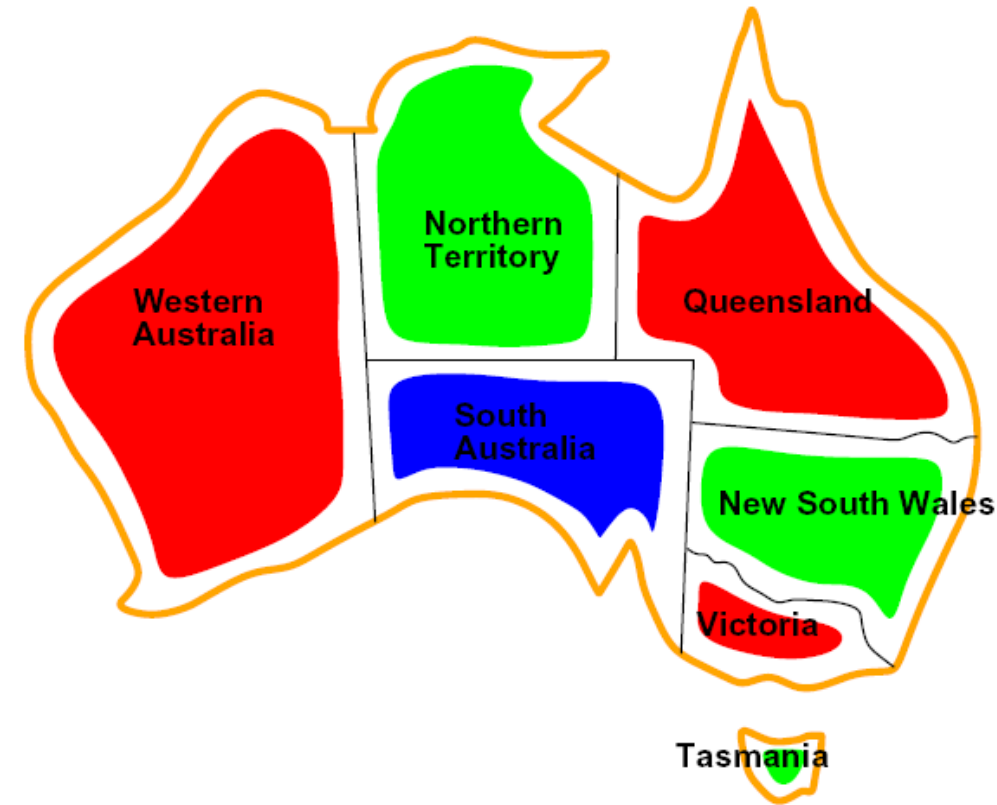
$D = \{\text{red}, \text{green}, \text{blue}\}$

- Goal test is a **set of constraints** specifying allowable combinations of values for subsets of variables
- Allows useful general-purpose algorithms with more power than standard search algorithms

Example: Map Coloring

- Variables: WA, NT, Q, NSW, V, SA, T
- Domains: $D = \{\text{red, green, blue}\}$
- Constraints: adjacent regions must have different colors
 - Implicit: $WA \neq NT$
 - Explicit: $(WA, NT) \in \{(\text{red, green}), (\text{red, blue}), \dots\}$
- Solutions are assignments satisfying all constraints, e.g.:

$\{WA=\text{red}, NT=\text{green}, Q=\text{red}, NSW=\text{green}, V=\text{red}, SA=\text{blue}, T=\text{green}\}$



Example: N-Queens

- Formulation 1
 - Variables: X_{ij}
 - Domains: $\{0, 1\}$
 - Constraints

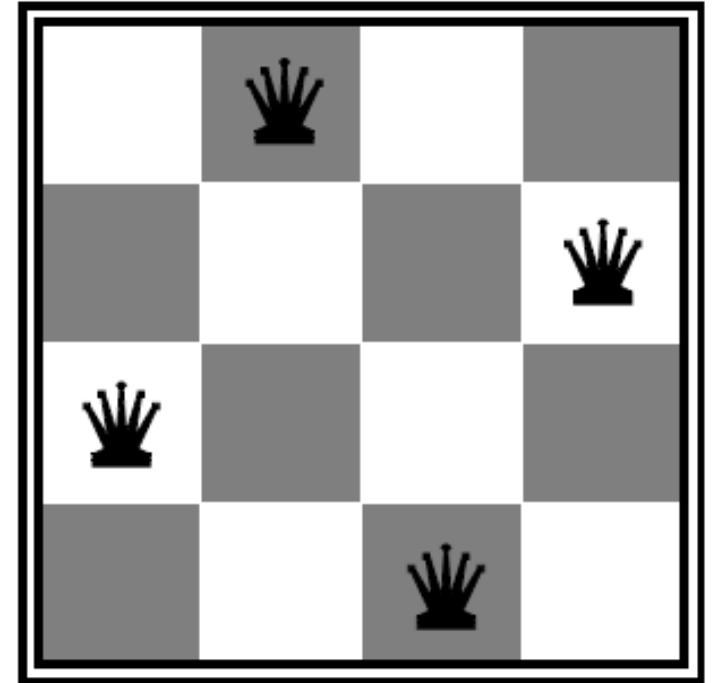
$$\forall i, j, k \quad (X_{ij}, X_{ik}) \in \{(0, 0), (0, 1), (1, 0)\}$$

$$\forall i, j, k \quad (X_{ij}, X_{kj}) \in \{(0, 0), (0, 1), (1, 0)\}$$

$$\forall i, j, k \quad (X_{ij}, X_{i+k, j+k}) \in \{(0, 0), (0, 1), (1, 0)\}$$

$$\forall i, j, k \quad (X_{ij}, X_{i+k, j-k}) \in \{(0, 0), (0, 1), (1, 0)\}$$

$$\sum_{i,j} X_{ij} = N$$



Example: N-Queens

- Formulation 2

- Variables: Q_k

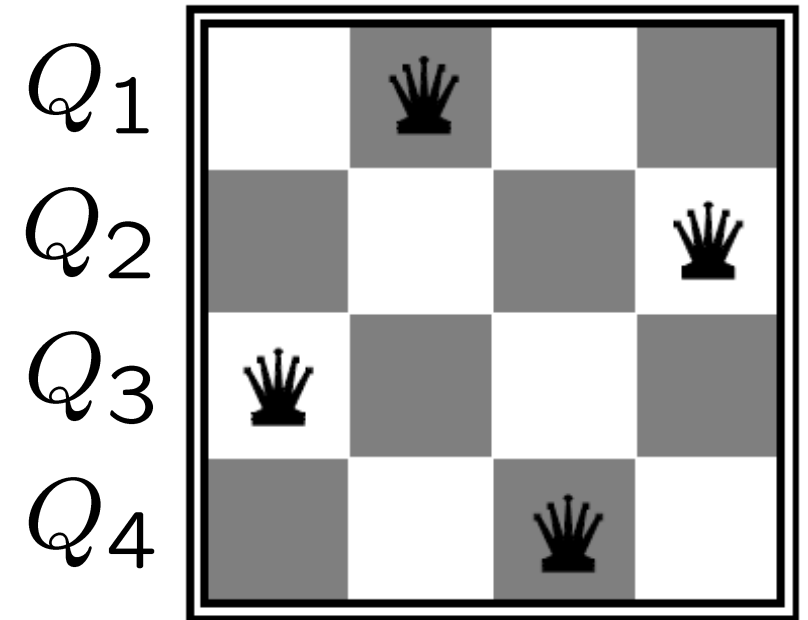
- Domains: $\{1, 2, 3, \dots, N\}$

- Constraints:

Implicit: $\forall i, j \text{ non-threatening}(Q_i, Q_j)$

Explicit: $(Q_1, Q_2) \in \{(1, 3), (1, 4), \dots\}$

...



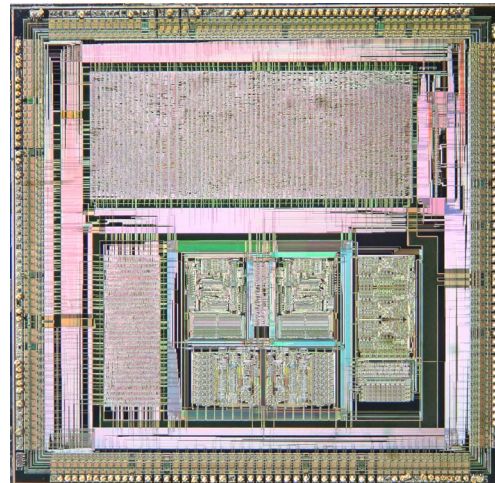
Example: Sudoku

					8			4
	8	4		1	6			
			5			1		
1		3	8			9		
6		8				4		3
		2			9	5		1
		7			2			
			7	8		2	6	
2			3					

- Variables:
Each (open) square
- Domains:
 $\{1,2,\dots,9\}$
- Constraints:
1-9 each column
1-9 each row
1-9 each region

Real-World CSPs

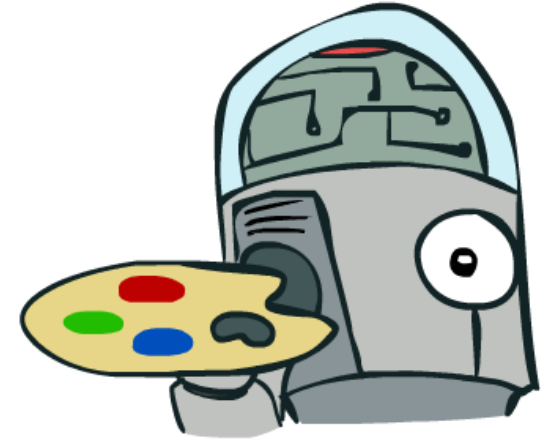
- Scheduling problems: e.g., when can we all meet?
- Timetabling problems: e.g., which class is offered when and where?
- Assignment problems: e.g., who teaches what class
- Transportation scheduling
- Factory scheduling
- Hardware configuration
- Circuit layout
- Fault diagnosis
- ... lots more!



MILL16	W	W21	W170-100	W105-170	W125-70	W589	W825-17								
MILL17A	W	W626-160	W61	W363-17	W446-30	W639-16	W611-150	W892-130	W52						
MILL17B	W	W143-40	W446-50	W591-170	W84	W848-20	W855-160								
MILL17C	W	W426-20	W52	W761-60	W558-20	W304-17	W586-170	W497-20	W384	W716-130					
MILL20	W	W171-80	W186-130	W215-10	W225-120	W481-100	W402-120	W472-7	W43						
MILL21	W	W87	W305-17	W257-13	W67	W215-70	W759-11	W874-60	W643-20	W887-20					
MILL22	W	W382-10	W47	W33	W63	W3	W1	W42	W767-17	W37	W365-60	W64	W311	W628-170	W2
MILL23	W	W485-130	W831-170	W552-40	W130-80	W304-10	W162-180	W400-40							
MILL24	W	W74	W716-50	W203-90	W584-17	W755	W798-11	W457-30	W757-100	W654-160					
MILL25	W				W210-50			W355-170							
MILL25A	W	W403-120	W690-80	W571-170	W472-140	W48	W340-70								
MILL27	W	W710-30	W415-70	W734-60	W134-150	W875-120	W671-14	W632-60	W610-170						
MILL28	W	W80	W21	W574-140	W603	W29	W672-130	W39	W747-120	W305-120					
MILL29	W	W118-180	W742-17	W663-40	W495-160	W871-80	W371-170								
MILL34	W	W399-90	W347-30	W47	W300-13	W861-120	W761-140	W66							
MILL35	W	W57	W685-60	W47	W235-170	W78	W462-90	W136	W578-7	W499-14					
MILL36	W	W627-180	W215-20	W599-80	W304-110	W597-160	W313-100								
MILL37	W	W298-60	W190-50	W235-170	W711-110	W531-120	W293-110	W459-170	W82						

Varieties of CSPs

- Discrete Variables
 - Finite domains
 - Size d means $O(d^n)$ complete assignments
 - E.g., Boolean CSPs, including Boolean satisfiability (NP-complete)
 - Infinite domains (integers, strings, etc.)
 - E.g., job scheduling, variables are start/end times for each job
 - Linear constraints solvable, nonlinear undecidable
- Continuous variables
 - E.g., start/end times for Hubble Telescope observations
 - Linear constraints solvable in polynomial time by LP methods (see cs170 for a bit of this theory)



Varieties of Constraints

- Varieties of Constraints
 - Unary constraints $SA \neq \text{green}$
 - Binary constraints $SA \neq WA$
 - Higher-order constraints
- Preferences (soft constraints):
 - E.g., red is better than green
 - Often representable by a cost for each variable assignment
 - Gives constrained optimization problems