

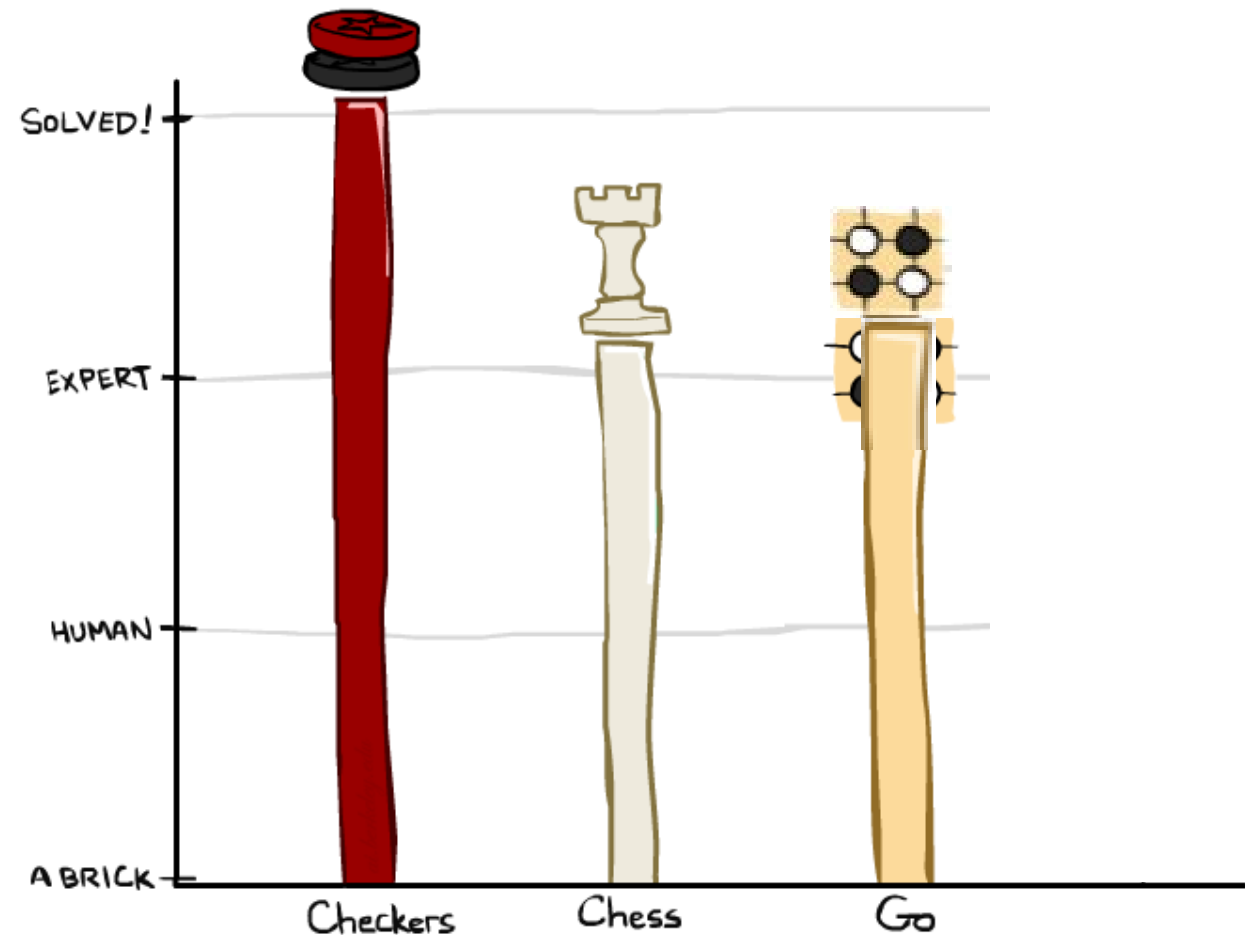
Search



Games: Adversarial Search

Game Playing State-of-the-Art

- **Checkers:** 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!
- **Chess:** 1997: Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.
- **Go:** 2016: Alpha GO defeats human champion. Uses Monte Carlo Tree Search, learned evaluation function.



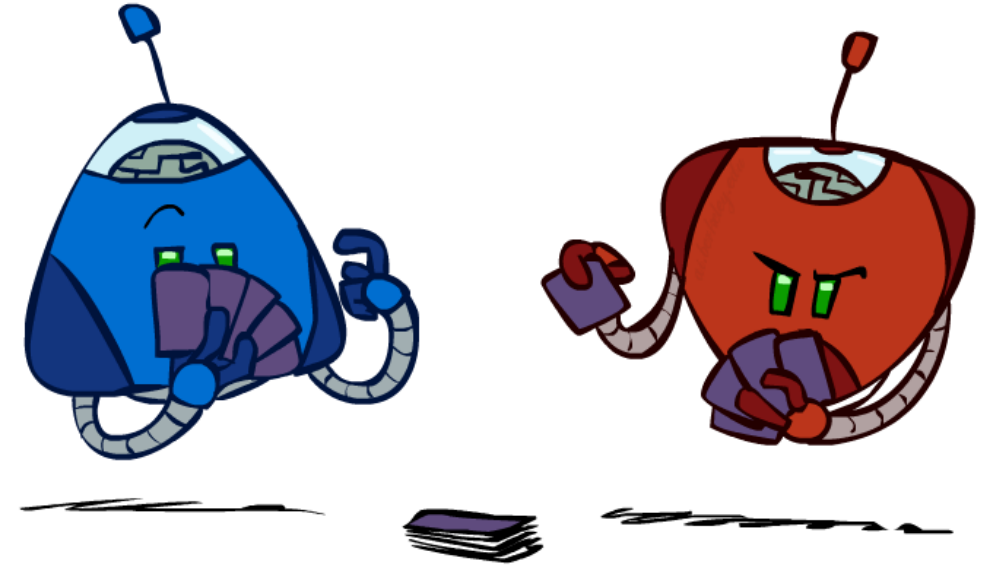
Computer can play complex games!



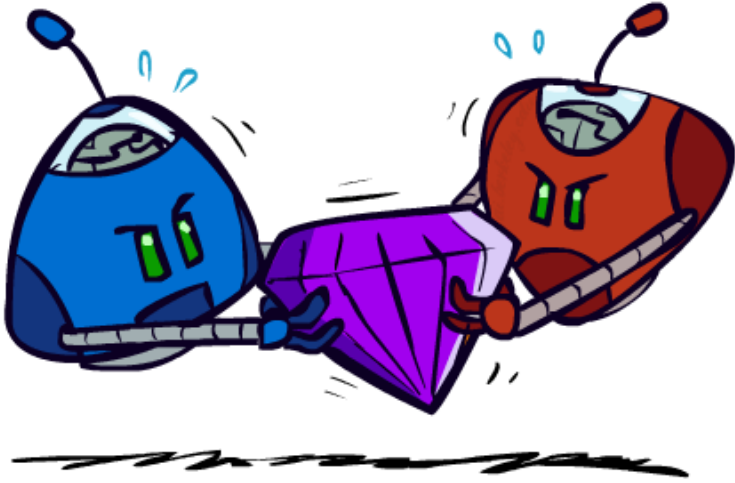
A Short Nature documentary on Alphastar https://youtu.be/6eiErYh_FeY

Types of Games

- Many different kinds of games!
- Axes:
 - Deterministic or stochastic?
 - One, two, or more players?
 - Zero sum?
 - Perfect information (can you see the state)?
- Want algorithms for calculating a **strategy (policy)** which recommends a move from each state

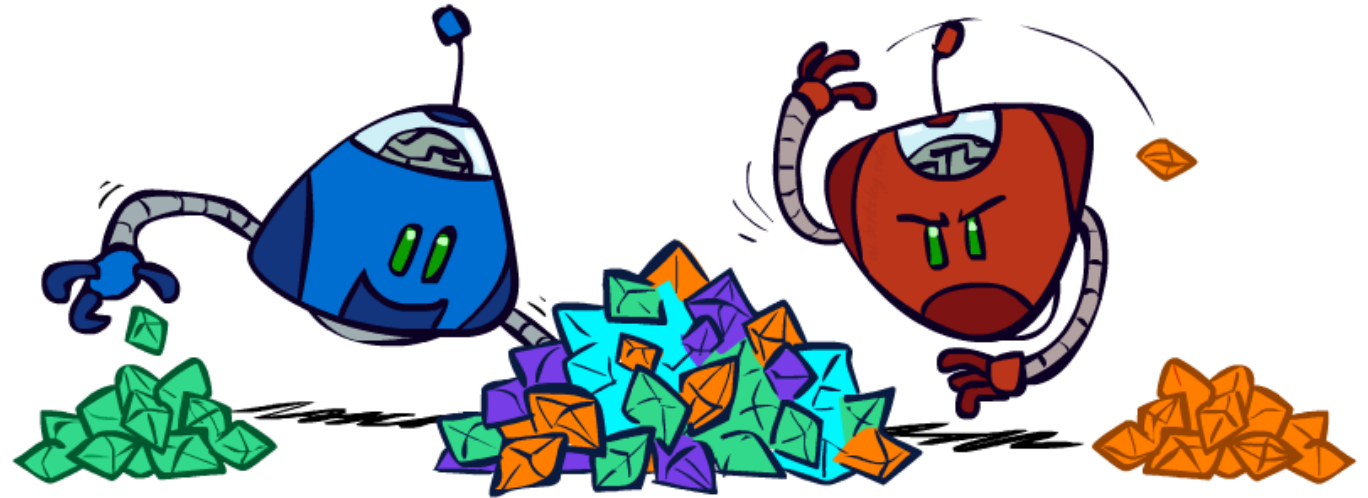


Zero-Sum Games



- Zero-Sum Games

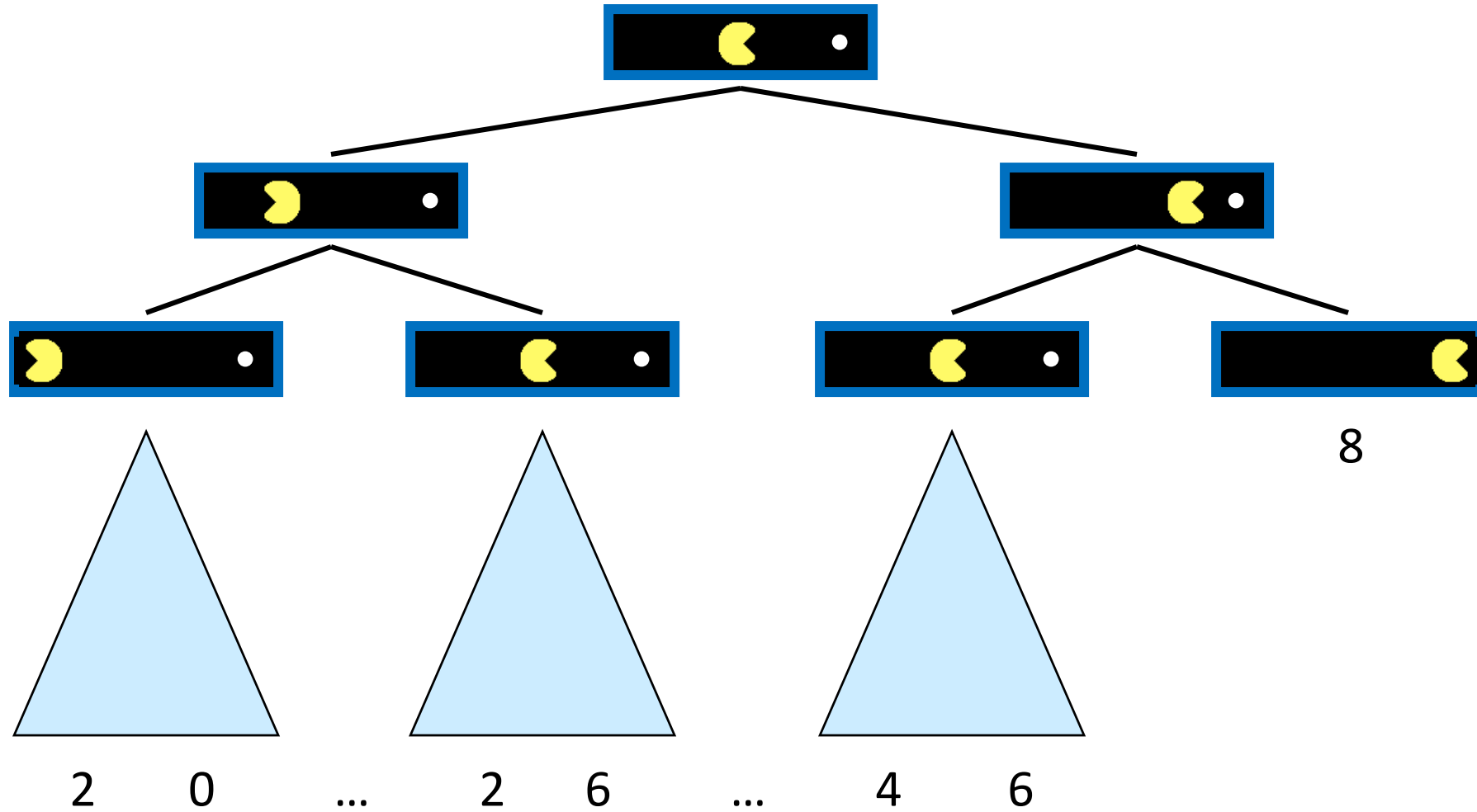
- Agents have opposite utilities (values on outcomes)
- Lets us think of a single value that one maximizes and the other minimizes
- Adversarial, pure competition



- General Games

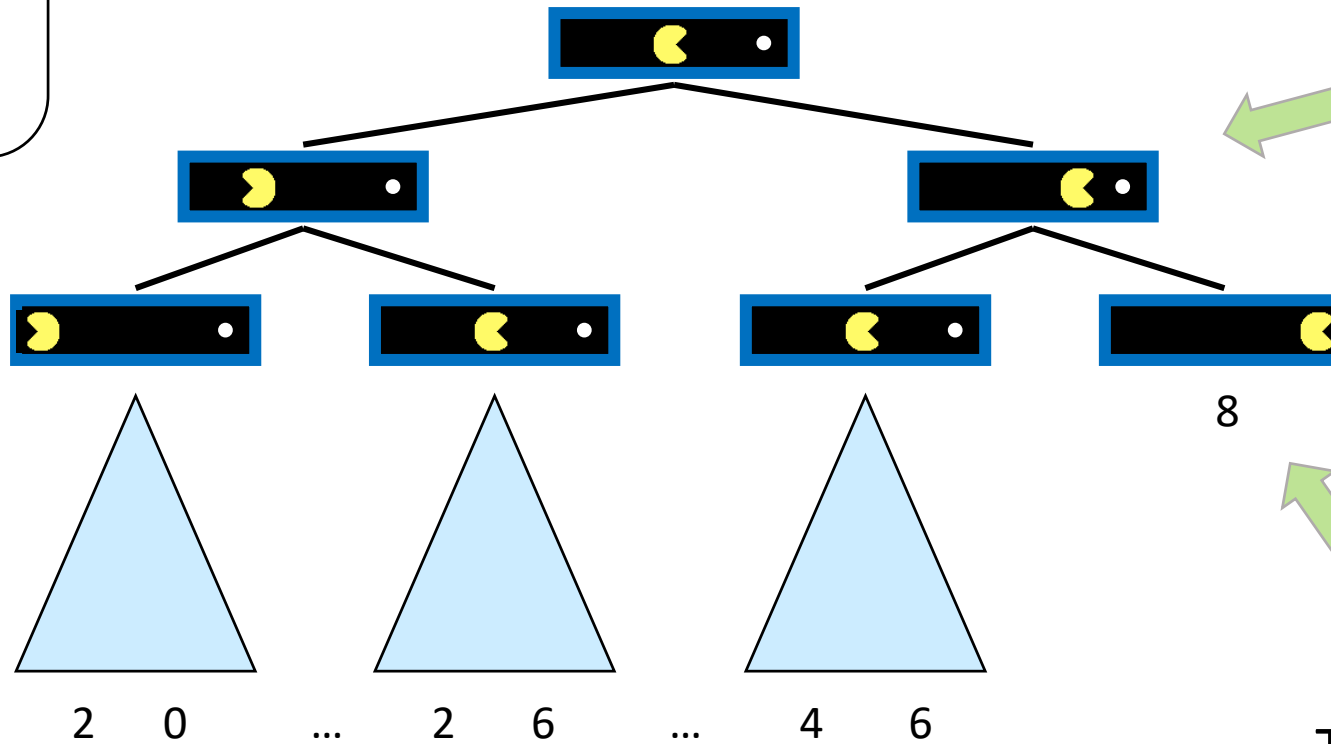
- Agents have independent utilities (values on outcomes)
- Cooperation, indifference, competition, and more are all possible

Single-Agent Trees



Value of a State

Value of a state:
The best achievable
outcome (utility)
from that state



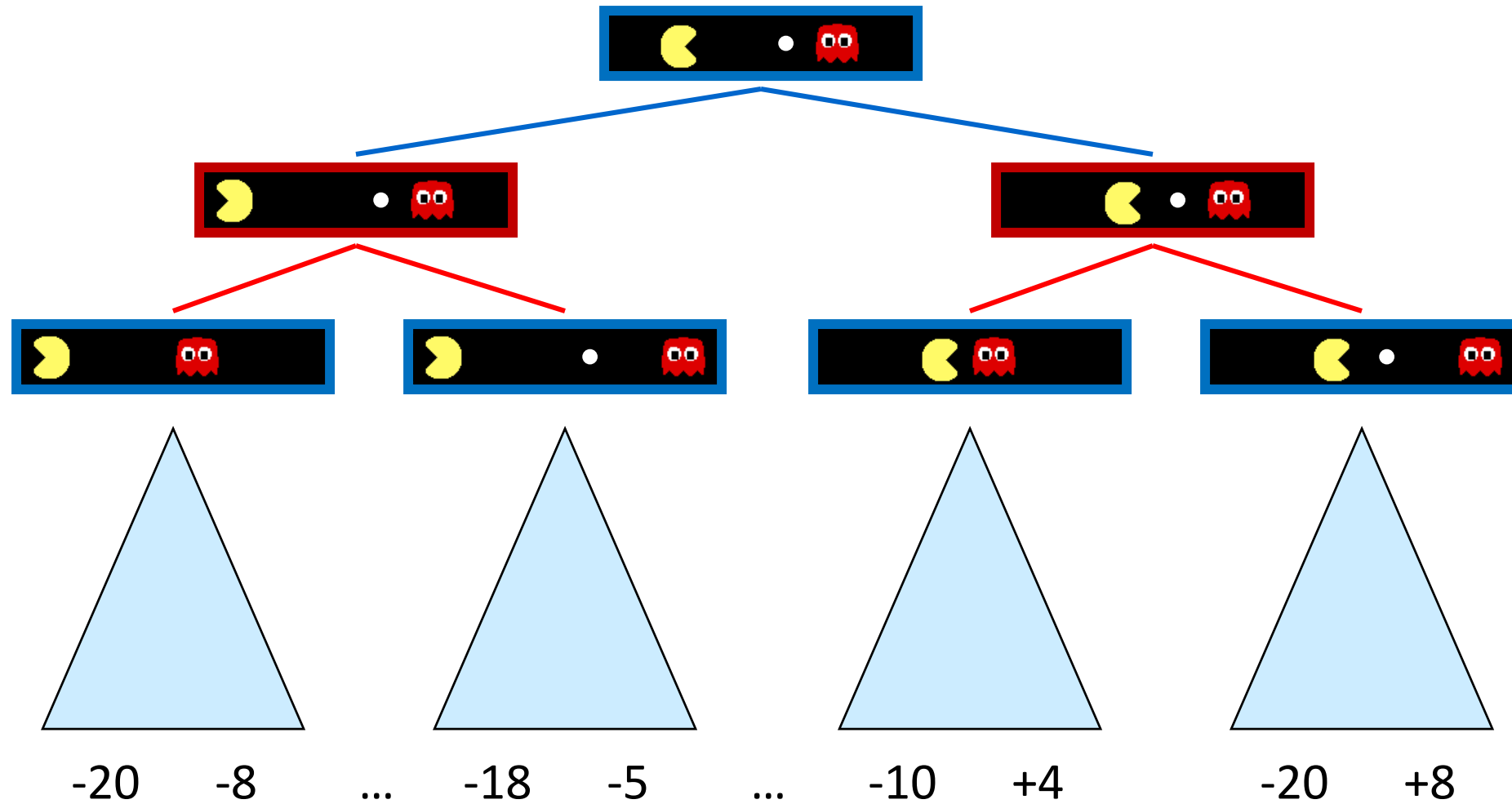
Non-Terminal States:

$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$

Terminal States:

$$V(s) = \text{known}$$

Adversarial Game Trees



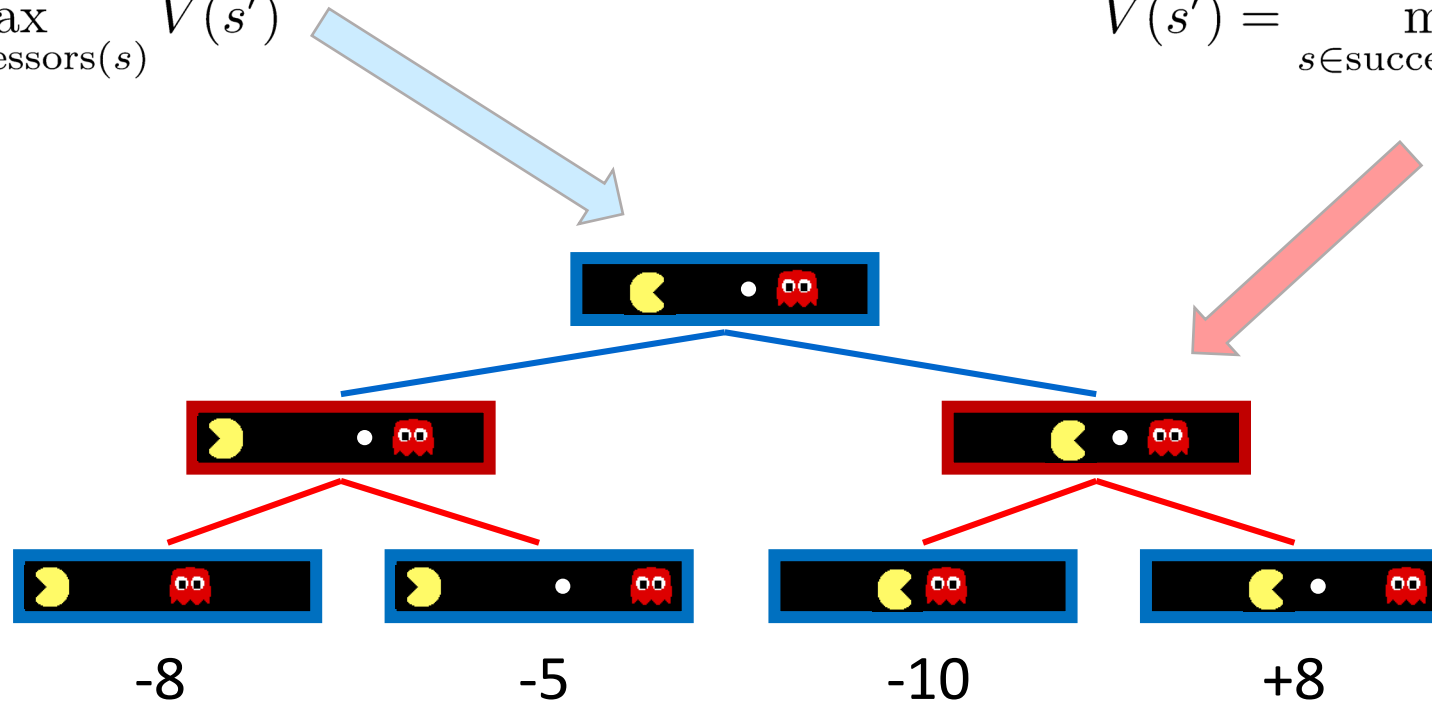
Minimax Values

States Under Agent's Control:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

States Under Opponent's Control:

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$



Terminal States:

$$V(s) = \text{known}$$

Tic-Tac-Toe Game Tree



MAX (X)



MIN (O)



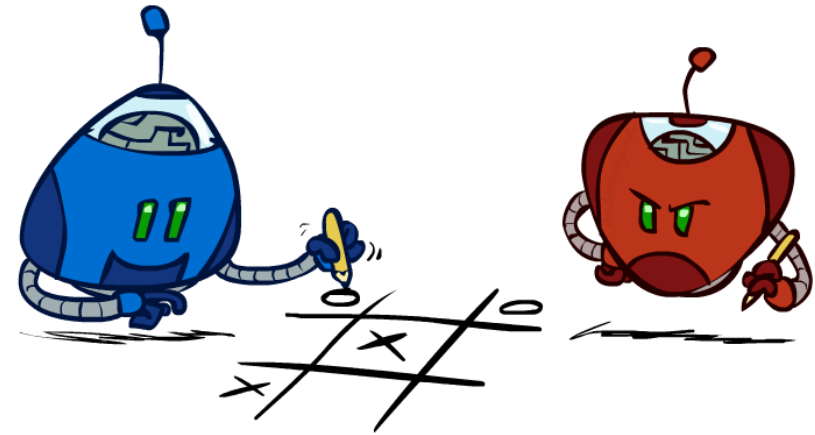
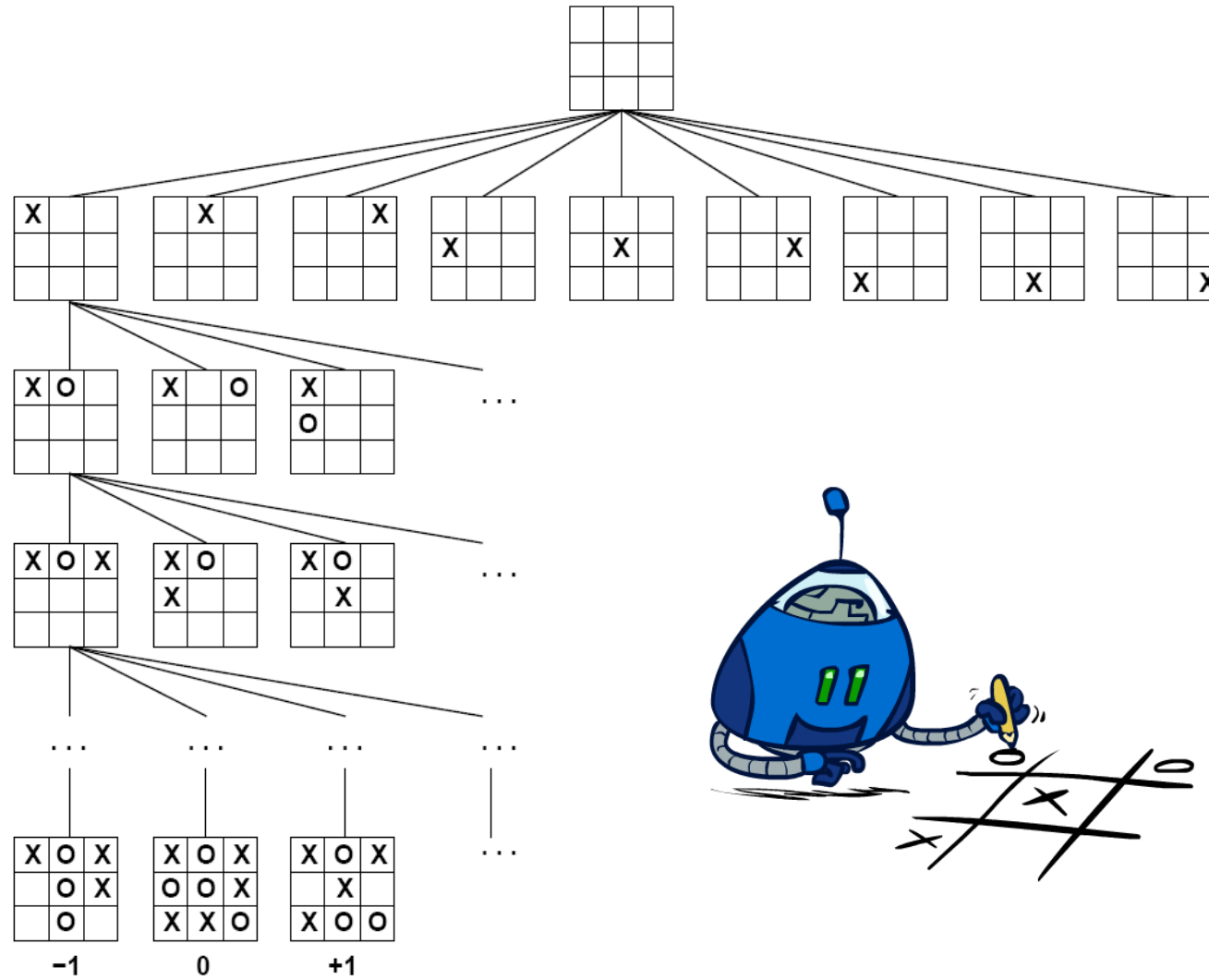
MAX (X)



MIN (O)

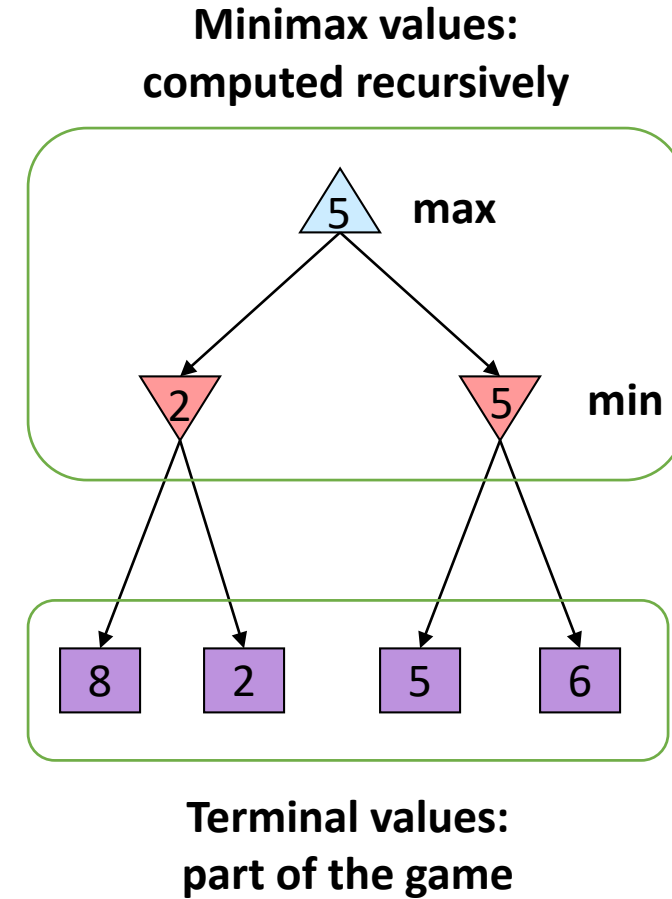
TERMINAL

Utility



Adversarial Search (Minimax)

- Deterministic, zero-sum games:
 - Tic-tac-toe, chess, checkers
 - One player maximizes result
 - The other minimizes result
- Minimax search:
 - A state-space search tree
 - Players alternate turns
 - Compute each node's **minimax value**: the best achievable utility against a rational (optimal) adversary



Minimax Implementation

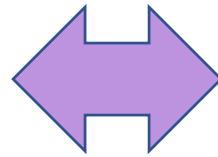
def max-value(state):

 initialize $v = -\infty$

 for each successor of state:

$v = \max(v, \text{min-value}(\text{successor}))$

 return v



def min-value(state):

 initialize $v = +\infty$

 for each successor of state:

$v = \min(v, \text{max-value}(\text{successor}))$

 return v

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

Minimax Implementation (Dispatch)

```
def value(state):
```

if the state is a terminal state: return the state's utility

if the next agent is MAX: return max-value(state)

if the next agent is MIN: return min-value(state)

```
def max-value(state):
```

initialize $v = -\infty$

for each successor of state:

$v = \max(v, \text{value}(\text{successor}))$

return v

```
def min-value(state):
```

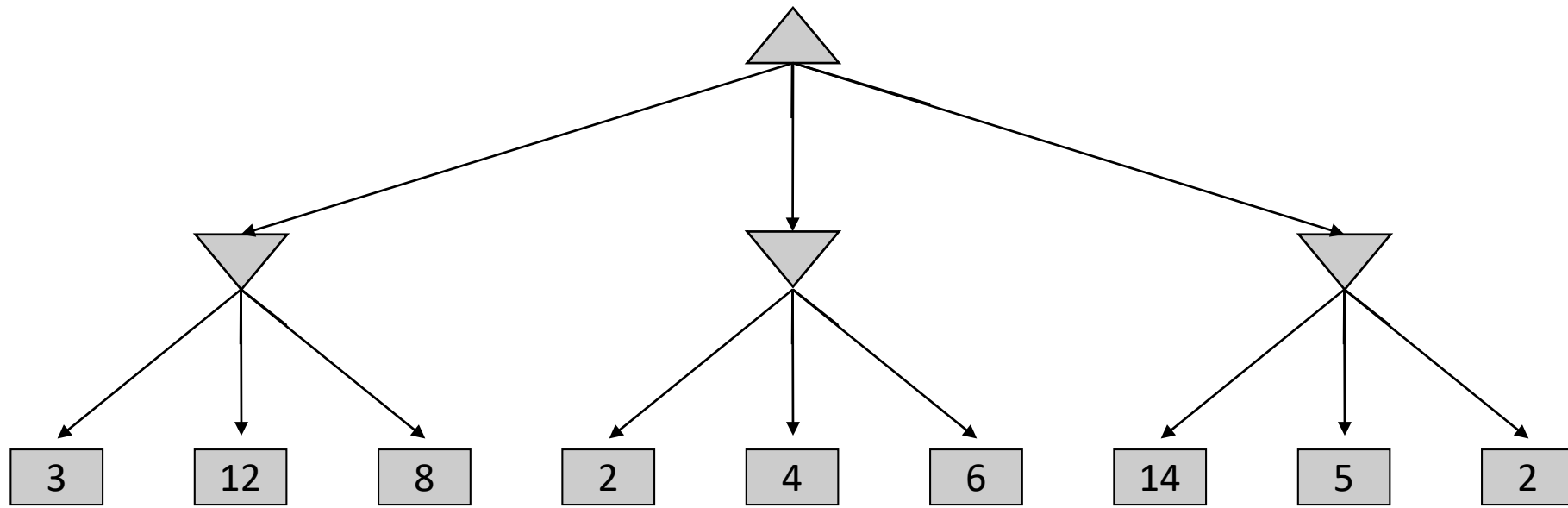
initialize $v = +\infty$

for each successor of state:

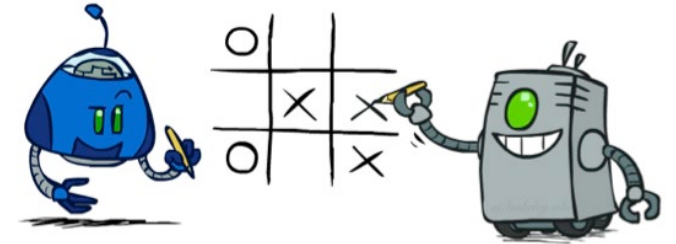
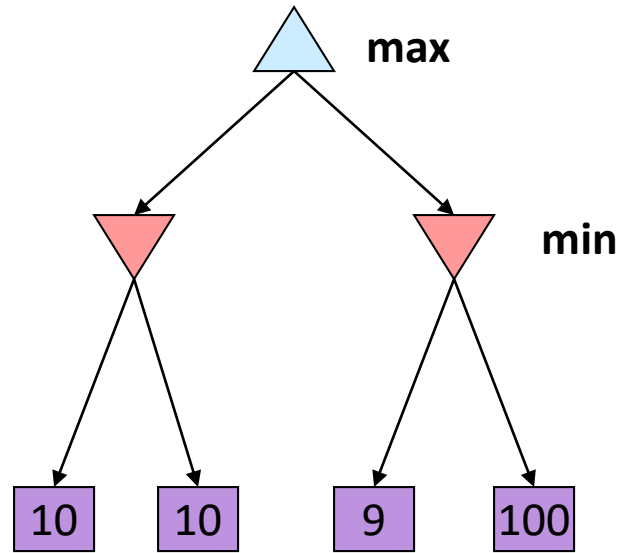
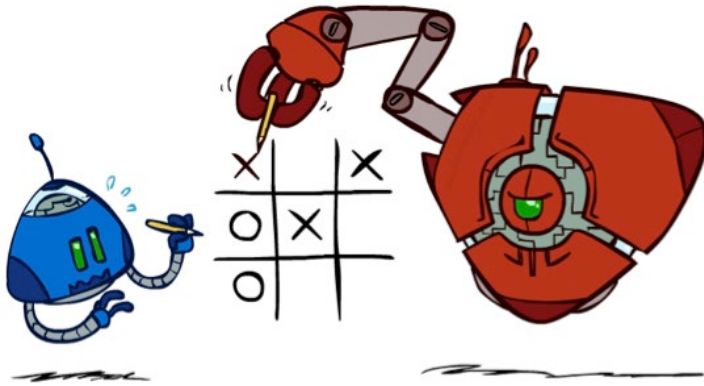
$v = \min(v, \text{value}(\text{successor}))$

return v

Minimax Example



Minimax Properties



Optimal against a perfect player. Otherwise?

Minimax Efficiency

- How efficient is minimax?
 - Just like (exhaustive) DFS
 - Time: $O(b^m)$
 - Space: $O(bm)$
- Example: For chess, $b \approx 35$, $m \approx 100$
 - Exact solution is completely infeasible
 - But, do we need to explore the whole tree?

