Induction and Recursion Mastery Workbook

I have neither given nor received unauthorized assitance______________________.

Warm-up (5 points)

1. With paper and pencil (or online tablet but not typing), add the numbers 1 - 100 any way you like. Do not research. Maybe look for patterns. Just do it. Describe whatever strategy you used. Include a photo in hw PDF.

2. (15pts) Understanding summation notation is essential in computer science. There will be an exam question about summation notation. Review/practice the exercises #29, #31, # 32 on page 169 in Rosen as needed. **You may do these in the Workbook but only select the graded portion below to submit.** Check your answers using https://www.mathsisfun.com/numbers/sigma-calculator.html)

Pick any ONE example from 29, 31, 32 and do the following **on a single page**.

- Write the summation example chosen in summation notation.
- Write out the terms and solution by hand.
- Write, in text, a short python function to calculate this sum **without using built in functions for sums.**
- Answer “How are summations and For Loops related?

3. Recurrence Relations (20 points)

Do #3, #9 and #13 as needed page 168 for practice. (not graded)

. Do #12 b) and c) with explanation on this page.

Induction Workout (Ungraded)

There will be an induction proof on the next exam. The proof will be similar to one of the 3 examples here.

Rewrite in your own words the examples **1, 2, and 3** from section 5.1 in the book by filling in the template below for each one. Write directly on the form. You may organize the algebra in a different way from the book.

There are several ways to organize each of these.

Feel free to post your rewrites in Piazza

Fill in the three following templates, then rewrite using our proof structure (line by line and with reasons for each step) to prepare for the exam question.

Mathematical Induction Proof Template

$\forall n \in \mathbb{N}\,(P(n))$

THEOREM: "For every $n \in \mathbb{N}$, $\underbrace{\boxed{}}_{P(n)}$

PROOF: By mathematical induction.

State and prove $P(1)$

Basis: $P(1)$ asserts that

Note: If appropriate, use P(0), P(2) or other value instead.

which is true because

*State $P(k)$ **(inductive hypothesis)***

Inductive step: Assume for an arbitrary $k \in \mathbb{N}$, $P(k)$ is true, i.e., namely:

State $P(k+1)$

We will now show that $P(k+1)$ is also true, i.e.:

Prove $P(k) \Rightarrow P(k+1)$

Proof of inductive step:

All done: wrap up proof

We thus have that $P(1)$ and $\forall k \in \mathbb{N}, P(k) \rightarrow P(k+1)$, so by the principle of mathematical induction, it follows that $P(n)$ is true for all natural numbers n.

Q.E.D.

Steps of a mathematical induction proof:
1) state the theorem, which is the proposition P(n)
2) show that P(base case) is true. Base case is usually P(1), but sometimes P(0) or P2) or other value is appropriate.
3) state the inductive hypothesis (substitute k for n)
4) state what must be proved (substitute k+1 for n)
5) state that you are beginning your proof of the inductive step, and proceed to manipulate the inductive hypothesis (which we assume is true) to find a link between the inductive hypothesis and the statement to be proven. Always state explicitly where you are invoking the inductive hypothesis.
6) finish your proof by invoking the principle of mathematical induction that allows you to infer that $P(n)$ is true for all natural numbers.

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Induction Proofs 4, 5 (10 points each)

- Please follow an induction structure from the video and/or book. You can use your notes from #4 above.
- Be careful not to assume the results of the induction STEP.
- **Do not "work both sides"** when proving the induction step (some videos on youtube do this) unless absolutely needed. You must make clear that the left side of the induction step = right side of the induction step at the end of the proof.
- ***BE SURE TO LABEL WHERE YOU USE THE induction hypothesis with **IH**. ***
- Also, justify each step in your proof.
- The set of natural numbers begins at 0.

<u>Grading Instructions for induction proofs over natural numbers (see grading quiz in Moodle)</u>

- **Says "proof by induction"**
- **Starts with n = 0 for Natural numbers, OR the correct start value.**
- **Proves the base case correctly**
- **States the Induction Hypothesis correctly (and says "induction hypothesis)**
- **If the induction step is used - it is correctly stated.**
- **Uses the Induction Hypothesis in the proof (and says so, or shows IH when used)**
- **Uses a single chain of reasoning to derive the result (does not "work both sides)**
- **Uses correct algebra in proof.**
- **Reasons given for each step in the proof.**
- **Says proved by induction.**

Please check your induction proofs with the grading instructions before submitting your Mastery Workbook.

4. For all positive integers n, $1 * 2 + 2 * 3 + \dots + n(n + 1) = n(n + 1)(n + 2)/3$

5. *For all positive integers n,* $1^3 + 2^3 + 3^3 + 4^3 + ... n^3 = (n(n+1)/2)^2$

Recursion Walk Thru

6. (20 points) You may use PythonTutor or Runestone's code lens to help you understand.

Do #3 (Ungraded on your own) Then answer here # 4 from page 370, by hand, show work.

Provide brief insight/explanation/notes about how this is different and/or the same as your RSA code.

7. (20 points)
Do #5 (Ungraded on your own) d
Do and show #6 page 370, by hand, show work.

Provide brief insight/explanation/notes about how this is different and/or the same as your RSA code.