# CSPB 2824 - Stade - Discrete Structures



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| **Started on** | Thursday, 9 November 2023, 5:10 PM |
| **State** | Finished |
| **Completed on** | Thursday, 9 November 2023, 5:10 PM |
| **Time taken** | 10 secs |
| **Marks** | 21.00/21.00 |
| **Grade** | **10.00** out of 10.00 (**100**%) |

Question **1**

Correct

Mark 1.00 out of 1.00

Suppose you can make a cake using either the Ketelsen Method or the Smith Way. In the Ketelsen Method, there are 2 steps: Step 1 can be done in any one of 3 ways, but Step 2 can only be done one way.

The Smith Way, on the other hand, requires 2 steps. Each of these steps may be completed in one of 3 ways.

How many ways are there to make a cake?

12

**Solution:**

There are two distinct alternative methods (Ketelsen Method **or** Smith Way), either one gets the job done. Thus, there is a sum rule between the two, and the total number of ways to make a cake is the number of ways using the Ketelsen Method *plus* the number of ways using the Smith Way.

The Ketelsen Method has two steps, both of which must be done, so there is a product rule to enumerate the number of ways the Ketelsen Method can be done. There are 3 ways to do Step 1 and 1 way to do Step 2. Thus there are 3 x 1 = 3 ways to do the Ketelsen Method.

The SmithWay has two steps, both of which must be done, so there is also a product rule to enumerate the number of ways to apply the Smith Way. There are 3 ways to do each of the 2 steps, so there are 3 x 3 = 9 ways to do the Smith Way.

Then, in total, there are 3 + 9 = 12 ways to make a cake.

Correct

Marks for this submission: 1.00/1.00.

Question **2**

Correct

Mark 1.00 out of 1.00

Suppose you can change a tire using either the Ketelsen Method or the Smith Way. In the Ketelsen Method, there are 2 steps: Step 1 can be done in any one of 5 ways, but Step 2 can only be done one way.

The Smith Way, on the other hand, requires 3 steps. Each of these steps may be completed in one of 3 ways.

How many ways are there to change a tire?

32

**Solution:**

There are two distinct alternative methods (Ketelsen Method **or** Smith Way), either one gets the job done. Thus, there is a sum rule between the two, and the total number of ways to change tire is the number of ways using the Ketelsen Method *plus* the number of ways using the Smith Way.

The Ketelsen Method has two steps, both of which must be done, so there is a product rule to enumerate the number of ways the Ketelsen Method can be done. There are 5 ways to do Step 1 and 1 way to do Step 2. Thus there are 5 x 1 = 5 ways to do the Ketelsen Method.

The Smith Way has three steps, all of which must be done, so there is also a product rule to enumerate the number of ways to apply the Smith Way. There are 3 ways to do each of the 3 steps, so there are 3 x 3 x 3 = 27 ways to do the Smith Way.

Then, in total, there are 5 + 27 = 32 ways change a tire.

Correct

Marks for this submission: 1.00/1.00.

Question **3**

Correct

Mark 1.00 out of 1.00

Your sweet new bicycle has 3 front sprockets and 7 rear sprockets.  Each distinct combination of a front sprocket with a rear sprocket constitutes a different gear on a bicycle.  How many gears does your bicycle have?

21

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Is it correct to say that a particular speed requires a particular front sprocket *or* a particular rear sprocket, or is it correct to say that a particular speed requires a particular front sprocket *and* a particular rear sprocket?

and

**Solution:**

In order to select a gear, you must select *both* a front sprocket **and** a rear sprocket.  Thus, this is a procedure requiring the product rule, and 3 x 7 = 21 gears total.

Correct

Marks for this submission: 1.00/1.00.

Question **4**

Correct

Mark 1.00 out of 1.00

A multiple-choice test contains 10 questions. There are four possible answers for each question.

a)  In how many ways can a student answer the questions on the test if the student answers every question?

1048576

b)  In how many ways can a student answer the questions on the test if the student can leave answers blank?

9765625

Your answer is correct.

Correct

Marks for this submission: 1.00/1.00.

Question **5**

Correct

Mark 1.00 out of 1.00

Consider the following predicate:

$A(n)$: *in* **any** *group of* $n$ *English words, there must be at least two that begin with the same letter.*

What is the minimum value of $n$ such that $A(n)$ is true?

Answer: 27 ✔

Correct.

There are 26 alphabets. We need a group of at least 26+1=27 alphabets where we can find two identical alphabets. Therefore we need at least 27 words.

The correct answer is: 27

Correct

Marks for this submission: 1.00/1.00.

Question **6**

Correct

Mark 1.00 out of 1.00

Consider the following predicate:

$P(n)$: *Among* **any** *group of* $n$ *people, at least two people have the same birthday.*

What is the minimum value of $n$ for which the predicate $P(n)$ is true?
(Assume each year has 365 days---ignore the leap year.)

Answer: 366 ✔

Correct.

This requires the use of pigeonhole principle:

*Let* $m$ *be a positive integer and* $n \geq m + 1$ *be an integer. If* $n$ *objects are distributed among* $m$ *bins, then at least 1 bin contains at least two objects.*

Note that, conversely, to ensure that at least 1 out of $n$ bin contains at least two objects, we need to distribute at least $n + 1$ objects.

The correct answer is: 366

Correct

Marks for this submission: 1.00/1.00.

Question **7**

Correct

Mark 1.00 out of 1.00

Let's say every student in a certain course is either an English major, a linguistic major or a joint major in these two subjects. How many students are there in this course if there are 38 English majors (including joint majors), 23 linguistic major (including joint majors) and 7 joint majors?

Answer: 54 ✔

This answer is correct.

Let $A = 38$ be the number of English majors, let $B = 23$ be the number of Linguistics majors, and let $A \cap B = 7$ be the number of joint majors. We can apply the inclusion-exclusion principle to obtain the number of students in total, which is $A \cup B$.

The correct answer is: 54

Correct

Marks for this submission: 1.00/1.00.

Question **8**

Correct

Mark 1.00 out of 1.00

Suppose shirts are one of 3 colors (red, blue and green) and pants are either black or brown. An outfit consists of a shirt and pants. What is the minimum number of people that need to be in a room together to guarantee that at least two of them are wearing same-colored outfits?

7

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What is the minimum number of people to ensure that at least 9 of them are dressed the same and can go play a game of baseball?

49

**Solution:**

The procedure of constructing an outfit requires both of two steps be completed: (1) pick a shirt (3 ways - red, blue or green) and (2) pick pants (2 ways - black or brown). Thus, the product rule gives the total number of ways to construct an outfit as 3 x 2 = 6.

The Pigeonhole Principle assures us that if there are 6+1 people in a room, that at least $\lceil \frac{7}{6} \rceil = 2$ of them must be wearing the same-colored outfit.

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The Generalized Pigeonhole Principle gives that $\lceil \frac{N}{6} \rceil \overset{\heartsuit}{=} 9$, so $N = 6 \times 8 + 1 = 49$ is the minimum number of people to guarantee that a baseball team can be formed.

Correct

Marks for this submission: 1.00/1.00.

Question **9**

Correct

Mark 1.00 out of 1.00

How many bit strings are there of length 6? Please provide the numerical answer.

Answer: 64

This answer is correct.

Using the product rule, the answer is $2^6$.

The correct answer is: 64

Correct

Marks for this submission: 1.00/1.00.

Question **10**

Correct

Mark 1.00 out of 1.00

How many bit strings are there of length between 1 and 10? Please provide the numerical answer.

(*between* is always inclusive by default)

Answer: 2046

This answer is correct.

the answer is $\sum_{j=1}^{(} 10)a_j$, where $a_j$ is the number of length-$j$ bit strings. Now note that $a_j = 2^j$. So the sum equals $2 \times \frac{2^{10}-1}{2-1}$.

The correct answer is: 2046

Correct

Marks for this submission: 1.00/1.00.

Question **11**

Correct

Mark 1.00 out of 1.00

How many bit strings of length 10 begin and end with a 1?

Answer: 256

This answer is correct.

Since the first and last bits are fixed, this problem is equivalent to counting the number of length-8 bit strings.

The correct answer is: 256

Correct

Marks for this submission: 1.00/1.00.

Question **12**

Correct

Mark 1.00 out of 1.00

How many bit strings of length 10 begin with three 1's and end with two 0's?

Answer: 32

This answer is correct.

Because the first three and last three bits are fixed, this problem is equivalent to counting the number of unique length-5 bit strings.

The correct answer is: 32

Correct

Marks for this submission: 1.00/1.00.

Question **13**

Correct

Mark 1.00 out of 1.00

How many bit strings of length 10 begin with two 1's or end with two 1's ? Note we have an "inclusive or" i.e, a string that begins with two 1s **and** ends with 1s is admissible.

Answer: 448

This answer is correct.

It is equivalent to counting the number of elements of $A \cup B$, where, $\begin{array}{l} A \triangleq \text{ the set of length-10 bit strings starting with two 1's,} \\ B \triangleq \text{ the set of length-10 bit strings ending with two 1's.} \end{array}$

Now note that: $|A| = 2^8, \quad |B| = 2^8, \quad |A \cap B| = 2^6$. Using the inclusion-exclusion principle, we obtain: $|A \cup B| = |A| + |B| - |A \cap B| = 448.$

The correct answer is: 448

Correct

Marks for this submission: 1.00/1.00.

Question **14**

Correct

Mark 1.00 out of 1.00

Suppose you are flying Northeast Airlines, a well-loved airline with a wildly permissive boarding procedure.

There are 180 passenger seats on a Northeast Airlines plane, and they are arranged in rows of 6, with a group of 3 on either side of the aisle (thus, there are 30 rows total). Passengers are assigned a **boarding number** between 1 and 180 with their ticket. Passengers board the plane in the order given by their boarding numbers, and may sit in any empty seat.

For example, the passenger with boarding number 1 may sit wherever they wish; the passenger with boarding number 2 may sit anywhere except where passenger 1 is sitting; and so on. You may assume that all of the 180 passengers are present, nobody swaps seats, and nobody is beaten and forcibly removed from the aircraft.

You and your business partner Rhonda are taking a trip to recruit more employees for your triangular-shaped business venture. Suppose your boarding number is $n$, and Rhonda's is $n+1$. What is the maximum boarding number $n$ you can have in order to still guarantee that you and Rhonda can sit in adjacent seats in the same row, not separated by the aisle?

60

✔

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What is the maximum boarding number you can have in order to guarantee you can get an aisle seat?

60

✔

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It's the holidays and lots of people are traveling. Things are getting out of control at baggage claim. Suppose everybody has exactly one bag, tagged with their first and last initial, which are both uppercase letters from the English alphabet.

What is the minimum number of people needed to guarantee that at least 3 luggage tags have the same 2 initials (in the same order) on them?

1353

✔

**Solution:**

There are 180 seats on the airplane, and they are arranged in groups of 3. Thus, there are 60 groups of 3 seats.

By the Pigeonhole Principle, at most the 60+1=61st passenger will need to sit in the same group of seats as someone else. If each of the first 60 passengers sits in a middle seat, then you and Rhonda cannot sit next to one another (without an aisle in between). Thus, you need to have a boarding number of **at most 60** in order to guarantee that you and Rhonda will secure adjacent seats.

Granted, in real life, it is highly unlikely that all of the first 60 people would take all 60 of the middle seats.

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There are only 60 aisle seats, so (again) you need to have boarding number of **at most 60** in order to guarantee you can get an aisle seat.

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There are $26 \times 26 = 676$ "bins" (2-initial combinations), and suppose there are $N$ people (with exactly one bag each). Using the Generalized Pigeonhole Principle, we know that if there must be at least one bin with at least $\lceil \frac{N}{676} \rceil$ people in it. We seek the minimum $N$ for which $\lceil \frac{N}{676} \rceil = 3$. This is $N = 2 \times 676 + 1 = 1353$

Correct

Marks for this submission: 1.00/1.00.

Question **15**

Correct

Mark 1.00 out of 1.00

In how many ways can a photographer at a wedding arrange 8 people in a row, including the bride and groom, so that the bride must be next to the groom?

Answer: 10080

This answer is correct.

If we think of the bride and groom as a single entity (bride on the left and groom on the right) instead of 2 people, the problem can be restated thus: how many ways can we permute 7 distinct entities? The answer would be \(7!=5140\). One final step: note that the bride can be on the left or right of the groom; to account for this we have to multiply \(7!\) by 2.

The correct answer is: 10080

Correct

Marks for this submission: 1.00/1.00.

Question **16**

Correct

Mark 1.00 out of 1.00

The Scooby Doo gang is going to the movies... again!  That's right, they haven't defriended Fred in real life (yet).

Enter your answers for this problem as integers.

Suppose a row of seats at the movies is 10 seats, and that this row will be filled up completely by the 5 members of the Scooby Doo gang, and 5 strangers.  How many arrangements of these 10 people are possible, such that the Scooby Doo gang can all sit adjacent to one another in this row?

86400

✔

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Now suppose Fred is being a huge jerk, so the Scooby Doo gang decides he has to sit on one of the ends of their group of 5 (not necessarily an end of the row). How many possible arrangements of the 10 people in the row are there now?

34560

✔

**Sanity check:** Should your answer for this part be greater than or less than the answer from the first part?

**Solution:**

The procedure of seating everyone within the row consists of 2 tasks, both of which must be completed.  So we must use the product rule.  The two tasks are (1) arrange the Scooby Doo Gang, and (2) arrange the whole row (placing them somewhere within it).

(1) There are 5 members of the Scooby Doo Gang, so there are 5! ways to arrange them.

(2) Treat the Scooby Doo Gang as a single entity, along with the 5 other people. Thus, there are 6 "people" total that must be arranged within the row, and there are 6! ways to do this.

Thus, the product rule gives $5! \times 6! = 86400$ total ways to arrange the row.

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Now the same two tasks must be accomplished, but they can be done in one of two distinct ways: either Fred sits on the right end of their group of 5, or he sits on the left end.  So we must use the sum rule as well as the product rule here.

If Fred sits on the right end:

(1) Need to arrange the rest of the Scooby Doo Gang members. There are 4 of them, so there are 4! ways to do this.

(2) Need to arrange the Scooby Doo Gang (treated again as a single unit), along with the 5 other people. Again, there are still 6! ways to do this.

So if Fred sits on the right end of their group of 5, then there are $4! \times 6!$ ways to arrange the whole row.

The same is true if Fred sits on the left end, so the total number of ways to arrange the row is $4! \times 6! + 4! \times 6! = 34560$

Correct

Marks for this submission: 1.00/1.00.

Question **17**

Correct

Mark 1.00 out of 1.00

How many strings of three decimal digits

a) do not contain the same digit three times?

990

b) begin with an odd digit?

500

c) have exactly two digits that are 4s?

27

Your answer is correct.

Correct

Marks for this submission: 1.00/1.00.

Question **18**

Correct

Mark 1.00 out of 1.00

Find the value of each of these quantities.

a) \(P(6,3)\)

120

✔

b) \(P(6,5)\)

720

✔

c) \(P(8,1)\)

8

✔

d) \(P(8,5)\)

6720

✔

e) \(P(8,8)\)

40320

✔

f) \(P(10,9)\)

3628800

✔

Your answer is correct.

Correct

Marks for this submission: 1.00/1.00.

Question **19**

Correct

Mark 1.00 out of 1.00

How many bit strings of length 10 contain

a) exactly four 1s?

210

b) at most four 1s?

386

c) at least four 1s?

848

d) an equal number of 0s and 1s?

252

Your answer is correct.

Correct

Marks for this submission: 1.00/1.00.

Question **20**

Correct

Mark 1.00 out of 1.00

A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes

a) are there in total?

1024

✔

b) contain exactly two heads?

45

✔

c) contain at most three tails?

176

✔

d) contain the same number of heads and tails?

252

✔

Your answer is correct.

Correct

Marks for this submission: 1.00/1.00.

Question **21**

Correct

Mark 1.00 out of 1.00

How many permutations of the letters ABCDEFG contain

a) the string BCD?

120

b) the string CFGA?

24

c) the strings BA and GF?

120

d) the strings ABC and DE?

24

e) the strings ABC and CDE?

6

f) the strings CBA and BED?

0

Your answer is correct.

Correct

Marks for this submission: 1.00/1.00.