logical equivalences, using one of the equivalences in Table 6 at a time, starting with $\neg(p \to q)$ and ending with $p \wedge \neg q$. We have the following equivalences.

$$
\begin{aligned}
\neg(p \to q) &\equiv \neg(\neg p \vee q) && \text{by Example 3}\\
&\equiv \neg(\neg p) \wedge \neg q && \text{by the second De Morgan law}\\
&\equiv p \wedge \neg q && \text{by the double negation law}
\end{aligned}
$$

◀

EXAMPLE 7 Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences.

↳ Kinda like equality

Solution: We will use one of the equivalences in Table 6 at a time, starting with $\neg(p \vee (\neg p \wedge q))$ and ending with $\neg p \wedge \neg q$. (*Note:* we could also easily establish this equivalence using a truth table.) We have the following equivalences.

Use properties from table 6 to prove things

→ Flips disj → conj, distributes ¬

$$
\begin{aligned}
\neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{by the second De Morgan law}\\
&\equiv \neg p \wedge [\neg(\neg p) \vee \neg q] && \text{by the first De Morgan law}\\
&\equiv \neg p \wedge (p \vee \neg q) && \text{by the double negation law}\\
&\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) && \text{by the second distributive law}\\
&\equiv \mathbf{F} \vee (\neg p \wedge \neg q) && \text{because } \neg p \wedge p \equiv \mathbf{F}\\
&\equiv (\neg p \wedge \neg q) \vee \mathbf{F} && \text{by the commutative law for disjunction}\\
&\equiv \neg p \wedge \neg q && \text{by the identity law for } \mathbf{F}
\end{aligned}
$$

↳ Original Exp.

→ De Morgan applied again

→ ¬(¬P) changes to P

→ Acts Similar to reg distr

→ Impossible for ¬x∧x ≡ T

→ Kinda like P∨q ≡ q∨P

This example is eerily Similar to solving systems of eqn. i.e.

x+2y=3 } y=x-0   3x=3
x+y=0 } x+2(x-0)=3   x=1, y=-1

→ "F" not important here

Consequently $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent. ◀

↳ Kinda like Saying QED or □

EXAMPLE 8 Show that $(p \wedge q) \to (p \vee q)$ is a tautology.

↳ use RBI ↳ Always true

Solution: To show that this statement is a tautology, we will use logical equivalences to demonstrate that it is logically equivalent to **T**. (*Note:* This could also be done using a truth table.)

→ using past result from T table

$$
\begin{aligned}
(p \wedge q) \to (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) && \text{by Example 3}\\
&\equiv (\neg p \vee \neg q) \vee (p \vee q) && \text{by the first De Morgan law}\\
&\equiv (\neg p \vee p) \vee (\neg q \vee q) && \text{by the associative and commutative laws for disjunction}\\
&\equiv \mathbf{T} \vee \mathbf{T} && \text{by Example 1 and the commutative law for disjunction}\\
&\equiv \mathbf{T} && \text{by the domination law}
\end{aligned}
$$

◀

→ Flipping conj to disj

→ All disj's, order doesn't matter

Tautological

Always true :)

Take away: · All disj's → re-arrange!
· All conj's → re-arrange!
· De Morgans is our friend
· RBI used most probs

Propositional Satisfiability

A compound proposition is **satisfiable** if there is an assignment of truth values to its variables that makes it true. When no such assignments exists, that is, when the compound proposition is false for all assignments of truth values to its variables, the compound proposition is **unsatisfiable**. Note that a compound proposition is unsatisfiable if and only if its negation is true for all assignments of truth values to the variables, that is, if and only if its negation is a tautology.

When we find a particular assignment of truth values that makes a compound proposition true, we have shown that it is satisfiable; such an assignment is called a **solution** of this particular