CSPB 3104 - Park - Algorithms

Started on	Tuesday, 27 February 2024, 11:40 AM
State	Finished
Completed on	Tuesday, 27 February 2024, 12:17 PM
Time taken	37 mins 32 secs
Marks	15.00/18.00
Grade	**8.33** out of 10.00 (**83**%)

Question 1

Correct

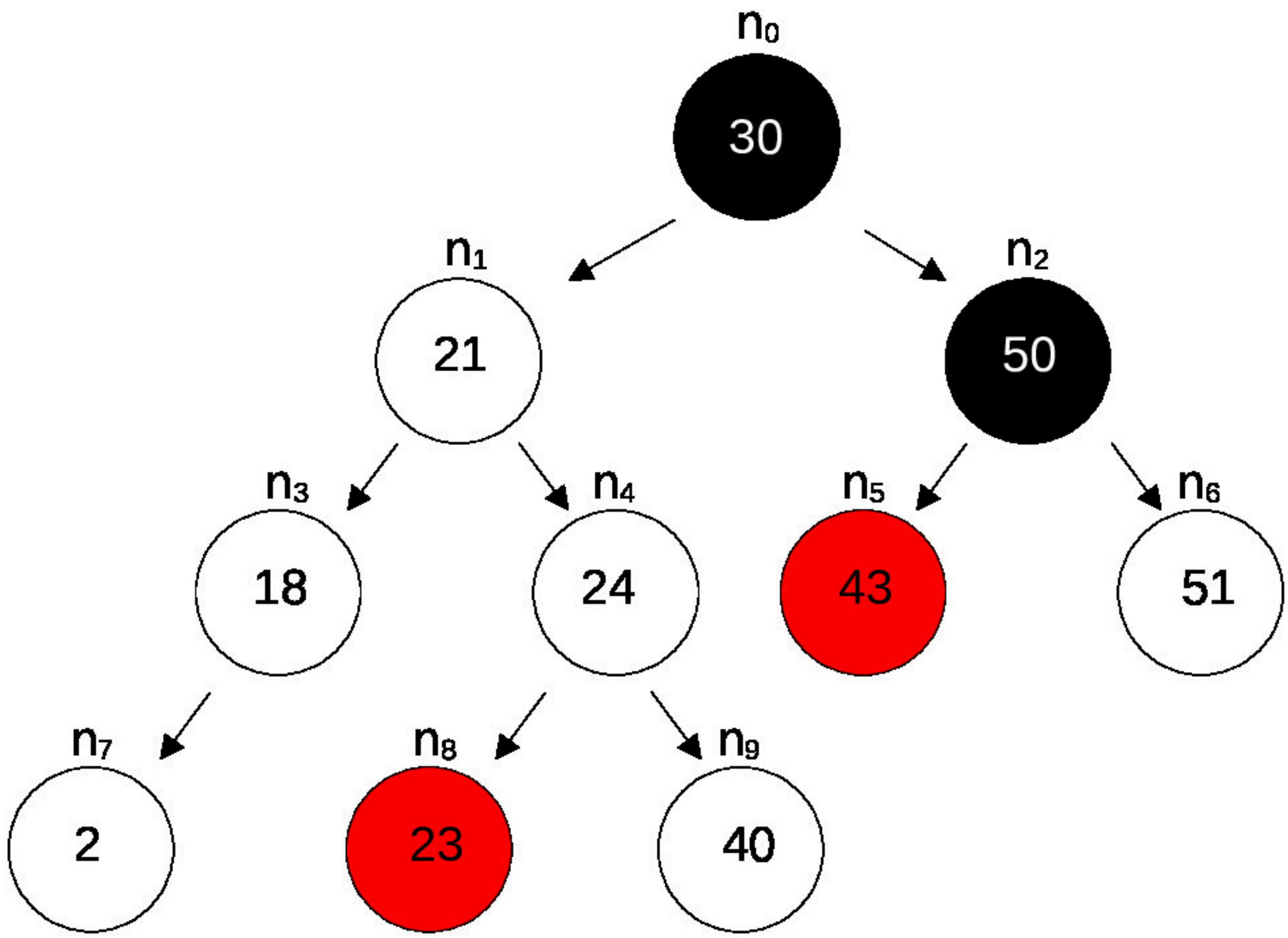
Mark 6.00 out of 6.00

Recall the rules of being a red-black tree:

a) root/leaves must be black (And NIL leaves are not shown in the diagram).

b) children of red nodes must be black.

c) For every node in the tree, the number of black nodes on every path from a node n to a leaf (not counting n but counting the leaf) must be the same.



Select the correct colors to make this a proper red black tree:

n1:

Red ✔

n3:

Black ✔

n4:

Black ✔

n6:

Red ✔

n7:

Red ✔

n9:

Red ✔

Question **2**

Partially correct

Mark 4.00 out of 5.00

Match each question with the appropriate answer.

What is the worst-case complexity of running quicksort with random pivot selection on an array of size n?	n^2 ✔
What is the worst-case complexity of running quickselect to find the median of an array with random pivot selection on an array of size n?	n log(n) ✖
What is the average-case complexity of running quicksort to find the median of an array with randomized pivot selection on an array of size n?	n log(n) ✔
What is the worst-case complexity of running quickselect to find the median of an array with "median of 5 medians" pivot selection on an array of size n?	n ✔
What is the average-case complexity of running quicksort to find the median of an array with "median of 5 medians" pivot selection on an array of size n?	n log(n) ✔

Your answer is partially correct.

You have correctly selected 4.
The correct answer is: What is the worst-case complexity of running quicksort with random pivot selection on an array of size n? → n^2, What is the worst-case complexity of running quickselect to find the median of an array with random pivot selection on an array of size n? → n^2, What is the average-case complexity of running quicksort to find the median of an array with randomized pivot selection on an array of size n? → n log(n), What is the worst-case complexity of running quickselect to find the median of an array with "median of 5 medians" pivot selection on an array of size n? → n, What is the average-case complexity of running quicksort to find the median of an array with "median of 5 medians" pivot selection on an array of size n? → n log(n)

Question **3**

Partially correct

Mark 4.00 out of 6.00

Suppose we wished to use the median of medians trick to find a pivot element for an array of size n.

(a) First divide the array into $\frac{n}{11}$ parts of size 11 each, finding the median of

each of the subarray of size 11 using insertion sort.

What is the time complexity of this step?

- ◯ $\Theta(n^2)$
- ◉ $\Theta(n)$ ✔ Correct
- ◯ $\Theta(n \log(n))$
- ◯ $\Theta(\log(n))$

Mark 1.00 out of 1.00

The correct answer is: $\Theta(n)$

(b) Next, we recursively invoke our algorithm to find the median of the medians found in the previous step.

What is the size of the array that is passed to this recursive step?

- ◯ n/10
- ◯ 5n/11
- ◉ 8n/11 ✖ Incorrect
- ◯ n/11

Mark 0.00 out of 1.00

The correct answer is: n/11

Let $T(n)$ denote the worst case running time of our median-of-medians median finding procedure. What is the running time of step (b)?

- ◯ T(n/11)
- ◯ n^2
- ◯ T(10n/11)
- ◉ n ✖ Inorrect

Mark 0.00 out of 1.00

The correct answer is: T(n/11)

(c) We use the median found by the recursive call in step (b). How many elements of the array are guaranteed to be smaller than this median?

- ◯ 9n/11
- ◯ 3n/5
- ◯ 0
- ◉ 6n/22 ✔ Correct

Mark 1.00 out of 1.00

The correct answer is: 6n/22

(d) If the median found in step (b) were used as a pivot to partition the array into two parts, what is the size of the larger of the two partitions in the worst case?

- 9n/22
- 7n/10
- 0
- 16n/22 ✔ Correct

Mark 1.00 out of 1.00

The correct answer is: 16n/22

(e) We will now write down a recurrence for finding the worst case running time $T(n)$ of finding a median using the median of 11 medians to find a pivot. Consider the options provided below:

A: $T(n) = T(n) + T(\frac{8n}{22}) + \Theta(n)$.

B: $T(n) = T(\frac{n}{11}) + \Theta(n^2)$.

C: $T(n) = T(\frac{n}{11}) + T(\frac{16n}{22}) + \Theta(n)$.

D: $T(n) = T(\frac{16n}{22}) + \Theta(n)$.

Which of the options above represents the correct recurrence for the algorithm?

- A
- B
- C ✔ Correct
- D

Mark 1.00 out of 1.00

The correct answer is: C

Question **4**

Correct

Mark 1.00 out of 1.00

A hashtable using chaining with $m = 7$ slots is built with a hash function that is chosen uniformly at random from the set of functions

$\{h_1(n) = (n^2 + 5) \mod 7,$

$h_2(n) = (n^2 + 2n) \mod 7,$

$h_3(n) = (n^2 + 5n + 2) \mod 7\}$

The elements $1, 3, 5$ and 6 are inserted into the table.

Calculate the expected (average over all instantiations) number of collisions?

Select one:

- a. 1
- b. 1/8
- c. 1/2
- d. 3/4
- e. 2/3
- f. 0
- g. 1/3 ✖
- h. 2

Your answer is correct.

Suppose h_1 is chosen

we have $h_1(1) = 6, h_1(3) = 0, h_1(5) = 2, h_1(6) = 6$ yielding one collision.

Suppose h_2 is chosen

we have $h_2(1) = 3, h_2(3) = 1, h_2(5) = 0, h_2(6) = 6$, yielding zero collisions

Suppose h_3 were chosen

we have $h_3(1) = 1, h_3(3) = 5, h_3(5) = 3, h_3(6) = 5$, yielding one collision.

Expected number of collisions is therefore $\frac{3}{3} = 1$.

The correct answer is: 2/3

Comment: