Proof Workout Mastery Workbook
I have neither given nor received unauthorized assistance________________________________.

Learning Goal to Master: Create logical well crafted proofs. In addition, this workbook tells a story about the challenges of proofs. What is it?

With simple Even/Odd proofs it is tempting to argue that for integers, even + even = even. However, as we learn to create rigorous proofs we will see this inductive (intuitive reasoning) hides some complexity.

For example, if asked to prove if k is even then n = 2 + 8 n is even, we cannot just say "Well n is even and even times and even is even etc... To see why, we will prove these elementary theorems (calle the Warm Up Thms). **In general, you will not be allowed to use the results of the Warm-Up Thms**, but you will use the *techniques* of proving them again and again.

***Notice these proofs are following the format of our logic equivalence (page 30). You must use this format.**

Warm Up THM 1

Prove, for all positive integers, A, B, if A and B are even, then C = A + B is even:

There is d such that A = 2 d	definition of even
There is h such that B = 2 h	definition of even (*notice d/ h cannot be the same, why?*)
C = A + B	premise
= 2 d + 2 h	substitution
= 2(d + h)	factoring
Define k integer as k = d + h	closure of the integers renaming d + h
= 2 (k)	substitution

● Thus we see that C = 2 (k), k is an integer, so by definition of even, C is even.

QED

Use Warm Up THMs 2-4 for ungraded practice.

Warm Up THM 2

For all positive integers, A, B, if A is even and B is odd, A + B = C is odd.

Warm Up THM 3

For all positive integers, A, B, if A is odd and B is odd, A + B = C is even.

Warm Up THM 4

Prove For all positive integers, A, B, if A and B are even, A * B = C is even.

Warm UP THM 5

Prove: For all positive integers, A, B, if A is even and B is odd, A * B = C is even.

#1 (Graded)

Warm Up THM 6

Prove:

For all positive integers, A, B, if A and B are odd, A * B = C is ______

#2 Prove that if x is odd, then 5 x + 3 is "...." two different ways.

- A direct proof by **applying the Warm - Up Thms** - this means just USE the RESULTS of what you have proved above.

- Using the *techniques* of the warm up Thm. and the formal definitions of even and odd. (this will look a lot like your WUT proofs)

Ungraded reflection: Explain the difference between using the RESULTS of a Theorem and using the TECHNIQUES of a theorem's proof.

You may NOT use Warm-Up THM results again unless specified - this includes exams.

Ungraded: Attempt to use a direct proof (similar to techniques of WUT THMs) :

If $3n + 7$ is odd, then n is even (n is an integer).

Why is this not working? (hint it should not work)

This is a good one to discuss on Piazza.

#3 Prove using a proof by contrapositive. (be sure to use the right format)

If $3 n + 7$ is odd, then n is even (n is an integer).

Why does this work, when a direct proof did not?

#4 Prove for positive integers, if n^2 is even, then n is even . <careful, careful, careful>

#5 Prove that $13n + 3$ is even if and only if n is odd. n is an integer.(must prove both directions, why?)

Ungraded - try this in you own words.

Prove there is no largest integer z. Assume the domain of integers.

There is no integer z, for all n, z > n.

Use a proof by contradiction. Review the directions for a proof by contradiction. After you proof briefly discuss, - What does this imply about the nature of infinity? Can infinity be an integer?

Say "We prove by…"

State the negation -

Consider the negation.

Do something valid with the negation that shows something goes wrong (see the proof tutorial video or worksheet)

State "Contradiction!!!"

State "by contradiction we have …"

#6 Assume the domain of positive integers. Prove if n = ab, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$

Prove by contradiction (using required steps)

#7 We define an exponent b of a, (a is an element of the positive real numbers, and b is an element of the natural numbers) as:

a^b is a multiplied by itself b times. Example $a^4 = a\,a\,a\,a$
$a^0 = 1$

We will use only the basic principles of algebra (associativity, commutativity, etc), and the definition of exponents given above to prove the rules of exponents.

Assume a and b are elements of the positive real numbers. You must explain each step.

Notice the format of this example proof and follow this structure for #8 and #9. Do not "work both sides of the equation."

Notice how this proof does not use the rule it is trying to prove. **For #7, just fill in the blanks with the correct justification of each step.**

- Prove that $a^3 * a^5 = a^{(3+5)}$

$a^3 * a^5 = (aaa) * (aaaaa)$	Def. of exponents
$= (aaaaaaaa)$	____________ law
$= a^8$	Definition of ____________
$= a^{(3+5)}$	Substitution using 8 = 3 +5

QED

#8 Prove that $(a^{(3)})^5 = a^{3*5}$

#9 Prove that $a^5 b^5 = (a\,b)^5$

#10 Considering the above, when we add the definition of exponents to the algebra, are the "rules" of exponents really a new idea or just a consequence of the existing structure of algebra? Give a thoughtful and complete answer.

Notes - ungraded, add as many pages as you like.

Use these questions or other notes to help you organize your study and prepare for the exam.

1. Which ideas were new to you?
2. What questions might make good exam questions?
3. What ideas will you want to review later?
4. What were some “a-ah” moments, or big ideas you want to remember?