I have neither given nor received unauthorized assistance_

Learning Goal to Master: Create logical well crafted proofs. In addition, this workbook tells a story about the challenges of proofs. What is it?

With simple Even/Odd proofs it is tempting to argue that for integers, even + even = even. However, as we learn to create rigorous proofs we will see this inductive (intuitive reasoning) hides some complexity.

For example, if asked to prove if k is even then n = 2 + 8 n is even, we cannot just say "Well n is even and even times and even is even etc... To see why, we will prove these elementary theorems (calle the Warm Up Thms). **In general, you will not be allowed to use the results of the Warm-Up Thms**, but you will use the *techniques* of proving them again and again.

*Notice these proofs are following the format of our logic equivalence (page 30). You must use this format.

Warm Up THM 1

Prove, for all positive integers, A, B, if A and B are even, then C = A + B is even:

There is d such that A = 2 d definition of even

There is h such that B = 2 h definition of even (notice d/h cannot be the same, why?)

C = A + B premise

= 2 d + 2 h substitution

= 2(d + h) factoring

Define k integer as k = d + h closure of the integers renaming d + h

= 2 (k) substitution

• Thus we see that C = 2 (k), k is an integer, so by definition of even, C is even.

QED

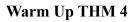
Use Warm Up THMs 2-4 for ungraded practice.

Warm Up THM 2

For all positive integers, A, B, if A is even and B is odd, A + B = C is odd.

Warm Up THM 3

For all positive integers, A, B, if A is odd and B is odd, A + B = C is even.



Prove For all positive integers, A, B, if A and B are even, A * B = C is even.

Warm UP THM 5

Prove: For all positive integers, A, B, if A is even and B is odd, A * B = C is even.

#1 (Graded)

Warm Up THM 6

Prove:

For all positive integers, A, B, if A and B are odd, A * B = C is _____

- #2 Prove that if x is odd, then 5 x + 3 is "..." two different ways.
 - A direct proof by **applying the Warm Up Thms** this means just USE the RESULTS of what you have proved above.

• Using the *techniques* of the warm up Thm. and the formal definitions of even and odd. (this will look a lot like your WUT proofs)

Ungraded reflection: Explain the difference between using the RESULTS of a Theorem and using the TECHNIQUES of a theorem's proof.

You may NOT use Warm-Up THM results again unless specified - this includes exams.

Ungraded: Attempt to use a direct proof (similar to techniques of WUT THMs):

If 3 n + 7 is odd, then n is even (n is an integer).

Why is this not working? (hint it should not work)

This is a good one to discuss on Piazza.

#3 Prove using a proof by contrapositive. (be sure to use the right format)
If $3 n + 7$ is odd, then n is even (n is an integer).
Why does this work, when a direct proof did not?

#4 Prove for positive integers, if n^2 is even, then n is even. <careful, careful, careful

#5 Prove that 13 n + 3 is even if and only if n is odd. n is an integer.(must prove both directions, why?)

Prove there is no largest integer z. Assume the domain of integers.
There is no integer z, for all $n, z > n$.
Use a proof by contradiction. Review the directions for a proof by contradiction. After you proof briefly discuss, - What does this imply about the nature of infinity? Can infinity be an integer?
Say "We prove by"
State the negation -
Consider the negation.
Do something valid with the negation that shows something goes wrong (see the proof tutorial video or worksheet)
State "Contradiction!!!"
State "by contradiction we have"

Ungraded - try this in you own words.

#6 Assume the domain of positive integers. Prove if n = ab, then $a \le \sqrt{n}$ or $b \le \sqrt{n}$ Prove by contradiction (using required steps) #7 We define an exponent b of a, (a is an element of the positive real numbers, and b is an element of the natural numbers) as:

$$a^b$$
 is a multiplied by itself b times. Example $a^4 = a a a a$

$$a^0 = 1$$

We will use only the basic principles of algebra (associativity, commutativity, etc), and the definition of exponents given above to prove the rules of exponents.

Assume a and b are elements of the positive real numbers. You must explain each step.

Notice the format of this example proof and follow this structure for #8 and #9. Do not "work both sides of the equation."

Notice how this proof does not use the rule it is trying to prove. For #7, just fill in the blanks with the correct justification of each step.

• Prove that
$$a^3 * a^5 = a^{(3+5)}$$

$$a^3 * a^5 = (aaa) * (aaaaa) Def. of exponents$$

$$= (aaaaaaaaa) law$$

$$= a^8 Definition of Definition of Substitution using $8 = 3 + 5$

$$QED$$$$

#8 Prove that $(a^{(3)})^5 = a^{3*5}$

#9 Prove that $a^5 b^5 = (a b)^5$

#10 Considering the above, when we add the definition of exponents to the algebra, are the "rules" of exponents really a new idea or just a consequence of the existing structure of algebra? Give a thoughtful and complete answer.

Notes - ungraded, add as many pages as you like.

Use these questions or other notes to help you organize your study and prepare for the exam.

- 1. Which ideas were new to you?
- 2. What questions might make good exam questions?
- 3. What ideas will you want to review later?
- 4. What were some "a-ah" moments, or big ideas you want to remember?