<u>Sets and Functions Problem Set</u>

<u>REQUIRED</u>
I have neither given nor received unauthorized assistance ________________________

1. (Ungraded) Prove the "other" DeMorgan's law following Chris' examples in the set video - slide 26 (similar to example 11 p. 131)

2. (20pts) From Rosen, page 136, do #1- 4, on this page (and next if needed) explain the important ideas of these problems with examples, insights and notes.

#additional space for #2 if needed.

3. (20pts) From Rosen, page 153, #8 (do #9 for ungraded practice). Add a few words about how the floor and ceiling functions work.

4. (10 pts) Prove the algebra rules for logs. Like exponents, logs are just an added definition, not a new set of rules. We define the log base a of a number x as: "The power of a that gives us x"

$$log_a(x) = y$$

We can use this **definition, the algebra properties, and the properties of exponents** to discover how log functions work - the log rules. Let's just consider log base 10 for simplicity

For example:

- Prove $log_{10}(a\,b) = log_{10}(a) + log_{10}(b)$?
    - Let's say:

$$log_{10}(a) = C$$

$$log_{10}(b) = D$$

    - By definition of log we know

$$10^C = a$$
$$10^D = b$$

$We\ start\ with\ the\ left\ side$ $\quad log_{10}(a\,b) = log_{10}(10^C \cdot 10^D)$ by substitution

$= log_{10}10^{(C+D)}$ exponent rules

$= C + D$ def. of log.

$= log_{10}(a) + log_{10}(b)$ substitute back

On the following page , now you discover/show/prove: $log_{10}(a^b) = b\,log_{10}a$ **using a <u>similar method.</u>**

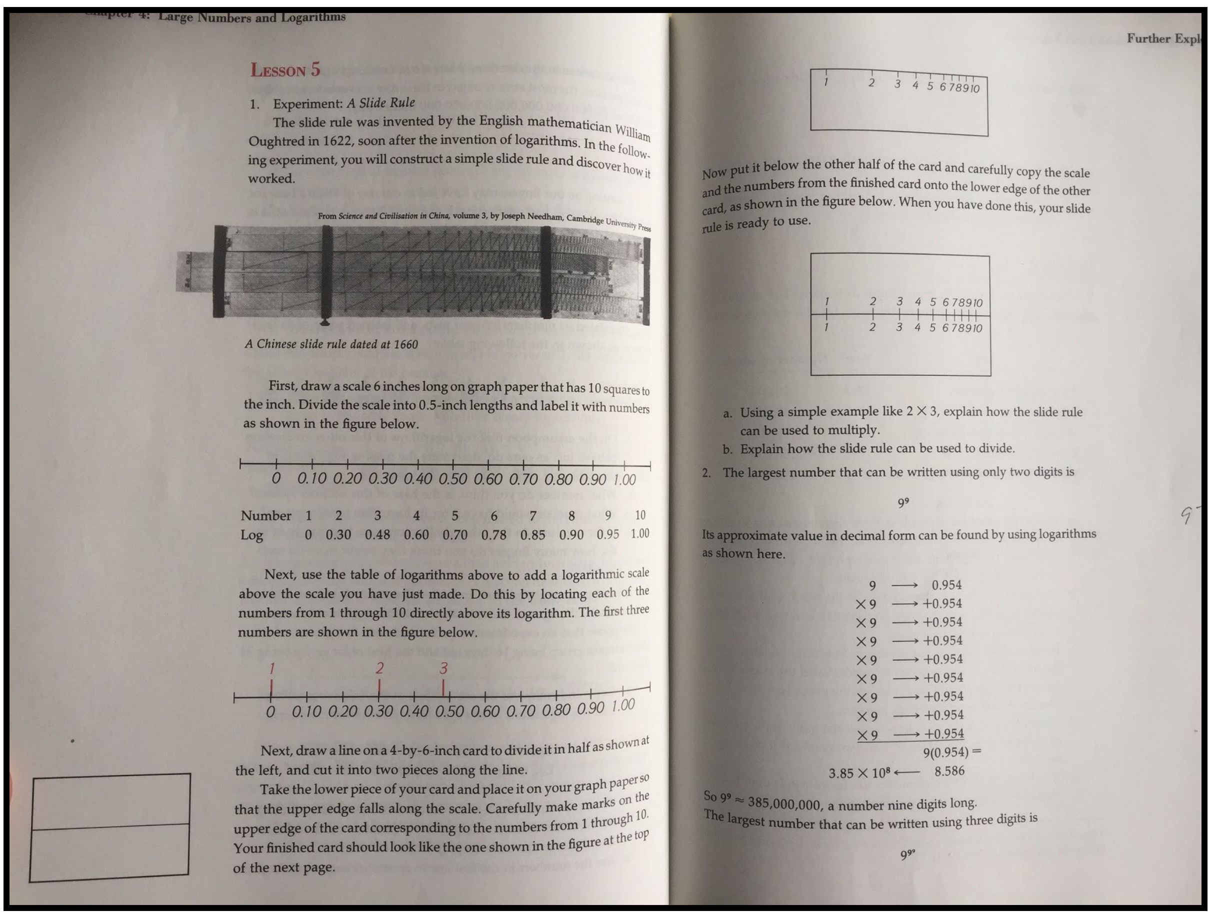
Convert a to something like 10 to a power (just like the example above).
Then start with the left side.
Make a substitution.
Use an exponent rule.
Convert back.
Get to the right hand side.

#4 prove on this page

5. (10 pts) Is $(log_{10}(a))^b = b\, log_{10} a$ **?**

Prove or disprove
State your method

6. (10 pts) Do the following slide rule activity from Jacobs page 240 -241, #1 only. I have included additional chapter pages in Moodle for you to do and review as needed to prepare. Post a photo of your result. **Answer a and b.**



Chapter 4: Large Numbers and Logarithms

LESSON 5

1. Experiment: *A Slide Rule*

The slide rule was invented by the English mathematician William Oughtred in 1622, soon after the invention of logarithms. In the following experiment, you will construct a simple slide rule and discover how it worked.

From *Science and Civilisation in China*, volume 3, by Joseph Needham, Cambridge University Press

A Chinese slide rule dated at 1660

First, draw a scale 6 inches long on graph paper that has 10 squares to the inch. Divide the scale into 0.5-inch lengths and label it with numbers as shown in the figure below.

0 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00

Number	1	2	3	4	5	6	7	8	9	10
Log	0	0.30	0.48	0.60	0.70	0.78	0.85	0.90	0.95	1.00

Next, use the table of logarithms above to add a logarithmic scale above the scale you have just made. Do this by locating each of the numbers from 1 through 10 directly above its logarithm. The first three numbers are shown in the figure below.

1 2 3

0 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00

Next, draw a line on a 4-by-6-inch card to divide it in half as shown at the left, and cut it into two pieces along the line.

Take the lower piece of your card and place it on your graph paper so that the upper edge falls along the scale. Carefully make marks on the upper edge of the card corresponding to the numbers from 1 through 10. Your finished card should look like the one shown in the figure at the top of the next page.

Further Expl

1 2 3 4 5 6 7 8 9 10

Now put it below the other half of the card and carefully copy the scale and the numbers from the finished card onto the lower edge of the other card, as shown in the figure below. When you have done this, your slide rule is ready to use.

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

a. Using a simple example like 2×3, explain how the slide rule can be used to multiply.

b. Explain how the slide rule can be used to divide.

2. The largest number that can be written using only two digits is

$$9^9$$

Its approximate value in decimal form can be found by using logarithms as shown here.

$$\begin{array}{rcl} 9 & \longrightarrow & 0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \hline & & 9(0.954) = \\ 3.85 \times 10^8 & \longleftarrow & 8.586 \end{array}$$

So $9^9 \approx 385{,}000{,}000$, a number nine digits long.

The largest number that can be written using three digits is

$$9^{9^9}$$

#6Photo and answers here

#6 if needed

7. (ungraded) Do #20/21 from Rosen p. 244 by hand and check your answers in the back or with a calculator.

8. (10pts) Do as many or #30 - 33 p. 245 as needed (if you want to include your work in the MW add it at the end) - *these use the multiplication and adding rules on page 242*. Use this shortcut. Then pick ONE SINGLE (for example 30 (b) or 33 (a)) and on this page do the calculations 2 ways, one way doing each part (long way) and another by using the shortcuts from page 242. Hopefully you get the same answers!

9. (20 pts) On the following page(s) explore the following with screenshots and explanations:

Use 1-3 pages to answer. Your answer can be notes on graphs, but do answer completely.

DO NOT USE CALCULUS

- Open up Desmos.com
- Graph, imagine these are functions that show runtime (y) as a function of inputs (x)
    - f(x) = x
    - g(x) = 1000 log (x)

    - From looking at how these two graphs grow, as x gets larger and larger, which function seems to be showing a longer runtime?
    - Is there a place on the graph where g(x) = 1000 log(x) is showing a longer runtime than f(x) = x?
    - Does which function is 'better' change as x gets big?
    - Where on the x-axis does this happen?
    - Where does it become clear that one of these is better (faster) runtime than the other?
- Experiment with other values of the log function:
    - p(x) = 10,000 log (x)
    - t(x) = 100,000 log(x)

  Is f(x) really 'better' than these log functions as x gets large? Why or why not?

  (not a proof, but what are you seeing? This is a HUGE and IMPORTANT computer science concept. Know and remember).