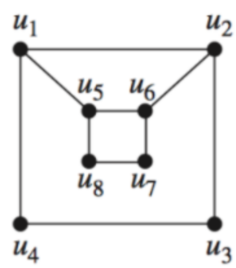
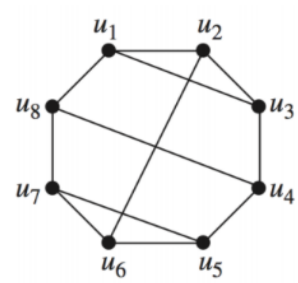
1. For each of the degree sequences shown below, determine whether they represent a valid undirected graph with no self-loops. Justify your reasoning.
   (a) $4, 3, 2, 1, 0$
   (b) $2, 2, 2, 2, 2$
   (c) $1, 1, 1, 1, 1$
   (d) $4, 4, 3, 2, 1$

2. For each of the graphs below:

   i. Determine if the graph has an Eulerian Tour. Justify your reasoning. If you conclude that the graph does not have a Eulerian Tour, can you **add** a small number of edges so that it does?

   ii. Determine if the graph is Bipartite. If you conclude that the graph is Bipartite, specify a two-coloring of the vertices. If you determine that it is not Bipartite, explain why.





3. Recall that for a graph G, the chromatic number $\chi(G)$ is the *minimum* number of colors necessary to color the vertices of G. We saw in lecture that the Greedy Graph Coloring Algorithm does not always find the minimum coloring. For each of the following graphs, your goal is to make a **clear** argument that zeros in on $\chi(G)$. Can you prove a definitive value for $\chi(G)$? If not, can you say that $\chi(G)$ is at least some number and at most some other number?