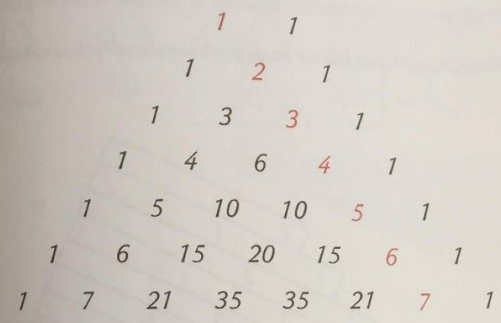
Combinatorics Supplement for the Problem Set Quiz

1. Find the values of 11^2, 11^3, and 11^4.
2. What do these numbers have to do with Pascal's triangle?
3. Now find the value of 11^5. Does it fit the pattern?

One sloping row of numbers in Pascal's triangle is shown in color in the figure below.



4. Starting at the top of this row, what is the sum of:

   its first two numbers?
   its first three numbers?
   its first four numbers?
   its first five numbers?
   its first six numbers?

5. What do you notice about these answers?

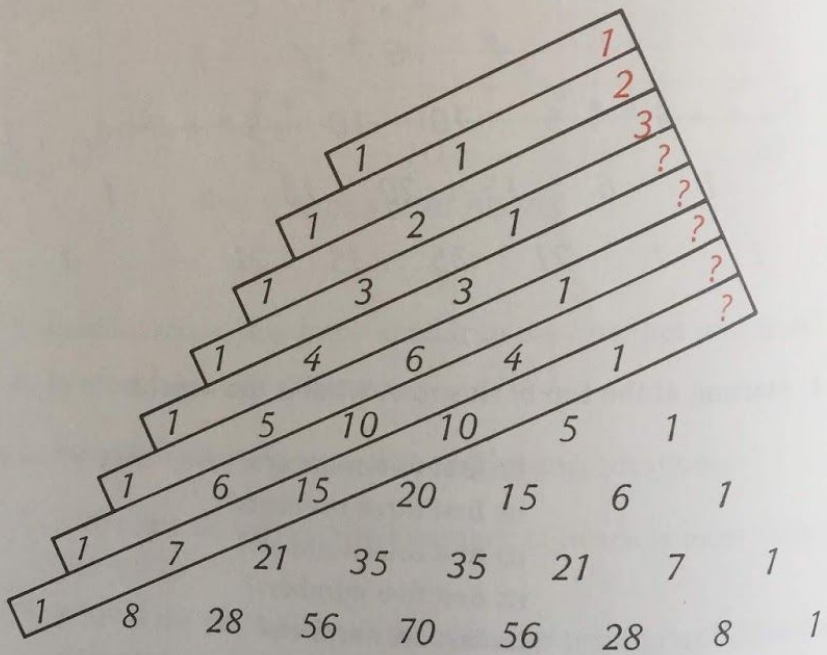
Another sloping row of numbers is shown in color in the figure below.

6. Starting at the top of this row, what is the sum of:

   its first and second numbers?
   its second and third numbers?
   its third and fourth numbers?
   its fourth and fifth numbers?
   its fifth and sixth numbers?

7. These sums form a certain number sequence. What sequence is it?

The numbers in the figure below have been separated into a set of sloping rows.



The sums of the numbers in the first three rows are 1, 2, and 3.

8. What are the sums of the numbers in the next five rows?
9. These eight sums form a certain number sequence. What sequence is it?

Wide World Photos

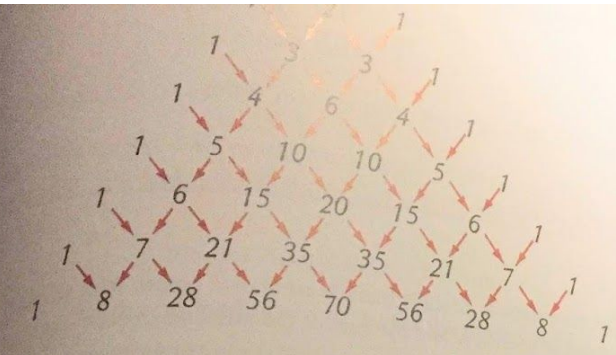
LESSON

5 Pascal's Triangle

All eight children in this remarkable family are girls. Because the probabilities of a child being a boy or girl are about the same, it would seem that in a family of this size, four girls and four boys would be much more likely. Exactly how do the probabilities of a family having eight girls and a family having four girls and four boys compare?

Although this could be figured out from a tree diagram, there is an easier way. It is based on a pattern of numbers so old that no one knows who first discovered it. The pattern, called "Pascal's triangle," is named after the great seventeenth-century mathematician Blaise Pascal. One of the originators of probability theory, Pascal wrote a book about the triangle and its properties. Pascal's triangle looks like the figure at the top of the next page.

Each number within the triangle is found by adding the pair of numbers in the row above it at the left and right. The triangle can be



continued indefinitely by writing a 1 at both ends of each new row and then adding each pair of numbers in the preceding row.

Each row of the triangle contains the numbers of ways of getting each possible outcome for a situation in which the probabilities are binomial. The *fourth* row, for example, contains the numbers of ways in which a coin can turn up when it is tossed *four* times.*

Number of heads	0	1	2	3	4
Number of ways	1	4	6	4	1

The question about the probabilities for a family with *eight* children can be answered by looking at the *eighth* row of the triangle.

Number of girls	0	1	2	3	4	5	6	7	8
Number of ways	1	8	28	56	70	56	28	8	1

If we assume that the probabilities of a child being a boy or girl are each $\frac{1}{2}$, the probability of boys and girls in any particular sequence of eight children is

$$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{256}$$

There is only one sequence of eight girls, so the probability of a family having eight girls is $1\times\frac{1}{256}$, or approximately 0.4%. There are 70 sequences of four girls and four boys, each having probability $\frac{1}{256}$, so the

*The ways are illustrated in the tree diagram on page 477.

"...probability of a family having four girls and four boys is:

70 * (1/256) = 70/256 or about 27.2%"