University of Colorado **Boulder**

Department of Computer Science

CSCI 2824: Discrete Structures

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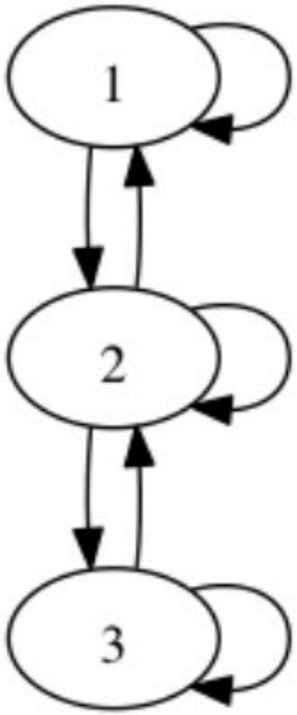
Relations
Properties of Relations

Reflexive Relations

Def: A relation is *reflexive* iff $(a, a) \in R$ for all $a \in A$

Graph Analogy: All nodes in the graph have self-loops

Example: R_2 **is** reflexive because each vertex has a self-loop

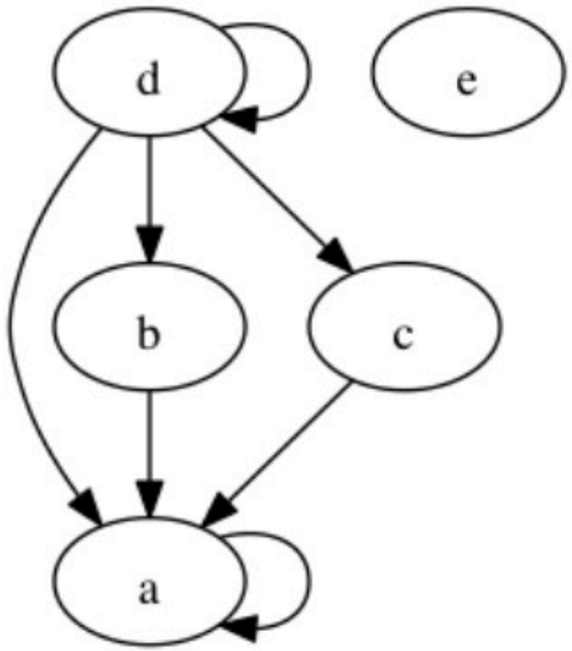


Reflexive Relations

Def: A relation is *reflexive* iff $(a, a) \in R$ for all $a \in A$

Graph Analogy: All nodes in the graph have self-loops

Example: R_1 is not reflexive. Vertices b, d, e don't have self-loops



Reflexive Relations

Def: A relation is *reflexive* iff $(a, a) \in R$ for all $a \in A$

Example: The Facebooks *friends* relation **is / is NOT** reflexive

Example: The Twitter *follower of* relation **is / is NOT** reflexive

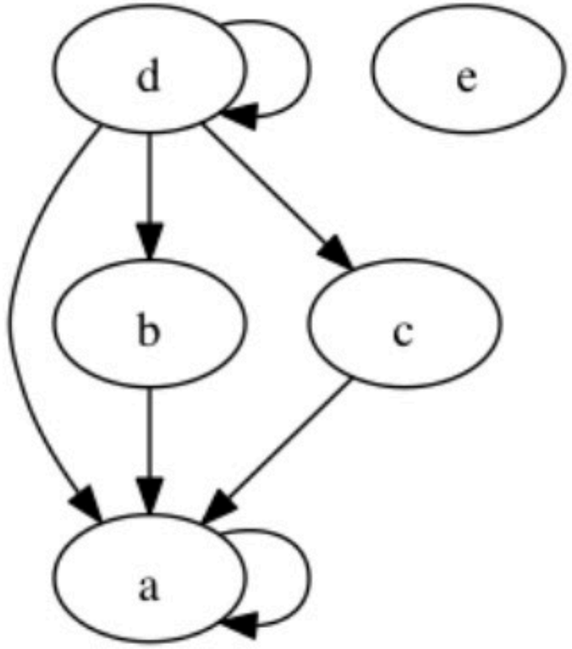
Example: The *at least as tall as* relation **is / is NOT** reflexive

Symmetric Relations

Def: A relation is *symmetric* iff for all $(a, b) \in R$ also $(b, a) \in R$

Graph Analogy: If there is an edge from a to b then there is also an edge from b to a

Example: R_1 is not symmetrc. Note $(d, c) \in R_1$ but not $(c, d) \in R_1$

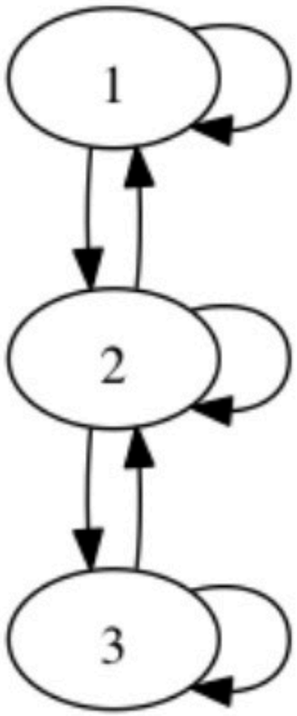


Symmetric Relations

Def: A relation is *symmetric* iff for all $(a, b) \in R$ also $(b, a) \in R$

Graph Analogy: If there is an edge from a to b then there is also an edge from b to a

Example: R_2 is symmetrc.



Symmetric Relations

Def: A relation is *symmetric* iff for all $(a, b) \in R$ also $(b, a) \in R$

Example: The Facebooks *friends* relation **is / is NOT** symmetric

Example: The Twitter *follower of* relation **is / is NOT** symmetric

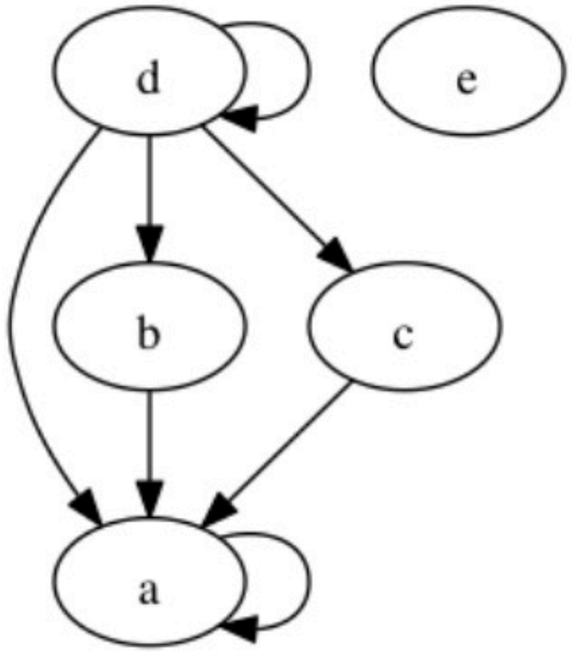
Example: The *at least as tall as* relation **is / is NOT** symmetric

Anti-Symmetric Relations

Def: A relation is *anti-symmetric* iff whenever $(a, b) \in R$ **AND** $(b, a) \in R$ **THEN** $a = b$.

Graph Analogy: If there is an edge from *distinct* a to b then there is **NOT** an edge from b to a

Example: R_1 is anti-symmetrc.



Anti-Symmetric Relations

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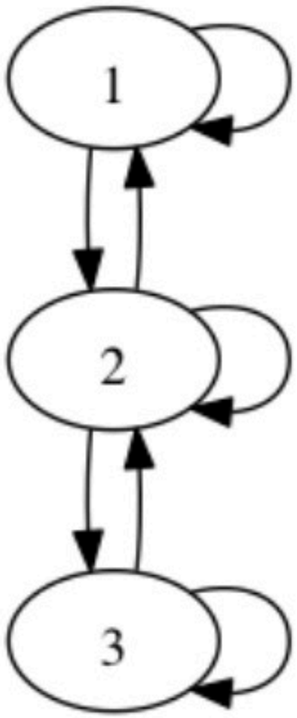
Graph Analogy: If there is an edge from *distinct* a to b then there is **NOT** an edge from b to a

Example: The following relation is **NOT** anti-symmetrc.

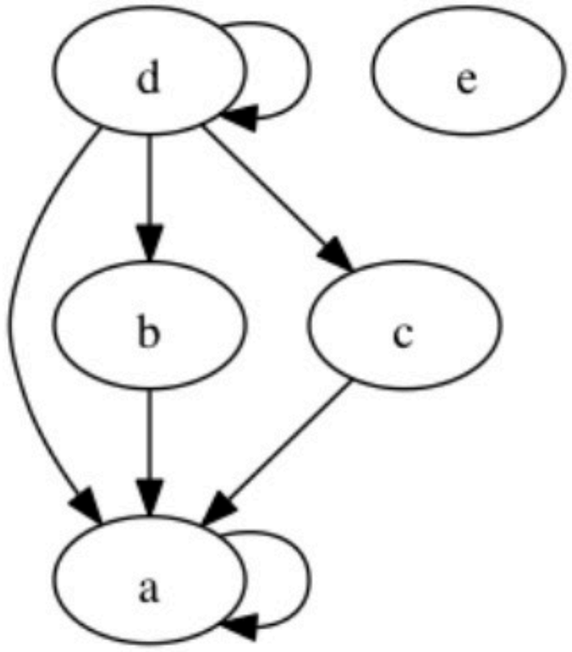


Transitive Relations

Def: A relation is *transitive* iff for all a, b, c, **IF** $(a, b) \in R$ **AND** $(b, c) \in R$ **THEN** $(a, c) \in R$

Graph Analogy: If there is an edge from a to b and an edge from b to c then there is also an edge from a to c

Example: R_1 **is** transitive.

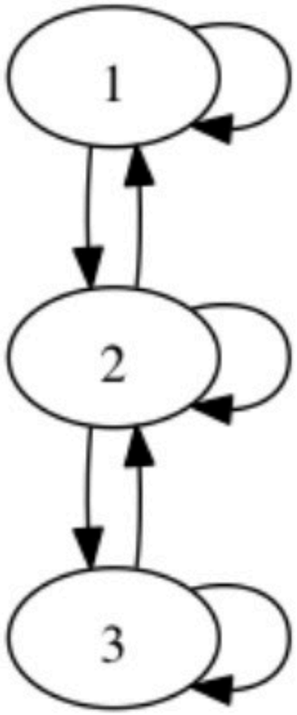


Transitive Relations

Def: A relation is *transitive* iff for all a, b, c, **IF** $(a, b) \in R$ **AND** $(b, c) \in R$ **THEN** $(a, c) \in R$

Graph Analogy: If there is an edge from a to b and an edge from b to c then there is also an edge from a to c

Example: R_2 is **not** transitive. Note: $1 \to 2$ and $2 \to 3$, but $1 \not\to 3$



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Example: The *at least as tall as* relation **is / is NOT** transitive