ROSEN PROOFS - Section 1.7

Example 1:

If n is an odd integer, then n^2 is odd.

n = (2k + 1)		by def. of odd
n^2	= (2k+1)^2	squaring n
	= 4k^2 + 4k + 1	algebra/FOIL
	= 2(k^2 + 2k) + 1	factoring
	= 2Q +1	Let Q = k^2 + 2k

Thus n^2 = 2Q +1 and therefore by the definition of odd, n^2 is odd.

QED

Example 2:

If n and m are both perfect squares then n*m is a perfect square.

n = s^2		Given, and def. of perfect square
m = t^2		Given, and def. of perfect square
mn	= s^2 t^2	Given
	= s*s*t*t	Def. of square
	= s*t*s*t	Commutativity
	= (s*t)*(s*t)	Associativity
		Let s*t =u by substitution.
	= u * u	By definitions above
	= (u)^2	Def. of square

Thus mn is a perfect square.

QED

Example 3:

If n is an integer and (3n +2) is odd, then n is odd.

We propose a proof by contrapositive.

The contrapositive is equivalent to the original proposition, if we succeed in proving the contrapositive we will know the original proposition is true.

The contrapositive is:

If n is an integer, and n is not odd, then (3n +2) is not odd.

Or, since, not odd = even,

If n is an integer, and n is even, then (3n +2) is even.

We commence the proof.

n = 2*a	By def. of even

3n + 2	= 3(2a) + 2	By given and substitution
	= 6a +2	By algebra
	= 2(3a + 1)	factoring
	= 2*Z	Let Z = (3a + 1)

Thus we see 3n + 2 = 2*Z and by the def. of even, 3n + 2 is even.

The contrapositive is proved and hence the original proposition is proved as well.

QED

Example 8: Prove if n^2 is odd, then n is odd.

This goes badly with a direct proof - (why?)
Prove by contrapositive
The contrapositive is, *If n is even, then n^2 is even*.

n	$= 2b$	def. of even
n^2	$= (2b)^2$	substitution
	$= 4b^2$	algebra
	$= 2(2b^2)$	factoring
		Let $k = (2b^2)$
	$= 2k$	substitution

Thus by def. Of even, n^2 is even.
The contrapositive is proved and so is the original statement.
QED

Example 9:

Show that at least 4 of any given 22 days must fall on the same day of the week (Sunday, Monday, etc..).

We prove by contradiction.

The *negation* of the proposition P is:

Not P = Less than 4 of any given 22 days must fall on the same day of the week.

We take the - P to be true

If Less than 4 of any given collection of 22 days must fall on the same day of the week, then **at mos**t 3 of any 22 days can be the same day of the week.

Consider such a collection of 22 days which includes **at mos**t 3 of any 22 days of the same day of the week.

Since there are 7 days in a week, we can see that it is possible to have 3 sets of days of the week in a set of 21 days.

To complete a collection of 22 days, the final day must then repeat one of the given 7 days of the week.

This leaves us with 4 days in the collection that fall on the same day of the week.

However, we assumed we can only have at most 3 days fall on the same day of the week (-P)

-P is then not true.

Hence by contradiction we see P is true.

*also note: we showed (- - P) is true.   - - P == P, thus P is true.

QED