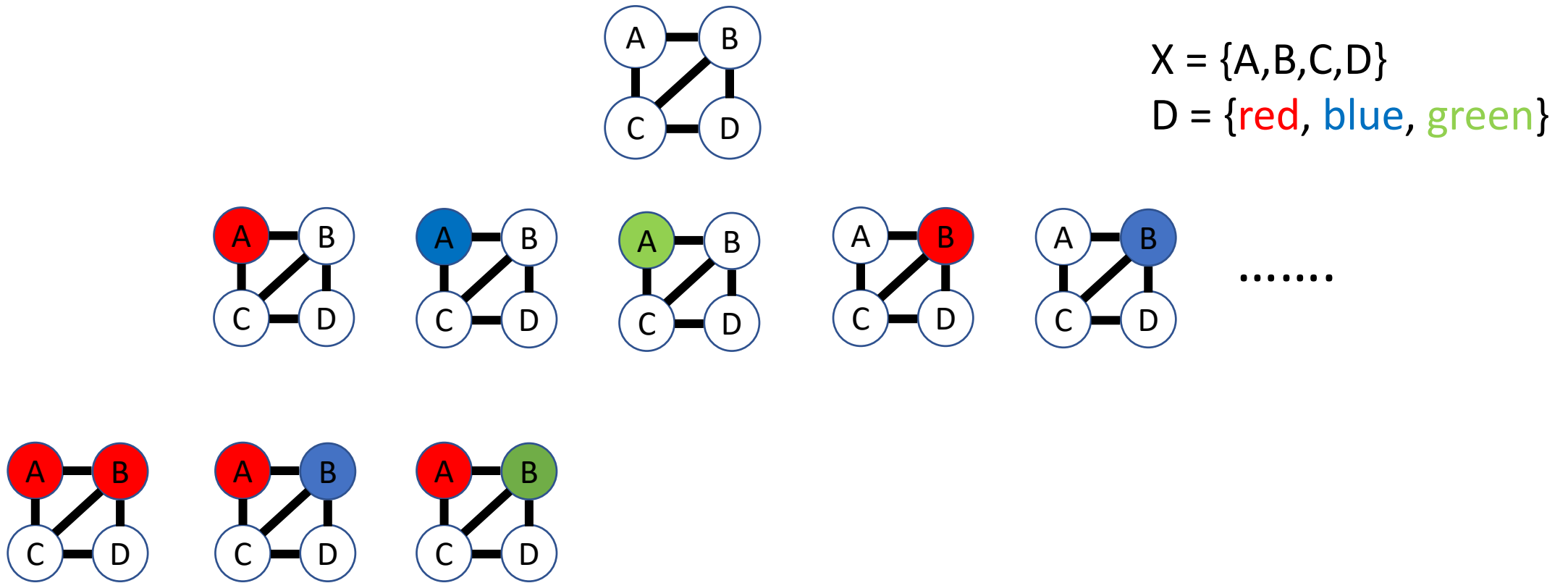


Search

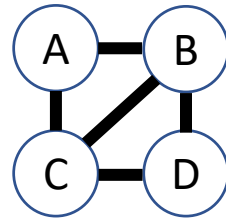


Constraint Satisfaction Problems

Solving CSP- Naïve Search: BFS

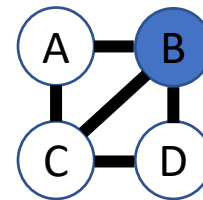
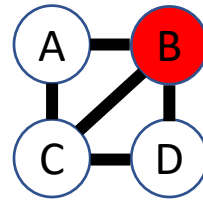
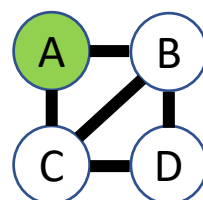
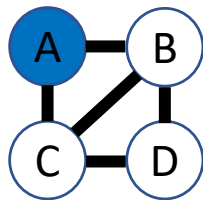
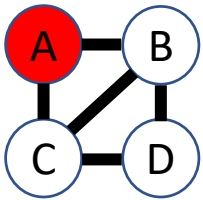


Solving CSP- Naïve Search: DFS

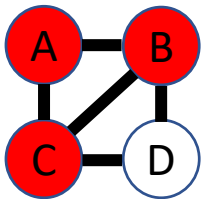
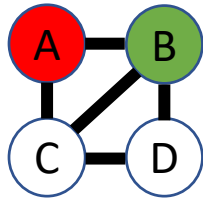
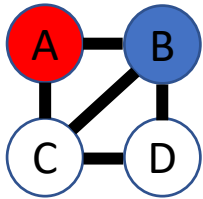
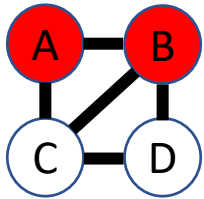


$X = \{A, B, C, D\}$

$D = \{\text{red}, \text{blue}, \text{green}\}$



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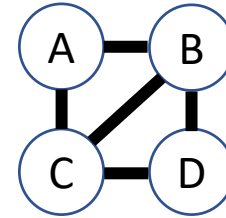
Backtracking

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
  return RECURSIVE-BACKTRACKING( $\{ \}$ , csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
  if assignment is complete then return assignment
  var  $\leftarrow$  SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
  for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
    if value is consistent with assignment given CONSTRAINTS[csp] then
      add  $\{var = value\}$  to assignment
      result  $\leftarrow$  RECURSIVE-BACKTRACKING(assignment, csp)
      if result  $\neq$  failure then return result
      remove  $\{var = value\}$  from assignment
  return failure
```

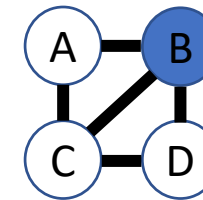
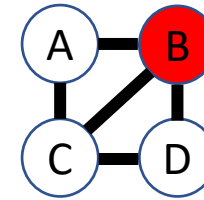
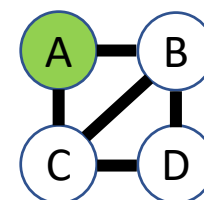
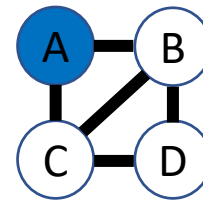
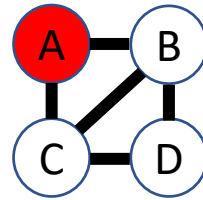
Backtracking

Idea: Check the constraints as we go

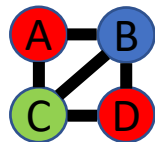
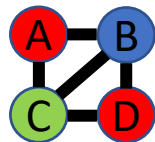
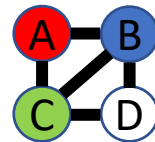
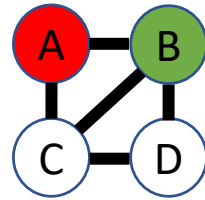
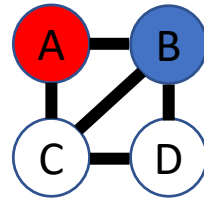
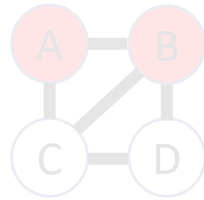


$X = \{A, B, C, D\}$

$D = \{\text{red}, \text{blue}, \text{green}\}$



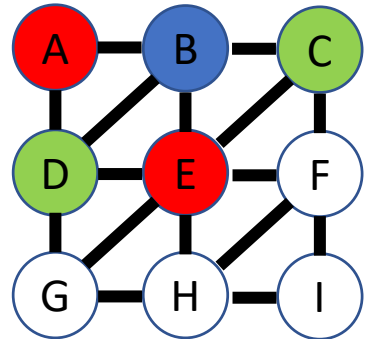
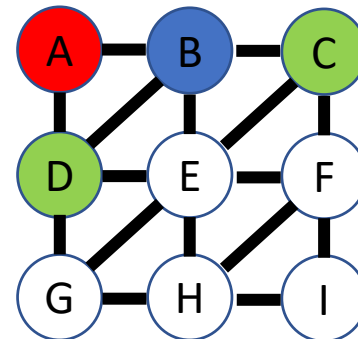
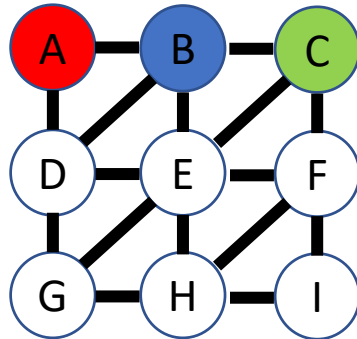
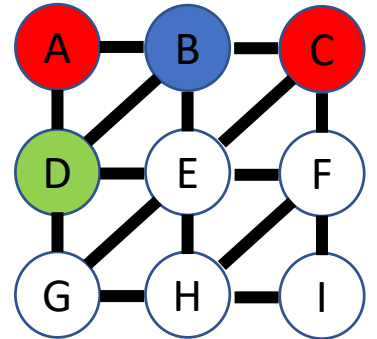
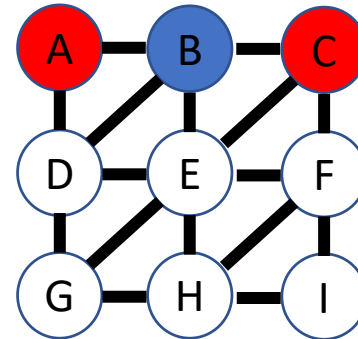
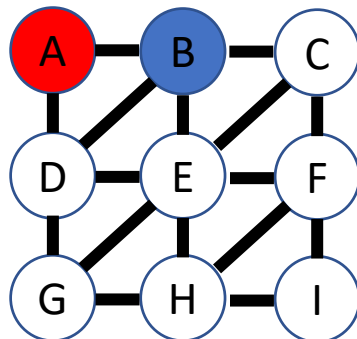
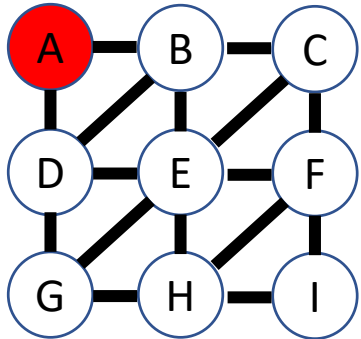
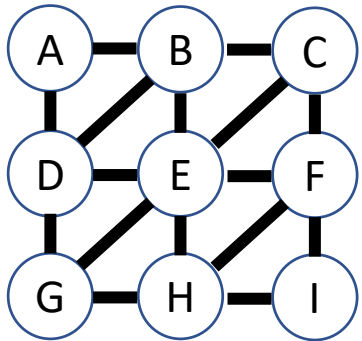
.....



Backtracking

$X = \{A, B, C, D, \dots\}$

$D = \{\text{red}, \text{blue}, \text{green}\}$



Improving backtracking

- **Filtering** : Check if there is any violation in the future
- **Ordering** : Pick next variable wisely
- **Structure** : Can we take advantage of certain structure of the problem?

Filtering

Idea : Check if there is any violation in the future
(Domain reduction)

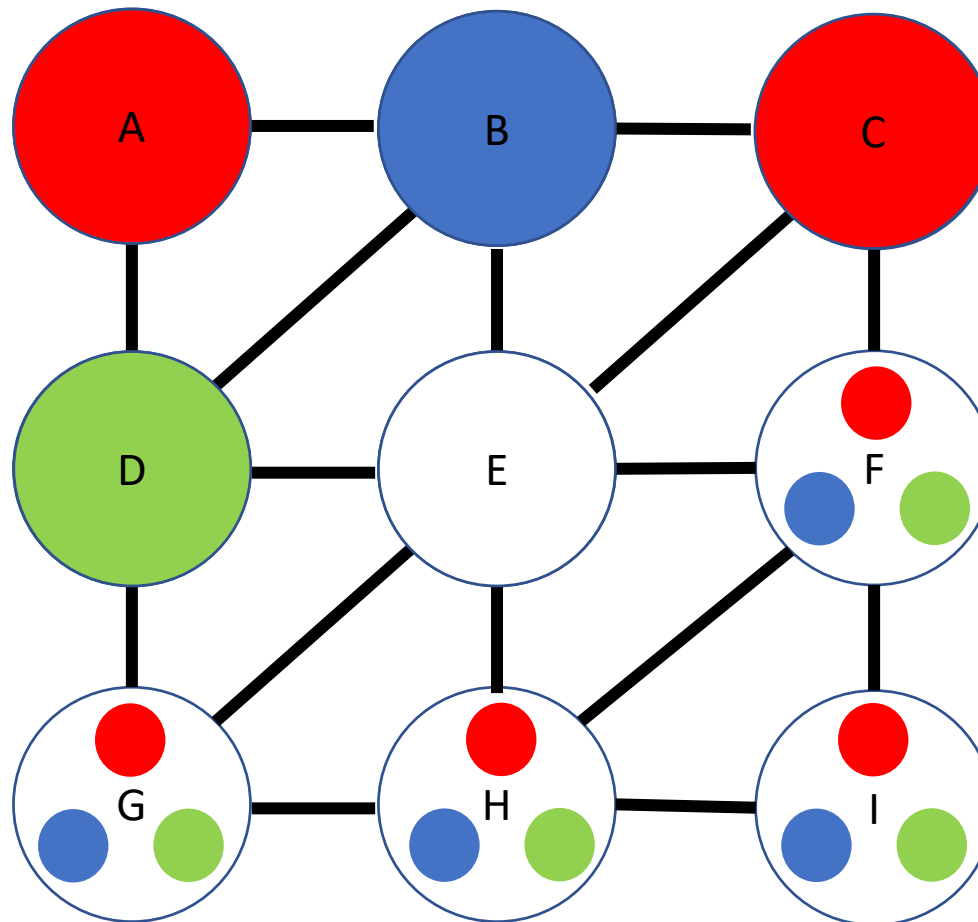
- **Forward Checking:** Inferencing domain reduction on neighboring variables
- **Arc Consistency:** Propagate the inference/constraints to the entire graph

Forward checking

Idea: cross off non-valid options in the $\{D\}$ for X_i

$X = \{A, B, C, D, \dots\}$

$D = \{\text{red}, \text{blue}, \text{green}\}$



Forward checking can
still have a problem!

Arc Consistency- AC3

function AC-3(*csp*) **returns** the CSP, possibly with reduced domains

inputs: *csp*, a binary CSP with variables $\{X_1, X_2, \dots, X_n\}$

local variables: *queue*, a queue of arcs, initially all the arcs in *csp*

while *queue* is not empty **do**

$(X_i, X_j) \leftarrow \text{REMOVE-FIRST}(\textit{queue})$

if REMOVE-INCONSISTENT-VALUES(X_i, X_j) **then**

for each X_k **in** NEIGHBORS[X_i] **do**

 add (X_k, X_i) to *queue*

function REMOVE-INCONSISTENT-VALUES(X_i, X_j) **returns** true iff succeeds

removed \leftarrow false

for each x **in** DOMAIN[X_i] **do**

if no value y in DOMAIN[X_j] allows (x, y) to satisfy the constraint $X_i \leftrightarrow X_j$

then delete x from DOMAIN[X_i]; *removed* \leftarrow true

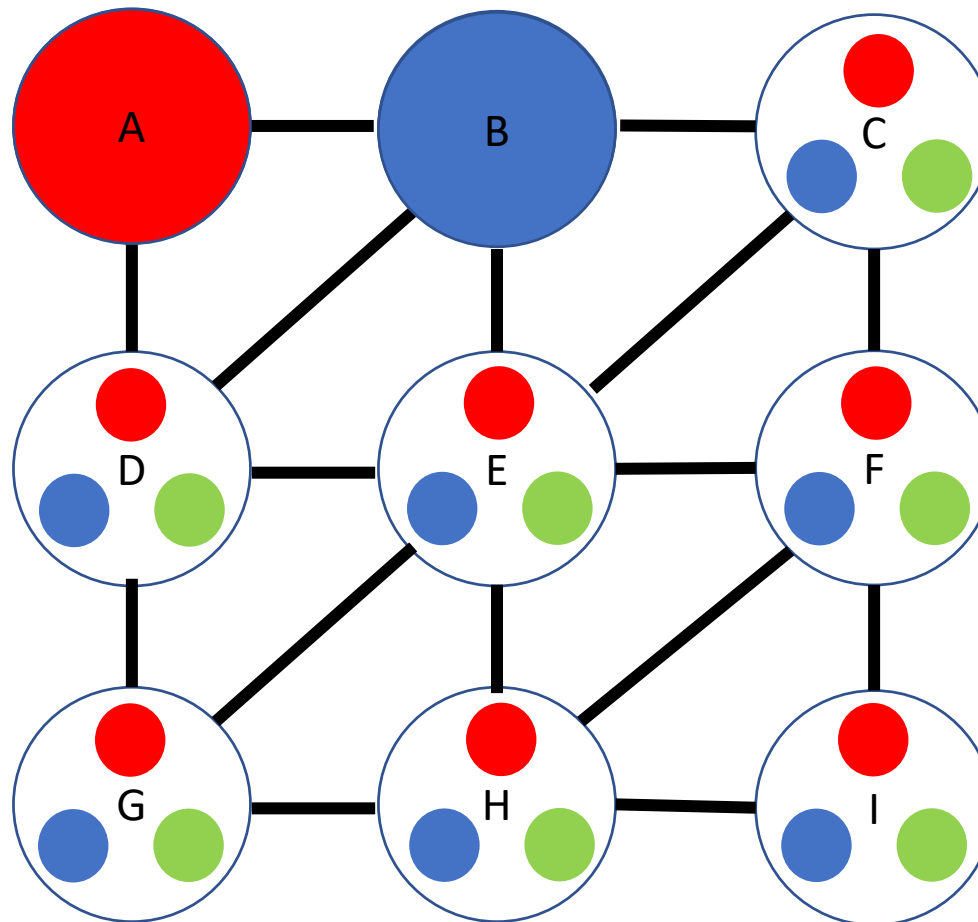
return *removed*

Arc Consistency- AC3

Idea: cross off non-valid options in the $\{D\}$ for all X

$X = \{A, B, C, D, \dots\}$

$D = \{\text{red}, \text{blue}, \text{green}\}$

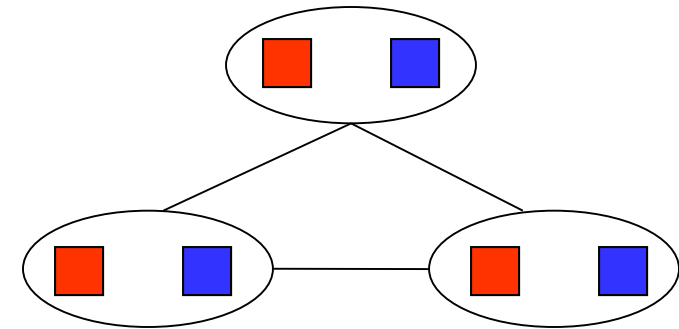
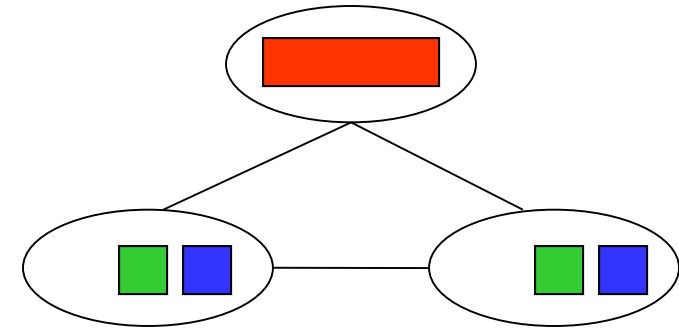


RULE 1: If a domain is removed/assigned, check all neighbors and neighbors or neighbors recursively!

RULE 2: Arc is consistent if there is at least a valid pair of assignments.

Limitations of Arc Consistency

- After enforcing arc consistency:
 - Can have one solution left
 - Can have multiple solutions left
 - Can have no solutions left (and not know it)
- Arc consistency still may need backtracking



*What went
wrong here?*