CSPB 2820 - Truong - Linear Algebra with Computer Science Applications

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Started on	Wednesday, 27 September 2023, 4:42 PM
State	Finished
Completed on	Wednesday, 27 September 2023, 4:57 PM
Time taken	14 mins 35 secs
Grade	Not yet graded

Question **1**

Correct

Mark 0.90 out of 1.00

Let $\mathbf{a} = (-5, \alpha, -6)$ and $\mathbf{b} = (\beta, 2, -10)$. Choose values for the variables α and β such that the set $\{\mathbf{a}, \mathbf{b}\}$ is linearly dependent.

$\alpha =$ 6/5

$\beta =$ -25/3

Your answer is correct!Vastauksesi on oikein!
Marks for this submission: 1.00/1.00. Accounting for previous tries, this gives **0.90/1.00**.

Worked solution:
Two vectors are known to be linearly dependent if and only if they are parallel. Two vectors being parallel means that they are scalar multiples of each other which means that there exists a scalar k such that $\mathbf{v}_1 = k\mathbf{v}_2$. Let's investigate the question using the definition

$$\mathbf{a} = k\mathbf{b}$$

with vectors $\mathbf{a}$ and $\mathbf{b}$. $\Leftrightarrow (-5,\ \alpha,\ -6) = k\,(\beta,\ 2,\ -10)$ $\Leftrightarrow (-5,\ \alpha,\ -6) = (\beta \cdot k,\ 2 \cdot k,\ -10 \cdot k)$ We may express this as a system of three equations:

$\begin{cases} -5 = \beta \cdot k \\ \alpha = 2 \cdot k \\ -6 = -10 \cdot k \end{cases}$. From the third equation we can solve $k = \frac{3}{5}$. Now the system simplifies to $\begin{cases} -5 = \dfrac{3 \cdot \beta}{5} \\ \alpha = \dfrac{6}{5} \end{cases}$ from which we may solve $\begin{cases} \beta = -\dfrac{25}{3} \\ \alpha = \dfrac{6}{5} \end{cases}$. Now $k\mathbf{b} = \frac{3}{5}\left(-\frac{25}{3},\ 2,\ -10\right) = \left(-5,\ \frac{6}{5},\ -6\right) = \mathbf{a}$ which means that the vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel and consequently linearly dependent

Question **2**

Correct

Mark 1.00 out of 1.00

Let $\mathbf{a} = (-5, \alpha, -3)$ and $\mathbf{b} = (\beta, -9, 2)$ where α and β are unknown real constants. We know that the vectors $\mathbf{a}$ and $\mathbf{b}$ are linearly independent. (x_1 and x_2 are scalars)

What can we say about the solutions to equation $x_1\mathbf{a} + x_2\mathbf{b} = \mathbf{0}$ based on the given information?

1. The equation has no solution.
2. There are no real solutions but at least one complex solution can be found.
3. The only solution to the equation is $(x_1, x_2) = (0, 0)$.
4. The equation has infinitely many solutions.
5. One solution to the equation is $x_1 = \mathbf{a}^{-1}, x_2 = -\mathbf{b}^{-1}$.

Give the number corresponding to the correct statement.

Answer:

3

Also give values to the constants α and β such that the vectors $\mathbf{a}$ and $\mathbf{b}$ truly are linearly independent.

$\alpha =$ 9

$\beta =$ 5

Your answer is correct!Vastauksesi on oikein!

Your answer is correct!Vastauksesi on oikein!
You have chosen the correct statement. Valitsit oikean väittämän monivalinnasta.
Marks for this submission: 0.50/0.50.

Your answer is correct!Vastauksesi on oikein!
You have chosen appropriate values for α and β.

Valitsit sopivat arvot vakioille α ja β.

Marks for this submission: 0.50/0.50.

Worked solution:
According to the definition vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are linearly independent if and only if equation $x_1\mathbf{v}_1 + \ldots + x_n\mathbf{v}_n = \sum_{i=1}^{n} x_i\mathbf{v}_i = \mathbf{o}$ has no other solution except for the trivial solution $x_1 = x_2 = \ldots = x_n = 0$.

Let's write the definition for the vectors in question. There are only two vectors so the equation is $x_1\mathbf{a} + x_2\mathbf{b} = \mathbf{0}$. It was mentioned in the assignment that vectors $\mathbf{a}$ and $\mathbf{b}$ are known to be linearly independent. Now according to the definition the equation has only the trivial solution $x_1 = x_2 = 0$.

In the case of two vectors linear independence means that the vectors are not parallel ergo they are not scalar multiples of each other. So now we should pick values for α and β such that the vectors are not parallel. Let's try to pick such values that the vectors are perpendicular to each other because in that case we know that they are not parallel. Two vectors are perpendicular if their dot product is zero so let's investigate the dot product of the given vectors. $\begin{aligned}\mathbf{a}\cdot\mathbf{b} &= 0\\ \Leftrightarrow -5\cdot\beta - 9\cdot\alpha - 6 &= 0\end{aligned}$ Solving this equation for α we get $\alpha = \frac{5\cdot\beta+6}{-9}$. Now let's pick $\beta = 0$. Now we see that $\alpha = -\frac{2}{3}$. Now we may also check that the dot product is indeed zero: $\mathbf{a}\cdot\mathbf{b} = 0 + 6 - 6 = 0$.

Question 3

Correct

Mark 1.00 out of 1.00

We run the Gram–Schmidt algorithm once on a given set of vectors $a_1, \ldots, a_k$ (we assume this is successful), which gives the vectors:

$q_1, \ldots, q_k$.

Then we run the Gram–Schmidt algorithm on the vectors $q_1, \ldots, q_k$,

which produces the vectors $z_1, \ldots, z_k$.

What can you say about $z_1, \ldots, z_k$?

Select one or more:

- [ ] a.
  While the ultimate conclusion will be the same, the algorithm will produce a new set of vectors yi, y_i != z_i
- [ ] b.
  Since the vectors are orthonormal, the inner product $z_i^T zi = q_i$
- [x] c.
  Since the vectors qi have
  norm one, the normalization step also does nothing: $z_i = q_i$. ✔
- [ ] d. Since the vectors are orthonormal, the inner product $q_i^T q_i = z_i$
- [x] e.
  Since the original list of vectors is orthogonal, the orthogonalization step in
  the Gram–Schmidt algorithm does nothing; ✔

Your answer is correct.

Correct

Marks for this submission: 1.00/1.00.

Question **4**

Correct

Mark 1.00 out of 1.00

When the Gram–Schmidt algorithm is run on a particular list of 10 15-vectors, a_1, a_2, a_3, ...a_{10}. It terminates in iteration 5 (since $q_5 = 0$).

Which of the following must be true?

Select one or more:

- [x] a_2, a_3, a_4 are linearly independent. ✔

  As a subset of the the linear independent vectors a_1, a_2, a_3, a_4, then a_2, a_3, a_4 must be linear independent as well.

- [ ] None of these are true.
- [x] a_1, a_2, a_3, a_4, a_5 are linearly dependent. ✔
- [ ] a is a basis
- [ ] a_1, a_2, a_5 are linearly dependent.

Your answer is correct.

Correct

Marks for this submission: 1.00/1.00.

Question **5**

Complete

Marked out of 3.00

Verify the Gram - Schmidt Algorithm with the book example from page 100, using the code given in section 5.4 f the Python Companion.

(Recommended that you verify additional results from section 5 as well)

Do the above on your own.

Now for this question, come up with your own vector (you can make up one or more, or find an interesting source).

Run GS on your new example (can be a simple example) and **verify the result.**

Respond below:

- Enter your vector(s) below with some context (this is a 5- vector etc..)
- Enter the result of running the algorithm.
- Enter some words about how you checked the results from the algorithm are correct.

The vector that I am using is a 3-Vector of 3-Vectors:

a = np.array([[1, -1, 1], [1, 0, 1], [1, 1, 2]])

The result of running the algorithm then produces:

Vectors are linearly independent. [array([0.57735027, -0.57735027, 0.57735027]), array([0.40824829, 0.81649658, 0.40824829]),array([-0.70710678, 0. , 0.70710678])]

Verifying this by hand we originally have:

$v_1 = (1, -1, 1)$

$v_2 = (\frac{1}{3}, \frac{2}{3}, \frac{1}{3})$

$v_3 = (-\frac{1}{2}, 0, \frac{1}{2})$

Normalizing these vectors we then have: (q represents the normalized vectors)

\(q_{1} = (\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, \frac{3}{3}) \)

\(q_{2} = (\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6}) \)

\(q_{3} = (-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}) \)

Verifying the results by hand shows that the algorithm is working as designed.