CSPB 2820 - Truong - Linear Algebra with Computer Science Applications

Started on	Sunday, 26 November 2023, 8:58 PM
State	Finished
Completed on	Sunday, 26 November 2023, 9:14 PM
Time taken	16 mins 10 secs
Marks	26.00/26.00
Grade	**10.00** out of 10.00 (**100**%)

Question **1**

Correct

Mark 1.00 out of 1.00

Which of the following are true?

Select one or more:

- [x] The inverse of a matrix A with orthonormal columns is A^T ✔
- [x] Any non-zero n-vector considered as a matrix is left invertible. ✔
- [ ] A 1x1 matrix does not have an inverse.
- [ ] Left inverses must be unique.

Your answer is correct.

Correct

Marks for this submission: 1.00/1.00.

Question **2**

Correct

Mark 1.00 out of 1.00

A matrix has a left inverse if and only if it columns are linearly independent.

Select one:

- True ✔
- False

Correct

Marks for this submission: 1.00/1.00.

Question **3**

Correct

Mark 1.00 out of 1.00

Suppose a matrix A is wide.

What can we say about A?

Select one or more:

- a. It is not left invertible. ✔
- b. The columns of A are linearly dependent. ✔
- c. None of these, more information is required.
- d. The columns of A are linearly independent.
- e. It is left invertible.

Your answer is correct.

Correct

Marks for this submission: 1.00/1.00.

Question **4**

Correct

Mark 1.00 out of 1.00

A tall matrix A, may or may not have a right inverse.

Select one:

- True
- False ✔

A tall matrix cannot have a right inverse.

Correct

Marks for this submission: 1.00/1.00.

Question **5**

Correct

Mark 1.00 out of 1.00

A matrix is right invertible if and only if its rows are linearly independent.

Select one:

- True ✔
- False

Correct

Marks for this submission: 1.00/1.00.

Question **6**

Correct

Mark 1.00 out of 1.00

Which of the following are true?

Select one or more:

- [x] a.
  If A has a left inverse C, then C^T is a right inverse of A^T ✔
- [x] b.
  A tall matrix A cannot have right inverse. ✔
- [x] c.
  If A has a right inverse B, then B^T is a left inverse of A^T ✔

  See page 201 for why.
- [x] d.
  Any right inverse X, of a right-invertible matrix A, must have the same dimensions as A^T ✔
- [ ] e.
  A tall matrix A cannot have left inverse.
- [ ] f.
  Any right inverse of X, of a right-invertible matrix A, has different dimensions from A^T

Your answer is correct.

Correct

Marks for this submission: 1.00/1.00.

Question **7**

Correct

Mark 1.00 out of 1.00

If A is right invertible, then the linear equations A x = b can be solved for any vector b.

Select one:

- (•) True ✔
- ( ) False

Correct

Marks for this submission: 1.00/1.00.

Question **8**

Correct

Mark 1.00 out of 1.00

Why is it the case that if A is right invertible, then the linear equations A x = b can be solved for any vector b?

- a. Because of the independence-dimension property.
- b. Because A x = A(Bb) = (AB)b = I b = b, and x = Bb is a solution. ✔
- c. Because A x = (AB)b = (B)b A= I b = b, and x = bB is a solution.
- d. Because math
- e. This is actually not correct. This only correct of left-invertible matrices.

Your answer is correct.

Correct

Marks for this submission: 1.00/1.00.

Question **9**

Correct

Mark 1.00 out of 1.00

Invertible matrices must be square

Select one:

- True ✔
- False

Correct

Marks for this submission: 1.00/1.00.

Question **10**

Correct

Mark 1.00 out of 1.00

Which of the following are true if A is invertible?

Select one or more:

- [ ] a.
  None of these.

b.
The rows of A are linearly independent. ✔

c.
The columns of A are linearly independent. ✔

d.
A has a left inverse. ✔

e.
A has a right inverse. ✔

Your answer is correct.

Correct

Marks for this submission: 1.00/1.00.

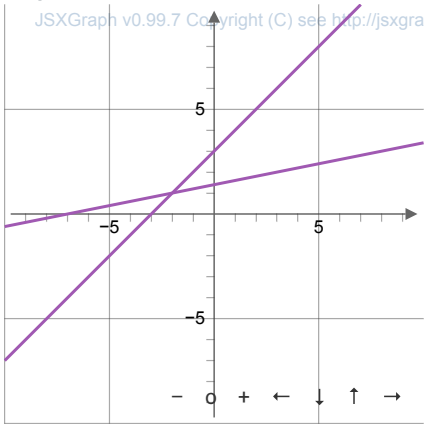
Question **11**

Correct

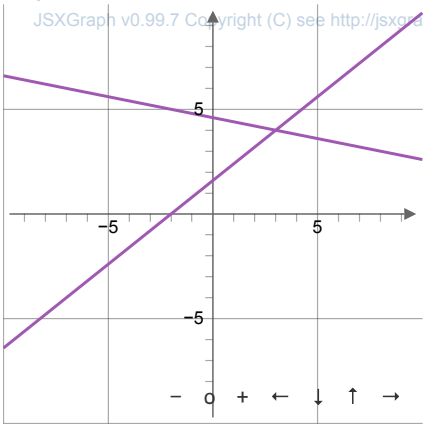
Mark 1.00 out of 1.00

Examine the pair of equations $\begin{cases} -5 \cdot y - x = -23 \\ 5 \cdot y - 4 \cdot x = 8. \end{cases}$ Which of the following graphs corresponds to it?

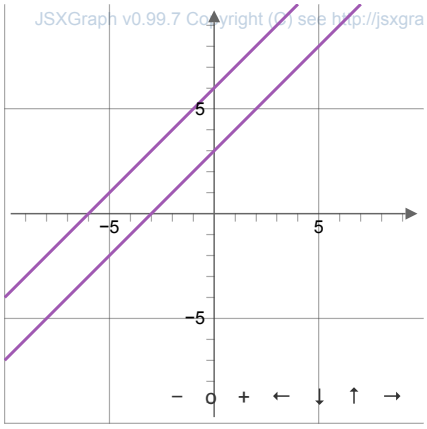
Graph 1:



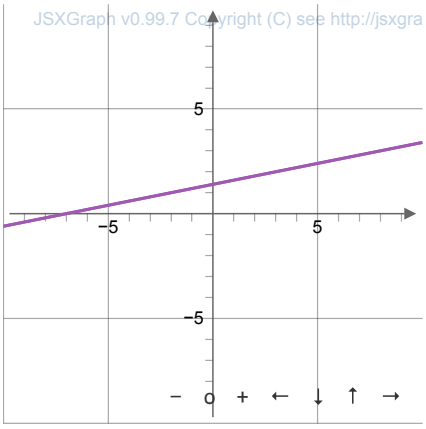
Graph 2:



Graph 3:



Graph 4:



Input the number of the corresponding graph.

Answer:

2

Your answer is correct!Vastauksesi on oikein!
Marks for this submission: 1.00/1.00.

Worked solution:

Let's solve both equations for y to see the slopes and intercepts of the lines. $\begin{cases} -5 \cdot y - x = -23 \\ 5 \cdot y - 4 \cdot x = 8 \end{cases}$ simplifies to $\begin{cases} y = \frac{23}{5} - \frac{x}{5} \\ y = \frac{4 \cdot x}{5} + \frac{8}{5} \end{cases}$

From this we can see that the slope of the first line is $k_1 = -\frac{1}{5}$ with it's intercept being $b_1 = \frac{23}{5}$ and the slope and intercept of the second line are $k_2 = \frac{4}{5}$ and $b_2 = \frac{8}{5}$. Based on this information the corresponding graph must be number 2.

Mallivastaus:

Ratkaistaan molemmista yhtälöparin $\begin{cases} -5 \cdot y - x = -23 \\ 5 \cdot y - 4 \cdot x = 8 \end{cases}$ yhtälöistä muuttuja y, jolloin näemme, minkälaisia suoria yhtälöt kuvaavat. $\begin{cases} y = \frac{23}{5} - \frac{x}{5} \\ y = \frac{4 \cdot x}{5} + \frac{8}{5} \end{cases}$ Tästä näemme, että ensimmäisen suoran kulmakerroin $k_1 = -\frac{1}{5}$ ja vakiotermi $b_1 = \frac{23}{5}$ sekä vastaavasti toisen suoran kulmakerroin $k_2 = \frac{4}{5}$ ja vakiotermi $b_2 = \frac{8}{5}$. Näiden tietojen perusteella yhtälöparin täytyy vastata kuvaajaa 2.

Question **12**

Correct

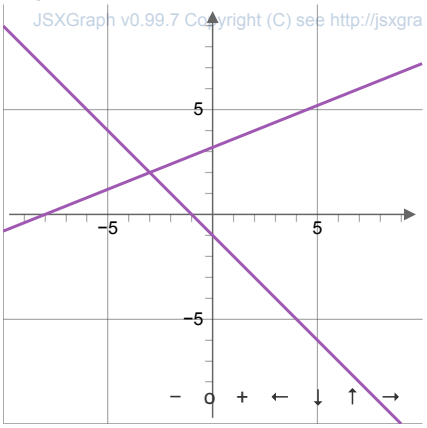
Mark 1.00 out of 1.00

Every pair of equations can be geometrically interpreted as a graph of two straight lines. Here we have four pairs of equations and four graphs. Connect each pair of equations to the correct graph.

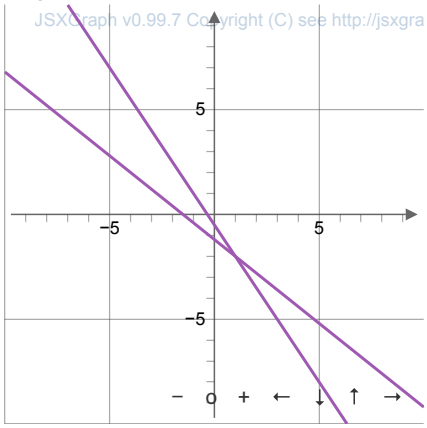
a) $\begin{cases} 5 \cdot y + 4 \cdot x = -6 \\ -2 \cdot y - 3 \cdot x = 1 \end{cases}$ **b)** $\begin{cases} 4 \cdot y + 4 \cdot x = -4 \\ 12 \cdot y + 12 \cdot x = -12 \end{cases}$

c) $\begin{cases} 2 \cdot x - 5 \cdot y = -16 \\ 4 \cdot y + 4 \cdot x = -4 \end{cases}$ **d)** $\begin{cases} 20 \cdot y - 8 \cdot x = 64 \\ 2 \cdot x - 5 \cdot y = -36 \end{cases}$

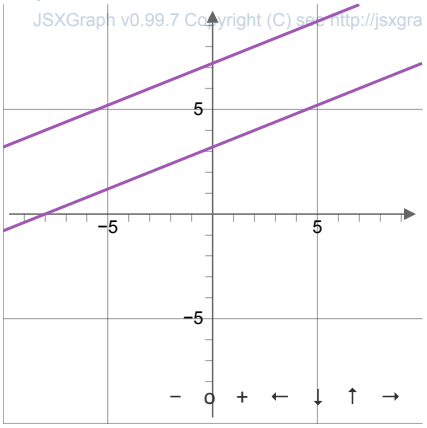
Graph 1:



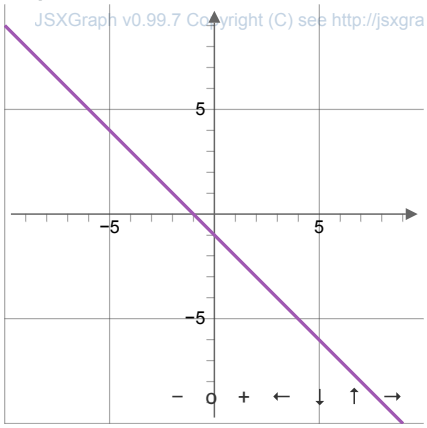
Graph 2:



Graph 3:



Graph 4:



Input the number of the graph corresponding to each pair of equations.

a)

2

b)

4

c)

1

d)

3

Your answer is correct!Vastauksesi on oikein!

Your answer is correct!Vastauksesi on oikein!
The graph corresponding to the pair of equations **a)** is correct! Yhtälöparin **a)** kuvaaja on valittu oikein!
Marks for this submission: 0.25/0.25.

Your answer is correct!Vastauksesi on oikein!
The graph corresponding to the pair of equations **b)** is correct! Yhtälöparin **b)** kuvaaja on valittu oikein!
Marks for this submission: 0.25/0.25.

Your answer is correct!Vastauksesi on oikein!
The graph corresponding to the pair of equations **c)** is correct! Yhtälöparin **c)** kuvaaja on valittu oikein!
Marks for this submission: 0.25/0.25.

Your answer is correct!Vastauksesi on oikein!
The graph corresponding to the pair of equations **d)** is correct! Yhtälöparin **d)** kuvaaja on valittu oikein!
Marks for this submission: 0.25/0.25.

Worked solution:
Let's take a look at the first pair of equations. First we simplify the equations to determine what the slopes and intercepts of the lines are. This will make it easier to see which lines correspond with the equations. $\begin{cases} 5 \cdot y + 4 \cdot x = -6 \\ -2 \cdot y - 3 \cdot x = 1 \end{cases}$ simplifies to $\begin{cases} y = -\frac{4 \cdot x}{5} - \frac{6}{5} \\ y = -\frac{3 \cdot x}{2} - \frac{1}{2}. \end{cases}$ Here we see that the slope of the first line is $k_1 = -\frac{4}{5}$ and the intercept $b_1 = -\frac{6}{5}$ and for the second line the slope is $k_2 = -\frac{3}{2}$ and the intercept $b_2 = -\frac{1}{2}$. Based on this information the pair of equations **a)** corresponds to the graph 2.

The remaining cases may be determined similarly.

Mallivastaus:
Tutkitaan yksitellen mitä kuvaajaa mikäkin yhtälöpari vastaa. Aloitetaan yhtälöparista **a)**. Sievennetään yhtälöparia ensin siten, että näemme helposti yhtälöitä vastaavien suorien kulmakertoimet ja vakiotermit. Yhtälöpari $\begin{cases} 5 \cdot y + 4 \cdot x = -6 \\ -2 \cdot y - 3 \cdot x = 1 \end{cases}$ sievenee muotoon $\begin{cases} y = -\frac{4 \cdot x}{5} - \frac{6}{5} \\ y = -\frac{3 \cdot x}{2} - \frac{1}{2}. \end{cases}$ Tästähän näemme suoraan, että ensimmäisen suoran kulmakerroin $k_1 = -\frac{4}{5}$ ja vakiotermi $b_1 = -\frac{6}{5}$ sekä vastaavasti toisen suoran kulmakerroin $k_2 = -\frac{3}{2}$ ja vakiotermi $b_2 = -\frac{1}{2}$. Näiden tietojen perusteella yhtälöparin täytyy vastata kuvaajaa 2.

Vastaavalla tavalla saamme tutkittua muut tapaukset.

Question **13**

Correct

Mark 1.00 out of 1.00

This is not explicitly covered in VMLS, however, you can use the given formula to solve.

Compute the inverse of $\mathrm{A} = \begin{bmatrix} -4 & -6 \\ 6 & 3 \end{bmatrix}$ using the following formula.

If matrix $\mathrm{X} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, its inverse $\mathrm{X}^{-1} = \frac{1}{\det(\mathrm{X})} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

First compute the determinant of A.

$\det(\mathrm{A}) =$ 24

Your last answer was interpreted as follows: 24

Next compute the inverse A^{-1}.

1/8	1/4
-1/4	-1/6

Your last answer was interpreted as follows: $\begin{bmatrix} \frac{1}{8} & \frac{1}{4} \\ \frac{-1}{4} & \frac{-1}{6} \end{bmatrix}$

Your answer is correct!Vastauksesi on oikein!

Your answer is correct!Vastauksesi on oikein!
The determinant is correct! Laskit determinantin oikein!
Marks for this submission: 0.50/0.50.

Your answer is correct!Vastauksesi on oikein!
The inverse A^{-1} is correct! Syötit käänteismatriisin A^{-1} oikein!
Marks for this submission: 0.50/0.50.

Worked solution:

First we need to calculate the determinant of A. The formula for the determinant of a 2×2 matrix is $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc.$

Determinant is sometimes also denoted $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Now $\det(\mathrm{A}) = \begin{vmatrix} -4 & -6 \\ 6 & 3 \end{vmatrix} = -4 \cdot 3 - 6 \cdot (-6) = 24.$ The determinant is nonzero so the matrix is invertible and we can compute the inverse using the given formula:

$$\mathrm{A}^{-1} = \frac{1}{\det(\mathrm{A})} \begin{bmatrix} 3 & 6 \\ -6 & -4 \end{bmatrix} = \frac{1}{24} \begin{bmatrix} 3 & 6 \\ -6 & -4 \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{6} \end{bmatrix}$$

Mallivastaus:

Kaavan käyttämistä varten meidän täytyy ensin laskea matriisin A determinantti. Determinantti lasketaan 2x2-matriisille kaavalla $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc.$ Determinanttia merkataan toisinaan myös $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{vmatrix} a & b \\ c & d \end{vmatrix}.$ Nyt siis

$\det(A) = \begin{vmatrix} -4 & -6 \\ 6 & 3 \end{vmatrix} = -4 \cdot 3 - 6 \cdot (-6) = 24.$ Determinantti on erisuuri kuin nolla, joten tiedämme varmuudella, että matriisi on kääntyvä.

Käytettyä kaavaakaan ei olisi määritelty, jos determinantti olisi nolla. Lasketaan nyt käänteismatriisi kaavalla:

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} 3 & 6 \\ -6 & -4 \end{bmatrix} = \frac{1}{24} \begin{bmatrix} 3 & 6 \\ -6 & -4 \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{6} \end{bmatrix}$$

Question **14**

Correct

Mark 12.00 out of 12.00

Define a function back_solve that takes as input an n x n upper triangular matrix R with nonzero diagonal entries, and an n-vector b, and returns a vector x such that \(R x = b\).

The quiz environment uses python3 with "**import numpy as np**" included. Thus you can call any numpy API using '**np.**'

R, b, and x are numpy arrays as stated above.

Template:

```
def back_solve(R, b):
    # Your code here, declare and calculate x.
    return x
```

Answer: (penalty regime: 0 %)

```
def back_solve(R,b):
    x = np.linalg.inv(R) @ b
    return x
```

	Test	Expected	Got	

✔	`import numpy as np`
	`R = np.array([[-1, 0, 3], [0, 4, 4], [0, 0, -2]])`
	`b = np.array([11, -0.5, -1])`
	`x = back_solve(R, b)`
	`print(np.allclose(R @ x, b))`
		True	True	✔

✔	`import numpy as np`
	`R = np.random.random((16,16))`
	`b = np.random.random(16)`
	`for i in range(16):`
	`  for j in range(i):`
	`  R[i, j] = 0`
	`x = back_solve(R, b)`
	`print(np.allclose(R @ x, b))`
		True	True	✔

Passed all tests! ✔

Correct

Marks for this submission: 12.00/12.00.

Question **15**

Correct

Mark 1.00 out of 1.00

Suppose the n × n matrices A and B are both invertible.

For each of the matrices given below - it is true that it must be invertible, without any further assumptions about A and B?

This would be a good one to discuss in Piazza.

\(\begin{bmatrix} A & A+B \\ 0 & B \end{bmatrix} \)	True ✔
A + B	False ✔
\(\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \)	True ✔
\(A B A \)	True ✔

Your answer is correct.

Correct

Marks for this submission: 1.00/1.00.