Propositional Logic Mastery Workbook

REQUIRED
I have neither given nor received unauthorized assistance ________________________.

1. (10pts) Before anything else, **annotate** examples 7 and 8 from page 30. *You **MUST** use this **EXACT** format when showing logical equivalences (and proofs) to get credit for your work in this course.*

Solution: We will use one of the equivalences in Table 6 at a time, starting with $\neg(p \vee (\neg p \wedge q))$ and ending with $\neg p \wedge \neg q$. (*Note:* we could also easily establish this equivalence using a truth table.) We have the following equivalences.

$$\begin{aligned}
\neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{by the second De Morgan law}\\
&\equiv \neg p \wedge [\neg(\neg p) \vee \neg q] && \text{by the first De Morgan law}\\
&\equiv \neg p \wedge (p \vee \neg q) && \text{by the double negation law}\\
&\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) && \text{by the second distributive law}\\
&\equiv \mathbf{F} \vee (\neg p \wedge \neg q) && \text{because } \neg p \wedge p \equiv \mathbf{F}\\
&\equiv (\neg p \wedge \neg q) \vee \mathbf{F} && \text{by the commutative law for disjunction}\\
&\equiv \neg p \wedge \neg q && \text{by the identity law for } \mathbf{F}
\end{aligned}$$

Consequently $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent. ◀

Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

Solution: To show that this statement is a tautology, we will use logical equivalences to demonstrate that it is logically equivalent to $\mathbf{T}$. (*Note:* This could also be done using a truth table.)

$$\begin{aligned}
(p \wedge q) \rightarrow (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) && \text{by Example 3}\\
&\equiv (\neg p \vee \neg q) \vee (p \vee q) && \text{by the first De Morgan law}\\
&\equiv (\neg p \vee p) \vee (\neg q \vee q) && \text{by the associative and commutative laws for disjunction}\\
&\equiv \mathbf{T} \vee \mathbf{T} && \text{by Example 1 and the commutative law for disjunction}\\
&\equiv \mathbf{T} && \text{by the domination law}
\end{aligned}$$

◀

2. (25pts) We will prove the first 5 logical equivalences in Chart 7 page 28

    - USE only RESULTS FROM **Chart 2 and Chart 6** to justify steps.
    - When making logical chains of equivalences use the REQUIRED format.
    - Prove these in this order and **you may use the results as you go**.

- This first one is what Chris calls RBI and it is essential because it converts a conditional into a disjunction. **Prove this with just a truth table.**

$$p \rightarrow q \equiv \neg p \vee q$$

- Carefully repeat the proof of the following using the method without a truth table from the video. Take note of what tricks Chris uses - as we will need these as we go. (Challenge - can you prove it another way?)

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

- Third one (hint: RBI works both ways.)

$$p \lor q \equiv \neg p \rightarrow q$$

- Fourth one, (think about which rule “flips” conjunctions and disjunctions)

$$p \land q \equiv \neg(p \rightarrow \neg q)$$

- Finally, for the fifth one, see if you can do it on your own.

$$\neg(p \rightarrow q) \equiv p \land \neg q$$

3. (10pts) We can use Table 6 p.27 to prove new "rules." Let's prove a Logic version of the algebrica version FOIL. You will need to use a compound substitution with an equivalence from Table 6 twice. (optional/recommended - create a truth table as well).

$$(a \land b) \lor (c \land d) \equiv (a \lor c) \land (b \lor c) \land (a \lor d) \land (b \lor d)$$

4. (10 pts) Rosen, page 53. Do #8 as warm-up on your own. Do #9 below. Add notes and insight for full credit.

5. (10pts) #33 section 1.5, page 67 (add insights and comments for full credit).

6. (35 pts) Now let's go back to section 1.1. Read the examples on page 11 and 12 to see an important computer science application. Do each of #43 and check your answers in the back of the book. Add notes and insights.

Notes - ungraded, add as many pages as you like.

Use these questions or other notes to help you organize your study and prepare for the exam.

1. Which ideas were new to you?
2. What questions might make good exam questions?
3. What ideas will you want to review later?
4. What were some “a-ah” moments, or big ideas you want to remember?