University of Colorado **Boulder**

Department of Computer Science
CSCI 2824: Discrete Structures
Chris Ketelsen

Propositional Logic (Lecture 1)

Conjunctions and Disjunctions

Propositional Logic

- The rules of logic give precise meaning to mathematical statements.

- These rules are then used to distinguish between **valid** and **invalid** mathematical arguments

Propositional Logic

Def: The basic building block of logic is the **proposition.** A proposition is a declarative sentence that is either true or false, but not both

Example: The following are propositions

1. Boulder is a city in Colorado
2. Boulder is the capital of Colorado
3. $2+2=5$
4. $2+2=4$

Def: The **truth value** of a proposition is **true**, denoted T, if it is a true proposition, and the truth value of a proposition is **false**, denoted by F, if it is a false proposition

Propositional Logic

Def: The basic building block of logic is the **proposition.** A proposition is a declarative sentence that is either true or false, but not both

Example: The following are **not** propositions

1. Study hard.
2. When is the homework due?
3. $x + 1 = 8$
4. $x + y = z$

Propositional Logic

Often times we replace propositions with symbols, usually p, q, r, s

Example: Let

- p represent *Boulder is a city in Colorado*
- q represent $2 + 2 = 5$

Then p has truth value T

And q has truth value F

Negation

Once we have simple propositions like p and q we want to modify them and combine them in fancy ways

Def: Let p be a proposition. The **negation** of p, denoted $\neg p$, is the statement "It is not the case that p". The truth value of $\neg p$ is the opposite of the truth value of p

Example: Let p denote the proposition "*Denver is at least 5000 ft above sea level*"

The negation of p is the proposition "*It is not the case that Denver is at least 5000 ft above sea level*"

Negation

When modifying or combining propositions, it's often convenient to tabulate all of the possible truth values that could occur. This is done in a **truth table**.

Given simple propositions like p and q, the truth table allows us to enumerate all possible truth values of combinations of p and q

The truth table for the negation operation is

p	$\neg p$
T	F
F	T

Logical Connectives

Now we'll start combining propositions. Two propositions can be combined using logical operators called **connectives**. Some examples of connectives are

- **conjunction** - "and" - denoted $\wedge$
- **disjunction** - "or" - denoted $\vee$
- **conditional** - "if - then" - denoted $\rightarrow$
- **biconditional** - "if and only if" - denoted $\leftrightarrow$

Logical Connectives

Now we'll start combining propositions. Two propositions can be combined using logical operators called **connectives.** Some examples of connectives are

- **conjunction** - "and" - denoted $\wedge$
- **disjunction** - "or" - denoted $\vee$
- **conditional** - "if - then" - denoted $\rightarrow$
- **biconditional** - "if and only if" - denoted $\leftrightarrow$

- Today: **conjunction** and **disjunction**
- Next Time: **conditionals** and **biconditionals**

Conjunction

Def: Let p and q be propositions. The **conjunction** of p and q, denoted by $p \land q$, is the proposition "p and q". The conjunction $p \land q$ is true when *both* p and q are true and false otherwise

Example:

- "Today for lunch I had a cheeseburger and I had a soda"

Conjunction

Def: Let p and q be propositions. The **conjunction** of p and q, denoted by $p \wedge q$, is the proposition "p and q". The conjunction $p \wedge q$ is true when *both* p and q are true and false otherwise

Example:

- "Today for lunch I had a cheeseburger and I had a soda"
- "Chris is fine but his neighbor Tony has the flu"

Conjunction

To write down the truth table for **conjunction** we need to list all possible combinations of truth values for the propositions being connected. Since we have two variables now, we need more rows in our truth table

p	q	$p \land q$
T	T	
T	F	
F	T	
F	F	

Disjunction

Def: Let p and q be propositions. The **disjunction** of p and q, denoted by $p \vee q$, is the proposition "p or q". The disjunction $p \vee q$ is false when *both* p and q are false and is true otherwise.

Note: This is an *inclusive or*, i.e. it's fine if both p and q are true.

Example:

- "Students who have taken Linear Algebra or Prob-Stats can take Machine Learning"

Disjunction

The truth table for **disjunction** looks as follows

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

Disjunction

In the previous example, the inclusive *or* made perfect sense.

Example: But what about this?

- "The burger comes with either fries or a salad"

Disjunction

In the previous example, the inclusive *or* made perfect sense.

Example: But what about this?

- "The burger comes with either fries or a salad"

If you think this is the same as the previous example, try getting fries and a salad (with no upcharge) next time you're at a restaurant.

This is an example of the *exclusive or*

Disjunction

Example: But what about this?

- "The burger comes with either fries or a salad"

This is an example of the *exclusive or*

Def: Let p and q be propositions. The **exclusive or** of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

Disjunction

The truth table for **exclusive or** looks as follows

p	q	$p \oplus q$
T	T	
T	F	
F	T	
F	F	

Conjunction - Disjunction Summary

p	q	$p \wedge q$	$p \vee q$	$p \oplus q$
T	T	T	T	F
T	F	F	T	T
F	T	F	T	T
F	F	F	F	F

Truth Tables of Compound Propositions

Often times we want to evaluate the possible truth values of more complicated compound propositions

Example: Determine the truth values of $\neg(p \vee q) \wedge q$

Strategy: Determine truth values of constituent propositions first

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg(p \vee q) \wedge q$
T	T			
T	F			
F	T			
F	F			

Application: Logic Puzzles

On the island there are two types of inhabitants: **Knights**, who always tell the truth, and **Knaves**, who always lie.

Example: You encounter two people, who we'll call A and B. A tells you that "B is a Knight" and B tells you that "The two of us are of opposite types"? What type of people are A and B?

We will solve this with a truth table

Let's consider all possible combinations of the types of people A and B could be, and see what that means for the truth value of their statements

Application: Logic Puzzles

Example: You encounter two people, who we'll call A and B. A tells you that "B is a Knight" and B tells you that "The two of us are of opposite types"? What type of people are A and B?

A is Knight	B is Knight	A's statement	B's statement
T	T		
T	F		
F	T		
F	F		

Translate Statements into Symbols:

- Let p be the proposition "A is a Knight"
- Let q be the proposition "B is a Knight"

Application: Logic Puzzles

Example: You encounter two people, who we'll call A and B. A tells you that "B is a Knight" and B tells you that "The two of us are of opposite types"? What type of people are A and B?

p (A is Knight)	q (B is Knight)	A's statement	B's statement
T	T		
T	F		
F	T		
F	F		

Translate Statements into Symbols:

- Then A's statement can by translated as: ____________
- And B's statement can by translated as: ____________

Application: Logic Puzzles

Example: You encounter two people, who we'll call A and B. A tells you that "B is a Knight" and B tells you that "The two of us are of opposite types"? What type of people are A and B?

p (A is Knight)	q (B is Knight)	q (A says)	$p \oplus q$ (B says)
T	T		
T	F		
F	T		
F	F		

Application: Logic Puzzles

p (A is Knight)	q (B is Knight)	q (A says)	$p \oplus q$ (B says)
T	T	T	F
T	F	F	T
F	T	T	T
F	F	F	F

Question: Is there a row (or rows) where A and B's type both make sense with the truth value of their statements?

Application: Logic Puzzles

p (A is Knight)	q (B is Knight)	q (A says)	$p \oplus q$ (B says)
T	T	T	F
T	F	F	T
F	T	T	T
F	F	F	F

Conclusion: A and B are both **Knaves** (corresponding to Row 4)

Application: Logic Puzzles

Example: On the island you encounter two people, A and B. A tells you "I am a Knave or B is a Knight". B says nothing. Use a truth table to determine what type of people A and B are.

Application: Logic Puzzles

Example: On the island you encounter two people, A and B. A tells you "I am a Knave or B is a Knight". B says nothing. Use a truth table to determine what type of people A and B are.

p	q	$\neg p$	$\neg p \vee q$ (A says)
T	T		
T	F		
F	T		
F	F		

Application: Logic Puzzles

Example: On the island you encounter two people, A and B. A tells you "I am a Knave or B is a Knight". B says nothing. Use a truth table to determine what type of people A and B are.

p	q	$\neg p$	$\neg p \vee q$ (A says)
T	T	F	**T**
T	F	F	F
F	T	T	T
F	F	T	T

Only row 1 is consistent with A's nature and the truth value of his statement. So we **conclude** that A and B are **both** Knights.