

# 12.1 Set abstract data type

## Set abstract data type

A **set** is a collection of distinct elements. A set **add** operation adds an element to the set, provided an equal element doesn't already exist in the set. A set is an unordered collection. Ex: The set with integers 3, 7, and 9 is equivalent to the set with integers 9, 3 and 7.

### PARTICIPATION ACTIVITY

#### 12.1.1: Set abstract data type.



### Animation content:

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### Animation captions:

1. Adding 67, 91, and 14 produces a set with 3 elements.
2. Because 91 already exists in the set, adding 91 any number of additional times has no effect.
3. Set 2 is built by adding the same numbers in a different order.
4. Because order does not matter in a set, the 2 sets are equivalent.

### PARTICIPATION ACTIVITY

#### 12.1.2: Set abstract data type.



1) Which of the following is not a valid set?



- ☐ { 78, 32, 46, 57, 82 }
- ☐ { 34, 8, 92 }
- ☐ { 78, 28, 91, 28, 15 }

2) How many elements are in a set that is built by adding the element 28 6 times, then the element 54 9 times?



- ☐ 1
- ☐ 2
- ☐ 15

3) Which 2 sets are equivalent?



- ☐ { 56, 19, 71 } and { 19, 65, 71, 56 }
- ☐ { 88, 54, 81 } and { 81, 88, 54 }
- ☐ { 39, 56, 14, 11 } and { 14, 56, 93, 11 }

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## Element keys and removal

Set elements may be primitive data values, such as numbers or strings, or objects with numerous data members. When storing objects, set implementations commonly distinguish elements based on an element's **key value**: A primitive data value that serves as a unique identifier for the element. Ex: An object for a student at a university may store information such as name, phone number, and ID number. No two students will have the same ID number, so the ID number can be used as the student object's key.

Sets are commonly implemented to use keys for all element types. When storing objects, the set retrieves an object's key via an external function or predetermined knowledge of which object property is the key value. When storing primitive data values, each primitive data value's key is itself.

Given a key, a set **remove** operation removes the element with the specified key from the set.

### PARTICIPATION ACTIVITY

12.1.3: Element keys and removal.



### Animation content:

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### Animation captions:

1. Different students at the same university may have the same name or phone number, but each student has a unique ID number.
2. A set for the course roster uses the student ID as the key value, since the exact same student cannot enroll twice in the same course.
3. The call to remove Student C provides only the student ID.

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12.1.4: Element keys and removal.



Refer to the example in the animation above.

1) If the student objects contained a field for GPA, then GPA could be used as the key value instead of student ID.

- ☐ True  
☐ False

2) `SetRemove(courseRosterSet, "Student D")` would remove Student D from the set.

- ☐ True  
☐ False

3) `SetRemove` will not operate properly on an empty set.

- ☐ True  
☐ False

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## Searching and subsets

Given a key, a set **search** operation returns the set element with the specified key, or null if no such element exists. The search operation can be used to implement a subset test. A set X is a **subset** of set Y only if every element of X is also an element of Y.

### PARTICIPATION ACTIVITY

12.1.5: SetIsSubset algorithm.

### Animation content:

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### Animation captions:

1. To test if set2 is a subset of set1, each element of set 2 is searched for in set1. Elements 19, 22, and 26 are found in set1.
2. Element 34 is in set2 but not set1, so set2 is not a subset of set1.
3. The first element in set3, 88, is not in set1, so set3 is not a subset of set1.
4. All elements of set4 are in set1, so set4 is a subset of set1.
5. No other set is a subset of another.
6. But each set is always a subset of itself.

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### PARTICIPATION ACTIVITY

12.1.6: Searching and subsets.

1) Every set is a subset of itself.

- ☐ True  
☐ False

2) For X to be a subset of Y, the number of elements in Y must be greater than or equal to the number of elements in X.

- ☐ True  
☐ False

3) The loop in SetIsSubset always performs N iterations, where N is the number of elements in subsetCandidate.

- ☐ True  
☐ False

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12.1.1: Set abstract data type.

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Start

Given an empty set numSet, what is numSet after the following operations?

SetAdd(numSet, 89)  
SetAdd(numSet, 43)  
SetAdd(numSet, 89)  
SetAdd(numSet, 89)  
SetAdd(numSet, 43)  
SetAdd(numSet, 50)

{ Ex: 1, 2, 3 }

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1

2

3

Check

Next

# 12.2 Set operations

## Union, intersection, and difference

The **union** of sets  $X$  and  $Y$ , denoted as  $X \cup Y$ , is a set that contains every element from  $X$ , every element from  $Y$ , and no additional elements. Ex:  $\{54, 19, 75\} \cup \{75, 12\} = \{12, 19, 54, 75\}$ .

The **intersection** of sets  $X$  and  $Y$ , denoted as  $X \cap Y$ , is a set that contains every element that is in both  $X$  and  $Y$ , and no additional elements. Ex:  $\{54, 19, 75\} \cap \{75, 12\} = \{75\}$ .

The **difference** of sets  $X$  and  $Y$ , denoted as  $X \setminus Y$ , is a set that contains every element that is in  $X$  but not in  $Y$ , and no additional elements. Ex:  $\{54, 19, 75\} \setminus \{75, 12\} = \{54, 19\}$ .

The union and intersection operations are commutative, so  $X \cup Y = Y \cup X$  and  $X \cap Y = Y \cap X$ . The difference operation is not commutative.

### PARTICIPATION ACTIVITY

12.2.1: Set union, intersection, and difference.



### Animation content:

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### Animation captions:

1. The union operation begins by adding all elements from set1.
2. Each element from set2 is added. Adding elements 82 and 93 has no effect, since 82 and 93 already exist in the result set.
3. The intersection operation iterates through each element in set1. Each element that is also in set2 is added to the result.
4. The difference of set1 and set2, denoted  $\text{set1} \setminus \text{set2}$ , iterates through all elements in set1. Only elements 61 and 76 are added to the result, since these elements are not in set2.
5. Set difference is not commutative.  $\text{SetDifference}(\text{set2}, \text{set1})$  produces a result containing only 23 and 46, since those elements are in set2 but not in set1.

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12.2.2: Union, intersection, and difference.

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- 1) How many elements are in the set  $\{83, 5\} \cup \{9, 77, 83\}$ ?

☐ 2

☐ 4



☐ 5

2) How many elements are in the set  $\{83, 5\} \cap \{9, 77, 83\}$ ?

☐ 1

☐ 2

☐ 3

3)  $\{83, 5\} \setminus \{9, 77, 83\} = ?$

☐  $\{83\}$

☐  $\{5\}$

☐  $\{83, 5\}$

4)  $\{9, 77, 83\} \setminus \{83, 5\} = ?$

☐  $\{9, 77\}$

☐  $\{9, 77, 83\}$

☐  $\{5\}$

5) Which set operation is not commutative?

☐ Union

☐ Intersection

☐ Difference

6) When  $X$  and  $Y$  do not have any elements in common, which is always true?

☐  $X \cup Y = X \cap Y$

☐  $X \cap Y = X \setminus Y$

☐  $X \setminus Y = X$

7) Which is true for any set  $X$ ?

☐  $X \cup X = X \cap X$

☐  $X \cup X = X \setminus X$

☐  $X \setminus X = X \cap X$

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## Filter and map

A **filter** operation on set  $X$  produces a subset containing only elements from  $X$  that satisfy a particular condition. The condition for filtering is commonly represented by a **filter predicate**: A function that takes an element as an argument and returns a Boolean value indicating whether or not that element will be in the filtered subset.

A **map** operation on set  $X$  produces a new set by applying some function  $F$  to each element. Ex: If  $X = \{18, 44, 38, 6\}$  and  $F$  is a function that divides a value by 2, then  $\text{SetMap}(X, F) = \{9, 22, 19, 3\}$ .

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#### 12.2.3: SetFilter and SetMap algorithms.

### Animation content:

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### Animation captions:

1. SetFilter is called with the EvenPredicate function passed as the second argument.
2. SetFilter calls EvenPredicate for each element. EvenPredicate returns true for each even element, and false for each odd element.
3. Every element for which the predicate returned true is added to the result, producing the set of even numbers from set1.
4. SetFilter(set1, Above90Predicate) produces the set with all elements from set1 that are greater than 90.
5. SetMap is called with the OnesDigit function passed as the first argument. Like SetFilter, SetMap calls the function for each element.
6. The returned value from each OnesDigit call is added to the result set, producing the set of distinct ones digit values.
7. SetMap(set1, StringifyElement) produces a set of strings from a set of numbers.

#### PARTICIPATION ACTIVITY

#### 12.2.4: Using SetFilter with a set of strings.



Suppose  $\text{stringSet} = \{\text{"zyBooks"}, \text{"Computer science"}, \text{"Data structures"}, \text{"set"}, \text{"filter"}, \text{"map"}\}$ . Filter predicates are defined below. Match each SetFilter call to the resulting set.

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```

StartsWithCapital(string) {
    if (string starts with capital letter) {
        return true
    }
    else {
        return false
    }
}

Has6OrFewerCharacters(string) {
    if (length of string <= 6) {
        return true
    }
    else {
        return false
    }
}

EndsInS(string) {
    if (string ends in "S" or "s") {
        return true
    }
    else {
        return false
    }
}

```

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If unable to drag and drop, refresh the page.

**SetFilter(stringSet, StartsWithCapital)**

**SetFilter(stringSet, EndsInS)**

**SetFilter(stringSet, Has6OrFewerCharacters)**

{ "zyBooks", "Data structures" }

{ "Computer science", "Data structures"  
}

{ "set", "filter", "map" }

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**Reset**

#### PARTICIPATION ACTIVITY

12.2.5: Using SetMap with a set of numbers.

Suppose numbersSet = { 6.5, 4.2, 7.3, 9.0, 8.7 }. Map functions are defined below. Match each SetMap call to the resulting set.



```
MultiplyBy10(number) {  
  return number * 10.0  
}  
  
Floor(number) {  
  return floor(number)  
}  
  
Round(number) {  
  return round(number)  
}
```

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SetMap(numbersSet, Round)

SetMap(numbersSet, MultiplyBy10)

SetMap(numbersSet, Floor)

{ 65.0, 42.0, 73.0, 90.0, 87.0 }

{ 7.0, 4.0, 9.0 }

{ 6.0, 4.0, 7.0, 9.0, 8.0 }

Reset

**PARTICIPATION  
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12.2.6: SetFilter and SetMap algorithm concepts.

- 1) A filter predicate must return true for elements that are to be added to the resulting set, and false for elements that are not to be added.  
☐ True  
☐ False
- 2) Calling SetFilter on set X always produces a set with the same number of elements as X.  
☐ True  
☐ False

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3) Calling SetMap on set X always produces a set with the same number of elements as X.

- ☐ True  
☐ False

4) Both SetFilter and SetMap will call the function passed as the second argument for every element in the set.

- ☐ True  
☐ False

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12.2.1: Set operations.

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Start

Given:

setA = { 39, 27, 22 }

setB = { 70, 27, 81 }

What is SetUnion(setA, setB)?

{ Ex: 1, 2, 3 }

What is SetIntersection(setA, setB)?

{ }

1

2

3

4

Check

Next

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## 12.3 Static and dynamic set operations

A **dynamic set** is a set that can change after being constructed. A **static set** is a set that doesn't change after being constructed. A collection of elements is commonly provided during construction of a static set, each of which is added to the set. Ex: A static set constructed from the list of integers (19, 67, 77, 67, 59, 19) would be { 19, 67, 77, 59 }.

Static sets support most set operations by returning a new set representing the operation's result. The table below summarizes the common operations for static and dynamic sets.

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Table 12.3.1: Static and dynamic set operations.

Operation	Dynamic set support?	Static set support?
Construction from a collection of values	Yes	Yes
Count number of elements	Yes	Yes
Search	Yes	Yes
Add element	Yes	No
Remove element	Yes	No
Union (returns new set)	Yes	Yes
Intersection (returns new set)	Yes	Yes
Difference (returns new set)	Yes	Yes
Filter (returns new set)	Yes	Yes
Map (returns new set)	Yes	Yes

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12.3.1: Static and dynamic set operations.

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1) Static sets do not support union or intersection, since these operations require changing the set.

☐ True

☐ False

2) A static set constructed from the list of integers (20, 12, 87, 12) would be { 20, 12, 87, 12 }.

- ☐ True  
☐ False

3) Suppose a dynamic set has N elements. Adding any element X and then removing element X will always result in the set still having N elements.

- ☐ True  
☐ False

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12.3.2: Choosing static or dynamic sets for real-world datasets.

For each real-world dataset, select whether a program should use a static or dynamic set.

1) Periodic table of elements

- ☐ Static  
☐ Dynamic

2) Collection of names of all countries on the planet

- ☐ Static  
☐ Dynamic

3) List of contacts for a user

- ☐ Static  
☐ Dynamic

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