**<u>Logic and Reasoning Mastery Workbook</u>**

REQUIRED
**Print your name below to acknowledge,**

I have neither given nor received unauthorized assistance.

Learning Goals to Master:

1. Understand and apply the fundamentals of logic.

2. Use the fundamentals of logic to explore the patterns and fundamental structure of algebra.

3. Explore inductive and deductive reasoning through applications of logic.

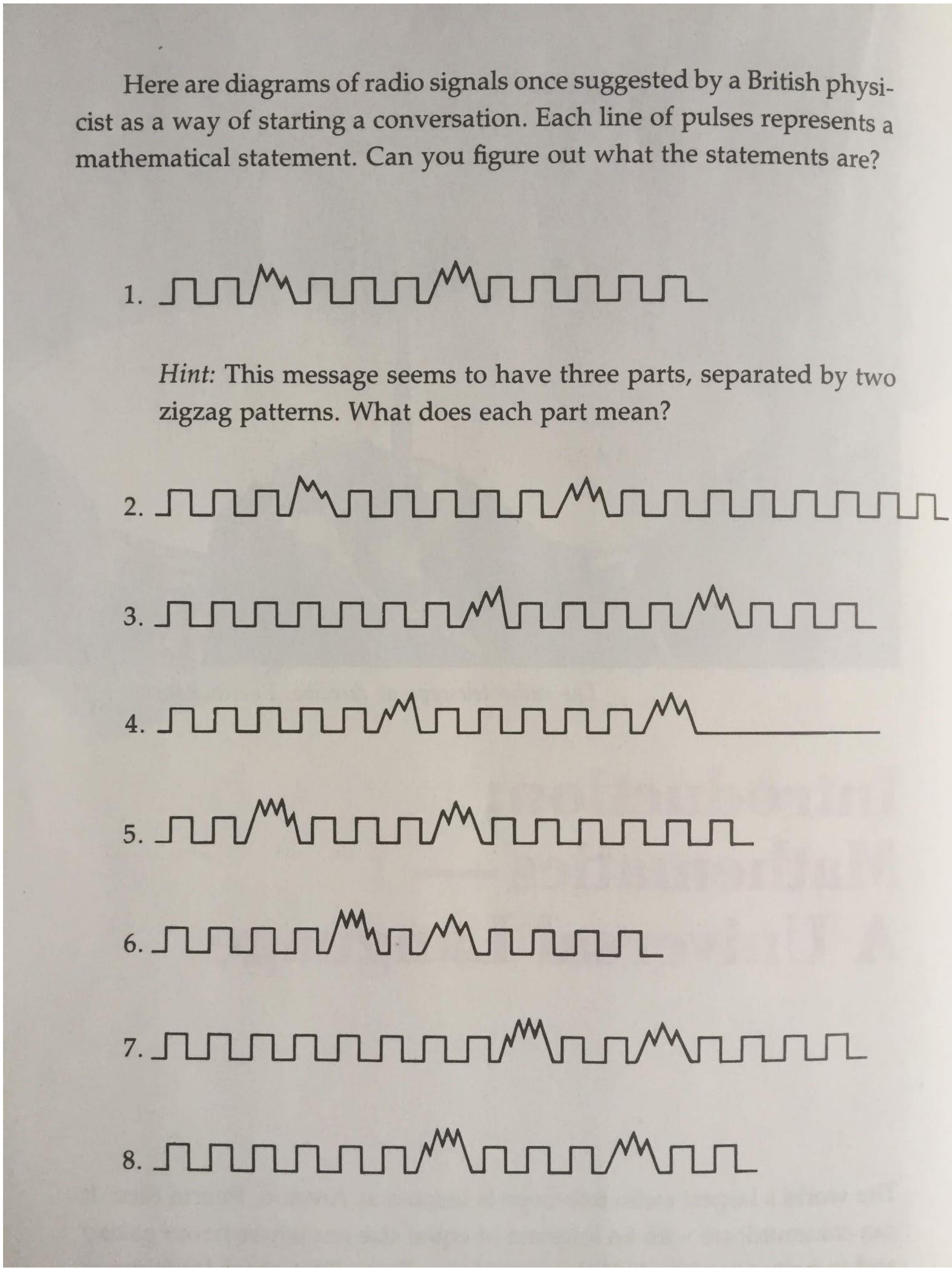
For this first workbook you have puzzles, some book problems, and a proof. The goal of these logic problems is for you to think about and explore your own reasoning process - not for me to see how "smart you are." In fact, you will get more out of the exercises if you don't see the solutions right away. In Piazza, you may discuss how you approached the problems, but do NOT post the answers. Also, be sure to take some time to explore on your own before asking for help. **NOTE: the questions I am asking are "How do you figure this out? How do you think?"**

**Inductive reasoning** - is the method of drawing general conclusions from a limited set of observations. It is reasoning from the particular to the general.
**Deductive reasoning** - is the method of using logic to draw conclusions from statements we accept as true.

Our goal is to become adept at using deductive reasoning by learning logic and proofs. However, we will continue to use inductive reasoning to provide essential intuition, tips, and lucky guesses that will help us find our way to a rigorous proof.

Puzzles - you do not need to use formal logic to solve, but we may discuss how you can in Piazza. Be sure to describe any "A-ha" moments.

1. (ungraded practice)
    - Solve the following
    - Describe, as carefully as you can, your process for solving.
    - Are you using inductive or deductive reasoning?
    - Can you be sure your answer is correct?



Here are diagrams of radio signals once suggested by a British physicist as a way of starting a conversation. Each line of pulses represents a mathematical statement. Can you figure out what the statements are?

1.

*Hint:* This message seems to have three parts, separated by two zigzag patterns. What does each part mean?

2.

3.

4.

5.

6.

7.

8.

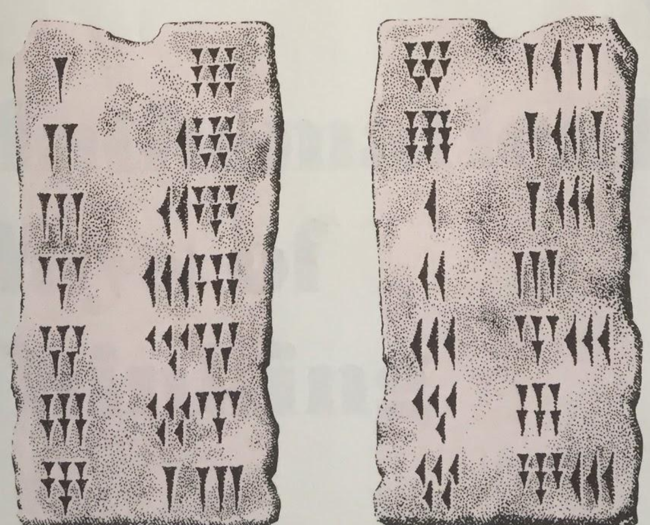
Answer here:

2. (20pts) Translate the tablets below. Describe your process of solving in detail. Are you using inductive or deductive reasoning? Can you be sure your answer is correct?

University Museum, University of Pennsylvania

behind thousands of clay tablets, some of which reveal their number system and their discoveries in algebra and geometry. The photograph at the top of the next page is of a Babylonian tablet of about 1800 B.C. The wedge-shaped writing, called cuneiform, was made by a stylus on wet clay.

A copy of the front and back of another tablet, dug up in the late-nineteenth century, is shown below. Can you translate the groups of wedge-shaped symbols into familiar symbols and explain what the tablet is about? *Hint:* Figure out what all the symbols in the left-hand columns mean before working on the right-hand columns.



#2 Solution here:    answer all parts.

3. (10pts) Imagine you are given 3 matchboxes, labeled “Red and White,” “Red and Red,” and “White and White.” Each box contains 2 marbles which can be either red or white. *The labels on each box are incorrect (although the labels would be correct if you switched them around).* You are allowed to peek inside one box and look at exactly one marble in that box - and see if it is red or white.

    - After this, can you figure out what is in each box and fix the labels to be correct?
    - Does it matter which box you choose to open at the start?
    - Are you using inductive or deductive reasoning?
    - Can you be sure of your answer?

    Explain.

    **Use pictures if you like**.

    You do not need to create a truth table (easier if you don’t)

    Solution should be on the next page.

#3 Solution Here

4. (10pts) At a famous pizza place, Albert, Bob, and Jose are asked by a waiter, “Does everybody want water?” Albert says “I don’t know.”  Bob then says, “I don’t know.” Jose says, “Not everyone wants water.” The waiter returns and brings correctly each person what they wanted.  Which people got water?

*Be sure to explain how you reached your conclusion. **HINTS:** Pay close attention to the specific question the waiter asks.  Assume everyone involved has a firm grasp on propositional logic. And that no one knows what the others want before they speak. Also, you do not need to use truth tables, but you must use a clear deductive argument.*

#4 Solution here:

5. (10pts) Do the the problem from Rosen page 13 #8 - **Include insights and notes for full credit.**

6. (10pts) Rosen page 15 #31,  Try  all of these and, and **just submit d)** on this page. ***Include notes and/or insights for full credit (did you get right the first time? Was it easy or hard?)*** You may check your answers in the back of the book.

Next, on Rosen page 8, buried in a bunch of words are some essential ideas **we will use again and again** this semester. You should know these and be ready.

7. (10pts) Write the definitions of converse, contrapositive, and the inverse in a format that is useful for you to reference - make a poster, index cards, or even make notes in your book. **Include a photo of this in your homework on this page (small is fine).**

**8.** (10 pts) Write up your own conditional example, like “If it is raining, the grass is wet.” but more interesting and memorable. Write the converse, contrapositive, and the inverse of your example.

9. (10 pts) Let's do a simple deductive proof that a negative number times a negative number is a positive number.

You may assume:

- Basic principles of algebra as defined in Appendix A and videos
- FOIL as proved in the reading quiz
- A negative number times a positive number is negative.

I'll prove this first: **(-50) (-2) = 100**

I start with a thing I know is true

| | | |
|---|---|---|
| 0 | = (-50 + 50) (-2 + 2) | *by def. of inverses* |
| | | ***Everyone asks - how do you know to start this way? In a word, practice. Proofs are an art, not a recipe. And you will need a bag of tricks. Using a tricky zero is a common trick.*** |
| ***Now*** | | |
| | = (-50)(-2) + 50(-2) + (-50)2 + 2 (50) | *FOIL proved before in Moodle Quiz* |
| | = (-50)(-2) -100 -100 + 100 | *We are given "neg. X positive is negative."* |
| | =( -50)(-2) - 100 | *Adding* |
| | **0 = (-50) (-2) - 100** | *From our chain of reasoning* |
| | *100 = (-50)(-2)* | *Add 100 to both sides* |

***We have proved a specific example that the product of 2 negative numbers is positive.

Hooray!

Now it's your turn to prove the general rule. On the next page, use the above as a template to prove **in general** that a negative number x a negative number is positive.

Do the proof for #9 here.

- **Prove (-a) (-b) = a b where a, and b are positive non-negative integers.**

10. (10 pts) Compare the proof in #9 to the car example video in Moodle.
    - Which method is inductive reasoning?
    - Which is deductive?
    - Which method is more convincing to you?

Answer here:

Notes - ungraded, add as many pages as you like.

**Hints for #3 and #4**

After attempting #3 on your own or if you are super stuck, consider how we might solve this with truth tables.

First, let's get organized.

On the table below, what represents the mislabeled boxes?
What represents the possible combinations of marbles that could be in each box?

| | Box 1 | Box 2 | Box 3 |
|---|---|---|---|
| Label | WR | WW | RR |
| | WW | WR | WR |
| | RR | RR | WW |

Experiment with seeing a red or white marble in a chosen box, and exam the truth table for possible solutions.

Can you figure out where the correct labels go?

| Box 1 is WW | Box 2 is WR | Box3 is WR |
|---|---|---|
| T | T | T |
| T | T | F |
| T | F | F |
| T | F | T |
| F | F | F |
| F | F | T |
| F | T | F |
| F | T | T |

#4 You can set up a similar chart for number 4.

First consider how customers can answer *the exact question given by the waiter* (yes, no, I don’t know). Consider carefully what each of these answers mean and why they might say them.

Assume they always respond to the question in the same order.

Now consider, what are states or each customer with regards to water? (water/no water) - would you construct  similar table to the one used in #3? (if fact you can use the same table and change the labels)

Notes - ungraded, add as many pages as you like.

**It can be helpful to use these questions or other notes to help you organize your study and prepare for the exam.**

1. Which ideas were new to you?
2. What questions might make good exam questions?
3. What ideas will you want to review later?
4. What were some “a-ah”  moments, or big ideas you want to remember?