18.1 Huffman compression

Basic compression idea

Given data represented as some quantity of bits, ***compression*** transforms the data to use fewer bits. Compressed data uses less storage and can be communicated faster than uncompressed data.

The basic idea of compression is to encode frequently-occurring items using fewer bits. Ex: Uncompressed ASCII characters use 8 bits each, but compression uses fewer than 8 bits for more frequently occurring characters.

PARTICIPATION ACTIVITY 18.1.1: The basic idea of compression is to use fewer bits for frequent items (and more bits for less-frequent items).

Animation content:

undefined

Animation captions:

1. The text "AAA Go" as ASCII would use 6 * 8 = 48 bits. Such data is uncompressed.
2. Compression uses a dictionary of codes specific to the data. Frequent items get shorter codes. Here, A (which is most frequent) is 0, space 10, G 110, and o 111.
3. Thus, "AAA Go" is compressed as 0 0 0 10 110 111. The compressed data uses only eleven bits, much fewer than the 48 bits uncompressed.

PARTICIPATION ACTIVITY 18.1.2: Basic compression.

Given the following dictionary:
00000000: 00
11111111: 01
00000010: 10
00000011: 110
00000100: 111

1) Compress the following: 00000000 00000000 11111111 00000100

Check Show answer

2) Compress the following: 00000011 00000010

Check Show answer

3) Decompress the following: 00 01 00

Check Show answer

4) Is any code in the dictionary a prefix of another code? Type yes or no.

Check Show answer

5) Decompress the following, in which the spaces that were inserted above for reading convenience are absent: 0011000.

Check Show answer

Building a character frequency table

Prior to compression, a character frequency table must be built for an input string. Such a table contains each distinct character from the input string and each character's number of occurrences.

Programming languages commonly provide a dictionary or map object to store the character frequency table.

PARTICIPATION ACTIVITY | 18.1.3: Building a character frequency table.

Animation content:

Static figure: A code block and a character frequency table.
Begin pseudocode:

```
BuildCharacterFrequencyTable(inputString) {
  table = new Dictionary()
  for (i = 0; i < inputString⇢length; i++) {
    currentCharacter = inputString[i]
    if (table has key for currentCharacter) {
      table[currentCharacter] = table[currentCharacter] + 1
    }
    else {
      table[currentCharacter] = 1
    }
  }
  return table
}

BuildCharacterFrequencyTable("APPLES AND BANANAS")
```

End pseudocode.

Step 1: A new dictionary is created for the character frequency table and iteration through the input string's characters begins.
The code BuildCharacterFrequencyTable("APPLES AND BANANAS") is highlighted. Then, the code BuildCharacterFrequencyTable(inputString) { is highlighted, followed by the code table = new Dictionary(). The text table: (empty) appears below the code block. The code for (i = 0; i < inputString⇢length; i++) { is highlighted. The text i: 0 appears below the code block. The code currentCharacter = inputString[i] is highlighted. The text currentCharacter: A appears under the text i: 0.

Step 2: The current character, A, is not in the dictionary. So A is added to the dictionary with a frequency of 1.
The code if (table has key for currentCharacter) { is highlighted. The text not in table appears next to the text currentCharacter: A. The code table[currentCharacter] = 1 is highlighted. The text (empty) disappears after the word table under the code block. A new entry, A 1, appears in the table.

Step 3: The next character, P, is also not in the dictionary and is added with a frequency of 1.
The code for (i = 0; i < inputString⇢length; i++) { is highlighted, followed by the code currentCharacter = inputString[i]. The letter A is replaced with a P after the text currentCharacter. The code if (table has key for currentCharacter) { is highlighted. The text not in table appears next to the text currentCharacter: P. The code table[currentCharacter] = 1 is highlighted. A new entry, P 1, appears in the table.

Step 4: The next character, P, is already in the table, so the existing frequency is incremented from 1 to 2.
The code currentCharacter = inputString[i] is highlighted, and the P after the text currentCharacter is highlighted. The code if (table has key for currentCharacter) { is highlighted. The text in table appears next to the text currentCharacter: P. The code table[currentCharacter] = table[currentCharacter] + 1 is highlighted. The P entry in the table is updated from 1 to 2.

Step 5: For remaining characters, first occurrences set the frequency to 1 and subsequent occurrences increment the existing frequency.
The entire for loop is highlighted. The frequency table is filled in: A 5, P 2, L 1, E 1, S 2, (space) 2, N 3, D 1, B 1.

Animation captions:

1. A new dictionary is created for the character frequency table and iteration through the input string's characters begins.
2. The current character, A, is not in the dictionary. So A is added to the dictionary with a frequency of 1.
3. The next character, P, is also not in the dictionary and is added with a frequency of 1.
4. The next character, P, is already in the table, so the existing frequency is incremented from 1 to 2.
5. For remaining characters, first occurrences set the frequency to 1 and subsequent occurrences increment the existing frequency.

PARTICIPATION ACTIVITY 18.1.4: Character frequency counts.

Given the text "seems he fleed", indicate the frequency counts.

1) s

- 1
- 2
- 3

2) e

- 2
- 5
- 6

3) Each m, h, f, l, and d

- 1
- 2

○ 3

4) (space)

○ 1

○ 2

○ 3

Huffman coding

Huffman coding is a common compression technique that assigns fewer bits to frequent items, using a binary tree.

PARTICIPATION ACTIVITY 18.1.5: A binary tree can be used to determine the Huffman coding.

Animation captions:

1. Huffman coding first determines the frequencies of each item. Here, a occurs 4 times, b 3, c 2, and d 1. (Total is 10).
2. Each item is a "leaf node" in a tree. The pair of nodes yielding the lowest sum is found, and merged into a new node formed with that sum. Here, c and d yield 2 + 1 = 3.
3. The merging continues. The lowest sum is b's 3 plus the new node's 3, yielding 6. (Note that c and d are no longer eligible nodes). The merging ends when only 1 node exists.
4. Each leaf node's encoding is obtained by traversing from the top node to the leaf. Each left branch appends a 0, and each right branch appends a 1, to the code.

PARTICIPATION ACTIVITY 18.1.6: Huffman coding example: Merging nodes.

A 100-character text has these character frequencies:
A: 50
C: 40
B: 4
D: 3
E: 3

1) What is the first merge?

○ D and E: 6

○ B and D: 7

○ B and D and E: 10

2) What is the second merge?

- B and D: 7
- DE and B: 10
- C and A: 90

3) What is the third merge?

- DEB and C: 40
- DEB and C: 50

4) What is the fourth merge?

- None
- DEBC and A: 100

5) What is the fifth merge?

- None
- DEBCA and F

6) What is the code for A?

- 0
- 1

7) What is the code for C?

- 1
- 01
- 10

8) What is the code for B?

- 001
- 110

9) What is the code for D?

- 1110
- 1111

10) What is the code for E?

- 1110
- 1111

11) 5 unique characters (A, B, C, D, E) can each be uniquely encoded in 3 bits (like 000, 001, 010, 011, and 100). With such a fixed-length code, how many bits are needed for the 100-character text?

- 100
- 300

12) For the Huffman code determined in the above questions, the number of bits per character is A: 1, C: 2, B: 3, D: 4, and E: 4. Recalling the frequencies in the instructions, how many bits are needed for the 100-character text?

- 14
- 166
- 300

Note: For Huffman encoded data, the dictionary must be included along with the compressed data, to enable decompression. That dictionary adds to the total bits used. However, typically only large data files get compressed, so the dictionary is usually a tiny fraction of the total size.

Building a Huffman tree

The data members in a Huffman tree node depend on the node type.

- Leaf nodes have two data members: a character from the input and an integer frequency for that character.
- Internal nodes have left and right child nodes, along with an integer frequency value that represents the sum of the left and right child frequencies.

A Huffman tree can be built from a character frequency table.

PARTICIPATION ACTIVITY 18.1.7: Building a Huffman tree.

Animation content:

undefined

Animation captions:

1. The character frequency table is built for the input string, BANANAS.
2. A leaf node is built for each table entry and enqueued in a priority queue. Lower frequencies have higher priority.
3. Leaf nodes for S and B are removed from the queue. A parent is built with the sum of the frequencies and is enqueued into the priority queue.
4. The two nodes with frequency 2 are dequeued and the parent with frequency 2 + 2 = 4 is built.
5. The remaining 2 nodes are dequeued and given a parent.
6. When the priority queue has 1 node remaining, that node is the tree's root. The root is dequeued and returned.

PARTICIPATION ACTIVITY 18.1.8: HuffmanBuildTree function.

Assume `HuffmanBuildTree("zyBooks")` is called.

1) The character frequency table has _____ entries.

- 5
- 6
- 7

2) The leaf node at the back of the priority queue, `nodes`, before the while loop begins is _____.

- z | 1
- B | 1
- o | 2

3) The parent node for nodes `B` and `k` has a frequency of _____.

- 1
- 2
- 4

Implementing with 1 node structure

Implementations commonly use the same node structure for leaf and internal nodes, instead of two distinct structures. Each node has a frequency, character, and 2 child *pointers. The child pointers are set to null for leaves and the character is set to 0 for internal nodes.*

Getting Huffman codes

Huffman codes for each character are built from a Huffman tree. Each character corresponds to a leaf node. The Huffman code for a character is built by tracing a path from the root to that character's leaf node, appending 0 when branching left or 1 when branching right.

PARTICIPATION ACTIVITY | 18.1.9: Getting Huffman codes.

Animation content:

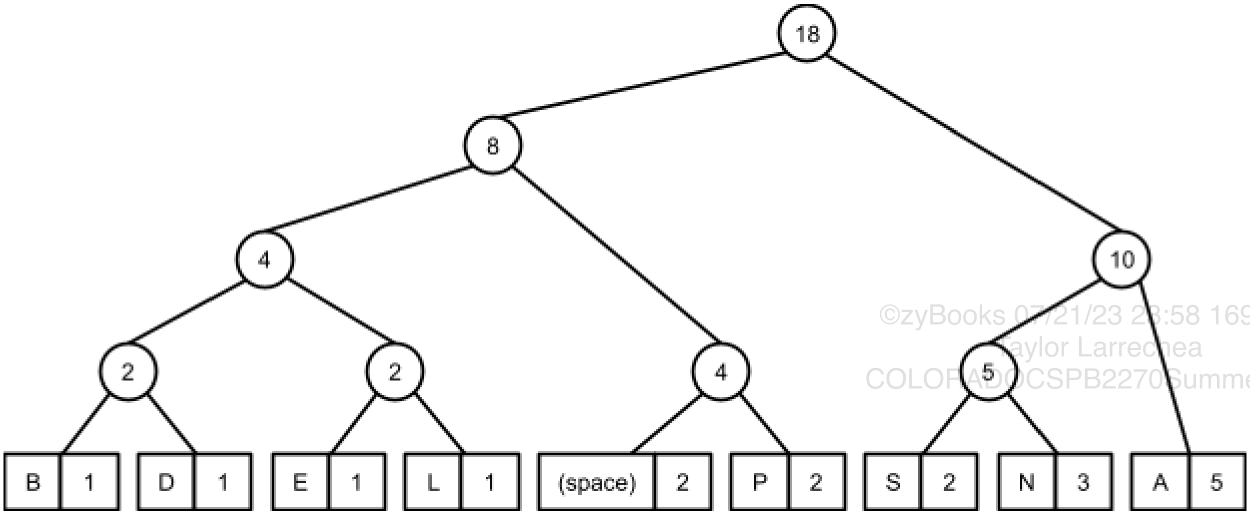
undefined

Animation captions:

1. Huffman codes are built from a Huffman tree. Left branches add a 0 to the code, right branches add a 1.
2. HuffmanGetCodes starts at the root node with an empty prefix.
3. A recursive call is made on the root's left child. The node is a leaf and A's code is set to the current prefix, 0.
4. The first recursive call completes. The next recursive call is made for node 4 and a prefix of "1".
5. Node 4 and node 2 are not leaves, so additional recursive calls are made.
6. B's code is set to 100.
7. The remaining recursive calls set codes for S and N.
8. Each distinct character has a code. Characters B and S have lower frequencies than A and N and thus have longer codes.

PARTICIPATION ACTIVITY | 18.1.10: Huffman codes.

Below is the Huffman tree for "APPLES AND BANANAS"



1) What is the Huffman code for A?

- 10
- 11
- 101

2) What is the Huffman code for P?

- 01
- 0000
- 011

3) What is the length of the longest Huffman code?

- 3
- 4
- 5

Compressing data

To compress an input string, the Huffman codes are first obtained for each character. Then each character of the input string is processed, and corresponding bit codes are concatenated to produce the compressed result.

Figure 18.1.1: HuffmanCompress function.

```
HuffmanCompress(inputString) {
   // Build the Huffman tree
   root = HuffmanBuildTree(inputString)

   // Get the compression codes from the tree
   codes = HuffmanGetCodes(root, "", new
Dictionary())

   // Build the compressed result
   result = ""
   for c in inputString {
      result += codes[c]
   }
   return result
}
```


PARTICIPATION ACTIVITY | 18.1.11: Compressing data.

Match each compression call to the result. Spaces are added between character codes for clarity, but would not exist in the actual compressed data.

If unable to drag and drop, refresh the page.

HuffmanCompress("BANANAS") HuffmanCompress("aabbbac")

HuffmanCompress("zyBooks")

	100 0 11 0 11 0 101
	00 101 010 11 11 011 100
	11 11 0 0 0 11 10

Reset

Decompressing Huffman coded data

To decompress Huffman code data, one can use a Huffman tree and trace the branches for each bit, starting at the root. When the final node of the branch is reached, the result has been found. The process continues until the entire item is decompressed.

PARTICIPATION ACTIVITY | 18.1.12: Decompressing Huffman code.

Animation captions:

1. The Huffman code is decompressed by first starting at the root. The branches are followed for each bit.
2. When the final node of the branch is reached, the result has been found.
3. Once the final node is reached decoding restarts at the root node.
4. The process continues until the entire item is decompressed.

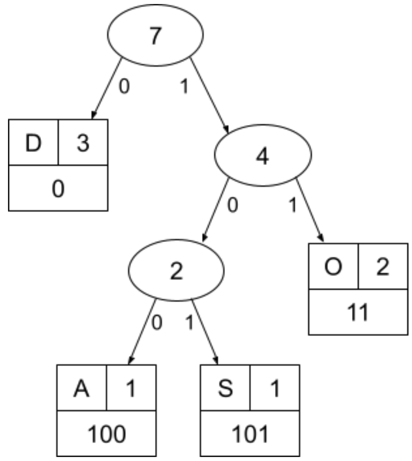
Figure 18.1.2: HuffmanDecompress function.

```
HuffmanDecompress(compressedString, treeRoot) {
   node = treeRoot
   result = ""
   for (bit in compressedString) {
      // Go to left or right child based on bit value
      if (bit == 0)
         node = node⇢left
      else
         node = node⇢right

      // If the node is a leaf, add the character to
the
      // decompressed result and go back to the root
node
      if (node is a leaf) {
         result += node⇢character
         node = treeRoot
      }
   }
   return result
}
```

PARTICIPATION ACTIVITY | 18.1.13: Decompressing Huffman code.

Use the tree below to decompress 0111101000101.



1) What is the first decoded character?

Check Show answer

2) What is the second decoded character?

Check Show answer

3) 11 yields the third character O.
0 yields the fourth character D.

What is the next decoded character?

Check Show answer

4) What is the decoded text?

Check Show answer

CHALLENGE ACTIVITY 18.1.1: Huffman compression.

489394.3384924.qx3zqy7

Start

Complete the character frequency table for the string "BEES AND TREES".

Character	Frequency
(space)	2
A	Ex: 4
B	
D	
E	
N	
R	
S	
T	

1 2 3

Check Next

18.2 Heuristics

This section has been set as optional by your instructor.

Heuristics

In practice, solving a problem in the optimal or most accurate way may require more computational resources than are available or feasible. Algorithms implemented for such problems often use a ***heuristic***: A technique that willingly accepts a non-optimal or less accurate solution in order to improve execution speed.

PARTICIPATION ACTIVITY | 18.2.1: Introduction to the knapsack problem.

Animation content:

undefined

Animation captions:

1. A knapsack is a container for items, much like a backpack or bag. Suppose a particular knapsack can carry at most 30 pounds worth of items.
2. Each item has a weight and value. The goal is to put items in the knapsack such that the weight <= 30 pounds and the value is maximized.
3. Taking a 20 pound item with an 8 pound item is an option, worth $142.
4. If more than 1 of each item can be taken, 2 of item 1 and 1 of item 4 provide a better option, worth $145.
5. Trying all combinations will give an optimal answer, but is time consuming. A heuristic algorithm may choose a simpler, but non-optimal approach.

PARTICIPATION ACTIVITY | 18.2.2: Heuristics.

1) A heuristic is a way of producing an optimal solution to a problem.

- True
- False

2) A heuristic technique used for numerical computation may sacrifice accuracy to gain speed.

- True
- False

PARTICIPATION ACTIVITY | 18.2.3: The knapsack problem.

Refer to the example in the animation above.

1) Which of the following options provides the best value?

- 5 6-pound items

- 2 6-pound items and 1 18-pound item
- 3 8-pound items and 1 6-pound item.

2) The optimal solution has a value of $162 and has one of each item: 6-pound, 8-pound, and 18-pound.
   - True
   - False

3) Which approach would guarantee finding an optimal solution?
   - Taking the largest item that fits in the knapsack repeatedly until no more items will fit.
   - Taking the smallest item repeatedly until no more items will fit in the knapsack.
   - Trying all combinations of items and picking the one with maximum value.

Heuristic optimization

A ***heuristic algorithm*** is an algorithm that quickly determines a near optimal or approximate solution. Such an algorithm can be designed to solve the ***0-1 knapsack problem***: The knapsack problem with the quantity of each item limited to 1.

A heuristic algorithm to solve the 0-1 knapsack problem can choose to always take the most valuable item that fits in the knapsack's remaining space. Such an algorithm uses the heuristic of choosing the highest value item, without considering the impact on the remaining choices. While the algorithm's simple choices are aimed at optimizing the total value, the final result may not be optimal.

PARTICIPATION ACTIVITY 18.2.4: Non-optimal, heuristic algorithm to solve the 0-1 knapsack.

Animation content:

undefined

Animation captions:

1. The item list is sorted and the most valuable item is put into the knapsack first.
2. No remaining items will fit in the knapsack.
3. The resulting value of $95 is inferior to taking the 12 and 8 pound items, collectively worth $102.
4. The heuristic algorithm sacrifices optimality for efficiency and simplicity.

PARTICIPATION ACTIVITY | 18.2.5: Heuristic algorithm and the 0-1 knapsack problem.

1) Which is not commonly sacrificed by a heuristic algorithm?

- speed
- optimality
- accuracy

2) What restriction does the 0-1 knapsack problem have, in comparison with the regular knapsack problem?

- The knapsack's weight limit cannot be exceeded.
- At most 1 of each item can be taken.
- The value of each item must be less than the item's weight.

3) Under what circumstance would the Knapsack01 function not put the most valuable item into the knapsack?

- The item list contains only 1 item.
- The weight of the most valuable item is greater than the knapsack's limit.

Self-adjusting heuristic

A ***self-adjusting heuristic*** is an algorithm that modifies a data structure based on how that data structure is used. Ex: Many self-adjusting data structures, such as red-black trees and AVL trees, use a self-adjusting heuristic to keep the tree balanced. Tree balancing organizes data to allow for faster access.

Ex: A self-adjusting heuristic can be used to speed up searches for frequently-searched-for list items by moving a list item to the front of the list when that item is searched for. This heuristic is self-adjusting because the list items are adjusted when a search is performed.

PARTICIPATION ACTIVITY | 18.2.6: Move-to-front self-adjusting heuristic.

Animation content:

undefined

Animation captions:

1. 42 is at the end of a list with 8 items. A linear search for 42 compares against 8 items.
2. The move-to-front heuristic moves 42 to the front after the search.
3. Another search for 42 now only requires 1 comparison. 42 is left at the front of the list.
4. A search for 64 compares against 8 items and moves 64 to the front of the list.
5. 42 is no longer at the list's front, but a search for 42 need only compare against 2 items.

PARTICIPATION ACTIVITY | 18.2.7: Move-to-front self-adjusting heuristic.

Suppose a move-to-front heuristic is used on a list that starts as (56, 11, 92, 81, 68, 44).

1) A first search for 81 compares against how many list items?

- 0
- 3
- 4

2) A subsequent search for 81 compares against how many list items?

- 1
- 2
- 4

3) Which scenario results in faster searches?

- Back-to-back searches for the same key.
- Every search is for a key different than the previous search.

18.3 Greedy algorithms

This section has been set as optional by your instructor.

Greedy algorithm

A ***greedy algorithm*** is an algorithm that, when presented with a list of options, chooses the option that is optimal at that point in time. The choice of option does not consider additional subsequent options, and may or may not lead to an optimal solution.

PARTICIPATION ACTIVITY 18.3.1: MakeChange greedy algorithm.

Animation content:

undefined

Animation captions:

1. The change making algorithm uses quarters, dimes, nickels, and pennies to make change equaling the specified amount.
2. The algorithm chooses quarters as the optimal coins, as long as the remaining amount is >= 25.
3. Dimes offer the next largest amount per coin, and are chosen while the amount is >= 10.
4. Nickels are chosen next. The algorithm is greedy because the largest coin <= the amount is always chosen.
5. Adding one penny makes 91 cents.
6. This greedy algorithm is optimal and minimizes the total number of coins, although not all greedy algorithms are optimal.

PARTICIPATION ACTIVITY | 18.3.2: Greedy algorithms.

1) If the MakeChange function were to make change for 101, what would be the result?

- 101 pennies
- 4 quarters and 1 penny
- 3 quarters, 2 dimes, 1 nickel, and 1 penny

2) A greedy algorithm is attempting to minimize costs and has a choice between two items with equivalent functionality: the first costing $5 and the second costing $7. Which will be chosen?

- The $5 item
- The $7 item
- The algorithm needs more information to choose

3) A greedy algorithm always finds an optimal solution.

- True
- False

Fractional knapsack problem

The ***fractional knapsack problem*** is the knapsack problem with the potential to take each item a fractional number of times, provided the fraction is in the range [0.0, 1.0]. Ex: A 4 pound, $10 item could be taken 0.5 times to fill a knapsack with a 2 pound weight limit. The resulting knapsack would be worth $5.

While a greedy solution to the 0-1 knapsack problem is not necessarily optimal, a greedy solution to the fractional knapsack problem is optimal. First, items are sorted in descending order based on the value-to-weight ratio. Next, one of each item is taken from the item list, in order, until taking 1 of the next item would exceed the weight limit. Then a fraction of the next item in the list is taken to fill the remaining weight.

Figure 18.3.1: FractionalKnapsack algorithm.

```
FractionalKnapsack(knapsack, itemList, itemListSize) {
   Sort itemList descending by item's (value / weight)
ratio
   remaining = knapsack⇢maximumWeight
   for each item in itemList {
      if (item⇢weight <= remaining) {
         Put item in knapsack
         remaining = remaining - item⇢weight
      }
      else if (remaining > 0) {
         fraction = remaining / item⇢weight
         Put (fraction * item) in knapsack
         break
      }
   }
}
```


PARTICIPATION ACTIVITY 18.3.3: Fractional knapsack problem.

Suppose the following items are available: 40 pounds worth $80, 12 pounds worth $18, and 8 pounds worth $8.

1) Which item has the highest value-to-weight ratio?
   - 40 pounds worth $80
   - 12 pounds worth $18
   - 8 pounds worth $8

2) What would FractionalKnapsack put in a 20-pound knapsack?
   - One 12-pound item and one 8-pound item
   - One 40-pound item
   - Half of a 40-pound item

3) What would FractionalKnapsack put in a 48-pound knapsack?
   - One 40-pound item and one 8-pound item
   - One 40-pound item and 2/3 of a 12-pound item

Activity selection problem

The ***activity selection problem*** is a problem where 1 or more activities are available, each with a start and finish time, and the goal is to build the largest possible set of activities without time conflicts. Ex: When on vacation, various activities such as museum tours or mountain hikes may be available. Since vacation time is limited, the desire is often to engage in the maximum possible number of activities per day.

A greedy algorithm provides the optimal solution to the activity selection problem. First, an empty set of chosen activities is allocated. Activities are then sorted in ascending order by finish time. The first activity in the sorted list is marked as the current activity and added to the set of chosen activities. The algorithm then iterates through all activities after the first, looking for a next activity that starts after the current activity ends. When such a next activity is found, the next activity is added to the set of chosen activities, and the next activity is reassigned as the current. After iterating through all activities, the chosen set of activities contains the maximum possible number of non-conflicting activities from the activities list.

PARTICIPATION ACTIVITY 18.3.4: Activity selection problem algorithm.

Animation content:

undefined

Animation captions:

1. Activities are first sorted in ascending order by finish time. The set of chosen activities initially has the activity that finishes first.
2. The morning mountain hike does not start after the history museum tour finishes and is not added to the chosen set of activities.
3. The boat tour is the first activity to start after the history museum tour finishes, and is the "greedy" choice.
4. Hang gliding and the fireworks show are chosen as 2 additional activities.
5. The maximum possible number of non-conflicting activities is 4, and 4 have been chosen.

PARTICIPATION ACTIVITY 18.3.5: ActivitySelection algorithm.

1) The fireworks show and the night movie both finish at 9 PM, so the sorting algorithm could have swapped the order of the 2. If the 2 were swapped, the number of chosen activities would not be affected.

- True
- False

2) Changing snorkeling's _____ would cause snorkeling to be added to the chosen activities.

- start time from 3 PM to 4 PM
- finish time from 5 PM to 4 PM

3) Regardless of any changes to the activity list, the activity with the _____ will always be in the result.

- earliest start time
- earliest finish time
- longest length

18.4 Dynamic programming

This section has been set as optional by your instructor.

Dynamic programming overview

Dynamic programming is a problem solving technique that splits a problem into smaller subproblems, computes and stores solutions to subproblems in memory, and then uses the stored solutions to solve the larger problem. Ex: Fibonacci numbers can be computed with an iterative approach that stores the 2 previous terms, instead of making recursive calls that recompute the same term many times over.

PARTICIPATION ACTIVITY | 18.4.1: FibonacciNumber algorithm: Recursion vs. dynamic programming.

Animation content:

undefined

Animation captions:

1. The recursive call hierarchy of FibonacciNumber(4) shows each call made to FibonacciNumber.
2. Several terms are computed more than once.
3. The iterative implementation uses dynamic programming and stores the previous 2 terms at a time.
4. For each iteration, the next term is computed by adding the previous 2 terms. The previous and current terms are also updated for the next iteration.
5. 3 loop iterations are needed to compute FibonacciNumber(4). Because the previous 2 terms are stored, no term is computed more than once.

PARTICIPATION ACTIVITY 18.4.2: FibonacciNumber implementation.

1) If the recursive version of FibonacciNumber(3) is called, how many times will FibonacciNumber(2) be called?

- ○ 1
- ○ 2
- ○ 3

2) Which version of FibonacciNumber is faster for large term indices?

- ○ Recursive version
- ○ Iterative version
- ○ Neither

3) Which version of FibonacciNumber is more accurate for large term indices?

- ○ Recursive version
- ○ Iterative version
- ○ Neither

4) The recursive version of FibonacciNumber has a runtime complexity of O( ). What is the runtime complexity of the iterative version?

- ○ O( )
- ○ O( )

O(   )

PARTICIPATION ACTIVITY | 18.4.3: Dynamic programming.

1) Dynamic programming avoids recomputing previously computed results by storing and reusing such results.

- True
- False

2) Any algorithm that splits a problem into smaller subproblems is using dynamic programming.

- True
- False

Longest common substring

The ***longest common substring*** algorithm takes 2 strings as input and determines the longest substring that exists in both strings. The algorithm uses dynamic programming. An N x M integer matrix keeps track of matching substrings, where N is the length of the first string and M the length of the second. Each row represents a character from the first string, and each column represents a character from the second string.

An entry at i, j in the matrix indicates the length of the longest common substring that ends at character i in the first string and character j in the second. An entry will be 0 only if the 2 characters the entry corresponds to are not the same.

The matrix is built one row at a time, from the top row to the bottom row. Each row's entries are filled from left to right. An entry is set to 0 if the two characters do not match. Otherwise, the entry at i, j is set to 1 plus the entry in i - 1, j - 1.

PARTICIPATION ACTIVITY | 18.4.4: Longest common substring algorithm.

Animation content:

undefined

Animation captions:

1. Comparing "Look" and "zyBooks" requires a 4x7 matrix.
2. In the first row, 0 is entered for each pair of mismatching characters.
3. In the next row, 'o' matches in 2 entries. In both cases the upper-left value is 0, and 1 is entered into the matrix.
4. Two matches for 'o' exist in the next row as well, with the second having a 1 in the upper-left entry.
5. The character 'k' matches once in the last row and an entry of 2 + 1 = 3 is entered.
6. The maximum entry in the matrix is the longest common substring's length. The maximum entry's row index is the substring ending index in the first string.

PARTICIPATION ACTIVITY 18.4.5: Longest common substring matrix.

Consider the matrix below for the two strings "Programming" and "Problem".

	P	r	o	g	r	a	m	m	i	n	g
P	1	0	0	0	0	0	0	0	0	0	0
r	0	2	0	0	?	0	0	0	0	0	0
o	0	0	?	0	0	0	0	0	0	0	0
b	0	0	0	0	0	0	0	0	0	0	0
l	0	0	0	0	0	0	0	0	0	0	0
e	0	0	0	0	0	0	0	0	0	0	0
m	0	0	0	0	0	0	1	?	0	0	0

1) What should be the value in the green cell?

- 0
- 1
- 2

2) What should be the value in the yellow cell?

- 1
- 2
- 3

3) What should be the value in the blue cell?

- 0
- 1

- 2

4) What is the longest common substring?
   - Pr
   - Pro
   - mm

Longest common substring algorithm complexity

The longest common substring algorithm operates on two strings of length N and M. For each of the N characters in the first string, M matrix entries are computed, making the runtime complexity O( ). Since an N x M integer matrix is built, the space complexity is also O( ).

Common substrings in DNA

A real-world application of the longest common substring algorithm is to find common substrings in DNA strings. Ex: Common substrings between 2 different DNA sequences may represent shared traits. DNA strings consist of characters C, T, A, and G.

PARTICIPATION ACTIVITY | 18.4.6: Finding longest common substrings in DNA.

Animation content:

undefined

Animation captions:

1. Finding common substrings in DNA strings can be used for detecting things such as genetic disorders or for tracing evolutionary lineages.
2. DNA strings are very long, often billions of characters. Dynamic programming is crucial to obtaining a reasonable runtime.
3. Optimizations can lower memory usage by keeping only the following in memory: previous row data and largest matrix entry information.

PARTICIPATION ACTIVITY 18.4.7: Common substrings in DNA.

1) Which cannot be a character in a DNA string?

- A
- B
- C

2) If an animal's DNA string is available, a genetic disorder might be found by finding the longest common substring from the DNA of another animal _____ the disorder.

- with
- without

3) When computing row X in the matrix, what information is needed, besides the 2 strings?

- Row X - 1
- Row X + 1
- All rows before X

PARTICIPATION ACTIVITY 18.4.8: Longest common substrings - critical thinking.

1) If the largest entry in the matrix were the only known value, what could be determined?

- The starting index of the longest common substring within either string
- The character contents of the common substring
- The length of the longest common substring

2) Suppose only the row and column indices for the largest entry in the matrix were known, and not the value

of the largest or any other matrix entry. What can be determined in O(1)?

- Only the longest common substring's ending index within either string
- The longest common substring's starting and ending indices within either string

Optimized longest common substring algorithm complexity

The longest common substring algorithm can be implemented such that only the previously computed row and the largest matrix entry's location and value are stored in memory. With this optimization, the space complexity is reduced to O( ). The runtime complexity remains O( ).