University of Colorado **Boulder**

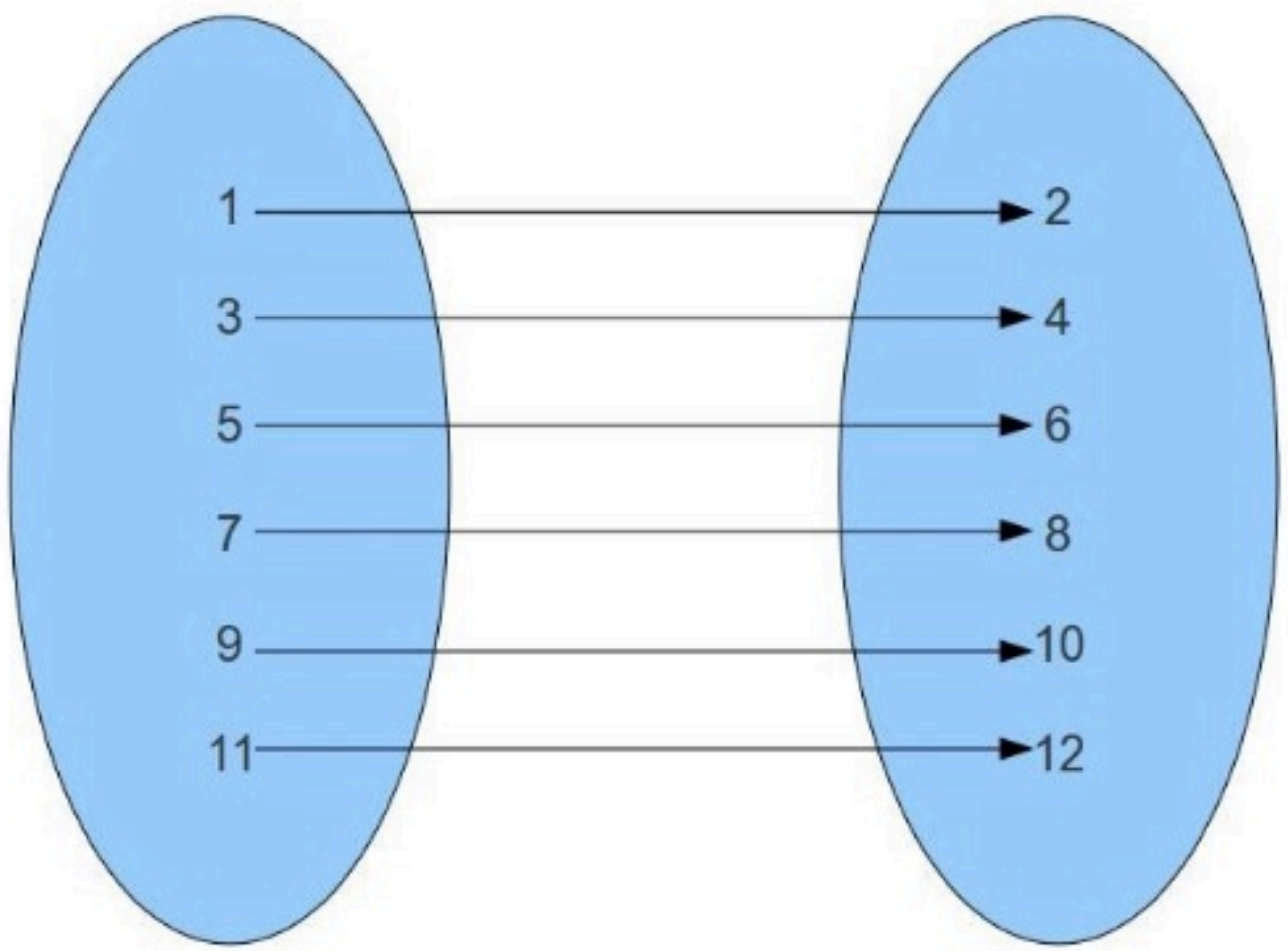
Department of Computer Science
CSCI 2824: Discrete Structures
Chris Ketelsen

Relations
Basic Relations and Their Representations

Basic Relations

Recall: A function f from a set A to a set B maps *every* element of A to some element of B. We write $f : A \to B$

Example: Let $A = \{1, 3, 5, 7, 11\}$ and $B = \{2, 4, 6, 8, 12\}$



Basic Relations

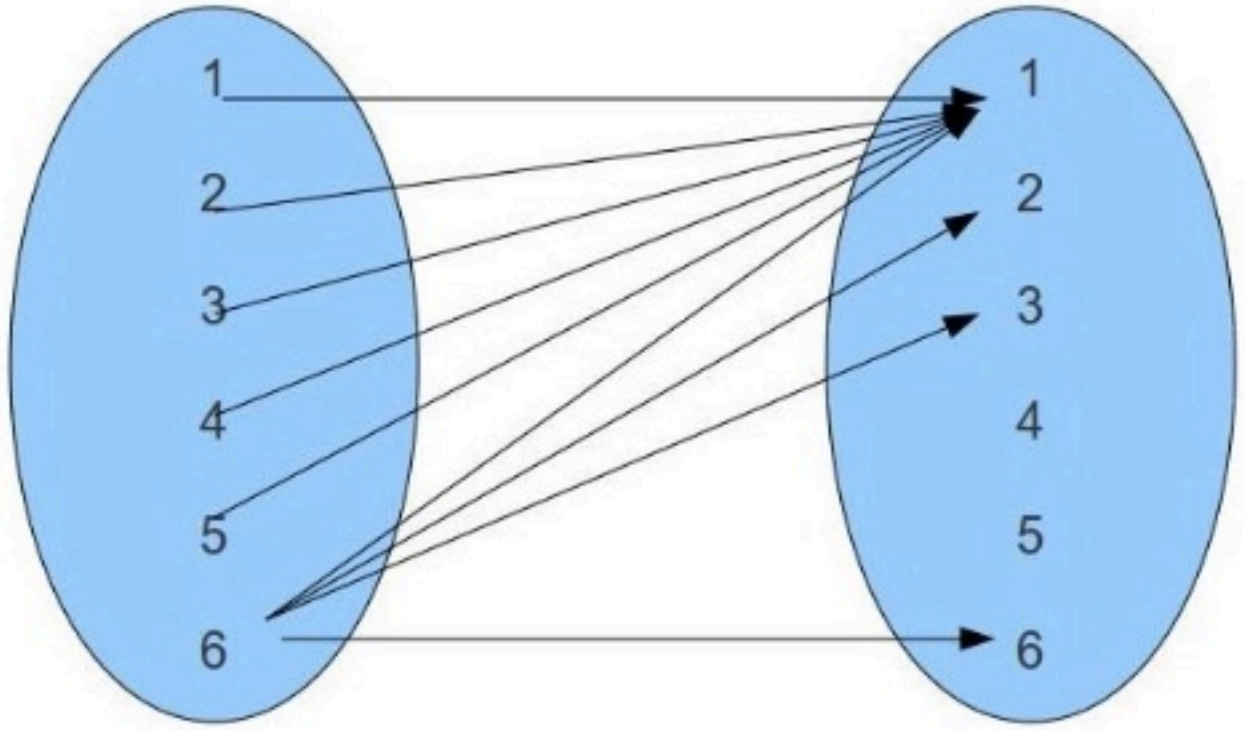
A **relation** is a generalization of a function

Def: A **relation** R is a subset of $A \times B$, i.e. $R \subseteq A \times B$

Example: Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{1, 2, 3, 4, 5, 6\}$

Consider the relation R defined by

$$R = \{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (6, 2), (6, 3), (6, 6)\}$$

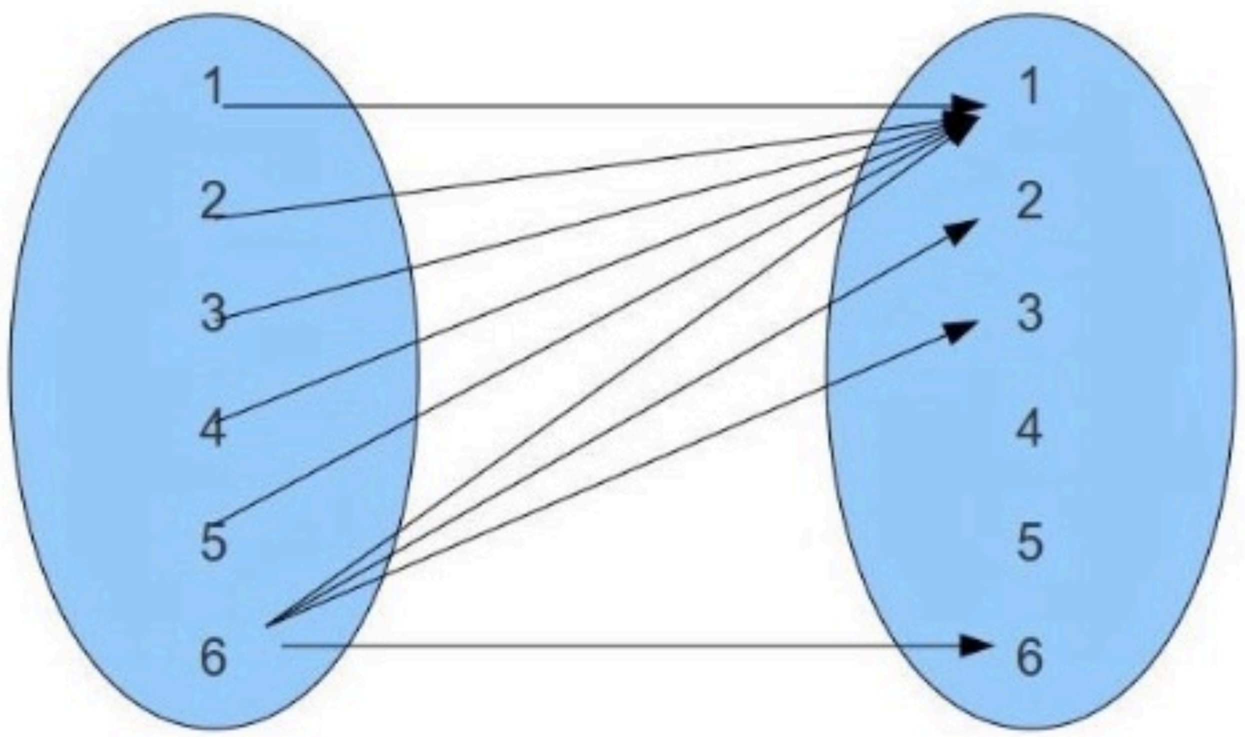


Basic Relations

Like a function, we can view a relation as a mapping from A to B

Unlike a function, in a relation it is possible that

- $x \in A$ is not related to any element in B
- $x \in A$ could be related to multiple elements in B



Rule: All functions are relations, but not all relations are functions

Basic Relations

Example: Let A be the set of students at CU, B be the set of CSCI courses, and R be the relation

$$R = \{(\textit{person}, \textit{class}) \mid \textit{person} \text{ is enrolled in } \textit{class}\}$$

Things that could be in R are

$$(\textit{Tommy}, \textit{Discrete}), (\textit{Bobby}, \textit{Intro CS}),$$
$$(\textit{Susie}, \textit{Data Structures}), (\textit{Linda}, \textit{Data Science}),$$
$$(\textit{Linda}, \textit{Data Viz})$$

Note that

- *Linda* gets mapped to multiple classes
- There's some *Ruth* in A that's not in any CSCI classes

Basic Relations

Example: Let $R \subseteq \mathbb{N} \times \mathbb{N}$ be $R = \{(m, n) \mid m \text{ divides } n\}$

- $(2, 4) \in R$ because 2 divides 4
- $(2, 6) \in R$ because 2 divides 6
- $(6, 2) \notin R$ because 6 doesn't divide 2

Basic Relations

Note that in the previous example the sets A and B are the same

Many useful relations are defined from a set to itself

Examples: Relations of the form $R \subseteq$ People $\times$ People

- Facebook *friends* relation
- Twitter *follower of* and *followed by* relations
- The *is least as tall as* relation

n-Ary Relations

We can have relations on more than two sets as well

Example: Relation involving a student's name, ID number, Major, and GPA

Example: Relation involving an airline, flight number, starting point, destination, departure time and arrival time

Def: Let $A_1, A_2, \ldots, A_n$ be sets. An **n-ary** relation on these sets is a subset of $A_1 \times A_2 \times \cdots \times A_n$. The number n is called the **degree** of the relation.

Example: Let R be the relation on $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ s.t. $(a, b, c) \in R$ if a, b, c are natural numbers satisfying $a < b < c$.

Relations as Graphs

We now restrict ourselves to relations of the form $R \subseteq A \times A$

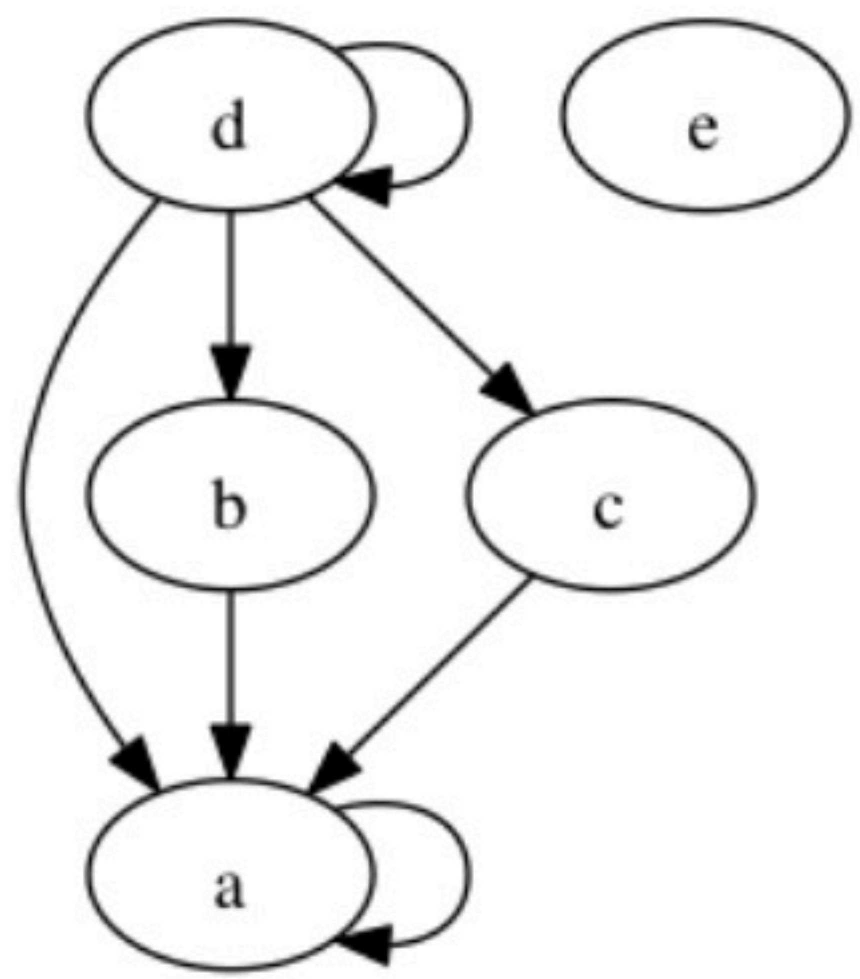
Def: A graph G consists of a set A of vertices and a relation $R \subseteq A \times A$ of edges. Each edge in a graph of the form $a \to b$ corresponds to a pair $(a, b) \in R$

Relations as Graphs

Example: Let $A = \{a, b, c, d, e\}$ be the set of vertices and R_1 be

$$R_1 = \{(a, a), (b, a), (c, a), (d, a), (d, b), (d, c), (d, d)\}$$

The graph representation looks as follows



Relations as Graphs

Example: Let $A = \{1, 2, 3\}$ be the set of vertices and R_2 be

$$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 3), (3, 2), (2, 2), (3, 3)\}$$

The graph representation looks as follows