University of Colorado **Boulder**

Department of Computer Science

CSCI 2824: Discrete Structures

Chris Ketelsen

Algorithms and Complexity

Sequences

Sequences

A sequence is a discrete structure used to represent an ordered list. It's one of the most fundamental structures in computer science.

Examples:

- $0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13, \dots$
- $2,\ 4,\ 8,\ 16,\ 32,\ 64, \dots$
- $3,\ 7,\ 11,\ 15,\ 19,\ 23, \dots$

Def: A **sequence** is a function that maps from a subset of the integers (usually $\{0, 1, 2, \dots\}$ or $\{1, 2, 3, \dots\}$) to a set S.

We use the notation $\{a_n\}$ to represent a sequence where a_n represents the n^{th} individual term in the sequence

Sequences

Example: Consider the sequence $\{a_n\}$ where $a_n = \frac{1}{n}$

In this case the sequence starts with $n = 1$

The first three terms several terms in the sequence are:

Geometric Sequences

Some sequential patterns are so common we have names for them

Example: A **geometric progression** is a sequence of the form

$$a, ar, ar^2, ar^3, \ldots, ar^n, \ldots$$

where the *initial term* a and *common ratio* r are real numbers

Example: What are a and r in the geometric sequence

$$3,\ 6,\ 12,\ 24,\ \ldots$$

Arithmetic Sequences

Example: Another common sequence is the arithmetic progression, which has the form

$$a,\ a+d,\ a+2d,\ a+3d, \ldots, a+nd, \ldots$$

which has initial term a and *common difference* d

Example: Find a and d in the arithmetic sequence

$$5,\ 9,\ 13,\ 17,\ 21,\ 25,\ \ \ldots$$

Sequences from Recurrences

Sometimes we don't give explicit formulas for a_n .

Instead define sequence by a **recurrance** specifying the first few terms, and then giving a formula to find additional terms

Example: Let $a_0 = 1$, $a_1 = 3$, and $a_n = 2a_{n-1} - a_{n-2}$

Sequences

Example: Show that $a_n = 4^n$ is a solution to $a_n = 8a_{n-1} - 16a_{n-2}$

Strategy: Plug in and show that both sides are equal

The Fibonacci Sequence

The most famous sequence ever is the following:

$$0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ 55$$

This is the **Fibonacci Sequence**

It's defined by the recurrence: $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$

Lots of amazing things hidden in Fibonacci Sequence:

- Hidden in Pascal's Triangle
- Ratio of terms approaches **Golden Ratio** $\frac{1+\sqrt{5}}{2}$
- Pattern found all over the place in nature

The Fibonacci Sequence

The **Fibonacci Sequence**

$$0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ 55$$

Notice any interesting patterns?

Parity Pattern: even, odd, odd, even, odd, odd, $\ldots$

Could we find the sequence of just the even Fibonacci numbers?

The Fibonacci Sequence

Example: Determine a recurrence for the even Fibonacci numbers

Strategy: Want a recurrence of the form $F_n = aF_{n-3} + bF_{n-6}$

The Fibonacci Sequence

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