CSPB3202 Artificial Intelligence

Linear Regression

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*Slide contents adopted from ISLR material

Example

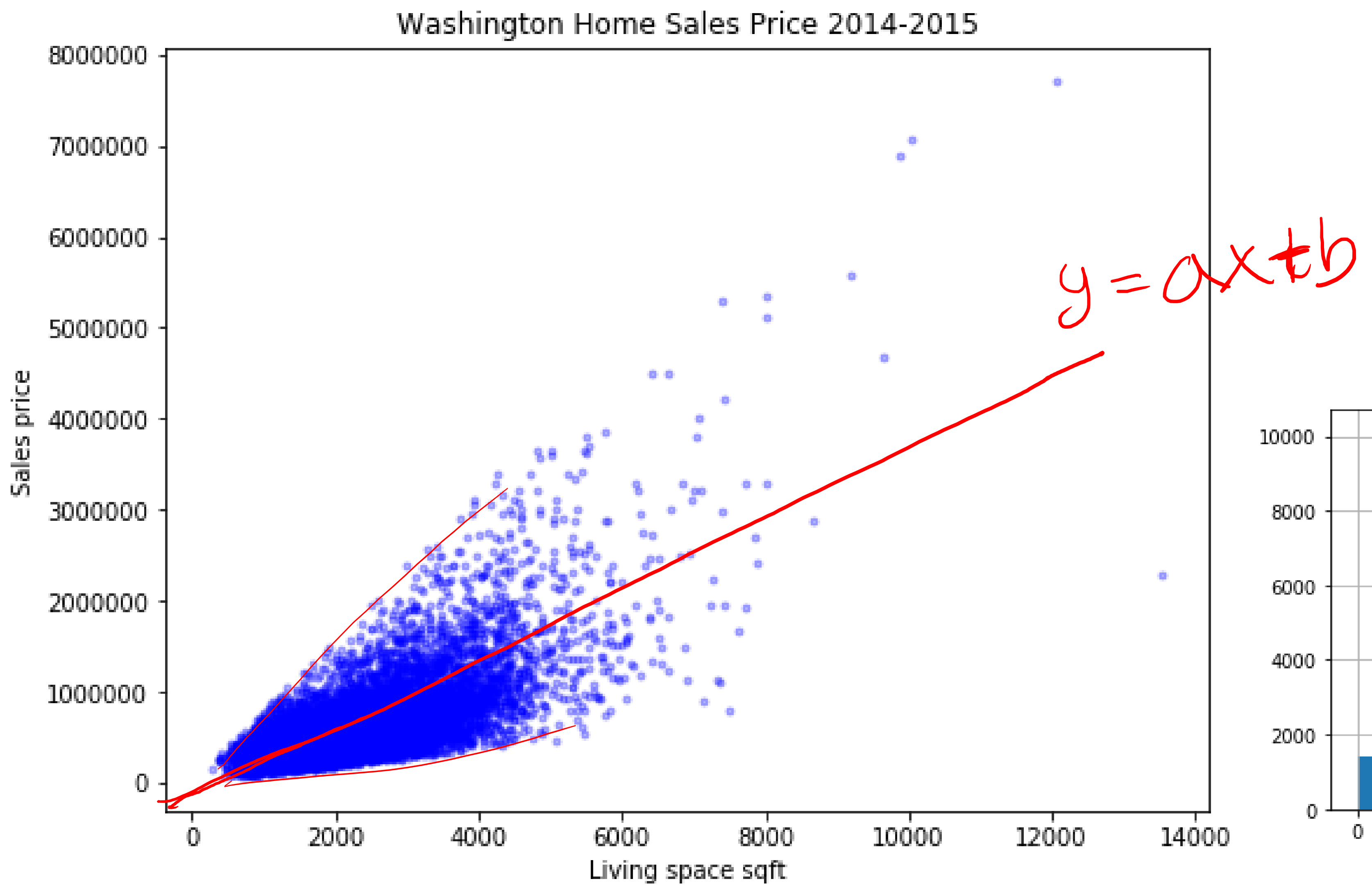
Predict

y	f_1	f_2	f_3	f_4	f_5
price	**bedrooms**	**bathrooms**	**sqft_living**	**sqft_lot**	**floors**
221900	3	1.00	1180	5650	1.0
538000	3	2.25	2570	7242	2.0
180000	2	1.00	770	10000	1.0
604000	4	3.00	1960	5000	1.0
510000	3	2.00	1680	8080	1.0

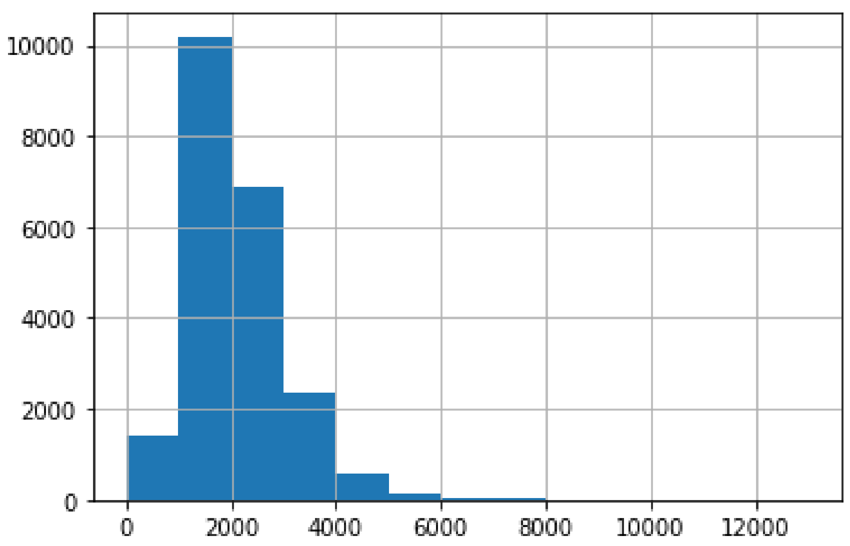
Pandas

```
Data columns (total 21 columns):
id               21613 non-null int64
date             21613 non-null object
price            21613 non-null int64
bedrooms         21613 non-null int64
bathrooms        21613 non-null float64
sqft_living      21613 non-null int64
sqft_lot         21613 non-null int64
floors           21613 non-null float64
waterfront       21613 non-null int64
view             21613 non-null int64
condition        21613 non-null int64
grade            21613 non-null int64
sqft_above       21613 non-null int64
sqft_basement    21613 non-null int64
yr_built         21613 non-null int64
yr_renovated     21613 non-null int64
zipcode          21613 non-null int64
lat              21613 non-null float64
long             21613 non-null float64
sqft_living15    21613 non-null int64
sqft_lot15       21613 non-null int64
```

Example

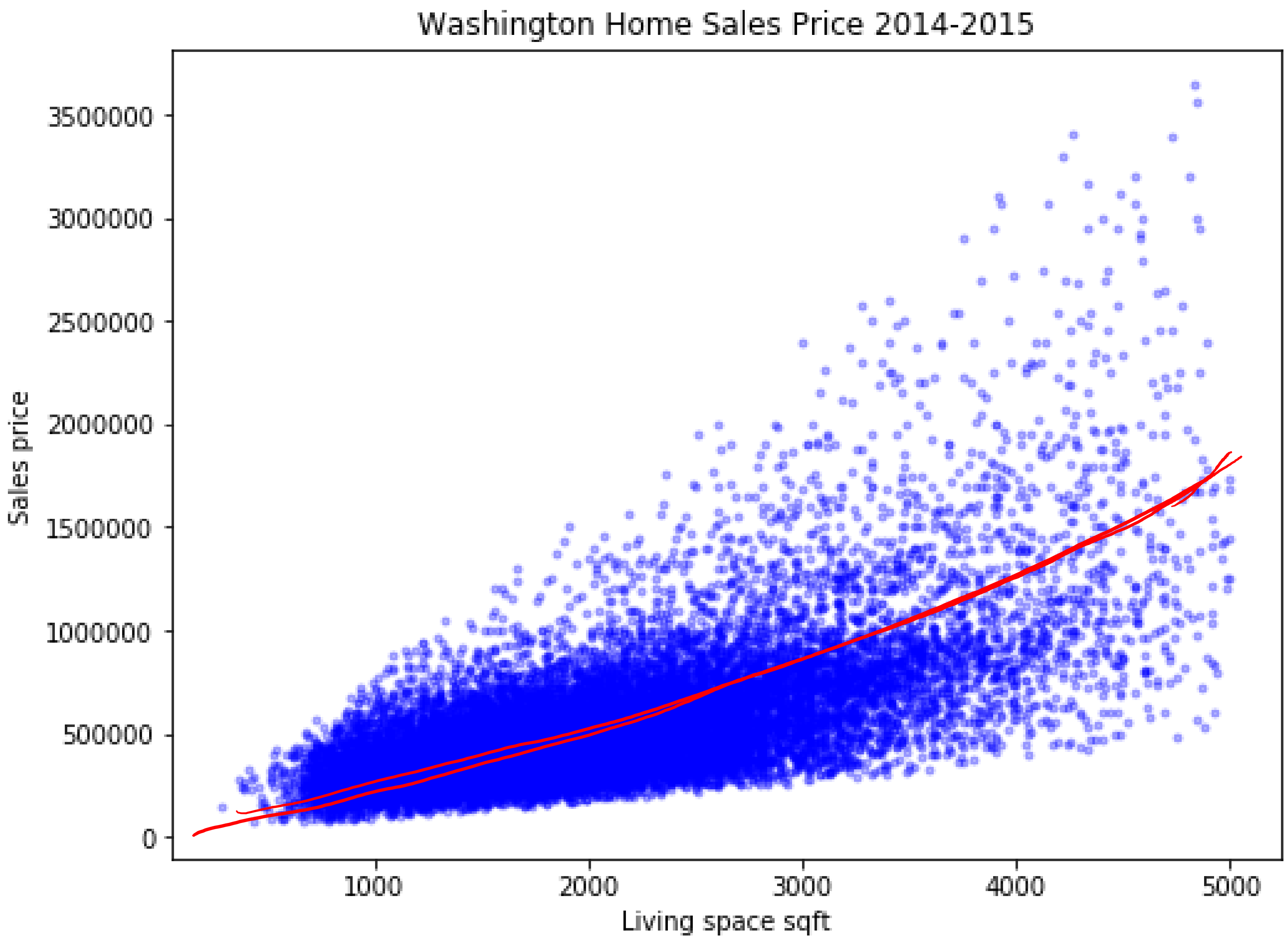


Washington Home Sales Price 2014-2015
Sales price
Living space sqft
8000000
7000000
6000000
5000000
4000000
3000000
2000000
1000000
0
2000
4000
6000
8000
10000
12000
14000
y=ax+b



10000
8000
6000
4000
2000
0
2000
4000
6000
8000
10000
12000

Example



$y = ax + bx^2 + c + \cdots$

Simple Linear Regression

Intercept　　Slope　　Residual

$$\underline{Y} = \underline{\beta_0}^{*} + \underline{\beta_1}\underline{X} + \underline{\epsilon}$$

Coefficients, or Parameters

$\beta_1 X_1 + \beta_2 X_2 + \ldots$

Estimated values

Model

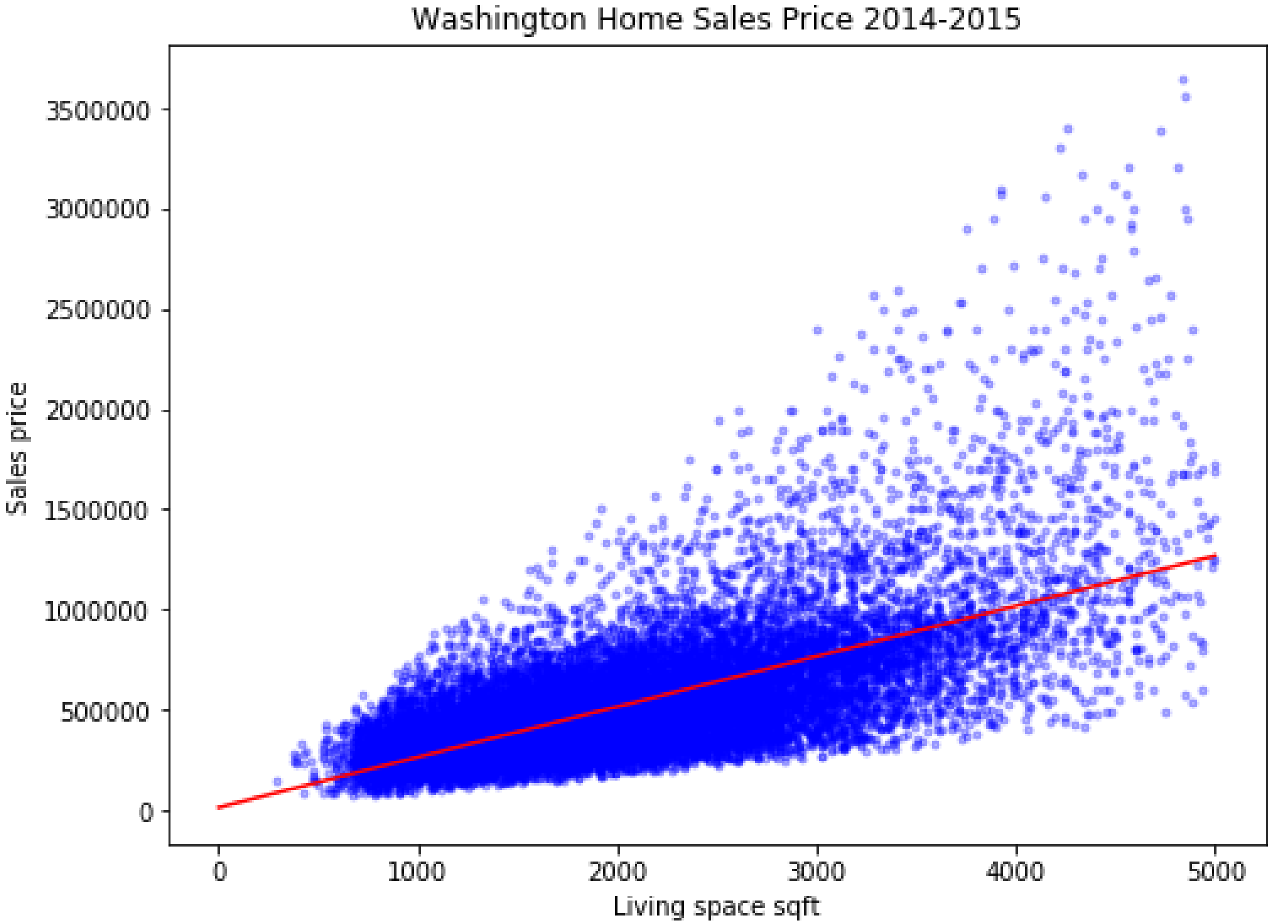
$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

Example

```python
import statsmodels.formula.api as smf
model = smf.ols(formula='price~sqft_living',data=sub).fit()
model.summary()
```

$y \sim x$

$y \sim x_1 + x_2$



OLS Regression Results

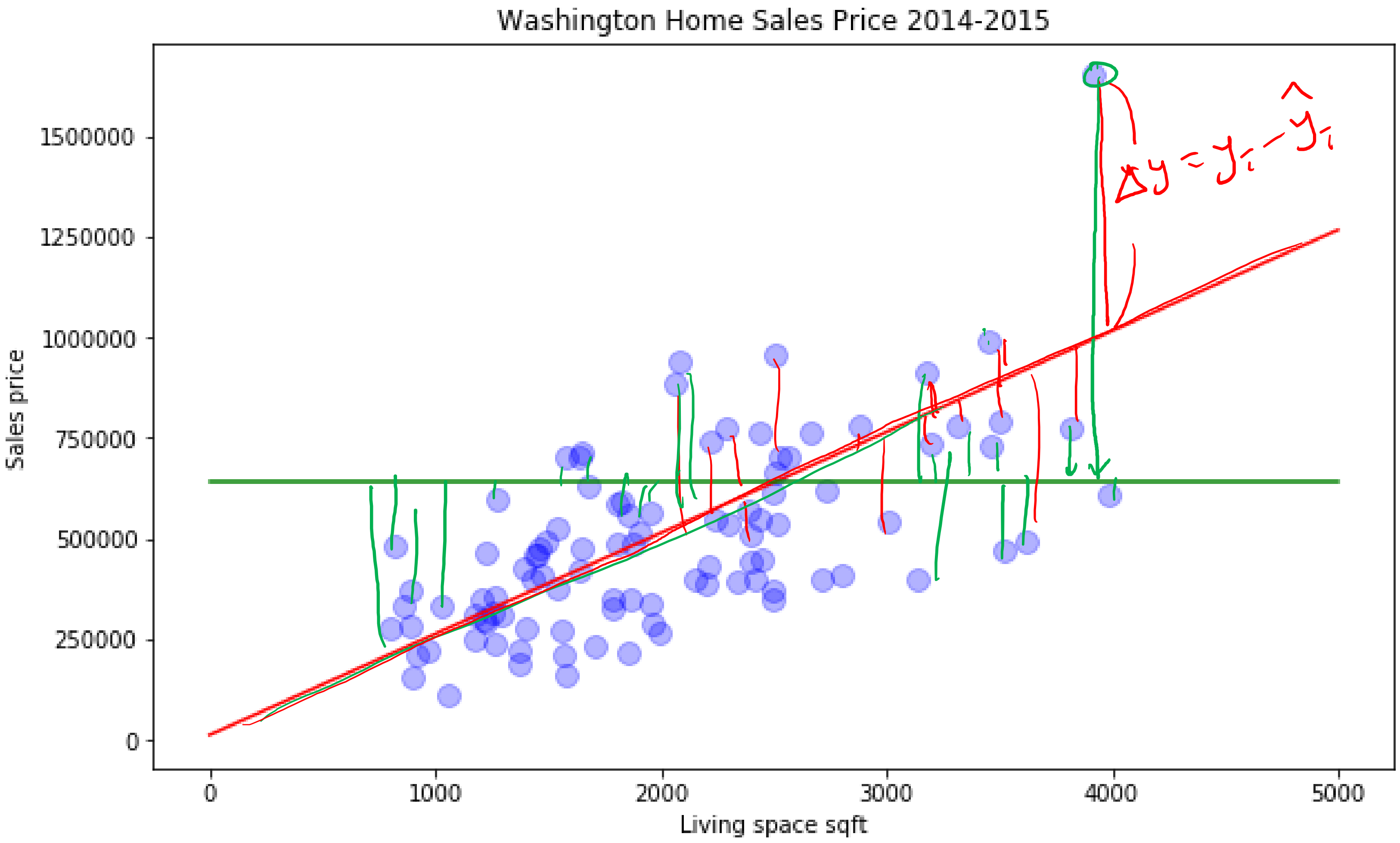
Dep. Variable:	price	**R-squared:**	0.436
Model:	OLS	**Adj. R-squared:**	0.436
Method:	Least Squares	**F-statistic:**	1.651e+04
Date:	Tue, 14 Jan 2020	**Prob (F-statistic):**	0.00
Time:	19:04:13	**Log-Likelihood:**	-2.9523e+05
No. Observations:	21402	**AIC:**	5.905e+05
Df Residuals:	21400	**BIC:**	5.905e+05
Df Model:	1		
Covariance Type:	nonrobust		

CI

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	1.24e+04	4305.280	2.881	0.004	3964.466	2.08e+04
sqft_living	251.0967	1.954	128.505	0.000	247.267	254.927

~ 0

R-squared



$i \to$ sample

$f(x_i)$

$\Sigma (y_i - \hat{y}_i)^2$

$$\boxed{RSS} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$TSS = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

fitness

~ 1

$$R^2 = 1 - \frac{RSS}{TSS}$$

p-value

$y \sim X$

Null hypothesis $$H_0 : \beta_1 = 0$$

Alternative hypothesis $$H_A : \beta_1 \neq 0$$

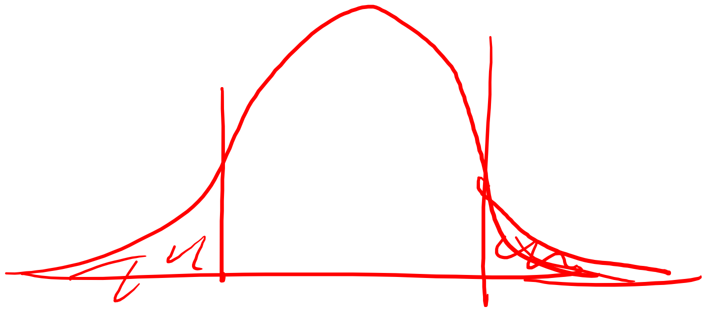
β

t-statistics

$$t = \frac{\hat{\beta}_1 - 0}{\mathrm{SE}(\hat{\beta}_1)}$$

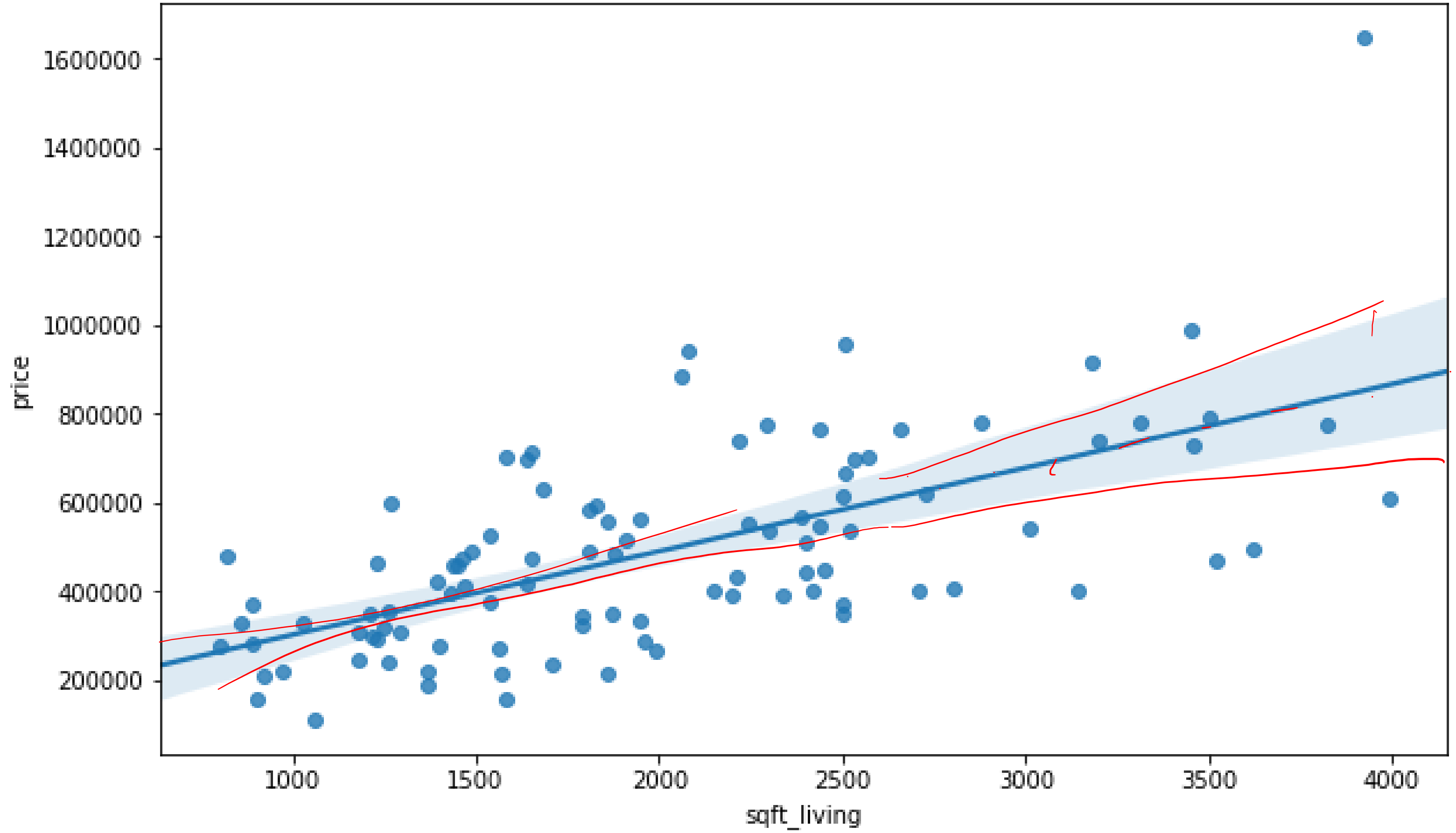
p-value

P > |*t*|



Accuracy of the coefficient estimates

```python
import seaborn as sns
plt.figure(figsize=(10,6))
sns.regplot(x="sqft_living", y = "price", data=x_sample, ci=95)
```

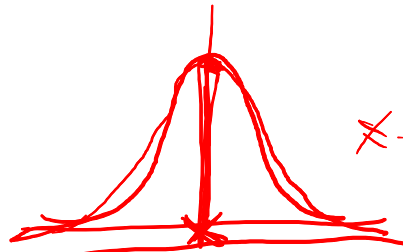


95% Confidence Interval

$$\left[\hat{\beta}_1 - 2 \cdot \mathrm{SE}(\hat{\beta}_1),\ \hat{\beta}_1 + 2 \cdot \mathrm{SE}(\hat{\beta}_1)\right]$$

Under the hood…

y = f(x)



$x_i\ y_i$

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Covariance Type:	nonrobust		

Maximum likelihood

$$\prod_{i=1}^{n} p(y_i|x_i;b_0,b_1,s^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{(y_i-(b_0+b_1x_i))^2}{2s^2}}$$

$y_i - f(x_i) = 0$

$$\begin{aligned} L(b_0,b_1,s^2) &= \log\prod_{i=1}^{n} p(y_i|x_i;b_0,b_1,s^2) \\ &= \sum_{i=1}^{n} \log p(y_i|x_i;b_0,b_1,s^2) \\ &= -\frac{n}{2}\log 2\pi - n\log s - \frac{1}{2s^2}\sum_{i=1}^{n}(y_i-(b_0+b_1x_i))^2 \end{aligned}$$

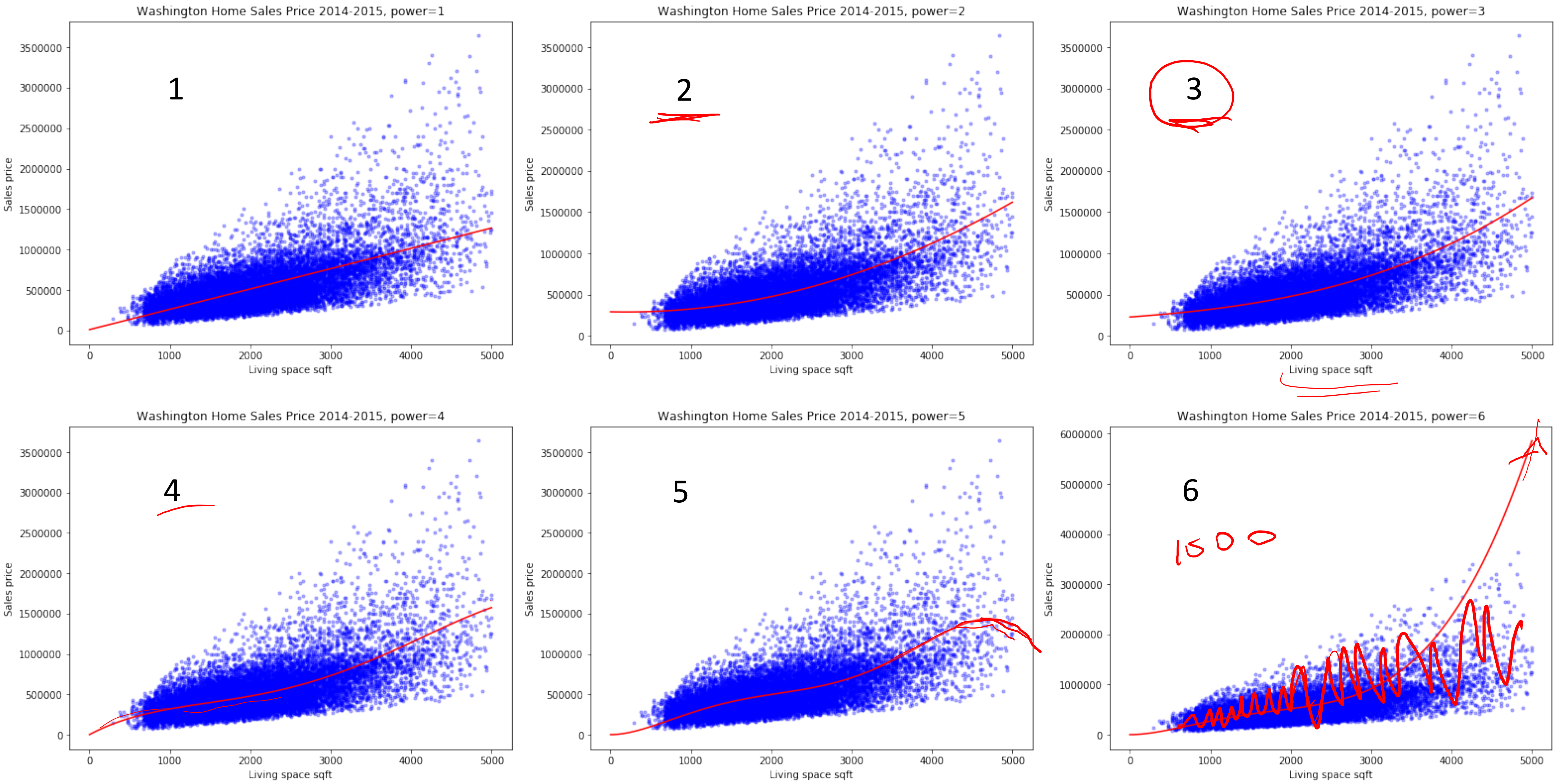
RSS ∝ MSE

Lk ↑

$\mathcal{L}$ ↓

Polynomial Regression

$y = aX + bx^2 + cx^3 + \dots$



Where to stop?

Labels

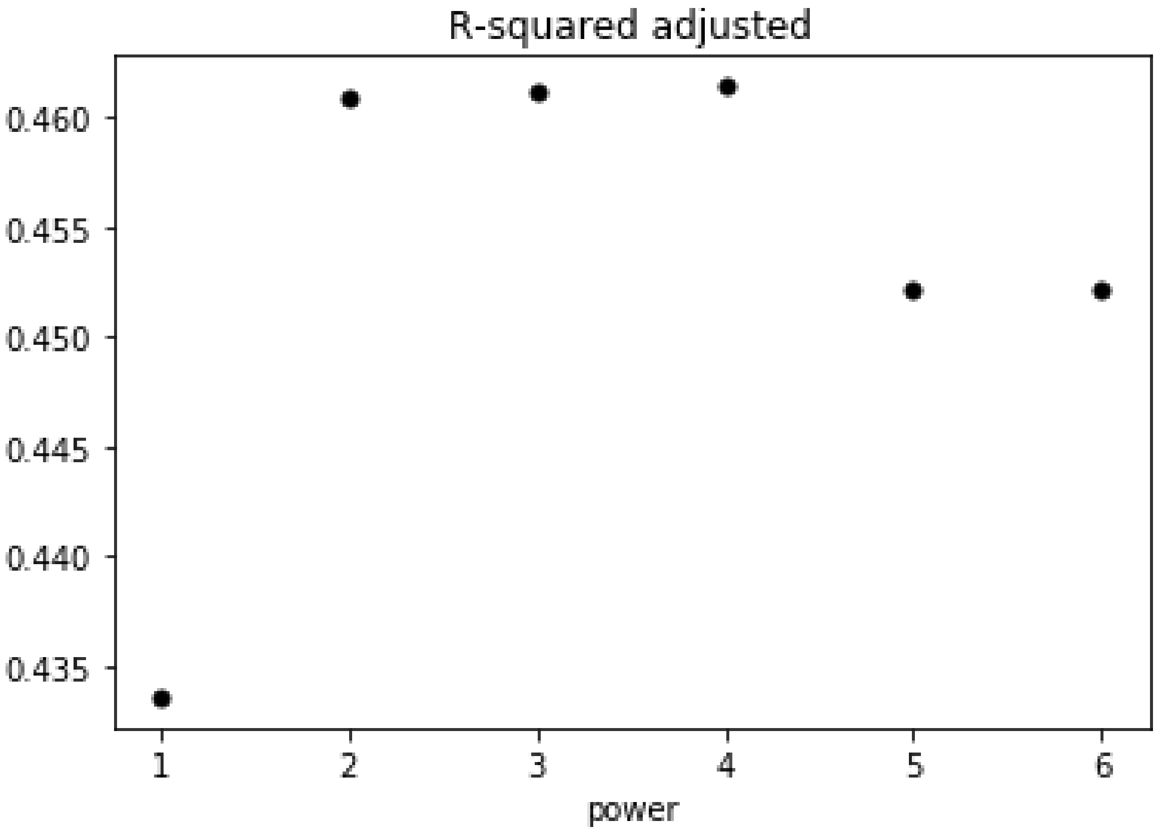
"test"

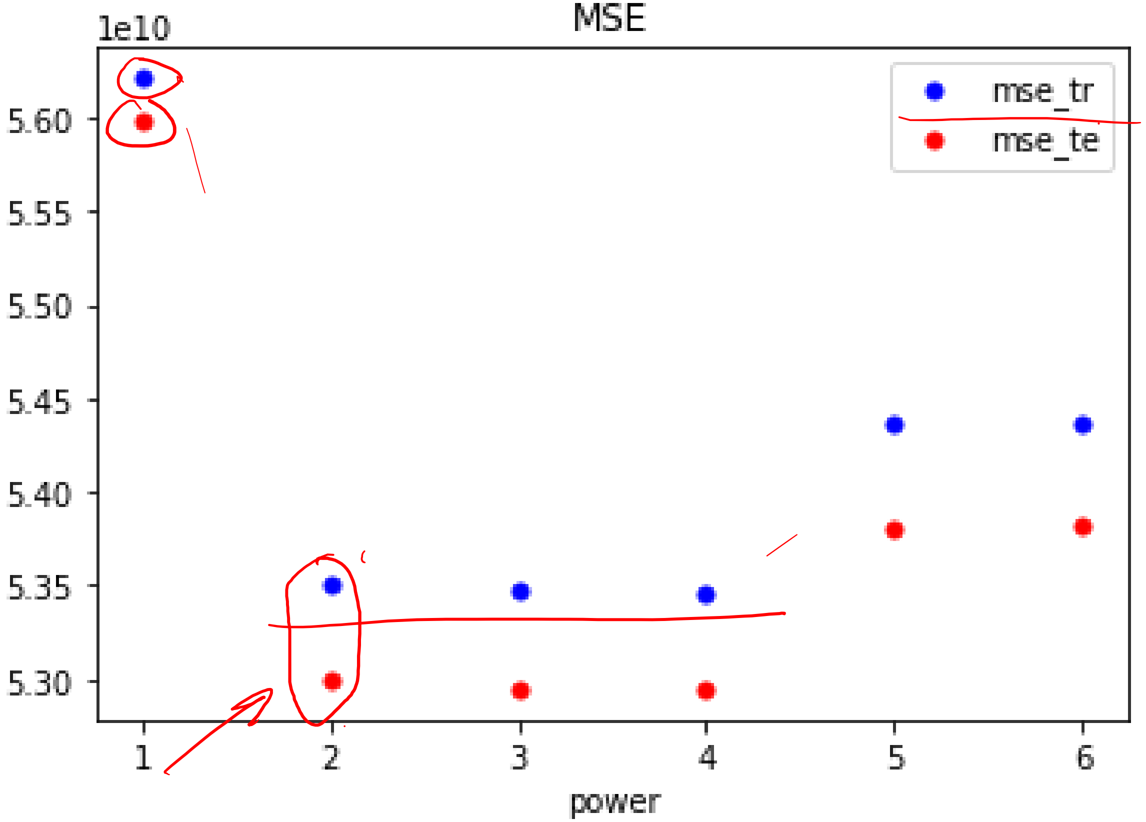
y

$f \to y_p$ → validation (test)

2

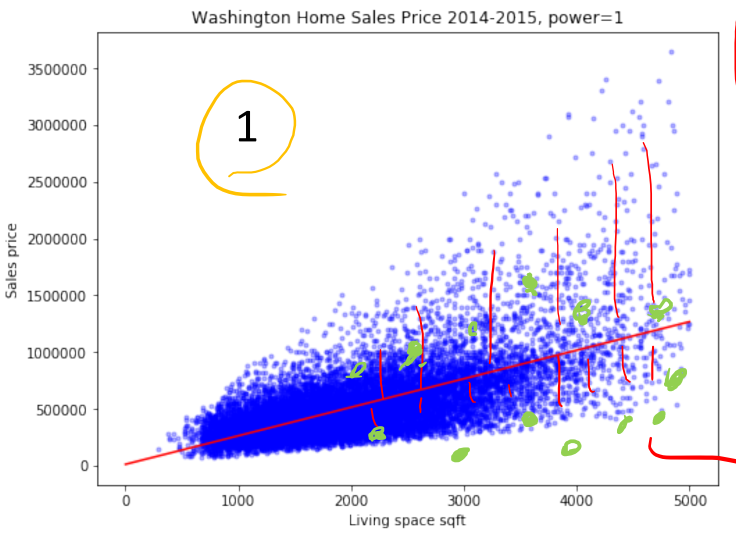
x^2

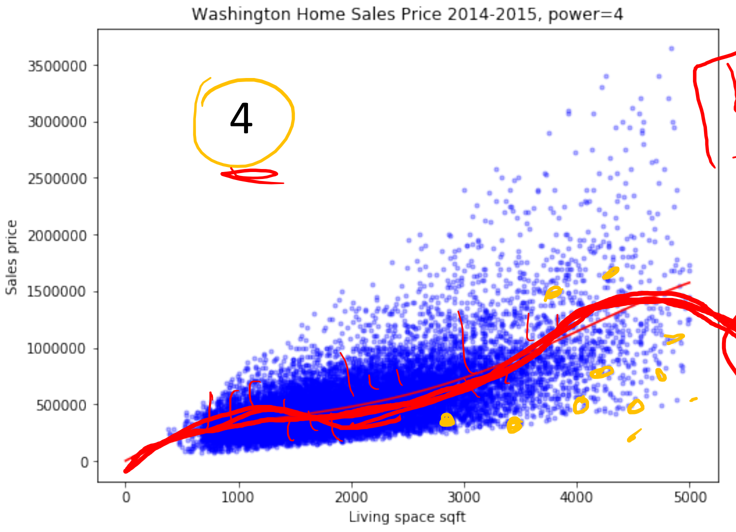


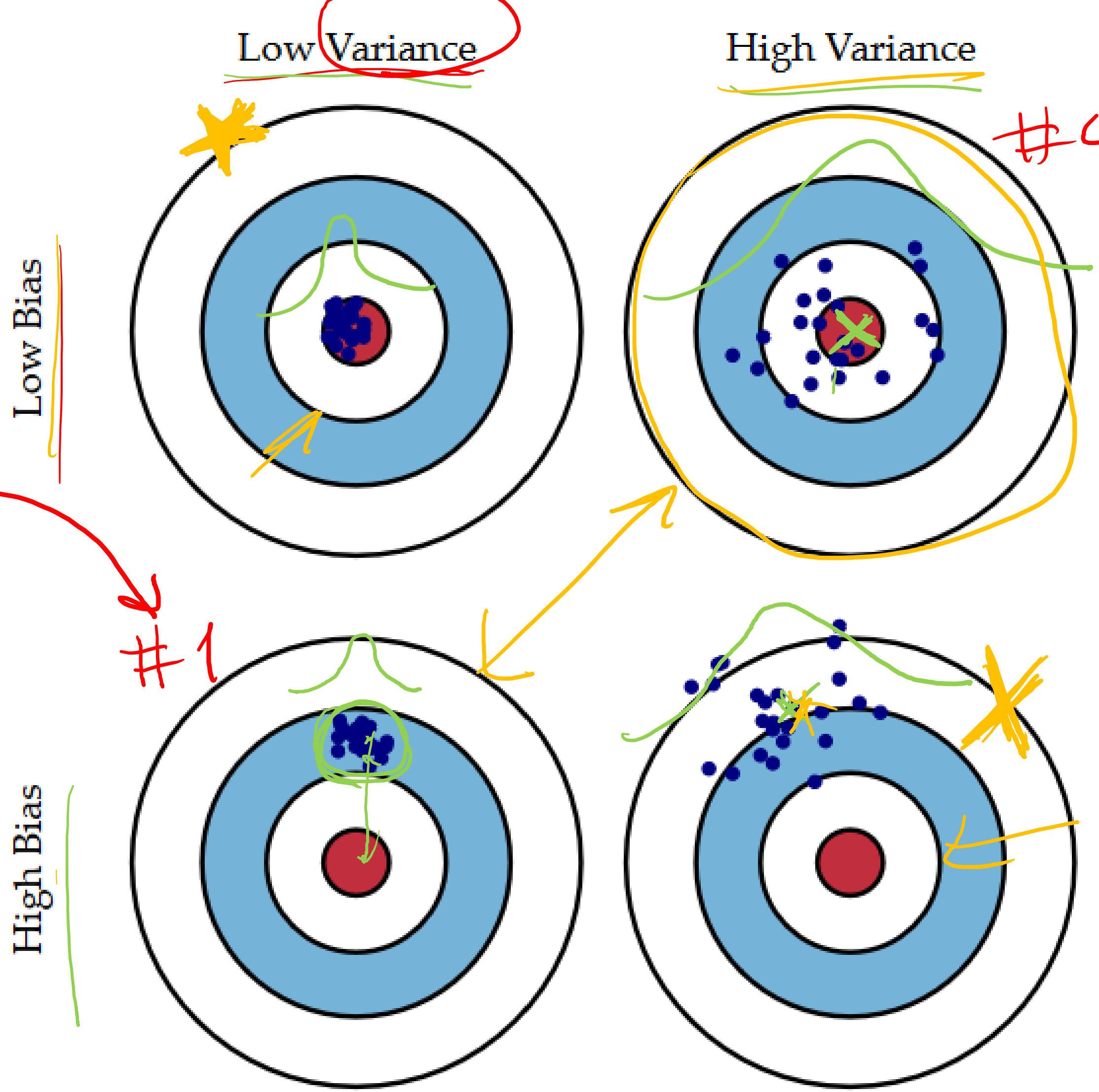


'Occam's razor'

Bias-Variance Trade-off







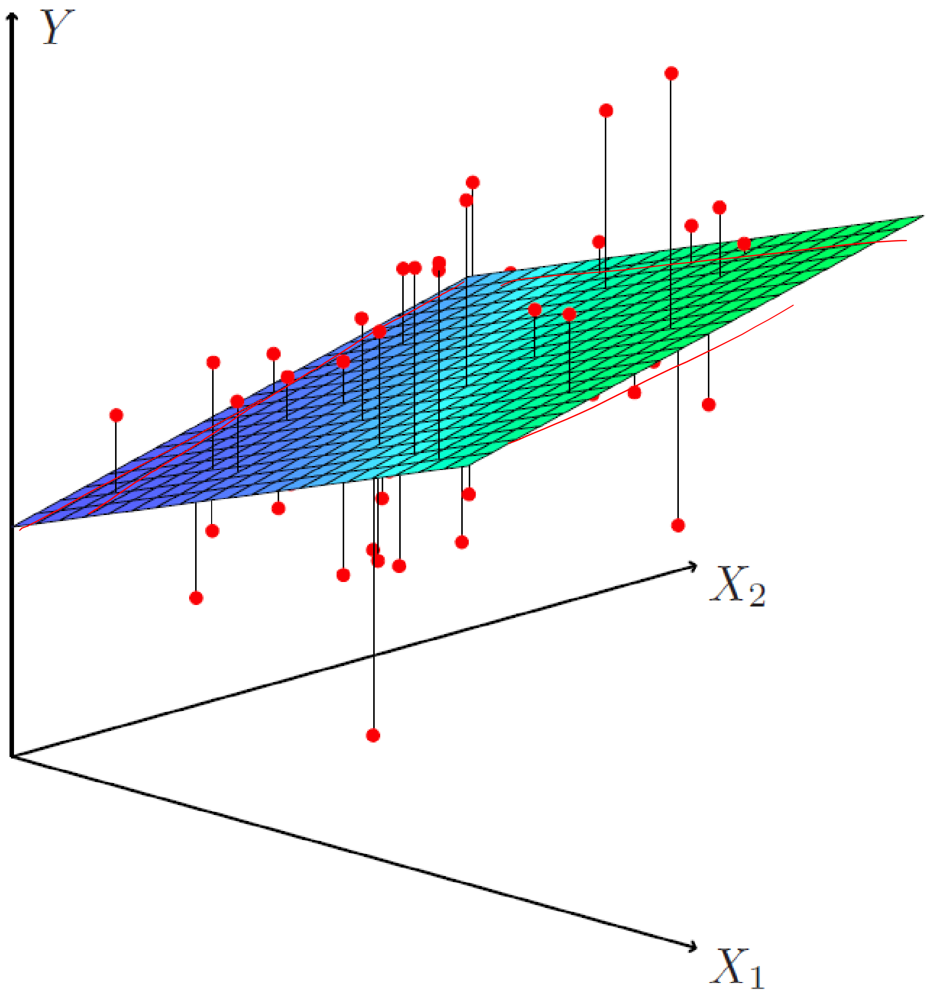
Multi-linear Regression

Multilinear regression model

$y = aX_1 + bX_1^2 + cX_1^3 + \cdots$

All predictors(variables) $X_1 \sim X_p$ are linear to Y

$$Y = \boxed{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p} + \epsilon$$



Multilinear regression model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \epsilon$$

β_j Average effect of X_j to Y when all other predictors fixed

Caution: In general,
predictors might be correlated

Collinearity

Caution: In general, predictors might be correlated

- When predictors are correlated, the coefficients estimates become inaccurate.
- Then it’s called (multi-)collinear.
- The interpretation of the coefficient as the variable’s contribution to the response becomes inaccurate.

Model selection

Parametric → $\boxed{\text{Linear regression}}$

SL

Regression Task

$y \in \mathbb{R}$

- Do I include all predictors or just a subset?

Forward Selection

step 1

```
y=b0
```

step 2

```
p=[]
for i=1,…,N
        y=b0+bi*Xi
        p.append(p_value(y))
k = argmin(p)
y = b0+bk*Xk
```

step 3

keep adding a predictor that gives the lowest p-value until it's satisfactory

· Polynomial order

features

Backward Selection

Parameters $\{\beta_0, \beta_1, \dots \beta_n\}$

step 1

```
y0=b0+b1*X1+b2*X2+…+bN*XN
```

RSS or MSE

Loss

step 2

```
p=[]
for i=1,…,N
        y=y0-bi*Xi
        p.append(p_value(y))
k = argmax(p)
y = y0-bk*Xk
```

$\arg\min_{\{\beta_0 \dots \beta_n\}} \mathcal{L}$

i) MLE $\prod P(y_i \mid \beta_0 \dots \beta_n, x_i)$

step 3

$\arg\max_{\{\beta_0 \dots \beta_n\}}$

keep removing a predictor that gives the largest p-value until it's satisfactory