

Entropy and Helmholtz free energy

The entropy $S(E, V, N)$ of an isolated system can never decrease: $\Delta S(E, V, N) \geq 0$.

$S(E, V, N)$ is a maximum at equilibrium. micro canonical ensemble:
isolated system

$$A = \underbrace{E}_{E=E(S,V,N)} - TS \quad \begin{array}{l} \text{"bring in"} \\ \text{"get rid of"} \end{array}$$

$A = A(T, V, N)$ — canonical ensemble:
system at constant T, V, N

The Helmholtz free energy is a minimum at equilibrium.

Helmholtz free energy $\hat{=}$ thermodynamic potential

$$\Delta A(T, V, N) \leq 0.$$

$$dA = dE - S dT - T dS$$

$$= \underbrace{(T dS - P dV + \mu dN)}_{dE} - S dT - T dS$$

$dE = T dS - P dV + \mu dN$
fundamental thermodynamic relation

$$= \underbrace{\left(-S\right)}_{\left(\frac{\partial A}{\partial T}\right)_{V,N}} dT + \underbrace{\left(-P\right)}_{\left(\frac{\partial A}{\partial V}\right)_{N,T}} dV + \underbrace{\left(\mu\right)}_{\left(\frac{\partial A}{\partial N}\right)_{T,V}} dN$$