Course Review

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Review

- State vector $|\alpha\rangle$.
- Dual correspondence $a^* \langle\alpha| \overset{\text{DC}}{\Longleftrightarrow} a\,|\alpha\rangle$.
- Normalization and positivity $\langle\alpha|\alpha\rangle \geq 0$.
- Observable $\tilde{\mathrm{A}} = \tilde{\mathrm{A}}^\dagger$.
  - Eigenvalues $\tilde{\mathrm{A}}\,|a_i\rangle = a_i\,|a_i\rangle$.
  - Eigenvalues real $a_i = a_i^*$.
  - Eigenvectors orthogonal $\langle a_i|a_j\rangle = \delta_{ij}$.
  - Completeness $\sum_i |a_i\rangle\langle a_i| = \tilde{1}$.
  - Expansion $|\alpha\rangle = \sum_i |a_i\rangle\,\langle a_i|\alpha\rangle$.
  - Compatible $\left[\tilde{\mathrm{A}}, \tilde{\mathrm{B}}\right] = 0$

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- Continuous eiegenvalues $\tilde{\mathbf{x}}\,|x\rangle = x\,|x\rangle$.
- Completeness $\int_a^b |x'\rangle\langle x'|\; dx' = \tilde{\mathbf{1}}$.
  - $\alpha(x) = \int \langle x\,|x'\rangle \langle x'\,|\alpha\rangle\; dx'$.
  - $\langle x'\,|x\rangle = \delta(x' - x)$.
  - $\langle x'\,|p'\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{ip'x'/\hbar}$.
  - $\alpha(p) = \langle p\,|\alpha\rangle = \int \langle p\,|x'\rangle \langle x'\,|\alpha\rangle\; dx'$

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- Unitary operator $\tilde{\mathbf{U}}(\phi) = e^{-i\tilde{\mathbf{A}}\phi}$ with $\tilde{\mathbf{A}} = \tilde{\mathbf{A}}^\dagger$
  - $\tilde{\mathbf{U}}\tilde{\mathbf{U}}^\dagger = \tilde{\mathbf{1}}$.
  - $e^{-ipx'/\hbar} |x\rangle = |x + x'\rangle$.
  - Infinitesimal operation $e^{-ip\delta x'/\hbar} = \tilde{\mathbf{1}} - ip\delta x'/\hbar$
- Time evolution $\tilde{\mathbf{U}} = e^{-i\tilde{\mathbf{H}}t/\hbar}$

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