5393 Quantum Mechanics
Homework 7

Reading Assignment	Sakurai	Chapter 2
Problems	Sakurai	Chapter 2 prob. 2.1, 2.3, 2.4, 2.9,
Date Due		Oct. 11, 2016 *by* 5:00 pm

Additional Problem:

1. Consider a physical system whose state space, which is three-dimensional, is spanned by the orthonormal basis formed by the three kets $|u_1\rangle$, $|u_2\rangle$ and $|u_3\rangle$. In this basis, the Hamiltonian operator $\tilde{\mathbf{H}}$ of the system and the two observable $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ are written:

$$\tilde{\mathbf{H}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad \tilde{\mathbf{A}} = a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \tilde{\mathbf{B}} = b \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   The physical system at time $t = 0$ is in the state

$$|\alpha, 0\rangle = \frac{1}{\sqrt{2}} |u_1\rangle + \frac{1}{2} |u_2\rangle + \frac{1}{2} |u_3\rangle$$

   (a) At time $t = 0$, the energy of the system is measured. What values can be found and with what probabilities? Calculate, for the system in the state $|\alpha, 0\rangle$, the mean value $\left\langle \tilde{\mathbf{H}} \right\rangle$ and the RMS deviation ΔH.

   (b) Instead of measuring $\tilde{\mathbf{H}}$ at time $t = 0$, one measures $\tilde{\mathbf{A}}$; what results can be found and with what probabilities? What is the state vector immediately after the measurement?

   (c) Calculate the state vector $|\alpha, 0; t\rangle$ of the system at time t.

   (d) Calculate the mean values $\left\langle \tilde{\mathbf{A}}(t) \right\rangle$ and $\left\langle \tilde{\mathbf{B}}(t) \right\rangle$ of $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ at time t. What comments can be made?

   (e) What results are obtained if the observable $\tilde{\mathbf{A}}$ is measured at time t? Same question for the observable $\tilde{\mathbf{B}}$. Interpret.