Key points 04/13 lecture

For fermions : $\langle n_{\vec{k}} \rangle = \dfrac{1}{z^{-1} e^{\beta \varepsilon_{\vec{k}}} + 1}$

$\langle n_{\vec{k}} \rangle \le 1$ and ≥ 0 $\Rightarrow$ $z \ge 0$

$$\frac{P}{kT} = \frac{1}{\lambda^3} f_{5/2}(z)$$

$$\frac{1}{v} = \frac{1}{\lambda^3} f_{3/2}(z)$$

$$f_m(z) = \frac{1}{\Gamma(m)} \int_0^\infty \frac{y^{m-1}}{e^{y-\nu}+1}\, dy = \sum_{\ell=1}^{\infty} (-1)^{\ell-1} \frac{z^\ell}{\ell^m}$$

$\nu = \log z = \frac{\mu}{kT}$

Fermi - Dirac function

properties of ideal Fermi gas can be expressed in terms of Fermi-Dirac functions

(note: these results are for 3D non-relativistic fermions without spin!)

In momentum space, the particles fill – at zero T – a sphere of radius p_F, the surface of which is called the Fermi surface: $p_F = \hbar k_F$; $\varepsilon_F = \frac{p_F^2}{2m}$.

For spin-s fermions: degeneracy $g = 2s+1$.

E.g.: $g \sum_{\vec{k}} \langle n_{\vec{k}} \rangle_{T=0} = N$.