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Statistical Mechanics

CH. 3 THE PROBLEM OF KINETIC THEORY LECTURE NOTES

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We now move on to examining Kinetic Theory treated classically.

Taking our system to be in equilibrium we aim to examine our system out of equilibrium

We begin studying collisions (cross section scattering)

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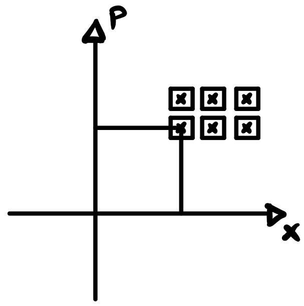
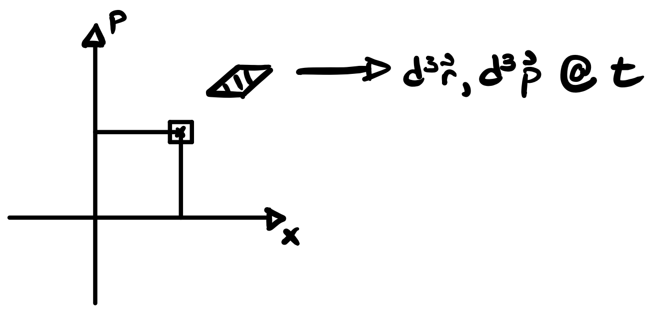
We have a density of particles in phase space that is $f(\vec{r},\vec{p},t)$ and will look like



The number of particles in this state can be found with

$$n(\vec{r},t) = \int f(\vec{r},\vec{p},t)\, d^3\vec{p}$$

If we take $(\vec{r},\vec{p},t) \xrightarrow{t+dt} \vec{r}(t) + \vec{v}(t)\,dt\ ,\ \vec{p}(t) + \vec{F}\,dt\ ,\ t+dt$ we can observe that our phase space will look like



Using the Louiville Theorem we can say

$$f(\vec{r},\vec{p},t)\,\cancel{d^3\vec{r}\,d^3\vec{p}} = f(\vec{r}(t+dt),\vec{p}(t+dt),t+dt)\,\cancel{d^3\vec{r}\,'\,d^3\vec{p}\,'}$$

Where we used the fact $d^3\vec{r}\,d^3\vec{p} = d^3\vec{r}\,'\,d^3\vec{p}\,'$. We can then say

$$\left(\frac{\partial}{\partial t} + \frac{d\vec{r}}{dt}\cdot\vec{\nabla}_{\vec{r}} + \frac{d\vec{p}}{dt}\cdot\vec{\nabla}_{\vec{p}}\right) f(\vec{r},\vec{p},t) = 0 \qquad (*)$$

Where the above is a seven dimensional partial differential equation.

For the RHS of (*) we define it to be

$$\left(\frac{\partial f}{\partial \tau}\right)$$

we then look at the duration of collision with $\delta\tau \ll \tau^*$. The RHS can be expressed as

$$\left(\frac{\partial f}{\partial \tau}\right) = \int \delta(\vec{p}' - \vec{p})\,\delta(E' - E)\,|T_{p_1}|^2 F^{(2)}(\vec{r}, \vec{p}_1', \vec{p}_2', t) F^{(2)}(\vec{r}, \vec{p}_1, \vec{p}_2, t)\, d^3\vec{p}_2'\, d^3\vec{p}_1'\, d^3\vec{p}_3' \quad (**)$$

If we are looking at two particles before and after a collision we label them as

t	$t+dt$
$\vec{r}_1$	$\vec{r}_1'$
$\vec{r}_2$	$\vec{r}_2'$
$\vec{p}_1$	$\vec{p}_1'$
$\vec{p}_2$	$\vec{p}_2'$

This then means the two particle distribution function is

$$F^{(2)}(\vec{r}, \vec{p}_1, \vec{p}_2, t) = f(\vec{r}, \vec{p}_1, t) f(\vec{r}, \vec{p}_2, t)$$

The above expression is frequently referred to as molecular chaos. It is important to note in (**) that

$$\vec{p} = \vec{p}_1 + \vec{p}_2 \quad , \quad \vec{p}' = \vec{p}_1' + \vec{p}_2'$$

we can finally write (**) as

$$\left(\frac{\partial f}{\partial \tau}\right) = \int \delta(\vec{p}' - \vec{p})\,\delta(E' - E)\,|T_{p_1}|^2 \left[f(\vec{r}, \vec{p}_1', t) f(\vec{r}, \vec{p}_2', t) - f(\vec{r}, \vec{p}_1, t) f(\vec{r}, \vec{p}_2, t)\right] d^3\vec{p}_2\, d^3\vec{p}_1\, d^3\vec{p}_3$$

The Boltzmann transport equation is then

$$\left(\frac{\partial}{\partial t} + \frac{d\vec{r}}{dt} \cdot \vec{\nabla}_{\vec{r}} + \frac{d\vec{p}}{dt} \cdot \vec{\nabla}_{\vec{p}}\right) f(\vec{r}, \vec{p}, t)$$

$$=$$

$$\int \delta(\vec{p}' - \vec{p})\,\delta(E' - E)\,|T_{p_1}|^2 \left[f(\vec{r}, \vec{p}_1', t) f(\vec{r}, \vec{p}_2', t) - f(\vec{r}, \vec{p}_1, t) f(\vec{r}, \vec{p}_2, t)\right] d^3\vec{p}_2\, d^3\vec{p}_1\, d^3\vec{p}_3$$

5-2-22

From the previous lecture we examined the Boltzmann transport equation, which describes how energy is exchanged in an interacting system.

The equilibrium solution to the BTE is

$$\exp\left(\frac{-(P^2/2m + U(\vec{r}))}{KT}\right)$$

If we let $t \to \infty$ we then have a solution of the form

$$f_0(\vec{r},\vec{p}) = \frac{n(\vec{r})}{(2\pi m k_B T)^{3/2}} e^{-\beta P^2/2m} \quad , \quad n(\vec{r}) = e^{\beta \vec{\nabla} \varphi}$$

we then choose to examine the mass density $\rho = m\,n(\vec{r},t)$ which then tells us

$$\frac{\partial \rho(\vec{r},t)}{\partial t} + \vec{\nabla} \cdot (\rho(\vec{r},t)\,u(\vec{r},t)) = 0$$

We can then go on to say that the potential energy is

$$u(\vec{r},t) = \langle \vec{v}(t) \rangle = \frac{\int f(\vec{r},\vec{p},t)\,\vec{v}\;d^3\vec{p}}{\int f(\vec{r},\vec{p},t)\,d^3\vec{p}}$$