5393 Quantum Mechanics
Homework 5

Reading Assignment	Sakurai	Sections 1.7, 2.1
Problems	Sakurai	Chapter 1 prob. 1.33, 1.34, 1.35, 1.36
Date Due		Oct. 5, 2021 *by* 5:00 pm

Q-1 Consider the Hamiltonian $\tilde{\mathbf{H}}$ of a particle in one-dimensional problem defined by:

$$\tilde{\mathbf{H}} = \frac{1}{2m}\tilde{\mathbf{P}}^2 + V(\tilde{\mathbf{X}}) \tag{1}$$

where $\tilde{\mathbf{P}}$ and $\tilde{\mathbf{X}}$ are momentum and position operators, respectively. These operators satisfy the commutation relation: $[\tilde{\mathbf{X}}, \tilde{\mathbf{P}}] = i\hbar$. The eigenvectors of $\tilde{\mathbf{H}}$ are denoted by $|\phi_n\rangle$ and satisfy the eigenvalue equation $\tilde{\mathbf{H}}\,|\phi_n\rangle = E_n\,|\phi_n\rangle$, where n is a discrete index.

(a) Show that:

$$\left\langle \phi_n \middle| \tilde{\mathbf{P}} \middle| \phi_{n'} \right\rangle = \alpha \left\langle \phi_n \middle| \tilde{\mathbf{X}} \middle| \phi_{n'} \right\rangle \tag{2}$$

where α is a coefficient that depends on the difference between E_n and $E_{n'}$. Calculate α.

(b) From this, deduce, using the completeness relation, the equation:

$$\sum_{n'} (E_n - E_{n'})^2 \left| \left\langle \phi_n \middle| \tilde{\mathbf{X}} \middle| \phi_{n'} \right\rangle \right|^2 = \frac{\hbar^2}{m^2} \left\langle \phi_n \middle| \tilde{\mathbf{P}}^2 \middle| \phi_n \right\rangle .$$