Homework Assignment #4

Math Methods

Homework Due: Wednesday, September 29th

Instructions:

Reading: Please re-read the rest of Chapter 1. There is no reading quiz this week. **Problems:** Below is a list of questions and problems from the texbook due by the time and date above. It is not sufficient to simply obtain the correct answer. You must also explain your calculation, and each step so that it is clear that you understand the material.

Homework should be written legibly, on standard size paper. Do not write your homework up on scrap paper. If your work is illegible, it will be given a zero.

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1. A rocket is fired horizontally off of a rooftop. As it leaves the rooftop it has an initial horizontal velocity v_0 and a constant horizontal acceleration a_0 in addition to the acceleration, g, downward due to gravity. What is the shape of its trajectory? (Hyperbola? Parabola? Straight line? Or something else?) *Hint:* This question can be answered without any need for calculation, if you think geometrically).

2. Byron and Fuller, Chapter 1, problem 1

3. Show that $\epsilon_{ijk}\epsilon_{ijk} = 6$.

4. Demonstrate algebraically whether or not the cross product is associative. That is, verify or falsify the following:

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$$

   If they are not in general equal, are they at least of equal magnitude?

5. Consider a two dimensional system where the vector $\vec{x}$ is given by

$$\vec{x} = x_1\hat{e}_1 + x_2\hat{e}_2$$

   and the x-coordinate is transformed into a different, non-orthogonal coordinates as:

$$\begin{aligned} x_1' &= \frac{1}{\sqrt{2}}(x_1 + x_2) \\ x_2' &= x_2 \end{aligned}$$

   The gradient of the scalar function is:

$$\vec{\nabla}\phi(\vec{x}) = \partial_1\phi\,\hat{e}_1 + \partial_2\phi\,\hat{e}_2$$

   Find the components of the gradient in the primed co-ordinates and show that it transforms as a covariant vector.

6. Show that for an orthogonal transformation, there is no distinction between a contravariant and a covariant vector.

Figure 1: Cartoon by R. Bolling, *'Super-Fun-Pak Comix".*