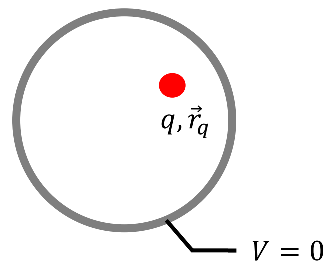
Physics 5573, Spring 2022
Test 2, Maxwell and Conductors

For Questions 1 – 3, you don't have to provide a complete solution to the problem, but you need to describe the approach you would take to solving the problem. Be sure to give the information requested and, if requested, explain your reasoning.

1) A grounded spherical conductor has a charge q inside at the position $\vec{r}_q$. You wish to determine the properties of this system everywhere inside the sphere.



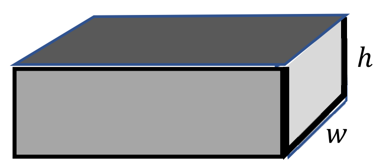
A) How would you setup your coordinates to simplify the solution to this problem? Draw a picture.

B) What general approach would you take to the solution to this problem? Explain, briefly, why you would take this approach to the problem?

C) What would be the general form of the solution for the potential inside the sphere, considering your approach to the problem? This doesn't mean you should solve the problem, just that you should give an expression that it would be possible to solve.

D) What approach would you take to go from the general solution from (C) to obtaining a complete solution to the problem? Give some details of how you would do this, but you don't need to do all the algebra to obtain a result.

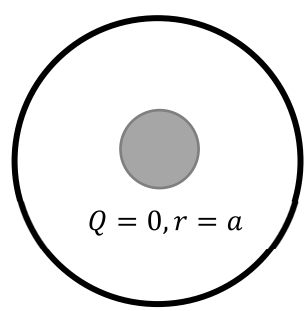
E) What questions would you ask yourself, what physics would you explore, and what calculations would you make to check your answer to this problem?



2) A very long (assume infinite) rectangular box has a height h and a width w have the left, right, and bottom walls all grounded ($V = 0$). The top wall of the box has an electric potential, $V_{Top}(x)$, where "x" is the coordinate across the box, perpendicular to the "very long" direction.

Repeat questions A – E from Problem 1 for this system.

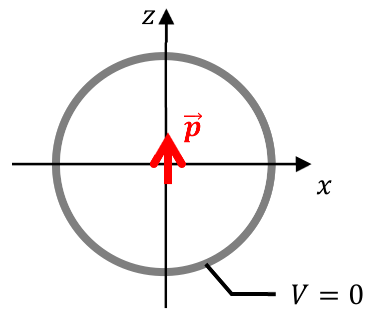
3) Consider two concentric spheres. The outer sphere is an insulator and has a radius $r = b$. The inner sphere is a solid conducting sphere with $r = a$ and $Q = 0$ (no net charge). You want to distribute charge on the outer sphere so that the potential is $V(b, \theta) = V_0 \cos^2 \theta$.



$V(b, \theta) = V_0 \cos^2 \theta, r = b$

Repeat questions A – E from Problem 1 for this system. Be sure to indicate how you could determine the charge distribution on the outer sphere that will give this potential. For question D, what terms do you expect to be non-zero?

4) A point dipole $\vec{p} = p\,\hat{z}$ is placed at the center of a grounded, conducting sphere of radius R. Solve for the electric properties inside the sphere.



A) The total potential is a sum of the dipole plus the sphere:

$$\phi_T(r,\theta) = \phi_p(r,\theta) + \phi_s(r,\theta)$$

Write the potential $\phi_p(r,\theta)$ in spherical coordinates. Remember

$$\phi = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

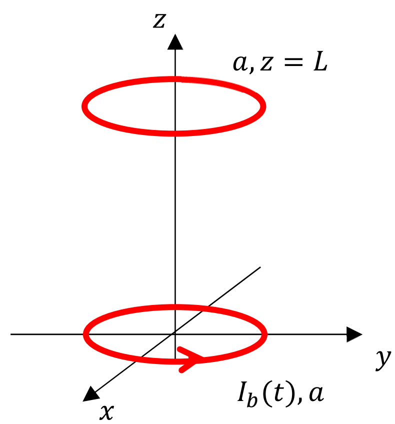
B) Give a general expression for $\phi_s(r,\theta)$ as an expansion (sum) in spherical coordinates. What are the boundary conditions that ϕ_s must satisfy?

C) Solve for $\phi_s(r,\theta)$ and $\phi_T(r,\theta)$. Show your work.

D) Solve for the electric field inside the sphere. Show that your solution gives the correct boundary conditions for the electric field.

E) What is the surface charge density on the sphere?

5) A pair of identical Helmholtz coils are centered on the z-axis. The coils have radius $r = a$, resistance R, and are separated by a distance $L \gg a$.



- Assume L is small enough that we can neglect the effects of the finite speed of light (retardation effects).
- Assume self-induction, the EMF in a coil due to the change in the magnetic field created by that coil, can be neglected

The current in the bottom coil is counterclockwise and increasing:

$$I_b(t) = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

A) What is the time-dependent magnetic moment of the bottom coil, $\vec{m}_b(t)$.

B) Treating the bottom coil as a magnetic dipole, what is the magnetic field in the center of the top coil, $\vec{B}_b(z = L, t)$?

C) Assume the field through the top loop is approximately a constant over the area of the loop at any time t (of course is changes in time):

$$\vec{B}_{bottom}(\vec{r},t)\cdot\hat{n}_{top} \approx \text{Constant(t)}$$

Use Faraday's law to determine the EMF and current in the top coil. Include the direction of the current.

D) Check the assumption made in Part C that the magnetic field through the top loop is approximately constant. You might, for example, determine how much the field changes from the middle to the edge of the coil as a function of the small parameter $\frac{a}{L}$.