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Classical Mechanics

PHYS 5153 HOMEWORK ASSIGNMENT #6

PROBLEMS: $\{1, 2, 3, 4\}$

Due: October 15, 2021 By: 6:00 PM

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PROFESSOR
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Problem 1

A particle of mass m moves in a spherically symmetric potential

$$V(r) = -\mathcal{C}\frac{e^{-\alpha r}}{r}, \tag{1}$$

with $\mathcal{C}$, $\alpha > 0$.

(a) Derive an *effective* one-dimensional potential that governs the qualitative motion of the particle. Sketch your potential.

We will first start with a Lagrangian: $L = T - V$, $= \frac{1}{2}m(\dot{x}^2+\dot{y}^2) + C\frac{e^{-\alpha r}}{r}$

using: $x = r\cos(\theta)$, $\dot{x} = \dot{r}\cos(\theta) - r\dot{\theta}\sin(\theta)$ & $y = r\sin(\theta)$, $\dot{y} = \dot{r}\sin(\theta) + r\dot{\theta}\cos(\theta)$

Then: $\dot{x}^2+\dot{y}^2 = \dot{r}^2\cos^2(\theta) - \cancel{2r\dot{r}\dot{\theta}\cos(\theta)\sin(\theta)} + r^2\dot{\theta}^2\sin^2(\theta) + \dot{r}^2\sin^2(\theta) + \cancel{2r\dot{r}\dot{\theta}\cos(\theta)\sin(\theta)} + r^2\dot{\theta}^2\cos^2(\theta)$

$= \dot{r}^2\underbrace{(\cos^2(\theta)+\sin^2(\theta))}_{1} + r^2\dot{\theta}^2\underbrace{(\sin^2(\theta)+\cos^2(\theta))}_{1} = \dot{r}^2 + r^2\dot{\theta}^2 \quad \therefore \quad T = \frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)$

So, L becomes:
$$L = \frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2) + \frac{C}{r}e^{-\alpha r}$$

Finding EOM:
$$\frac{\partial L}{\partial r} - \frac{d}{dt}\frac{\partial L}{\partial \dot{r}} = 0, \quad \frac{\partial L}{\partial \theta} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}}$$

$$\frac{\partial L}{\partial r} = mr\dot{\theta}^2 - C\alpha\frac{e^{-\alpha r}}{r} - C\frac{e^{-\alpha r}}{r^2}, \quad \frac{\partial L}{\partial \dot{r}} = m\dot{r}, \quad \frac{d}{dt}\frac{\partial L}{\partial \dot{r}} = m\ddot{r}$$
$$m\ddot{r} = mr\dot{\theta}^2 + V(r)(\alpha + 1/r)$$

$\frac{\partial L}{\partial \theta} = 0 \longrightarrow$ This means angular momentum is conserved, $\frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta}$

Calculating the energy function:

$$h = \dot{r}\frac{\partial L}{\partial \dot{r}} + \dot{\theta}\frac{\partial L}{\partial \dot{\theta}} - L = \dot{r}\cdot m\dot{r} + \dot{\theta}\cdot mr^2\dot{\theta} - \frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2) + V(r) = \frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2) + V(r)$$

E then becomes: $E = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + V(r)$, because θ is cyclic, $\dot{\theta}^2 = \frac{\ell^2}{m^2r^4}$

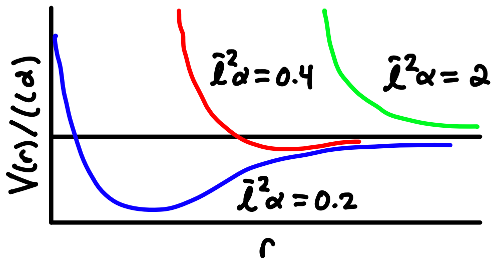
$E = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}\frac{\ell^2}{mr^2} + V(r)$, where $T = \frac{1}{2}m\dot{r}^2$ and $V_{eff} = \frac{1}{2}\frac{\ell^2}{mr^2} + V(r)$

$$\boxed{V_{eff} = \frac{1}{2}\frac{\ell^2}{mr^2} + V(r)}$$

sketches on next page

Problem 1 Continued

$$\frac{V_{eff}}{C\alpha} = \frac{\bar{l}^2\alpha}{\bar{r}^2} - \frac{e^{-\bar{r}}}{r} \quad w/ \ \bar{l}^2 = \frac{l^2}{2cm}, \quad \bar{r} = \alpha r$$



(b) Using your solution to (a), discuss and classify the expected motion of the particle as a function of the initial energy, E_0.

There are two main types of motion: i) Bound motion and ii) Scattering (Unbound) motion. Bound motion requires the potential to have a local minimum. We can see that the blue curve above is bound, and depending on where the particle is placed, it will either oscillate or rebound and leave. The red and green curves are unbound motion because no matter where the particle is placed, it will eventually leave the system.

Problem 1: Review

Procedure:

- Define generalized co-ordinates, in this case r and θ.
- Write out the kinetic and potential energies in terms of cartesian co-ordinates.
- Convert to polar co-ordinates r and θ.
- Write out the Lagrangian, and proceed to calculate the EOM with Euler Lagrange formalism.
- Since
$$\frac{\partial L}{\partial \theta} = 0$$
angular momentum is conserved and θ is cyclic. This means that we can write
$$\frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta}$$
- Proceed to calculate the energy function with
$$h = \sum_i^n \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L.$$
- Re-arrange terms and identify the effective potential.
- Solve for effective potential in a simpler form where one can determine curves of the potential easier.
- Comment on the behavior of our system, whether it is bound or unbound.

Key Concepts:

- Use the Euler Lagrange formalism to show that there is a cyclic co-ordinate. i.e. $\partial L/\partial q_k = 0$.
- Once a cyclic co-ordinate is defined, we can calculate the energy function.
- After the energy function is calculated, we can regroup terms to determine an effective potential.
- Bound motion is described by a potential curve that has a local minimum.
- Unbound motion is described by a potential curve that does not contain a local minimum.
- Depending on the location of the particle, it will either oscillate or rebound out.

Variations:

- The potential defined in the problem statement can change.
  - This would slightly change our problem, the math but not the over all procedure.

Problem 2

A typical potential describing the interaction between particles is very complicated and the analytic computation of scattering cross sections is difficult. However, we can often gain qualitative insight by approximating the precise form of the potential.

One common treatment of intermolecular scattering is the Lennard-Jones potential (which is in itself already an approximation):

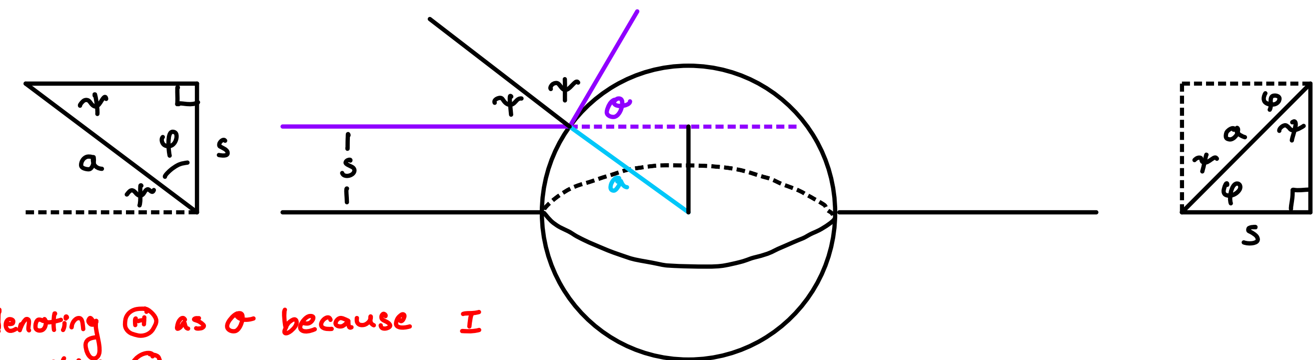
$$V_{LJ} = \mathcal{V}_0 \left[\left(\frac{a}{r}\right)^{12} - \left(\frac{a}{r}\right)^{6} \right]. \tag{2}$$

It features a divergent repulsive contribution at short ranges ($r << a$) and a long attractive tail ($r >> a$), with the minimum of the potential occurring at $r = 2^{1/6}a$.

(a) When the short-range repulsive contribution is expected to dominate the physics we can adopt a so-called "hard-core" interaction,

$$V_{hc}(r) = \begin{cases} \infty & \text{if } r \leq a_0 \\ 0 & \text{if } r > a_0 \end{cases}, \tag{3}$$

where a_0 characterizes the length scale of the interaction ($a_0 \neq a$). This potential is equivalent to the scattering of a particle of an impenetrable sphere. Compute the scattering cross section $\sigma(\Theta)$ and total cross section σ_T for this potential. Discuss and interpret both results.



I'm denoting Θ as θ because I hate writing Θ

$$2\psi + \theta = \pi \ , \ \psi + \varphi = \pi/2 \ \therefore \ 2\psi + 2\varphi = \pi \ : \ \cancel{2\psi} + \theta = \cancel{2\psi} + 2\varphi \ : \ \varphi = \theta/2$$

$$\cos(\theta/2) = \frac{s}{a} \ : \ s = a\cos(\theta/2)$$

The diffential coss section on θ

$$\sigma(\theta) = \frac{s}{\sin(\theta)} \left| \frac{ds}{d\theta} \right| \longrightarrow \text{EQ } (3.93)$$

$$\frac{ds}{d\theta} = -\frac{a}{2}\sin(\theta/2) \ : \ \left|\frac{ds}{d\theta}\right| = \frac{a}{2}\sin(\theta/2) \ \therefore \ \sigma(\theta) = \frac{s}{\sin(\theta)} \cdot \frac{a}{2}\sin(\theta/2)$$

$$\sigma(\theta) = \frac{a\cdot\cos(\theta/2)}{\sin(\theta)} \cdot \frac{a}{2}\sin(\theta/2) \ : \ \frac{1}{2}\sin(x)\cos(x) = \frac{1}{4}\sin(2x) \ : \ \sigma(\theta) = \frac{a^2}{4} \cdot \frac{\cancel{\sin(\theta)}}{\cancel{\sin(\theta)}} = \frac{a^2}{4}$$

$$\boxed{\sigma(\theta) = \frac{a^2}{4}}$$

Problem 2 Continued

Total scattering cross section

$$\sigma = \iint \sin\theta \, \sigma(\theta) \, d\phi d\theta$$
$$= \frac{a^2\pi}{2} \int \sin\theta \, d\theta$$
$$= a^2\pi$$

$$\boxed{\sigma = a^2\pi}$$

The differential cross section is not dependent upon the angle at which the particle strikes the sphere, rather it is only dependent upon the value of a.

The total cross section is reliant upon the area of which the particle strikes the sphere, and since this is a sphere the area of this section is the area of a circle.

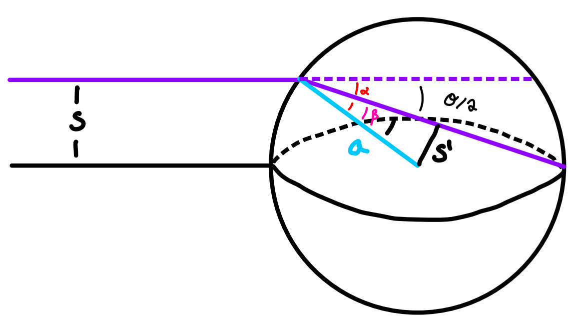
(b) When the attractive part of the interaction is important we instead adopt a so-called "soft-core" interaction,

$$V_{sc}(r) = \begin{cases} -V_0 & \text{if } r \leq a_0 \\ 0 & \text{if } r > a_0 \end{cases}. \tag{4}$$

Show that the impact parameter associated with this potential can be written as,

$$s = \frac{a \sin\Theta/2}{\sqrt{1 + \frac{1}{n^2} - \frac{2}{n}\cos(\Theta/2)}} \tag{5}$$

where $n = \sqrt{1 + V_0/E}$ and E is the energy of the incident particle. Hint: You might find it useful to consider energy and momentum conservation, and consider the associated expressions for $r \leq a_0$ and $r > a_0$ separately.



Energy outside : $E = T = \frac{1}{2}mv^2$, Angular momentum outside : $l = mvs$

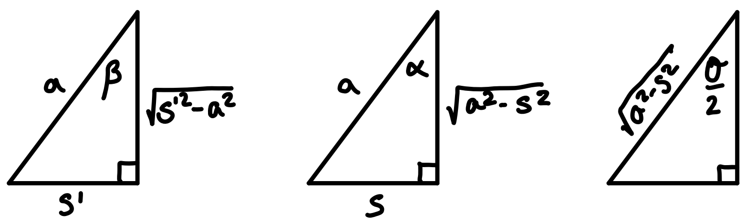
$v = \frac{l}{ms}$, $v^2 = \frac{l^2}{m^2 s^2}$ $\therefore$ $E = \frac{1}{2} m \cdot \frac{l^2}{m^2 s^2} = \frac{1}{2}\frac{l^2}{m s^2}$ $\therefore$ $l = s\sqrt{2Em}$

Energy Inside : $E = T + V = \frac{1}{2}m\tilde{v}^2 - V_0$: $\tilde{v}^2 = \frac{l^2}{m^2\tilde{s}^2}$: $E = \frac{1}{2}\frac{l^2}{m\tilde{s}^2} - V_0$

Problem 2 Continued

$$\sqrt{2m(E+V_0)}\,\tilde{s} = \ell \quad \therefore \quad \sqrt{2m(E+V_0)}\,\tilde{s} = s\sqrt{2Em}$$

$$\tilde{s} = s\,\frac{\sqrt{2Em}}{\sqrt{2m(E+V_0)}} = s\sqrt{\frac{E}{E+V_0}} \quad \therefore \quad s = \tilde{s}\sqrt{\frac{E+V_0}{E}} = \tilde{s}\sqrt{1+\frac{V_0}{E}} = \tilde{s}n$$



$$\sin(\beta) = \frac{\tilde{s}}{a} \quad \therefore \quad \tilde{s} = a\sin(\beta) \quad \text{w/} \quad \alpha = \frac{\sigma}{2} + \beta \quad \therefore \quad \beta = \alpha - \frac{\sigma}{2} \;:\; s = a\cdot\sin(\alpha)$$

$$\tilde{s} = a\left(\sin(\alpha)\cos(\sigma/2) - \cos(\alpha)\sin(\sigma/2)\right) = a\left(s/a\cos(\sigma/2) - \cos(\alpha)\sin(\sigma/2)\right)$$

$$\hat{s} = a\left(s/a\cos(\sigma/2) - \sqrt{a^2-s^2}/a\,\sin(\sigma/2)\right) = s\cos(\sigma/2) - \sqrt{a^2-s^2}\,\sin(\sigma/2)$$

$$s = \left(s\cos(\sigma/2) - \sqrt{a^2-s^2}\,\sin(\sigma/2)\right)n = s\cos(\sigma/2)n - \sqrt{a^2-s^2}\,\sin(\sigma/2)n$$

$$s - s\cos(\sigma/2)n = -\sqrt{a^2-s^2}\,\sin(\sigma/2)n \;:\; s(1-\cos(\sigma/2)n) = -\sqrt{a^2-s^2}\,\sin(\sigma/2)n$$

$$s^2(1-\cos(\sigma/2)n)^2 = (a^2-s^2)\sin^2(\sigma/2)n^2, \quad s^2(1-\cos(\sigma/2)n)^2 + s^2\sin^2(\sigma/2)n^2 = a^2\sin^2(\sigma/2)n^2$$

$$s^2\left(1 - 2\cos(\sigma/2)n + \cos^2(\sigma/2)n^2 + \sin^2(\sigma/2)n^2\right) = a^2\sin^2(\sigma/2)n^2$$

$$s^2\left(1+n^2-2\cos(\sigma/2)n\right) = a^2\sin^2(\sigma/2)n^2 \quad \therefore \quad s^2 = \frac{a^2n^2\sin^2(\sigma/2)}{1+n^2-2n\cos(\sigma/2)}$$

$$s^2 = \frac{a^2\sin^2(\sigma/2)}{1+1/n^2-2/n\cos(\sigma/2)} \quad \therefore \quad s = \frac{a\sin(\sigma/2)}{\sqrt{1+\frac{1}{n^2}-\frac{2}{n}\cos(\sigma/2)}}$$

$\blacksquare$

Problem 2: Review

Procedure:

- Begin by drawing out the hardcore potential scattering diagram.
- Determine the scattering angle in terms of the impact parameter s and the radius of the sphere a via geometric interpretations.
- Calculate the scattering angle via
$$\sigma(\theta) = \frac{s}{\sin(\theta)}\left|\frac{ds}{d\theta}\right|.$$
- Proceed to calculate the scattering cross section
$$\sigma = \int\int \sin\theta\,\sigma(\theta)\,d\phi\,d\theta.$$
- Draw the soft core potential scattering diagram.
- Use conservation of energy inside and outside the sphere along with conservation of angular momentum to solve for $s = \tilde{s}n$.
- Proceed to use geometric interpretations to solve for the relationship that is desired.

Key Concepts:

- Hard core potential scattering occurs when the particle rebounds off the sphere. The potential outside is 0 and inside is ∞.
- Soft core potential scattering occurs when the particle enters the sphere and has the trajectory diverted. The potential inside is constant and outside is 0.

Variations:

- The potential inside the sphere can be a different value.
  - If the potential is positive it will cause the particle to be repelled instead of attracted.
- We can be asked qualitative questions about the scattering.
  - Just use equations derived in the process of doing the problem.

Problem 3

Consider the central force field of the form $F(r) = k/r^3$.

(a) Using the formula,

$$\Theta(s,E) = \pi - 2\int_0^{u_{max}} \frac{s\ du}{\sqrt{1 - \frac{V(u)}{E} - s^2u^2}}, \tag{6}$$

with $u = 1/r$, show that,

$$\Theta(s,E) = \pi\left[1 - \frac{s\sqrt{2E}}{\sqrt{k + 2Es^2}}\right]. \tag{7}$$

In principle, one can obtain the scattering cross section from Eq. (7). However, we shall pursue a different route for this example. The motion of a particle in the central force can be written in terms of $u = 1/r$ as,

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{l^2}\frac{dV(u)}{du}. \tag{8}$$

$F(r) = k/r^3$: w/ $u = 1/r$: $F(u) = ku^3$, $F = -\partial V/\partial r$ $\therefore$ $V = -\int F\,dr$

$V = -\int kr^{-3}\,dr = \frac{k}{2r^2}$: w/ $u = 1/r$ $\therefore$ $V = \frac{k}{2}u^2$

$$\Theta(S,E) = \pi - 2\int_0^{u_{max}} \frac{s\,du}{\sqrt{1 - K/(2E)\,u^2 - S^2u^2}} = \pi - 2\int_0^{u_{max}} \frac{s\,du}{\sqrt{1 - u^2\,(K/2E + S^2)}}$$

w/ $a = K/2E + S^2$: $\Theta(S,E) = \pi - 2\int_0^{u_{max}} \frac{s\,du}{\sqrt{1 - u^2a}}$: $\int \frac{dx}{\sqrt{1 - x^2a}} = \frac{\arcsin(\sqrt{a}\,x)}{\sqrt{a}}$

$\pi - 2\int_0^{u_{max}} \frac{s\,du}{\sqrt{1-u^2a}} = \pi - 2\frac{\arcsin(\sqrt{a}\,\tilde{u})\cdot S}{\sqrt{a}}$, $\tilde{u} = \frac{1}{\sqrt{a}}$, $\pi - \frac{2\cdot\arcsin(1)S}{\sqrt{a}}$

$$\Theta(S,E) = \pi - \frac{2\cdot\pi/2\,S}{\sqrt{a}} = \pi\left(1 - \frac{S}{\sqrt{a}}\right) = \pi\left(1 - \frac{S\sqrt{2E}}{\sqrt{K + 2ES^2}}\right) \checkmark$$

■

(b) Show that a parametrization of the motion is,

$$u(\theta) = \alpha\cos\gamma\theta + \beta\sin\gamma\theta, \qquad \gamma = \sqrt{1 + \frac{mk}{l^2}}. \tag{9}$$

$\frac{\partial u}{\partial \theta} = -\alpha\gamma\sin(\gamma\theta) + \beta\gamma\cos(\gamma\theta)$, $\frac{\partial^2 u}{\partial \theta^2} = -\alpha\gamma^2\cos(\gamma\theta) - \beta\gamma^2\sin(\gamma\theta)$

$\frac{\partial^2 u}{\partial \theta^2} + u = -\alpha\gamma^2\cos(\gamma\theta) - \beta\gamma^2\sin(\gamma\theta) + \alpha\cos(\gamma\theta) + \beta\sin(\gamma\theta)$

Problem 3 Continued

$$\cos(\gamma\theta)\alpha(1-\gamma^2) + \sin(\gamma\theta)\beta(1-\gamma^2) = (1-\gamma^2)\overbrace{(\cos(\gamma\theta)\alpha + \sin(\gamma\theta)\beta)}^{u}$$

$$1-\gamma^2 = 1 - \left(1+\frac{mk}{\ell^2}\right) = -\frac{mk}{\ell^2} \quad \therefore \quad LHS = -\frac{mk}{\ell^2}u$$

$$\frac{dv}{du} = ku \quad \therefore \quad RHS = -\frac{mk}{\ell^2}u \quad \therefore \quad LHS = RHS$$

$\blacksquare$

(c) Use that the particle approaches from an initial angle $\theta_i = \pi$ to show that: i) $\alpha = -\beta\tan(\gamma\pi)$ and ii) $\gamma = \pi/(\Theta - \pi)$.

$$u(\theta) = \alpha\cos(\gamma\theta) + \beta\sin(\gamma\theta) \;:\; \text{w/ } u(\theta) = 0 \;:\; 0 = \alpha\cos(\gamma\theta) + \beta\sin(\gamma\theta) \;:\; 0 = \alpha + \beta\operatorname{Tan}(\gamma\theta)$$

$$\text{w/ } \theta = \pi \quad 0 = \alpha + \beta\operatorname{Tan}(\pi\gamma) \quad \therefore \quad \alpha = -\beta\operatorname{Tan}(\pi\gamma) \checkmark$$

$$u(\theta) = \alpha\cos(\gamma\theta) + \beta\sin(\gamma\theta) \quad \text{w/ } u(\theta) = 0 \;:\; \alpha\cos(\gamma\Theta) + \beta\sin(\gamma\Theta) = 0$$

$$-\beta\operatorname{Tan}(\pi\gamma)\cos(\gamma\Theta) + \beta\sin(\gamma\Theta) = 0 \;:\; -\operatorname{Tan}(\pi\gamma) + \operatorname{Tan}(\gamma\Theta) = 0 \quad \therefore \quad \pi\gamma = \gamma\Theta$$

$$\text{or} \quad -\operatorname{Tan}(\pi\gamma + \pi) + \operatorname{Tan}(\gamma\Theta) = 0 \;:\; \pi\gamma + \pi = \gamma\Theta \;:\; \pi = \gamma(\Theta - \pi) \quad \therefore \quad \gamma = \frac{\pi}{\Theta - \pi}$$

$\hookrightarrow$ Periodicity of Tan by π $\blacksquare$

(d) Finally, defining $x = \Theta/\pi$, show that the cross section can be obtained as,

$$\sigma(\Theta)d\Theta = \frac{k}{E}\frac{(1-x)dx}{x^2(2-x)^2\sin(\pi x)}. \tag{10}$$

$$\sigma(\theta)d\theta = \frac{s\,ds}{\sin(\theta)} \;,\; \frac{\theta}{\pi} = \left[1 - \frac{s\sqrt{2E}}{\sqrt{k+2Es^2}}\right] = x \;:\; 1-x = \frac{s\sqrt{2E}}{\sqrt{k+2Es^2}} \;:\; (1-x)^2 = \frac{s^2\cdot 2E}{k+2Es^2}$$

$$(1-x)^2(k+2Es^2) = s^2 2E \;:\; (1-x)^2k + (1-x)^2 2Es^2 = 2Es^2 \;:\; (1-x)^2k = 2Es^2 - (1-x)^2 2Es^2$$

$$(1-x)^2k = 2Es^2(1-(1-x)^2) \;:\; s^2 = \frac{k}{2E}\frac{(1-x)^2}{(1-(1-x)^2)} = \frac{k}{2E}\frac{(1-x)^2}{x(2-x)}$$

$$2s\cdot ds = \frac{k}{\cancel{2}E}\frac{\cancel{2}\cdot(x-1)}{x^2(x-2)^2}dx \;:\; s\,ds = \frac{k}{E}\frac{(x-1)}{x^2(x-2)^2}dx \;:\; \sigma(\theta) = \frac{s}{\sin\theta}\left|\frac{ds}{d\theta}\right|$$

$$\sigma(\theta)d\theta = \frac{s\,ds}{\sin(\theta)} = \frac{k}{E}\frac{(x-1)dx}{x^2(x-2)^2\sin(\pi x)} = \frac{k}{E}\frac{(1-x)dx}{x^2(2-x)^2\sin(\pi x)} \checkmark$$

$\blacksquare$

Problem 3: Review

Procedure:

- Begin by substituting $u = 1/r$ into the force equation $F(r)$.
- Insert this relationship into the integral $\Theta(s, E)$.
- Simplify the integral so that it can be solved by the use of an integral table.
- Solve the integral and proceed to show the relationship that is desired.
- Proceed to differentiate $u(\theta)$ by θ two times and insert the equation for γ into the result.
- Show that the LHS and RHS are equal.
- Use the $u(\theta)$ equation in equation (9) again and show that the relationships can be obtained by manipulating the equations.
- Show that equation (10) can be obtained by using the relationship in the problem statement along with equation (7).
- Solve this modified equation for s^2 and differentiate the result with respect to x. Manipulate this result and show that the result in equation (10) can be obtained.

Key Concepts:

- Integrals can be simplified by changes of variables such that the resulting integral can be easier to solve.
- One can calculate the scattering cross section via equation (6) in the form of an integral.

Variations:

- For a variation to occur, the entire problem would have to be changed. Thus this is the only way the current problem can be presented.

Problem 4

Read Secs. I and II of the article "Elastic scattering by a paraboloid of revolution" by Evan James at `https://doi.org/10.1119/1.15593` [Am. J. Phys. **56**, 423 (1988)]. Write a brief summary of the article, discussing the main conclusions and results of the work.

In this article, the author discusses scattering of particles that are incident on a hard paraboloid. The author derives a differential scattering cross section and then proceeds to use this result with others to define what the scattering cross section will be. The author also discusses how the distribution of angles by the scattered particles is the same as electrostatic scattering of charged particles. Using Rutherford scattering and previous equations that were derived, the author goes on to compare the cross section scattering with differential cross section scattering of a hard sphere. From this the author goes on to deduce that the total scattering of cross sections will be exactly the same. More so, the differential cross section scattering will be close to the same of large scattering angles. When these angles are reduced the difference slowly goes to zero.

Problem 4: Review

Procedure:

- Read the article and write out a summary for said article.