Problem:

(a) In an ideal classical gas of density n at temperature T, what is the total distance travelled by all the particles within a volume $\tilde{V}$ per unit time?

(b) Within a volume $\tilde{V}$ in a classical gas of spherical particles of diameter d, at what rate do collisions occur?

Note: Determine the rate using the Maxwell-Boltzmann distribution and the cross section.

(c) Using your results from (a) and (b), what is the average distance between collisions of a particle in an ideal classical gas?

(a)

$$\text{average speed } \bar{v} = 4\pi \int_0^\infty v \underbrace{n \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv^2}{2kT}}}_{\text{Maxwell Boltzmann distribution}} v^2 \, dv$$

$$= \left(\frac{8kT}{\pi m} \right)^{1/2}$$

N: # of particles $\rightarrow N = n \tilde{V}$

$\Rightarrow$ total distance traveled by all particles in $\tilde{V}$:

$$= N \bar{v} = \underbrace{n \tilde{V}}_{N} \underbrace{\left(\frac{8kT}{\pi m} \right)^{1/2}}_{\bar{v}}$$

(b)

the cross section is πd^2

$$\text{let } P(\bar{v}_1) = \underbrace{n \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{m \bar{v}_1^2}{2kT}}}_{\text{Maxwell Boltzmann distribution}}$$

$$R = \frac{1}{2}\pi d^2 \tilde{V} \int_{\text{all velocities}} P(\vec{v}_1)\, P(\vec{v}_2)\, |\vec{v}_1 - \vec{v}_2|\, d^3 v_1\, d^3 v_2$$

we are looking at all relative velocities: say $\vec{v}_1 = \vec{v}\,'$ and $\vec{v}_2 = \vec{v}\,''$ $\rightarrow$ we get the same $|\vec{v}_1 - \vec{v}_2|$ if $\vec{v}_1 = \vec{v}\,''$ and $\vec{v}_2 = \vec{v}\,'$; thus, we need a factor of $\frac{1}{2}$ to avoid double-counting

$\vec{v} = \vec{v}_1 - \vec{v}_2$ rel. velocity

$\vec{V} = (\vec{v}_1 + \vec{v}_2)/2$ CM velocity

$$\Rightarrow P(\vec{v}_1)\, P(\vec{v}_2) = n^2 \left(\frac{m}{2\pi kT}\right)^3 e^{-\frac{m\vec{v}^2}{4kT}}\, e^{-\frac{m\vec{V}^2}{kT}}$$

$$R = \frac{1}{2} n^2 \pi d^2 \tilde{V} \left(\frac{m}{kT}\right)^{-1/2} \frac{1}{\sqrt{\pi}}\, 4$$

$$\Rightarrow R = \frac{1}{2}\underbrace{\underbrace{\left(\frac{\pi kT}{m}\right)^{1/2}}_{\text{units of } \frac{\text{length}}{\text{time}}} \underbrace{4 \tilde{V} d^2 n^2}_{\text{units of } 1/\text{length}}}_{\text{units of } 1/\text{time}}$$

(c)

$$\begin{array}{c}\text{average distance} \\ \text{between collisions}\end{array} = \frac{\text{total distance traveled}}{\text{\# of collisions encountered}}$$

$$= \frac{n \tilde{V} \left(\frac{8kT}{\pi m}\right)^{1/2}}{2R} \quad \leftarrow \text{from (a)}$$

the reason that we have a 2 here is that each collision involves two particles

$$= \frac{n \tilde{V} \left(\frac{8kT}{\pi m}\right)^{1/2}}{2 \cdot 2 \tilde{V} n^2 d^2 \left(\frac{\pi k T}{m}\right)^{1/2}}$$

$$= \frac{\sqrt{2}}{2 n d^2 \pi} = \frac{1}{\sqrt{2}\, n d^2 \pi}$$