PHYS5153 Assignment 5

Due: 1:30pm on 10/06/2021.
Marking: Total of 10 marks (weighting of each question is indicated).
Fine print: Solutions should be presented legibly (handwritten or LaTeX is equally acceptable) so that the grader can follow your line of thinking and any mathematical working should be appropriately explained/described. If you provide only equations you will be marked zero. If you provide equations that are completely wrong but can demonstrate some accompanying logical reasoning then you increase your chances of receiving more than zero. If any of your solution has relied on a reference or material other than the textbook or lectures, please note this and provide details.

Question 1 (3 marks)

Consider a small marble placed at rest on top of an impenetrable sphere of radius R and subject only to gravity. Let's start the problem by being adventurous physicists and assume that the marble may be treated as a point particle of mass m.

(a) Determine the forces of constraint using the Lagrange equations of motion and Lagrange multipliers. (You may simplify your approach by considering that you will want to use it to solve (b)...).

(b) At what height (relative to the base of the sphere) does the marble fall off the sphere.

(c) In reality, the marble has a finite radius a. If this is accounted for, at what does the marble fall off the sphere? Assume it rolls without slipping while on the sphere.

Question 2 (3 marks)

Let us revisit a previous problem, Q3 on Assignment 3. We considered the situation in Fig. 1, where a pair of blocks of mass M and m, respectively, are connected by a massless string of length l. The former mass lies on top of a table while the latter hangs below, with the string passing through a small hole in the centre of the table. The motion of the hanging block is restricted to be only in the vertical direction (e.g., along $\hat{z}$ only), while the block on the table is restricted to move in a 2D plane (e.g., the $\hat{x} - \hat{y}$ plane defined by the table's surface).

(a) Write down the Lagrangian and equations of motion to obtain the generalized forces of constraint for the three co-ordinates r, φ and z, where the former are polar co-ordinates describing the motion of the mass on the table such that

$$\begin{aligned} x &= r\cos\varphi, \\ y &= r\sin\varphi, \end{aligned} \tag{1}$$

and z describes the vertical position of the hanging block. You should derive the equations of motion using Lagrange multipliers and then eliminate them to determine the generalized forces.

(b) Using the generalized forces you deduced in (a), show that the block on the table will not be pulled down through the hole if it is initially rotating with frequency,

$$\dot{\varphi} = \sqrt{\frac{Mg}{mr_0}} \tag{2}$$

where $r_0 = r(0)$ is the initial distance of the block lying on the table from the hole.

(c) Write an integral of the form $t(r) = \int_{r_0}^{r_f} ...dr'$ that in principle could be solved and inverted to provide an expression for the motion of $r(t)$.

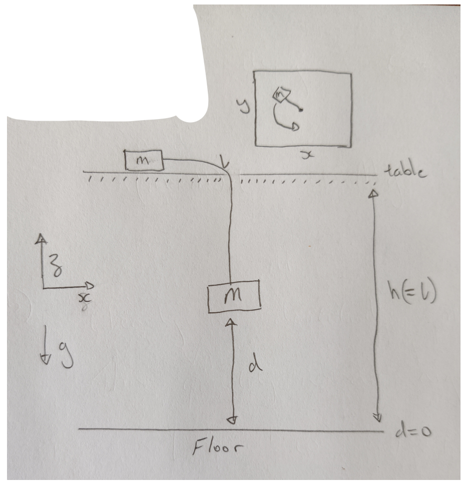

Figure 1: Physical system for question 2

Question 3 (3 marks)

Sometimes, the effects of dissipation or frictional forces on a system can be captured by a time-dependent Lagrangian. An example is the one-dimensional system described by,

$$L = e^{\gamma t}\left(\frac{m}{2}\dot{q}^2 - \frac{k}{2}q^2\right). \tag{3}$$

(a) Obtain the equation of motion for the system defined by Eq. (3). What does it appear to describe?

(b) Is the *energy function* associated with Eq. (3) conserved?

(c) Introduce the transformation $Q = e^{\gamma t/2}q$ and rewrite the Lagrangian (3) in terms of the new co-ordinate Q.

(d) Show that the Lagrangian of (c) features a conserved quantity. How does the form of the conserved quantity connect to your secondary answer to part (a).

Question 4 (1 mark)

A point particle of mass m is restricted to move on the surface of a cylinder with radius R. The particle is subject to a force $\mathbf{F} = -k\mathbf{r}$ where $\mathbf{r}$ is the position vector of the particle with respect to the center of the cylinder. For simplicity, take the cylinder to be infinitely long so the particle will never 'fall' off the top or bottom edge.

(e) Write an equation describing any constraint(s) in the system and thus identify an appropriate set of generalized co-ordinates.

(f) Compute the Lagrangian of the system and derive the equations of motion(s) for your chosen set of generalized co-ordinate(s). Do your equations enable you to identify any constants of motion associated with your Lagrangian? If so, what do they correspond to?

(g) Discuss the physical significance of the equation(s) you derived in (f) and the dynamics they describe.