Key points lecture 04/25/2022

Hamiltonian of photon gas: $\hat{\mathcal{H}} = \sum_{\vec{k},\varepsilon} \underbrace{\hbar c k}_{\hbar\omega} \hat{a}^{+}_{\vec{k},\varepsilon} \hat{a}_{\vec{k},\varepsilon}$

Energy of photon gas: $E = \sum_{\vec{k},\varepsilon} \hbar c k \, n_{\vec{k},\varepsilon}$, $n_{\vec{k},\varepsilon} = 0, 1, 2, \ldots$

$\uparrow$ 2 polarization directions ($\vec{k} \cdot \hat{\varepsilon} = 0$)

N (number of photons) = internal variable $\Rightarrow \mu = 0$

$$Q = \sum_{\{n_{\vec{k},\varepsilon}\}} e^{-\beta E\{n_{\vec{k},\varepsilon}\}} = \prod_{\vec{k},\varepsilon} \left(1 - e^{-\beta \hbar c k}\right)^{-1}$$

$$\Rightarrow \log Q = -2 \sum_{\vec{k}} \log\left(1 - e^{-\beta \hbar c k}\right)$$

$PV = \frac{1}{3} U$ (non-relativistic: $PV = \frac{2}{3} U$)

$$\frac{U}{V} \propto T^4 \quad (\text{Stefan's law}) \quad \rightsquigarrow \frac{U}{V} = \int_0^\infty \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\beta\hbar\omega} - 1} \, d\omega$$

max of this fct is realized at $\omega_{max} = 2.821 \frac{kT}{\hbar}$

(Wien's displacement law)

Phonons

Einstein: N_E distinct modes $\rightsquigarrow$ behavior at low T not correct

Debye: not all frequencies are equal $\rightarrow \int_0^{k_m} 3 \frac{1}{\left(\frac{2\pi}{L}\right)^3} 4\pi k^2 dk = 3N$

$\Rightarrow \frac{C_V}{Nk} \rightarrow T^3$ at low T and $\rightarrow 3$ at high T

$$\lambda_m = \frac{2\pi}{k_m} = \frac{2\pi c}{\omega_m} = \left(\frac{4}{3}\pi v\right)^{1/3}$$