COLLEGE OF ARTS AND SCIENCES
HOMER L. DODGE
DEPARTMENT OF PHYSICS AND ASTRONOMY
The UNIVERSITY of OKLAHOMA

Quantum Mechanics 1

PHYS 5393 HOMEWORK ASSIGNMENT #2

PROBLEMS: {1.9, 1.11, 1.17, 1.18, Q-1}

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STUDENT
Taylor Larrechea

PROFESSOR
Dr. Phillip Gutierrez

Problem 1: 1.9

Consider a ket space spanned by the eigenkets $\{|a'\rangle\}$ of a Hermitian operator $\tilde{\mathbf{A}}$. There is no degeneracy.

(a) Prove that

$$\prod_{a'}(\tilde{\mathbf{A}} - a')$$

is the null operator.

$\tilde{A}|a'\rangle = a'|a'\rangle$, $\tilde{A}|a''\rangle = a''|a''\rangle$, $\tilde{A}|a'''\rangle = a'''|a'''\rangle$, $\tilde{A}|a^4\rangle = a^4|a^4\rangle$

we need to first assume a vector $|\alpha\rangle$

$$\prod_{a'}(A-a')|\alpha\rangle = \sum_{a''}\prod_{a'}(A-a')|a''\rangle\langle a''|\alpha\rangle = \sum_{a''}|a''\rangle\langle a''|\prod_{a'}(A-a')|\alpha\rangle$$

Because $|a'\rangle, |a''\rangle, \ldots,$ are all eigenvalues of A;

$$\prod_{a'}(A-a')|\alpha\rangle = \sum_{a''}|a''\rangle\langle\alpha|\prod_{a'}(a''-a')|a''\rangle = |0\rangle$$

$$\boxed{\prod_{a'}(A-a') = |0\rangle}$$

(b) What is the significance of

$$\prod_{a''\neq a'}\frac{(\tilde{\mathbf{A}} - a'')}{a' - a''}?$$

$\tilde{A}|a'\rangle = a'|a'\rangle$, $\tilde{A}|a''\rangle = a''|a''\rangle$, $\tilde{A}|a'''\rangle = a'''|a'''\rangle$, $\tilde{A}|a^4\rangle = a^4|a^4\rangle$

we need to first assume a vector $|\alpha\rangle$

$$\prod_{a''\neq a'}\frac{(A-a'')}{(a'-a'')}|\alpha\rangle = \sum_{a'''}\prod_{a''\neq a'}\frac{(A-a'')}{(a'-a'')}|a'''\rangle\langle a'''|\alpha\rangle = \sum_{a'''}|a'''\rangle\langle a'''|\prod_{a''\neq a'}\frac{(A-a'')}{(a'-a'')}|\alpha\rangle$$

Because $|a'\rangle, |a''\rangle, \ldots,$ are all eigenvalues of A;

$$\prod_{a''\neq a'}\frac{(A-a'')}{(a'-a'')}|\alpha\rangle = \sum_{a'''}\langle a'''|\alpha\rangle\prod_{a''\neq a'}\frac{(a'''-a'')}{(a'-a'')}|a'''\rangle = \langle a'''|\alpha\rangle|a'''\rangle = |a'''\rangle\langle a'''|\alpha\rangle$$

$$\boxed{\prod_{a''\neq a'}\frac{(A-a'')}{(a'-a'')} = |a'\rangle\langle a'|\alpha\rangle}$$

$\tilde{P}_1 = |a'\rangle\langle a'|$, this operator is projecting on an eigenket.

Problem 1: 1.9 Continued

(c) Illustrate using (a) and (b) using $\tilde{\mathbf{A}}$ set equal to $\tilde{\mathbf{S}}_z$ of a spin 1/2 system.

$$\prod_{a'}(A-a') = \left(\tilde{S}_z - \frac{\hbar}{2}\right)\left(\tilde{S}_z + \frac{\hbar}{2}\right) = |0\rangle$$

$$\blacksquare$$

$$\boxed{\prod_{a''\neq a'} \frac{(A-a'')}{(a'-a'')} = \begin{cases} \dfrac{\tilde{S}_z + \hbar/2}{\hbar} \\[2ex] \dfrac{\tilde{S}_z - \hbar/2}{-\hbar} \end{cases}}$$

Problem 1: 1.9 Review

Procedure:

- Begin by expanding in a complete set, rearrange the equation and show that this is the null operator.
- Begin again by expanding in a complete set for part (b), rearrange the equation, and show that this is a projection operator acting on an eigenstate.
- For part (c), substitute $\tilde{\mathbf{S}}_z$ for $\tilde{\mathbf{A}}$ and show that the results for (a) and (b) can be obtained.

Key Concepts:

- Expand in a complete set of the eigenstates of an operator when we do not explicitly know how an operator acts on a state that is not an eigenstate of that operator.
- When we expand in a complete set we can show that the operator in (a) is a null operator and the operator in (b) is a projection operator.

Variations:

- The operator that we are given in (a) and (b) can change.
  - This then would change the final answer of our problem but we would still want to expand in a complete set to determine how the operator acts.
- We can be given a different Spin 1/2 operator in (c).
  - This would cause our problem to have different eigenvalues but should still result in the same answer in parts (a) and (b).

Problem 2: 1.11

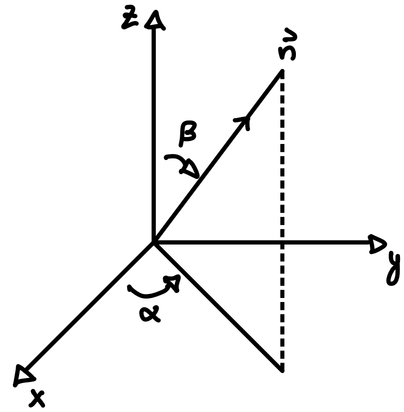
Construct $|\hat{S} \cdot \hat{n}; +\rangle$ such that

$$S \cdot \hat{n} \, |S \cdot \hat{n}; +\rangle = \left(\frac{\hbar}{2}\right) |S \cdot \hat{n}; +\rangle$$

where $\hat{n}$ is characterized by the angles shown in the figure. Express your answer as a linear combination of $|+\rangle$ and $|-\rangle$. Note: The answer is

$$\cos\frac{\beta}{2} |+\rangle + \sin\frac{\beta}{2} e^{-i\alpha} |-\rangle .$$

But do not just verify that this answer satisfies the above eigenvalue equation. Rather, treat the problem as a straight forward eigenvalue problem. Also do not use rotation operators, which we will introduce later in this book.



$$\hat{S} = (S_x + S_y + S_z) = \frac{\hbar}{2}\Big((|+\rangle\langle -| + |-\rangle\langle +|)\vec{x} + (-i|+\rangle\langle -| + |-\rangle\langle +|)\vec{y} + (|+\rangle\langle +| - |-\rangle\langle -|)\vec{z} \Big)$$

$$\vec{n} = (n_x + n_y + n_z) = (\cos(\alpha)\sin(\beta)\vec{x} + \sin(\alpha)\sin(\beta)\vec{y} + \cos(\beta)\vec{z})$$

$$|\hat{S}\cdot\vec{n};+\rangle = a|+\rangle + b|-\rangle$$

$$\hat{S}\cdot\vec{n} = \frac{\hbar}{2}\Big(\overset{\theta}{\cos(\alpha)\sin(\beta)}(|+\rangle\langle -| + |-\rangle\langle +|) + \overset{\phi}{\sin(\alpha)\sin(\beta)}(i|-\rangle\langle +| - i|+\rangle\langle -|) + \overset{\lambda}{\cos(\beta)}(|+\rangle\langle +| - |-\rangle\langle -|) \Big)$$

$$\hat{S}\cdot\vec{n}\,|\hat{S}\cdot\vec{n};+\rangle = \frac{\hbar}{2}\Big(a\theta(|+\rangle\langle -| + |-\rangle\langle +|)|+\rangle + a\phi(i|-\rangle\langle +| - i|+\rangle\langle -|)|+\rangle + a\lambda(|+\rangle\langle +| - |-\rangle\langle -|)|+\rangle +$$
$$b\theta(|+\rangle\langle -| + |-\rangle\langle +|)|-\rangle + b\phi(i|-\rangle\langle +| - i|+\rangle\langle -|)|-\rangle + b\lambda(|+\rangle\langle +| - |-\rangle\langle -|)|-\rangle \Big)$$

$$\hat{S}\cdot\vec{n}\,|\hat{S}\cdot\vec{n};+\rangle = \frac{\hbar}{2}\Big(a\theta|-\rangle + ia\phi|-\rangle + a\lambda|+\rangle + b\theta|+\rangle - ib\phi|+\rangle - b\lambda|-\rangle \Big)$$

$$\hat{S}\cdot\vec{n}\,|\hat{S}\cdot\vec{n};+\rangle = \frac{\hbar}{2}\Big((a\lambda + b\theta - ib\phi)|+\rangle + (a\theta + ia\phi - b\lambda)|-\rangle \Big)$$

$$\Big((a\lambda + b\theta - ib\phi)|+\rangle + (a\theta + ia\phi - b\lambda)|-\rangle \Big) = a|+\rangle + b|-\rangle$$

$$a = a\lambda + b\theta - ib\phi \longrightarrow a = a\cos(\beta) + b(\cos(\alpha)\sin(\beta) - i\sin(\alpha)\sin(\beta)) = a\cos(\beta) + b\sin(\beta)e^{-i\alpha}$$

$$b = a\theta + ia\phi - b\lambda \longrightarrow b = a(\cos(\alpha)\sin(\beta) + i\sin(\alpha)\sin(\beta)) - b\cos(\beta)) = a\sin(\beta)e^{i\alpha} - b\cos(\beta)$$

$$a = a\cos(\beta) + b\sin(\beta)e^{-i\alpha}, \quad a = \frac{b\sin(\beta)e^{-i\alpha}}{1-\cos(\beta)} \; : \; b = a\sin(\beta)e^{i\alpha} - b\cos(\beta), \quad b = \frac{a\sin(\beta)e^{i\alpha}}{1+\cos(\beta)}$$

Problem 2: 1.11 Continued

$$|a^2|+|b^2|=1 \;:\; a^2+\frac{a^2\sin^2(\beta)}{(1+\cos(\beta))^2}=1 \;:\; a^2\left(1+\frac{\sin^2(\beta)}{(1+\cos(\beta))^2}\right)=1$$

$$a^2\left(\frac{(1+\cos(\beta))^2+\sin^2(\beta)}{(1+\cos(\beta))^2}\right)=1 \;:\; a^2\left(\frac{1+2\cos(\beta)+\cos^2(\beta)+\sin^2(\beta)}{(1+\cos(\beta))^2}\right)=1$$

$$a^2\left(\frac{2+2\cos(\beta)}{(1+\cos(\beta))^2}\right)=1 \;:\; 2a^2\left(\frac{\cancel{(1+\cos(\beta))}}{(1+\cos(\beta))^{\cancel{2}}}\right)=1 \;:\; a^2\frac{1}{1+\cos(\beta)}=\frac{1}{2} \;:\; a^2=\frac{1}{2}(1+\cos(\beta))$$

$$\cos^2(\theta)=\frac{1}{2}(1+\cos(2\theta)) \quad w/\ \theta=\beta/2,\quad a^2=\cos^2(\beta/2),\quad a=\cos(\beta/2)$$

$$b=\frac{\cos(\beta/2)\sin(\beta)e^{i\alpha}}{(1+\cos(\beta))}=\frac{\cancel{\cos(\beta/2)}\sin(\beta)e^{i\alpha}}{2\cos^{\cancel{2}}(\beta/2)}=\frac{1}{2}\frac{\sin(\beta)e^{i\alpha}}{\cos(\beta/2)}$$

$$\sin(\beta)=2\sin(\beta/2)\cos(\beta/2)e^{i\alpha}$$

$$b=\frac{\cancel{2}\sin(\beta/2)\cancel{\cos(\beta/2)}e^{i\alpha}}{\cancel{2}\cancel{\cos(\beta/2)}}=\sin(\beta/2)e^{i\alpha}$$

with $a=\cos(\beta/2)$ and $b=\sin(\beta/2)e^{i\alpha}$

$$\boxed{|\hat{S}\cdot\vec{n};+\rangle=\cos(\beta/2)|+\rangle+\sin(\beta/2)e^{i\alpha}|-\rangle}$$

Problem 2: 1.11 Review

Procedure:

- Begin by writing out the spin operator like
$$\tilde{\mathbf{S}} = (\tilde{\mathbf{S}}_x + \tilde{\mathbf{S}}_y + \tilde{\mathbf{S}}_z)$$
where we have
$$\tilde{\mathbf{S}}_x = \frac{\hbar}{2}(\left|+\right\rangle \left\langle -\right| + \left|-\right\rangle \left\langle +\right|) \quad \tilde{\mathbf{S}}_y = \frac{\hbar}{2}(\left|-\right\rangle \left\langle +\right| - i \left|+\right\rangle \left\langle -\right|) \quad \tilde{\mathbf{S}}_z = \frac{\hbar}{2}(\left|+\right\rangle \left\langle +\right| - \left|-\right\rangle \left\langle -\right|).$$
- Use the relationship
$$\left|\tilde{\mathbf{S}} \cdot \vec{n}; +\right\rangle = a \left|a\right\rangle + b \left|-\right\rangle$$
where me have
$$\vec{n} = (n_x + n_y + n_z) = (\cos(\alpha)\sin(\beta)\vec{x} + \sin(\alpha)\sin(\beta)\vec{y} + \cos(\beta)\vec{z}).$$
- Proceed to use the above relationships and solve for the constants a and b.

Key Concepts:

- We can describe the total Spin 1/2 operator by summing the individual components. The same can be said for $\vec{n}$.
- The $\left|+\right\rangle$ and $\left|-\right\rangle$ are rotated by the angles β and α.

Variations:

- Since this problem is essentially a proof again, it cannot change that much.
- The problem can be changed where one of the angles is equal to 0.
  - This would make it very easy since each individual component would only be rotated by the same angle.

Problem 3: 1.17

Let $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ be observables. Suppose the simultaneous eigenkets of $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ $\{|a', b'\rangle\}$ form a complete orthonormal set of base kets. Can we always conclude that

$$[\tilde{\mathbf{A}}, \tilde{\mathbf{B}}] = 0?$$

If your answer is yes, prove the assertion. If your answer is no, give a counter-example.

Simultaneous means if an observable acts on one of two states, it will not affect the other state.

$$\tilde{A}|a',b'\rangle = a|a',b'\rangle \ , \ \tilde{B}|a',b'\rangle = b|a',b'\rangle \ : \ [\tilde{A},\tilde{B}] = \tilde{A}\tilde{B} - \tilde{B}\tilde{A}$$

To show $[\tilde{A},\tilde{B}] = 0$ we will do the following : $[\tilde{A},\tilde{B}]|a',b'\rangle$

$$[\tilde{A},\tilde{B}]|a',b'\rangle = [\tilde{A}\tilde{B} - \tilde{B}\tilde{A}]|a',b'\rangle = \tilde{A}\tilde{B}|a',b'\rangle - \tilde{B}\tilde{A}|a',b'\rangle$$

$$\tilde{A}\tilde{B}|a',b'\rangle = a\tilde{B}|a',b'\rangle = ab|a',b'\rangle \ : \ \tilde{B}\tilde{A}|a',b'\rangle = b\tilde{A}|a',b'\rangle = ba|a',b'\rangle$$

Scalar products are commutative therefore $a \cdot b = b \cdot a$, and thus

$$[\tilde{A},\tilde{B}]|a',b'\rangle = ab|a',b'\rangle - ba|b',a'\rangle = ab|a',b'\rangle - ab|a',b'\rangle = 0$$

From this we can conclude that simultaneous eigenkets of A and B that form a complete orthornormal set of base kets will always have

$$\boxed{[A,B] = 0}$$

Problem 3: 1.17 Review

Procedure:

- Take the commutation of $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ and apply it on an arbitrary eigenstate of $|a', b'\rangle$.
- From the above we can deduce that the coefficients ab must be equal to ba.

Key Concepts:

- Simultaneous eigenkets mean if an observable acts on two states, it will not affect the other state. It also means that the order in which they are applied is arbitrary and will produce the same result.
- For the observables to commute this means that $ab = ba$.
- If two observables are simultaneous this means that the their eigenstates form a complete orthonormal set of base kets.

Variations:

- We could be asked to prove this for the anticommutator.
  - We would use the same procedure and deduce what the eigenvalues would have to be relative to one another.

Problem 4: 1.18

Two Hermitian operators anticommute:

$$\{\tilde{\mathbf{A}}, \tilde{\mathbf{B}}\} = \tilde{\mathbf{A}}\tilde{\mathbf{B}} + \tilde{\mathbf{B}}\tilde{\mathbf{A}} = 0.$$

Is it possible to have a simultaneous (that is, common) eigenket of $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$? Prove or illustrate your assertion.

Simultaneous means if an observable acts on one of two states, it will not affect the other state.

$$\tilde{A}|a',b'\rangle = a|a',b'\rangle \;,\; \tilde{B}|a',b'\rangle = b|a',b'\rangle \;,\; \{\tilde{A},\tilde{B}\} = \tilde{A}\tilde{B}+\tilde{B}\tilde{A}=0$$

To prove this we will have to test $\{\tilde{A},\tilde{B}\}|a',b'\rangle$

$$\{\tilde{A},\tilde{B}\}|a',b'\rangle = [\tilde{A}\tilde{B}+\tilde{B}\tilde{A}]|a',b'\rangle = \tilde{A}\tilde{B}|a',b'\rangle + \tilde{B}\hat{A}|a',b'\rangle$$

$$\tilde{A}\tilde{B}|a',b'\rangle = a\tilde{B}|a',b'\rangle = ab|a',b'\rangle \;:\; \tilde{B}\tilde{A}|a',b'\rangle = b\tilde{A}|a',b'\rangle = ba|a',b'\rangle$$

Scalar products are commutative therefore $a \cdot b = b \cdot a$, and thus

This then yields the result: $\{\tilde{A},\tilde{B}\}|a',b'\rangle = ab|a',b'\rangle + ab|a',b'\rangle = 2ab|a',b'\rangle$

The only way for $\{\tilde{A},\tilde{B}\}=0$ would be for one of the eigenvalues (at least one) to be equal to zero. i.e. a or b equal to zero.

Problem 4: 1.18 Review

Procedure:

- Begin by applying an arbitrary simultaneous state to the anti commutator of $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$.
- Carry out the algebra and show that the only way the above is true is if $ab = ba = 0$. This means one of the eigenvalues is zero.

Key Concepts:

- The only way the above is true is if one of the eigenvalues is zero.
- Because of the above the only way that $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ can have a simultaneous eigenstate is if one of the states is the null vector.

Variations:

- We could be asked to prove this for a commutation relation instead.
  - We would use the same procedure and deduce what the eigenvalues would have to be relative to one another.

Problem 5: Q-1

The state space of a certain physical system is 3-dimensional. Let $\{|u_1\rangle, |u_2, |u_3\rangle\rangle\}$ be an orthonormal basis of this space. The kets $|\psi_0\rangle$ and $|\psi_1\rangle$ are defined by:

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}|u_1\rangle + \frac{i}{2}|u_2\rangle + \frac{1}{2}|u_3\rangle$$
$$|\psi_1\rangle = \frac{1}{\sqrt{3}}|u_1\rangle + \frac{i}{3}|u_2\rangle$$

(a) Are these kets normalized?

Normalization : $\langle a|a\rangle = 1$

$$\langle\psi_0|\psi_0\rangle = \left(\frac{1}{\sqrt{2}}\langle u_1| - \frac{i}{2}\langle u_2| + \frac{1}{2}\langle u_3|\right)\left(\frac{1}{\sqrt{2}}|u_1\rangle + \frac{i}{2}|u_2\rangle + \frac{1}{2}|u_3\rangle\right)$$

$$\langle\psi_0|\psi_0\rangle = \frac{1}{2}\overbrace{\cancel{\langle u_1|u_1\rangle}}^{1} - \frac{i^2}{4}\overbrace{\cancel{\langle u_2|u_2\rangle}}^{1} + \frac{1}{4}\overbrace{\cancel{\langle u_3|u_3\rangle}}^{1} = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$$

$$\boxed{\psi_0 \text{ is normalized}}$$

$$\langle\psi_1|\psi_1\rangle = \left(\frac{1}{\sqrt{3}}\langle u_1| - \frac{i}{\sqrt{3}}\langle u_2|\right)\left(\frac{1}{\sqrt{3}}|u_1\rangle + \frac{i}{\sqrt{3}}|u_2\rangle\right)$$

$$\langle\psi_1|\psi_1\rangle = \frac{1}{3}\overbrace{\cancel{\langle u_1|u_1\rangle}}^{1} - \frac{i^2}{3}\overbrace{\cancel{\langle u_2|u_2\rangle}}^{1} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3} \neq 1$$

$$\boxed{\psi_1 \text{ is not normalized}}$$

(b) Calculate the matrices $\tilde{\mathbf{P}}_0$ and $\tilde{\mathbf{P}}_1$ representing, in the above given basis set, the projection operators onto the state $|\psi_0\rangle$ and $|\psi\rangle$. Verify that the matrices are Hermitian.

$$\tilde{P} = \sum_j |\psi_j\rangle\langle\psi_j| : \tilde{P}_0 = |\psi_0\rangle\langle\psi_0| \quad : \quad \tilde{P}_1 = |\psi_1\rangle\langle\psi_1|$$

Hermiticity requires $\tilde{P}^* = \tilde{P}$: $\tilde{P}_i|\psi_i\rangle = \langle\psi_i|\tilde{P}_i^*$

$$\tilde{P}_0 = \left(\frac{1}{\sqrt{2}}|u_1\rangle + \frac{i}{2}|u_2\rangle + \frac{1}{2}|u_3\rangle\right)\left(\frac{1}{\sqrt{2}}\langle u_1| - \frac{i}{2}\langle u_2| + \frac{1}{2}\langle u_3|\right)$$

$$\tilde{P}_0 = \frac{1}{2}|u_1\rangle\langle u_1| - \frac{i}{2\sqrt{2}}|u_1\rangle\langle u_2| + \frac{1}{2\sqrt{2}}|u_1\rangle\langle u_3| + \frac{i}{2\sqrt{2}}|u_2\rangle\langle u_1|$$
$$+ \frac{1}{4}|u_2\rangle\langle u_2| + \frac{i}{4}|u_2\rangle\langle u_3| + \frac{1}{2\sqrt{2}}|u_3\rangle\langle u_1| - \frac{i}{4}|u_3\rangle\langle u_2| + \frac{1}{4}|u_3\rangle\langle u_3|$$

$$\tilde{P}_1 = \left(\frac{1}{\sqrt{3}}|u_1\rangle + \frac{i}{\sqrt{3}}|u_2\rangle\right)\left(\frac{1}{\sqrt{3}}\langle u_1| - \frac{i}{\sqrt{3}}\langle u_2|\right)$$

Problem 5: Q-1 Continued

$$\tilde{P}_1 = \frac{1}{3}|u_1\rangle\langle u_1| - \frac{i}{3}|u_1\rangle\langle u_2| + \frac{i}{3}|u_2\rangle\langle u_1| + \frac{1}{3}|u_2\rangle\langle u_2|$$

$$\tilde{P}_1|\psi_1\rangle = \frac{1}{3\sqrt{3}}|u_1\rangle\cancelto{1}{\langle u_1|u_1\rangle} + \frac{1}{3\sqrt{3}}|u_1\rangle\cancelto{1}{\langle u_2|u_2\rangle} + \frac{i}{3\sqrt{3}}|u_2\rangle\cancelto{1}{\langle u_1|u_1\rangle} + \frac{i}{3\sqrt{3}}|u_2\rangle\cancelto{1}{\langle u_2|u_2\rangle}$$

$$\tilde{P}_1|\psi_1\rangle = \frac{2}{3\sqrt{3}}|u_1\rangle + \frac{2i}{3\sqrt{3}}|u_2\rangle = \frac{2}{3\sqrt{3}}\left(|u_1\rangle + i|u_2\rangle\right)$$

$$\langle\psi_1|\tilde{P}_1^* = \frac{1}{3\sqrt{3}}\cancelto{1}{\langle u_1|u_1\rangle}\langle u_1| + \frac{1}{3\sqrt{3}}\cancelto{1}{\langle u_2|u_2\rangle}\langle u_1| - \frac{i}{3\sqrt{3}}\cancelto{1}{\langle u_1|u_1\rangle}\langle u_2| - \frac{i}{3\sqrt{3}}\cancelto{1}{\langle u_2|u_2\rangle}\langle u_2|$$

$$\langle\psi_1|\tilde{P}_1^* = \frac{2}{3\sqrt{3}}\langle u_1| - \frac{2i}{3\sqrt{3}}\langle u_2| = \frac{2}{3\sqrt{3}}\left(\langle u_1| - i\langle u_2|\right)$$

Since $\langle\psi_1|\tilde{P}_1^*$ is the conjugate of $\tilde{P}_1|\psi_1\rangle$ we can conclude $\tilde{P}_1^* = \tilde{P}_1$ and thus $\tilde{P}_1$ is Hermition.

$$\tilde{P}_0|\psi_0\rangle = \frac{1}{2\sqrt{2}}|u_1\rangle\cancelto{1}{\langle u_1|u_1\rangle} + \frac{1}{4\sqrt{2}}|u_1\rangle\cancelto{1}{\langle u_2|u_2\rangle} + \frac{1}{4\sqrt{2}}|u_1\rangle\cancelto{1}{\langle u_3|u_3\rangle} + \frac{i}{4}|u_2\rangle\cancelto{1}{\langle u_1|u_1\rangle} + \frac{i}{8}|u_2\rangle\cancelto{1}{\langle u_2|u_2\rangle}$$
$$+ \frac{i}{2}|u_2\rangle\cancelto{1}{\langle u_3|u_3\rangle} + \frac{1}{4}|u_3\rangle\cancelto{1}{\langle u_1|u_1\rangle} + \frac{1}{8}|u_3\rangle\cancelto{1}{\langle u_2|u_2\rangle} + \frac{1}{8}|u_3\rangle\cancelto{1}{\langle u_3|u_3\rangle}$$

$$\tilde{P}_0|\psi_0\rangle = \frac{1}{\sqrt{2}}|u_1\rangle + \frac{7i}{8}|u_2\rangle + \frac{1}{2}|u_3\rangle$$

$$\langle\psi_0|\tilde{P}_0^* = \frac{1}{2\sqrt{2}}\cancelto{1}{\langle u_1|u_1\rangle}\langle u_1| + \frac{1}{4\sqrt{2}}\cancelto{1}{\langle u_2|u_2\rangle}\langle u_1| + \frac{1}{4\sqrt{2}}\cancelto{1}{\langle u_3|u_3\rangle}\langle u_1| - \frac{i}{4}\cancelto{1}{\langle u_1|u_1\rangle}\langle u_2| - \frac{i}{8}\cancelto{1}{\langle u_2|u_2\rangle}\langle u_2|$$
$$- \frac{i}{2}\cancelto{1}{\langle u_3|u_3\rangle}\langle u_2| + \frac{1}{4}\cancelto{1}{\langle u_1|u_1\rangle}\langle u_3| + \frac{1}{8}\cancelto{1}{\langle u_2|u_2\rangle}\langle u_3| + \frac{1}{8}\cancelto{1}{\langle u_3|u_3\rangle}\langle u_3|$$

$$\langle\psi_0|\tilde{P}_0^* = \frac{1}{\sqrt{2}}\langle u_1| - \frac{7i}{8}\langle u_2| + \frac{1}{2}\langle u_3|$$

Since $\langle\psi_0|\tilde{P}_0^*$ is the conjugate of $\tilde{P}_0|\psi_0\rangle$ we can conclude $\tilde{P}_0^* = \tilde{P}_0$ and thus $\tilde{P}_0$ is Hermition.

Problem 5: Q-1 Review

Procedure:

- Use the normalization check of $\langle \alpha | \alpha \rangle$ to check if the states are normalized.
- Use the formula for projection operators of
$$\tilde{\mathbf{P}} = \sum_i = |\psi_i\rangle \langle \psi_i|$$
and carry out the algebra for the above states that are given to us.

Key Concepts:

- States are normalized if their inner products are equal to 1.
- Projection operators are Hermitian.

Variations:

- We can be given different initial states.
    - The same procedure would be used for both parts.