PHYS5153 Assignment 1

Due: 1:30pm on 09/01/2021 (prior to class commencing).
Marking: Total of 10 marks (weighting of each question is indicated).
Fine print: Solutions should be presented legibly (handwritten or LaTeX is equally acceptable) so that the grader can follow your line of thinking and any mathematical working should be appropriately explained/described. If you provide only equations you will be marked zero. If you provide equations that are completely wrong but can demonstrate some accompanying logical reasoning then you increase your chances of receiving more than zero. If any of your solution has relied on a reference or material other than the textbook or lectures, please note this and provide details.

Question 1 (3 marks)

Consider a mass m attached by a massless spring to a piston, as illustrated in Fig. 1. The attachment point of the spring to the piston can be described by the co-ordinate X while the mass is described by co-ordinate x.

(a) If the motion of the mass is subject to a damping force $F_d = -m\nu\dot{x}$, show that it is described by the second-order differential equation

$$\ddot{x} + \nu\dot{x} + \omega_0^2 x = F_0(t), \tag{1}$$

where $\omega_0 = \sqrt{k/m}$ and $F_0(t) = \omega_0^2 X(t)$.

(b) Assume the piston is initialized at $X(0) = 0$ and is driven according to,

$$X(t) = X_0 e^{\alpha t}\cos(\omega t). \tag{2}$$

Determine the particular solution of Eq. (1) in this case.

(c) What is the resonance frequency ω_R? Does its existence depend on the sign of α?

Question 2 (3 marks)

When a car drives along a bumpy road, periodic ripples in the road surface can force the wheels to oscillate on the suspension (e.g., springs).

(a) If the spacing between ripples on the road is about 2 m, at what speed will the cars suspension be driven into resonance?

(b) Estimate a realistic damping constant provided by the shock absorbers so that the car's suspension does not catastrophically fall apart.

Note: This question has no unique quantitative solution. You will be marked primarily on your approach and the application of physical principles. However, your answer should provide reasonable estimates for relevant parameters to enable you to arrive at a concrete solution. Things to ponder include: How high are the ripples in the road? How far does a car drop in height when 4 adults hop inside?

Question 3 (4 marks)

Consider the nonlinear damped-driven system

$$\ddot{x} + (x^2 - \dot{x}^2 - 1)\dot{x} + x = 0, \tag{3}$$

that describes an harmonic oscillator with $m = \omega = 1$.

(a) Find an expression for the change in energy $\dot{E}$.

(b) For the polar co-ordinates (r, θ) given by the transformation $x = r\cos(\theta)$ and $\dot{x} = r\sin(\theta)$, derive the equations of motion:

$$\dot{r} = r(1 - r^2)\sin^2(\theta) \tag{4}$$

$$\dot{\theta} = (1 - r^2)\sin(\theta)\cos(\theta) - 1. \tag{5}$$

(c) Construct a rough phase portrait of the system in terms of the co-ordinates $(x, \dot{x})$. Your diagram should include at least three trajectories that characterize motion in the system, one of which is an attractor. Hint: Look at your solutions in (a) and (b) for insight.