Key points lectures 01/26/2022:

- Within statistical mechanics, using the microcanonical ensemble, the temperature is defined through

$$\frac{1}{T} = \left(\frac{\partial S(E,V,N)}{\partial E}\right)_{V,N} = k \frac{\partial}{\partial E}\left(\log(\Gamma(E,V,N))\right)_{V,N}$$

This definition can be used for systems of any size (it does not require $N \to \infty$ or large N).

- Stirling approximation: $\log(N!) \approx N \log N - N$

- For a two-level system consisting of N distinguishable particles the number of microstates is

$$\Gamma(E,N) = \frac{N!}{N_{ex}!\,(N-N_{ex})!},$$

where N_{ex} is the number of particles in the excited state.

- The calculation for the two-level system yields

$$\frac{1}{T} = \frac{k}{\varepsilon} \log\left(\frac{N}{N_{ex}} - 1\right)$$

where 0 and ε denote the single-particle energy of the two states