Occupation numbers: Example

↳ neglecting spin!

Let us consider three single-particle states $\varphi_1, \varphi_2, \varphi_3$

Let us look at what 3-particle wave functions we can construct, assuming our 3-particle energy is $E = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$

energy associated with φ_1

Bosons:

$$\Psi(x_1, x_2, x_3) = \frac{1}{\sqrt{3!}} \,\mathrm{per} \begin{pmatrix} \varphi_1(x_1) & \varphi_1(x_2) & \varphi_1(x_3) \\ \varphi_2(x_1) & \varphi_2(x_2) & \varphi_2(x_3) \\ \varphi_3(x_1) & \varphi_3(x_2) & \varphi_3(x_3) \end{pmatrix}$$

one fully symmetrized wave fct.

"per" stands for permanent – it's a determinant with the minus signs replaced by plus signs.

This state would be characterized by the occupation numbers $\{n_1 = 1, n_2 = 1, n_3 = 1\}$

⇝ each state holds one boson

Fermions:

$$\Psi(x_1, x_2, x_3) = \frac{1}{\sqrt{3!}} \det \begin{vmatrix} \varphi_1(x_1) & \varphi_1(x_2) & \varphi_1(x_3) \\ \varphi_2(x_1) & \varphi_2(x_2) & \varphi_2(x_3) \\ \varphi_3(x_1) & \varphi_3(x_2) & \varphi_3(x_3) \end{vmatrix}$$

one fully symmetrized state / wave fct.

This state is characterized by occupation numbers

$\{ n_1 = 1, \; n_2 = 1, \; n_3 = 1 \}$

$\rightsquigarrow$ each state holds one fermion

Boltzmann particles:

$\Psi = \varphi_1(x_1)\, \varphi_2(x_2)\, \varphi_3(x_3) \quad \rightsquigarrow \quad \{ n_1 = 1, \; n_2 = 1, \; n_3 = 1 \}$

$\Psi = \varphi_1(x_2)\, \varphi_2(x_1)\, \varphi_3(x_3) \quad \rightsquigarrow \quad \{ n_1 = 1, \; n_2 = 1, \; n_3 = 1 \}$

$\Psi = \varphi_1(x_1)\, \varphi_2(x_3)\, \varphi_3(x_2) \quad \rightsquigarrow$ same

$\Psi = \varphi_1(x_2)\, \varphi_2(x_3)\, \varphi_3(x_1) \quad \rightsquigarrow$ same

$\Psi = \varphi_1(x_3)\, \varphi_2(x_1)\, \varphi_3(x_2) \quad \rightsquigarrow$ same

$\Psi = \varphi_1(x_3)\, \varphi_2(x_2)\, \varphi_3(x_1) \quad \rightsquigarrow$ same

so, there exist $\dfrac{N!}{\prod_k (n_k!)}$

states with this set of

occupation numbers