Key points lecture 05/02/2022

The H-theorem states: If at a given time t the state of the gas satisfies the assumption of molecular chaos

(i.e., $F^{(2)}(\vec{r}, \vec{p}_1, \vec{p}_2, t) = \underbrace{f(\vec{r}, \vec{p}_1, t)\, f(\vec{r}, \vec{p}_2, t)}$),

momenta of the two particles are uncorrelated

then we have at $t+\varepsilon$ ($\varepsilon > 0$ and $\varepsilon \to 0$) : $\frac{dH(t)}{dt} \leq 0$

$\frac{dH(t)}{dt} = 0$ if and only if $f(\vec{r}, \vec{p}, t)$ is the Maxwell-Boltzmann distribution

$$H(t) = \int \underbrace{f(\vec{r}, \vec{p}, t)}_{\substack{f \text{ depends} \\ \text{on } \vec{r}, \vec{p}, t}} \log\left(f(\vec{r}, \vec{p}, t)\right) d^3\vec{p}\, d^3\vec{r}$$

H-fct.

implies: entropy never decreases (2nd law of thermodynamics)

this f satisfies the Boltzmann transport equation only when the assumption of molecular chaos happens to be valid.

Boltzmann transport equation leads to conservation laws. E.g., continuity equation for the mass: $\frac{\partial \rho(\vec{r}, t)}{\partial t} + \vec{\nabla} \cdot \left(\rho(\vec{r}, t)\, \underbrace{\vec{u}(\vec{r}, t)}\right) = 0$

$\rho = m\, n$
$=$ mass $\cdot$ density

$\langle \vec{v}(\vec{r}, t) \rangle$
$= \frac{1}{n} \int f(\vec{r}, \vec{p}, t)\, \vec{v}(\vec{r}, t)\, d^3\vec{p}$

$\left(n(\vec{r}, t) = \int f(\vec{r}, \vec{p}, t)\, d^3\vec{p}\right)$

Energy of classical Ising model (energy for specific arrangement of magnetic moments):

$$E\{s_i\} = -J \sum_{\langle ij \rangle} s_i s_j - \overset{H}{\mu B} \sum_i s_i$$

$\langle ij \rangle \leftarrow$ nearest neighbors

$s_i \in (-1, +1)$