Homework Assignment #9

Math Methods

Due: Monday, November 15th, midnight

Instructions:

Reading Quiz #6 on the last part of chapter 5 was supposed to be due last Wednesday - but I never opened it up! So I have made it due for Wednesday of this week.
Reading Quiz #7 on chapter 5 is due on Wednesday of next week.

Below is the a list of questions and problems. It is not sufficient to simply obtain the correct answer. You must also explain your calculation, and each step so that it is clear that you understand the material.
Homework should be written legibly, on standard size paper. Do not write your homework up on scrap paper. If your work is illegible, it will be given a zero.

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1. **Non-degenerate Perturbation Theory:** Consider the matrix

$$\mathcal{A} = \mathcal{A}_0 + \epsilon \mathcal{A}_1$$

where

$$\mathcal{A}_0 = \begin{pmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix}$$

and

$$\mathcal{A}_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

(a) The matrix A has eigenvalues λ_i with eigenvectors x_i, so that

$$\mathcal{A} x_i = \lambda_i x_i.$$

Solve for the eigenvalues and normalized eigenvectors of $\mathcal{A}_0$ explicitly, that is for $\lambda_i^{(0)}$ and $x_i^{(0)}$ such that

$$\mathcal{A}_0 x_i^{(0)} = \lambda_i^{(0)} x_i^{(0)}$$

Verify your answers.

(b) Using the results of (a), calculate the first order correction to the eigenvalues.

(c) Calculate the first order corrections to the eigenvectors.

(d) Calculate the second order corrections to the eigenvalues.

Please do all of the above *by hand*, and not using MATHEMATICA or any other software.

2. Repeat the above problem using MATHEMATICA or any other software.

3. You might question the wisdom of using perturbation theory in the days of computers, when we can solve many problems quickly. In this problem you may use Mathematica to either do perturbation theory or solve the problem exactly.

   Extend the above problem so $\mathcal{A}_0$ is a 20×20 tridiagonal matrix, and $\mathcal{A}_1$ is a 20×20 matrix with 1 in the far upper right and lower left corners. Determine the numerical value of the eigenvalue with the smallest magnitude in the unperturbed problem, and the numerical coefficient of the first and second order correction to the lowest eigenvalue. That is, find the best polynomial to second order in ϵ that fits the lowest eigenvalue.

   When you calculate the eigenvectors of $\mathcal{A}_0$, be sure that they are normalized. The eigenvalues and eigenvectors will all be real, so you don't have to worry about complex conjugation.

4. **Degenerate Perturbation Theory:** Consider the matrix

$$\mathcal{A} = \mathcal{A}_0 + \epsilon \mathcal{A}_1$$

   where

$$\mathcal{A}_0 = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$$

   and

$$\mathcal{A}_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

   the matrix A has eigenvalues λ_i with eigenvectors x_i, so that

$$\mathcal{A} x_i = \lambda_i x_i.$$

   (a) Solve for the eigenvalues and normalized eigenvectors of $\mathcal{A}_0$ explicitly, that is for $\lambda_i^{(0)}$ and $x_i^{(0)}$ such that

$$\mathcal{A}_0 x_i^{(0)} = \lambda_i^{(0)} x_i^{(0)}$$

   Verify your answers.

   (b) Using the results of (a), calculate the first order correction to the eigenvalues.

   (c) Calculate the second order corrections to the eigenvalues.

   Note that you are not asked to explicitly calculate the first order corrections to the eigenvectors.