5163, Homework Assignment 1
due on Friday, 01/28/2022, at 6pm (to be uploaded to Canvas)

This homework set consists of four problems.

Problem 1:
Three fun, independent parts related to probabilities.
(a) A game show contestant is presented with three closed doors. Behind two of the doors are goats and behind the third is a fancy new car. The contestant would prefer getting the car...
The game goes as follows:

1. The contestant goes to a door but does not open it.
2. The host, who knows which door has the car behind it, goes to one of the other doors, opens it, and reveals a goat.
3. The contestant is given the choice to either stay with the door first picked or to switch.

What is the contestant's probability to win the car when they stay and when they switch?
(b) Imagine you take a blue painted cube and cut it into 64 equal pieces. What is the probability P_n ($n = 0, 1, 2, 3$) that a little cube, picked at random, has n painted faces?
(c) You are given four coins, each having equal and independent probability to be a quarter, nickel, dime, or penny. What is the probability that you just made 37 cents?

Problem 2:
Make sure to carefully define your notation.
(a) Write down the classical and quantum mechanical ideal gas Hamiltonian for a system consisting of K point particles (atoms or molecules).
(b) Write down the classical and quantum mechanical Hamiltonian for a system that consists of eight structureless particles of mass m and five structureless particles of mass M. Each mass-m particle interacts with all other mass-m particles through the two-body potential V_{mm}. Each mass-M particle interacts with all other mass-M particles through the two-body potential V_{MM}. Each mass-m particle interacts with each mass-M particle through the two-body potential V_{mM}.
(c) Consider a system that consists of an equal number of electrons and protons.
(ci) Write down the classical and quantum mechanical Hamiltonian for this system.
(cii) What equilibrium state are you expecting the system to be in in the low temperature regime?
(ciii) What equilibrium state are you expecting the system to be in in the high temperature regime?
(civ) What equilibrium state are you expecting the system to be in at room temperature?

Problem 3:

Consider a non-interacting classical gas of three-level atoms with energy levels 0, ϵ, and 10ϵ. Further assume that there exists some "magic" mechanism by which the number of atoms in the single-particle state with energy 0 is equal to the number of atoms in the single-particle state with energy ϵ.

(a) Treating N, E, and V as macro-variables, derive a condition that ensures that the system is characterized by a negative temperature.

(b) Provide a physical interpretation of the existence of the negative temperature in this system of three-level atoms.

(c) Can negative temperatures be realized in any system? If your answer is "yes", explain. If your answer is "no", provide a necessary condition for negative temperatures to be realized?

Hint: Stirling's formula: $N! \approx N\log N - N$.

Problem 4:

1200 particles are to be distributed among three energy levels with energies ϵ_1, ϵ_2, and ϵ_3: $\epsilon_1 = 1$ eV, $\epsilon_2 = 2$ eV, and $\epsilon_3 = 3$ eV. The total energy of the system is fixed at 2400 eV and each possible microstate is equally probable.

(a) Assume that the particles are distinguishable. What is the number of microstates? Express your result in terms of N_1, where N_1 denotes the number of particles in the energy level ϵ_1. What is the most probable value of N_1?

(b) Assume that the particles are identical bosons (quantum particles that are described by a fully symmetric many-body wave function). What is the number of microstates? What is the most probable value of N_1?

Note, even though part (b) mentions the many-body wave function, there is no need to explicitly construct wave functions to answer the questions.