

Task: (Non-interacting fermions \rightarrow canonical ensemble)

Calculate the partition function of $N=3$ identical fermions in equilibrium with a heat bath at temperature T . Assume that each particle can be in one of four possible microstates with energies $\epsilon_1, \epsilon_2, \epsilon_3$, and ϵ_4 .

Solution:

$$\begin{aligned} E &= \epsilon_1 + \epsilon_2 + \epsilon_3 \\ E &= \epsilon_1 + \epsilon_2 + \epsilon_4 \\ E &= \epsilon_2 + \epsilon_3 + \epsilon_4 \\ E &= \epsilon_1 + \epsilon_3 + \epsilon_4 \end{aligned} \quad \left. \vphantom{\begin{aligned} E &= \epsilon_1 + \epsilon_2 + \epsilon_3 \\ E &= \epsilon_1 + \epsilon_2 + \epsilon_4 \\ E &= \epsilon_2 + \epsilon_3 + \epsilon_4 \\ E &= \epsilon_1 + \epsilon_3 + \epsilon_4 \end{aligned}} \right\} \text{allowed energies}$$

in all cases, the three fermions are occupying different states

$$\begin{aligned} \Rightarrow Q(T) = & e^{-\beta(\epsilon_1 + \epsilon_2 + \epsilon_3)} + e^{-\beta(\epsilon_1 + \epsilon_2 + \epsilon_4)} \\ & + e^{-\beta(\epsilon_2 + \epsilon_3 + \epsilon_4)} + e^{-\beta(\epsilon_1 + \epsilon_3 + \epsilon_4)} \end{aligned}$$