



COLLEGE OF ARTS AND SCIENCES

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*The* UNIVERSITY *of* OKLAHOMA

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## Statistical Mechanics

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PHYS 5163 HOMEWORK ASSIGNMENT 6

PROBLEMS: {1,2,3,4}

Due: March 25, 2022 at 6:00 PM

STUDENT

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PROFESSOR

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**Problem 1:**

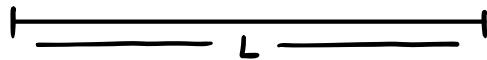
Consider a non-interacting one-dimensional non-relativistic spinless quantum gas confined to a "rod" of length  $L$ . Assuming periodic boundary conditions, calculate the density of states.

To calculate the density of states we use,

$$D(E) = \frac{dN(E)}{dE}$$

where the above is valid for periodic boundary conditions. We need to find  $N(E)$  to do this.

To find  $N(E)$ , we first consider our rod of length  $L$



Where when we talk about periodic boundary conditions, mathematically we mean

$$\Psi(x+L) = \Psi(x)$$

Where  $x$  is some arbitrary distance from the end of our rod. we now need to find a wave function that will describe our system so we can in turn find the energy for our system.

We begin by constructing a normalized wave function for one of these particles of the gas. Starting with the time-independent Schrödinger Equation

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = E\psi$$

Since we are in 1D and  $V=0$ , Schrödinger's equation becomes

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} = E\psi(x)$$

We then solve Schrödinger's equation to find  $\psi(x)$

$$\frac{\partial^2 \psi(x)}{\partial x^2} = -\frac{2mE}{\hbar^2} \psi(x)$$

We now impose our periodic boundary conditions of  $\psi(x+L) = \psi(x)$ ,

$$\psi(0) = \psi(L), \quad \psi'(0) = \psi'(L)$$

### Problem 1: Continued

We now know that our wave function will be

$$\Psi(x) = A e^{ikx}$$

where the normalized wave function is

$$\int_0^L \Psi^*(x) \Psi(x) dx = A^2 \int_0^L dx = A^2 L = 1 \Rightarrow \Psi(x) = \frac{e^{ikx}}{\sqrt{L}}$$

We then put this normalized wave function in the Time Independent Schrödinger Equation

$$-k^2 \Psi(x) = -\frac{2mE}{\hbar^2} \Psi(x) \Rightarrow k^2 = \frac{2mE}{\hbar^2}$$

Periodicity tells us that  $k_n = \frac{n\pi}{L}$  and therefore we then have

$$\left(\frac{n\pi}{L}\right)^2 n^2 = \frac{2mE}{\hbar^2} \Rightarrow n^2 = \frac{mL^2 E}{2\pi^2 \hbar^2} \Rightarrow N(E) = \sqrt{\frac{mL^2 E}{2\pi^2 \hbar^2}}$$

The density of states is then (multiplying by 2 for -k's)

$$D(E) = \sqrt{\frac{mL^2}{2\pi^2 \hbar^2 E}}$$

## Problem 1: Review

### Procedure:

- We first begin by finding the normalized wave function with periodic boundary conditions

$$\psi(x) = \frac{e^{ikx}}{\sqrt{L}}$$

- Take our wave function and put it into the time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m} \nabla^2 \psi = E\psi$$

where of course here  $V = 0$  and solve for  $k^2$

- Use the fact that  $k_n = \frac{2\pi}{L}n$  and solve for an expression for  $n$
- In the previous parts,  $n$  is essentially  $N$  and when it is a function of  $E$  it is  $N(E)$ . We then calculate the density of states with

$$D(E) = \frac{dN(E)}{dE}$$

where at the end we multiply by 2 to account for the negative  $k$  values

### Key Concepts:

- Periodic boundary conditions say

$$\psi(x + L) = \psi(x)$$

which in English means that if we shift our system by some arbitrary distance  $x$  we need to have the same wave function as if it is not shifted

- This is essentially a particle in a box that is confined to 1 dimension and this then means that the potential is zero. This is why our Schrödinger equation simplifies to  $\mathcal{H}\psi = E\psi$
- We apply boundary conditions and find our normalized wave function to then find the energy of our particle according to level
- We then can solve for an expression  $N(E)$  that then allows us to solve for the density of states
- Density of states tells us how populated in particles our states are

### Variations:

- We could be handed more dimensions to work with
  - \* This is essentially problem 3 of this assignment
- We could be given boundary conditions that are not periodic
  - \* This would then change what our wave function is but everything after would be the same procedure as before

**Problem 2:**

Consider a system consisting of  $N$  non-interacting particles each with isospin  $I = 3/2$ . The energies of the states with different projection quantum numbers  $m_I$  (the eigenvalues of  $\hat{I}_z$  are  $\hbar m_I$ ;  $I_z$  denotes the  $z$ -component of  $I$ ) are given by

$$E(m_1 = -3/2) = E_1, \quad (1)$$

$$E(m_1 = -1/2) = E_2, \quad (2)$$

$$E(m_1 = 1/2) = E_3, \quad (3)$$

$$E(m_1 = 3/2) = E_3, \quad (4)$$

with

$$E_1 < E_2 < E_3 \quad (5)$$

and

$$\Delta_{12} = E_2 - E_1 \ll \Delta_{23} = E_3 - E_2. \quad (6)$$

You may treat the particles as distinguishable throughout.

- (a) Without using the partition function, give the value of the total energy  $\langle E \rangle$  at temperatures (i)  $T = 0$  (ii)  $\Delta_{12} \ll kT \ll \Delta_{23}$  and (iii)  $\Delta_{23} \ll T$ . Provide a justification for your results. Sketch  $\langle E \rangle$  as a function of temperature.

*When  $T=0$ : we are essentially in the ground state of our system. So, the ensemble average is then*

$$\langle E \rangle = E_1$$

*When  $\Delta_{12} \ll kT \ll \Delta_{23}$ : we are looking at*

$$\underline{\hspace{2cm} E_3 }$$

$$\bullet \rightsquigarrow kT$$

$$\underline{\hspace{2cm} E_2 }$$

*Where we make the argument that our  $E_1$  and  $E_2$  states are equally populated and  $E_3$  is not populated due to it not having the excitation energy that is required. Therefore we say*

$$\langle E \rangle = \frac{N}{2}(E_1 + E_2)$$

*When  $\Delta_{23} \ll kT$ : we are looking at*

$$\bullet \rightsquigarrow kT$$

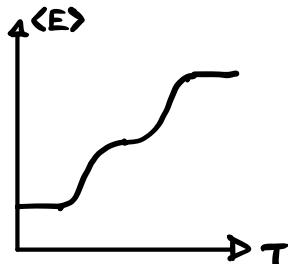
$$\underline{\hspace{2cm} E_3 }$$

## Problem 2: Continued

where in this case each state is as equally occupied as the other so we have

$$\langle E \rangle = \frac{E_1 + E_2 + 2E_3}{4}$$

Graphically this will look like



- (b) What is the occupation of the four different  $m_1$ -states in the  $T \rightarrow \infty$  limit. Without using the partition function, give a value of the specific heat in the  $T \rightarrow \infty$  limit. Provide a justification for your results.

As  $T$  goes towards infinity, our energy increases and the likely hood of being in a specific state is the same for all states. This then means each  $m_1$  state should be as equally probable as the other.

occupation of each  $m_1$ -state  
is the same

In this case, we are looking at a canonical ensemble where  $T$  is changing and going to infinity. Mathematically this means

$$\lim_{T \rightarrow \infty} C_p = \lim_{T \rightarrow \infty} \left( \frac{\partial E}{\partial T} \right)_p = \frac{1}{\infty} = 0 \quad \therefore C_p = 0$$

From this we can say the specific heat is 0.

$$C_p = 0$$

- (c) Without using the partition function, give the value of the average  $\langle \hat{I} \rangle$  of the isospin  $z$ -component per particle at temperatures (i)  $T = 0$ , (ii)  $\Delta_{12} \ll kT \ll \Delta_{23}$  and (iii)  $\Delta_{23} \ll T$ . Provide a justification for your results. Sketch  $\langle \hat{I} \rangle$  as a function of the temperature.

when  $T=0$ , we are in the Ground state of our system. This then means the only allowed value for  $m_1$  is  $m_1 = -3/2$  so the average  $\langle \hat{I} \rangle$  is

$$\langle \hat{I} \rangle = -3/2$$

## Problem 2: Continued

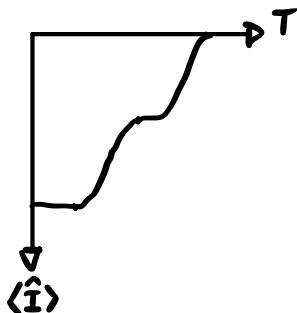
When we have  $\Delta_{12} \ll kT \ll \Delta_{23}$  we know that  $E_1$  and  $E_2$  are equally occupied and the probability of being in either state is equal. This then means the  $\langle \hat{I} \rangle$  is the average of the two  $m_i$  values ( $-3/2$  &  $-1/2$ )

$$\langle \hat{I} \rangle = -1$$

When we have  $\Delta_{23} \ll kT$  we know that the probability of being in either of the three states is equal. This then means we have  $\langle \hat{I} \rangle$  to be an average of the four  $m_i$  values ( $-3/2, -1/2, 1/2, 3/2$ )

$$\langle \hat{I} \rangle = 0$$

Graphically this looks like



(d) Using the partition function, compute  $\langle \hat{I} \rangle$  in the  $T \rightarrow \infty$  limit. How is your result related to the results in part (c)?

For the  $T \rightarrow \infty$  limit, we calculate the isospin average as

$$\langle \hat{I} \rangle = \frac{\sum_i I_i e^{-\beta E_i}}{\sum_i e^{-\beta E_i}}$$

where the average isospin value will be

$$\langle \hat{I} \rangle = \frac{I_1 e^{-\beta E_1} + I_2 e^{-\beta E_2} + I_3 e^{-\beta E_3}}{e^{-\beta E_1} + e^{-\beta E_2} + e^{-\beta E_3}} = \frac{I_1 e^0 + I_2 e^0 + I_3 e^0}{e^0 + e^0 + e^0} = \frac{-3/2 - 1/2 + 1/2 + 3/2}{3}$$

This then means that  $\langle \hat{I} \rangle = 0$ , which is the same as part c.)

$$\langle \hat{I} \rangle = 0$$

## Problem 2: Review

### Procedure:

- – For the following conditions consider the occupation each energy level and how they must be distributed
- – Apply the limit of  $T \rightarrow \infty$  to show that  $C_p = 0$
- – Perform the same analysis for (c) as in part (a)
- – Apply the limit to the partition function for  $\langle \hat{I} \rangle$  and show that it must go zero

### Key Concepts:

- – In the limits for  $kT$ , at specific points it will seem like certain energy levels do not exist. When these energy levels do not exist the particles will then distribute themselves evenly amongst the available energy levels
- – As our temperature goes to infinity, this causes our energy to go to infinity as well. Because of this each state will be occupied with the same number of particles and this causes the specific heat to be zero
- –  $\langle \hat{I} \rangle$  is the average spin of all of our particles, depending on the occupancy of our states this will change what the average spin value is
- – As  $T \rightarrow \infty$  we see that our partition function will go to zero and this causes our average spin to be zero as well

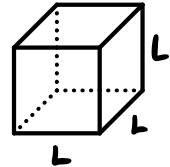
### Variations:

- – We can be given different initial values for our states
  - \* This changes what we can expect for occupancy of our states and then this changes our expected average values
- – We could be asked to evaluate a different limit
  - \* We then would apply this limit and then find what our occupancy after that would be
- – We could be given different initial spins
  - \* This would cause a different average value for each occupancy but the way we find that would be different
- – We could be asked to evaluate a different limit for our partition function
  - \* This then changes what our average value would evaluate to

**Problem 3:**

- (a) Calculate the density of states  $D(\epsilon)$  for a non-interacting three-dimensional gas of spin-0 particles confined to a cube of volume  $L^3$  with periodic boundary conditions.

Here we have a cube with volume  $V = L^3$



where the density of states is calculated with

$$D(E) = \frac{dN(E)}{dE} \quad (*)$$

Where the Hamiltonian for our system is

$$\frac{\partial^2}{\partial m} \quad \text{with} \quad p^2 = p_x^2 + p_y^2 + p_z^2 \quad (**)$$

If we follow the same procedure from problem 1, we know our normalized wave function is

$$\psi_n(x) = \frac{e^{i\vec{k} \cdot \vec{x}}}{\sqrt{L^3}} \quad (***)$$

Where we know that  $\vec{k}$  is

$$k_i = \frac{2\pi}{L} n_i \quad (****)$$

Taking this result and putting it into the Time Independent Schrödinger equation we then find our energy is

$$E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m} \left( \frac{2\pi}{L} \right)^2 n^2 = \frac{2\pi^2 \hbar^2}{m L^2} n^2 = \frac{2\pi^2 \hbar^2}{m L^2} (n_x^2 + n_y^2 + n_z^2)$$

We are then interested in finding the density of states with

$$dN(E) = D(E) dE \Rightarrow N(E) = \int D(E) dE$$

## Problem 3: Continued

Where we find  $D(E)$  by taking the term out of the integrand. To do this we first create a differential equation

$$dE = \frac{\hbar^2 k}{m} dk$$

Where we now have to find a way to relate  $k$ -space to  $n$ . This is done with (xxxx)

$$dk = \frac{2\pi}{L} dn \Rightarrow d^3k = \left(\frac{2\pi}{L}\right)^3 d^3n$$

If we remember,  $n \equiv N$ , we find  $N(k)$  to be

$$\int d^3n = \left(\frac{L}{2\pi}\right)^3 \int d^3k \Rightarrow N = \left(\frac{L}{2\pi}\right)^3 \int_0^{2\pi} \int_0^{2\pi} \int_0^\infty k^2 \sin(\alpha) dk d\alpha dp$$

Taking  $k = \sqrt{\frac{2mE}{\hbar^2}}$ , then  $N(E)$  becomes

$$N(E) = L^3 \cdot \frac{1}{(2\pi)^3} \cdot L \int_0^\infty k^2 \cdot \frac{m}{\hbar^2 k} dE = \frac{L^3}{2\pi^2} \int_0^\infty \frac{m}{\hbar^2} \sqrt{\frac{2mE}{\hbar^2}} dE$$

Referencing (\*) we can then say our density of states is

$$D(E) = \left(\frac{L}{\hbar}\right)^3 \frac{m \sqrt{\frac{2mE}{\hbar^2}}}{2\pi^2}$$

- (b) Calculate  $D(E)$  for a non-interacting three-dimensional gas of spin-0 particles confined to a cube of volume  $L^3$  with hard wall boundary conditions (i.e., boundary conditions such that the single-particle wave function vanishes at the edges of the box).

Since we are dealing with Hardwall Boundary Conditions we know that our wave function will follow the form of

$$\Psi(x, y, z) = X(x) Y(y) Z(z) \quad (*)$$

Where (\*) is separable partial differential equation. Our normalized wave functions for each direction are simply

$$\Psi(x) = \sqrt{\frac{2}{L}} \sin(k_x x), \quad \Psi(y) = \sqrt{\frac{2}{L}} \sin(k_y y), \quad \Psi(z) = \sqrt{\frac{2}{L}} \sin(k_z z)$$

### Problem 3: Continued

where when we put these into the Time Independent Schrödinger equation we find that our energy will end up being the same as the periodic boundary condition example

$$E = \frac{\hbar^2 k^2}{2m} = \frac{2\pi^2 \hbar^2}{mL^2} (n_x^2 + n_y^2 + n_z^2)$$

From this, we can then say that the density of states for the Hardwell potential example is same of the continuous boundary conditions. This is because when we are dealing with the Hardwell case  $k$  is discretized as

$$k_n = \frac{n\pi}{L}, \quad n=0,1,2,\dots$$

But since we are looking at only  $1/8^{\text{th}}$  of  $k$ -space when we use the Hardwell Boundary Conditions due to the constraint on  $n_x, n_y, n_z$  to be positive, we are also only looking at  $1/8^{\text{th}}$  the particles. This then tells us our density of states will be

$$D(E) = \left(\frac{L}{\hbar}\right)^3 \frac{m\sqrt{2mE}}{2\pi^2}$$

which is the exact same as the Periodic Boundary Conditions density of states. This is due to Hardwell Boundary Conditions being a subset to Periodic Boundary Conditions.

## Problem 3: Review

### Procedure:

- First begin by finding the normalized three dimensional wave function

$$\psi_n(\vec{x}) = \frac{e^{i\vec{k}\cdot\vec{x}}}{\sqrt{L^3}}$$

and determine the energies of our system in terms of  $k$

- We calculate the density of states with

$$D(E) = \frac{dN(E)}{dE}$$

where we must solve for  $N(E)$  by solving Schrödinger's equation in three dimensions

- We first solve for  $N(E)$  with

$$N = \left( \frac{L}{2\pi} \right)^3 \int d^3k$$

where we have to change the radial component of our integral by solving for the differential of  $E$  in terms of  $k$

- Constraining this problem to Hardwall Boundary Conditions will yield the same result as the periodic boundary conditions. This is because by applying Hardwall Boundary Conditions we essentially constrain our value of  $n_i$  and this in turn constrains what part of  $k$ -space we look at

### Key Concepts:

- In three dimensions we must find  $N$  to find our density of states. We do this by integrating over  $k$ -space that is constrained by our boundary conditions
- Since Hardwall Boundary Conditions are subsets of Periodic Boundary Conditions, the answer for (b) will be the same for (a)

### Variations:

- The only way this problem can drastically change is if we are looking at a different number of dimensions
  - \* We then would just restrict one dimension and it would simplify our equations

**Problem 4:**

This problem considers a quantum mechanical system that contains three non-interacting particles. The spatial degree of the particle can be in one of two states (the spatial degree is for simplicity assumed to be the x-coordinate):  $\psi_1(x)$  with single-particle energy  $E_1$  or  $\psi_2(x)$  with single-particle energy  $E_2$ , where  $E_1 < E_2$ . In some of the cases considered below, the particles have a spin  $s$ . Assume that the energy levels are independent of the projection quantum number  $m_s$ .

For each of the following cases, write down (i) the zero-temperature energy, (ii) the degeneracy, and (iii) the normalized zero-temperature wave function(s) of the three-particle quantum system.

- (a) Three spinless Boltzmann particles.

The Zero Temperature energy is essentially the ground state energy. With three Spinless Boltzmann particles the Zero Temperature Energy is

$$E_T = 3E_0$$

The degeneracy of this System is one and the Normalized wave Function is

$$\Psi_T = \Psi_0(1)\Psi_0(2)\Psi_0(3)$$

For this configuration of particles we finally have

$$E_T = 3E_0 , \quad \Psi_T = \Psi_0(1)\Psi_0(2)\Psi_0(3) , \quad \text{Degeneracy} = 1$$

- (b) Three identical spin-0 bosons.

The Zero Temperature energy for Spinless Bosons will be the same as the Spinless Boltzmann Zero Temperature energy

$$E_T = 3E_0$$

Where the difference between part a.) is that the Bosons are indistinguishable. But Since our Bosons are Spinless, this means the total wave function is the same as the Spinless Boltzmann since it only has a spatial part

$$\Psi = \Psi_0(1)\Psi_0(2)\Psi_0(3)$$

Therefore the degeneracy is one and therefore we have

$$E_T = 3E_0 , \quad \Psi_T = \Psi_0(1)\Psi_0(2)\Psi_0(3) , \quad \text{Degeneracy} = 1$$

## Problem 4: Continued

(c) Three identical spin-1/2 fermions.

For three identical spin-1/2 Fermions, the occupancy for each level is

$$\textcircled{1} = 2S+1 = 2(\frac{1}{2})+1 = 2 \Rightarrow \text{Two particles per level}$$

For this system we have two particles in the ground state and one in an excited state with energy  $E_1$ . This then means the zero temperature energy is

$$E_T = 2E_0 + E_1$$

where we now construct the normalized wave functions. Since Fermions are anti-symmetric under particle exchange we have our wave functions as

$$\Psi_T = \frac{1}{\sqrt{3!}} \det \begin{pmatrix} \gamma_0(1) \chi(\frac{1}{2}) & \gamma_0(2) \chi(\frac{1}{2}) & \gamma_0(3) \chi(\frac{1}{2}) \\ \gamma_0(1) \chi(-\frac{1}{2}) & \gamma_0(2) \chi(-\frac{1}{2}) & \gamma_0(3) \chi(-\frac{1}{2}) \\ \gamma_1(1) \chi(\frac{1}{2}) & \gamma_1(2) \chi(\frac{1}{2}) & \gamma_1(3) \chi(\frac{1}{2}) \end{pmatrix}$$

Where if we take the determinant of the above matrix to find our total wave function. The degeneracy of this system is two, so finally

$$\Psi_T = \frac{1}{\sqrt{3!}} \det \begin{pmatrix} \gamma_0(1) \chi(\frac{1}{2}) & \gamma_0(2) \chi(\frac{1}{2}) & \gamma_0(3) \chi(\frac{1}{2}) \\ \gamma_0(1) \chi(-\frac{1}{2}) & \gamma_0(2) \chi(-\frac{1}{2}) & \gamma_0(3) \chi(-\frac{1}{2}) \\ \gamma_1(1) \chi(\frac{1}{2}) & \gamma_1(2) \chi(\frac{1}{2}) & \gamma_1(3) \chi(\frac{1}{2}) \end{pmatrix}$$

$$E_T = 2E_0 + E_1, \text{ Degeneracy} = \text{two}$$

(d) Three identical spin-3/2 fermions.

For three identical spin-3/2 Fermions we are allowed

$$\textcircled{1} = 2S+1 = 2(\frac{3}{2})+1 = 4 \Rightarrow 4 \text{ particles per level}$$

4 particles max per energy level. But since we only have 3 spin-3/2 Fermions, there will be 3 in our ground state. This then means the zero temperature energy is

$$E_T = 3E_0$$

We then can go on to determining our wave function. Since we have our magnetic quantum number ranging from  $-\frac{3}{2}$  to  $\frac{3}{2}$ , this will produce 4 separate matrices that we must take a determinant of. This means our total wave function will look like

## Problem 4: Continued

$$\Psi_T = \sum_{i=1}^4 \Psi_i \quad (*)$$

Where we define  $\Psi_i$  to be wave functions that omit one of our magnetic spin quantum numbers. This is because we have 4 possible magnetic spin quantum numbers with only 3 total particles. If we say

$$M_i = [-3/2, -1/2, 1/2, 3/2] \quad \text{where } i \in [1, 4]$$

we can then go ahead and say

$$\Psi_1 = \frac{1}{\sqrt{3!}} \det \begin{pmatrix} \psi_0(1) \chi(-1/2) & \psi_0(2) \chi(-1/2) & \psi_0(3) \chi(-1/2) \\ \psi_0(1) \chi(1/2) & \psi_0(2) \chi(1/2) & \psi_0(3) \chi(1/2) \\ \psi_0(1) \chi(3/2) & \psi_0(2) \chi(3/2) & \psi_0(3) \chi(3/2) \end{pmatrix} \quad M_1 = -3/2 \Rightarrow \text{omitted}$$

$$\Psi_2 = \frac{1}{\sqrt{3!}} \det \begin{pmatrix} \psi_0(1) \chi(-3/2) & \psi_0(2) \chi(-1/2) & \psi_0(3) \chi(-1/2) \\ \psi_0(1) \chi(1/2) & \psi_0(2) \chi(1/2) & \psi_0(3) \chi(1/2) \\ \psi_0(1) \chi(3/2) & \psi_0(2) \chi(3/2) & \psi_0(3) \chi(3/2) \end{pmatrix} \quad M_2 = -1/2 \Rightarrow \text{omitted}$$

$$\Psi_3 = \frac{1}{\sqrt{3!}} \det \begin{pmatrix} \psi_0(1) \chi(-3/2) & \psi_0(2) \chi(-1/2) & \psi_0(3) \chi(1/2) \\ \psi_0(1) \chi(-1/2) & \psi_0(2) \chi(1/2) & \psi_0(3) \chi(1/2) \\ \psi_0(1) \chi(3/2) & \psi_0(2) \chi(3/2) & \psi_0(3) \chi(3/2) \end{pmatrix} \quad M_3 = 1/2 \Rightarrow \text{omitted}$$

$$\Psi_4 = \frac{1}{\sqrt{3!}} \det \begin{pmatrix} \psi_0(1) \chi(-3/2) & \psi_0(2) \chi(-1/2) & \psi_0(3) \chi(-1/2) \\ \psi_0(1) \chi(-1/2) & \psi_0(2) \chi(-1/2) & \psi_0(3) \chi(1/2) \\ \psi_0(1) \chi(1/2) & \psi_0(2) \chi(1/2) & \psi_0(3) \chi(1/2) \end{pmatrix} \quad M_4 = 3/2 \Rightarrow \text{omitted}$$

Referencing (\*) we can then of course say

$$\boxed{\Psi_T = \Psi_1 + \Psi_2 + \Psi_3 + \Psi_4}$$

This then just leaves the degeneracy. Since we have 4 possible magnetic spin quantum numbers, with 3 possible particles, we have a degeneracy of

$$\binom{4}{3} = \frac{4!}{(3!)(4-3)!} = 4$$

So we can finally say we have a degeneracy of

$$\boxed{\text{Degeneracy} = \text{Four}}$$

## Problem 4: Review

### Procedure:

- – Because our particles are at zero temperature, this implicates that they are all in the ground state. Knowing this we can write what our energy is for our system and we can write a total wave function for our Boltzmann particles
- – Because the only difference between Bosons and Boltzmann particles is that Boltzmann particles are distinguishable, we should expect the same total energy as well as the same total wave function as in part (a)
- – We use Slater's determinant to find the total wave function of our particles
- – This part is essentially the same as part (c) but instead we have to account for three spins at once. This will cause us to have four separate Slater determinants to find the total wave function with all the correct spins being accounted for

\* It should be noted that the notation

$$\psi_i(j)\chi(k)$$

means the energy level  $i$  for particle number  $j$  with spin value  $k$

### Key Concepts:

- – Because we are dealing with Boltzmann particles, they all can fit in the same energy level and this then means our degeneracy is 1
- The biggest difference between and Boltzmann particle and a Boson is that Boltzmann particles are distinguishable whereas Boson's are not
- – Like the Boltzmann particles, Boson's can all fit in the same energy level and this in turn means the degeneracy is 1 due to the Boson's having no spin
- – Because of the spin of our Fermions we know that only two can occupy one level by the occupancy rule. Because of this we know two will reside in the ground state and one particle will reside in the next excited state. Because the particles are Spin - 1/2 this means that we have a degeneracy of two
- We then use Slater's determinant to find the total wave function
- – The biggest difference between part (c) and (d) is that we have all four particles now being accounted for. This then creates four separate Slater determinants for the total wave function with a degeneracy of four due to the spin of the Fermions

### Variations:

- – We could be given a different particle for parts (a) through (d)
  - \* This then changes how we treat the problem but the same broad procedure would apply