COLLEGE OF ARTS AND SCIENCES
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Statistical Mechanics

CH. 4 THE EQUILIBRIUM STATE OF A DILUTE GAS LECTURE NOTES

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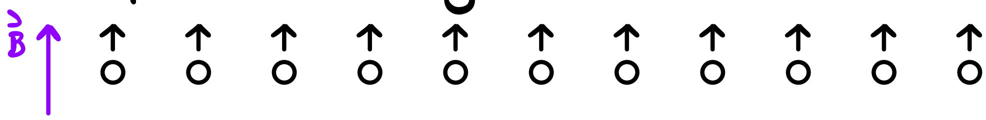
We start by examining the Ising model. We begin looking at a bunch of particles that are present in a magnetic field



We can calculate the magnetization $\mathcal{M}$ with

$$\mathcal{M} = \frac{1}{V}\left\langle \frac{\partial \mathcal{H}}{\partial B} \right\rangle = \left\langle \sum_{i=1}^{N} S_i \right\rangle$$

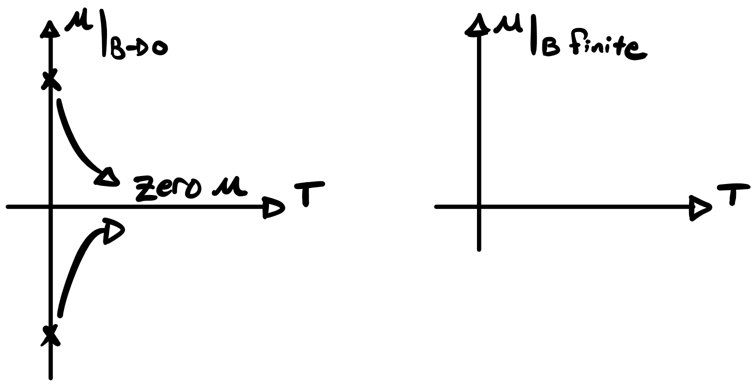
We then use the above to further say

$$E\{S_i\} = -\mathcal{J}\sum_{\langle i,j \rangle} S_i S_j - \mathcal{M}B\sum_{i=1}^{N} S_i$$

We can proceed to say if the spins are

$$(-1) \longrightarrow +\mathcal{M}|\vec{B}| \;,\quad (+1) \longrightarrow -\mathcal{M}|\vec{B}|$$

We then wish to examine $\mathcal{M}(B,T)|_{B\to 0}$. Graphically this looks like



We now want to look at the Helmholtz Free Energy

$$\Delta A = \Delta U - T\Delta S$$

When our system has more domain walls added, the Helmholtz free energy will have its energy decreased.

5-4-22

Working in the Ising model we have

$$\mathcal{H} = -\mathcal{J}' \underbrace{\sum_{\langle i,j \rangle}}_{\text{Nearest Neighbors}} \mathcal{M}_{i,z}\mathcal{M}_{j,z} - B\sum_{i=1}^{N}\mathcal{M}_{\hat{n},z}$$

$\mathcal{M}_{i,z} = \mathcal{M}S_{\hat{n}}$, $S_{\hat{n}} = \pm 1$

We can then re-write our Hamiltonian as

$$\mathcal{H} = -(J'\mu^2)\sum_{\langle i,j \rangle} S_i S_j - B\mu \sum_{i=1}^{N} S_i$$

We then make the assumption, $J > 0$. In our 1D case we have

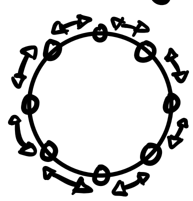
$$\Delta A = \Delta U - T\Delta S \quad , \quad \text{w/ } \Delta U = 2J \Rightarrow \Delta A = 2J - k_B T \log(N-1)$$

Where $\Delta S = k_B \log(N-1)$. In 2D we have a change in Helmholtz free energy of

$$\Delta A = 2JL - k_B T \log(L)$$

Where "L" is the length of our domain wall.

If we look at a 1D system



$$\Rightarrow \mathcal{H} = -J\sum_{i=1}^{N} S_i S_{i+1} - \frac{1}{2} H \sum_{i=1}^{N} (S_i + S_{i+1})$$

We can calculate the partition function to be

$$Q_N = \sum_{S_1 = \pm 1} \sum_{S_2 = \pm 1} \cdots\cdots \sum_{S_N = \pm 1} e^{\beta \sum_{i=1}^{N} (J S_i S_{i+1} + \frac{1}{2} H (S_i + S_{i+1}))} = \lambda_+^N + \lambda_-^N$$

We then define $\lambda_\pm^N$ to be

$$\lambda_\pm = e^{\beta J} \cosh(\beta H) \pm \sqrt{e^{-2\beta J} + e^{2\beta J} \sinh^2(\beta H)}$$

Using the above we can re-write the partition function to be

$$\frac{1}{N} \log(Q_N) = \log(\lambda_+)$$

We can then calculate the Magnetization with

$$\mu = kT \frac{\partial}{\partial \beta} \left(\frac{\log(Q_N)}{N} \right)$$

Plugging in Q_N into the equation for magnetization we find

$$M = \mu \frac{\sinh(\beta\mu B)}{(e^{-4\beta J} + \sinh(\beta\mu B))^{1/2}}$$

Going to the 2D example, we use something called Mean Field Theory. The Hamiltonian for this system is then

$$\mathcal{H} = -\sum_{i}\left(J\sum_{j=1} s_j + H\right)$$

We then say the effective Hamiltonian is

$$\mathcal{H}_{eff} = J\sum_{j}^{q} \langle s_j \rangle + H$$