

Key points of 03/21 lecture

- grandcanonical ensemble, quantum:

$$\mathcal{Q}(z, V, T) = \text{Tr}(e^{-\beta(\hat{H} - \mu \hat{N})}) = \sum_N \text{Tr}(e^{-\beta(\hat{H} - \mu N)}) = \sum_N z^N \underbrace{\text{Tr}(e^{-\beta \hat{H}})}_{\mathcal{Q}}$$

\hat{N} = number operator

Sum over all states

N = # of particles

$$\hat{\rho}_G = \frac{e^{-\beta(\hat{H} - \mu \hat{N})}}{\mathcal{Q}(z, V, T)}$$

- microcanonical ensemble, quantum:

$\mathcal{T}(E) \hat{=}$ number of states in energy interval $[E, E + \Delta E]$.

$$\hat{\rho}_{MC} = \sum_n \left(\frac{1}{\mathcal{T}(E)} \right) |\psi_n\rangle \langle \psi_n|$$

Sum goes over all eigenstates whose energy lies in $[E, E + \Delta E]$

all states contribute equally

- Quantum many-body system:
 - bosons (integer spin):
 - wave fct symmetric under exchange of any pair of particles
 - fermions (half-integer spin):
 - wave fct anti-symmetric under exchange of any pair of particles
- Boltzmann particles (theory construction): no exchange symmetry