Lecture Set 03

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Spin Precession

- $\tilde{\mathbf{H}} = \vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}} = -\left(\frac{e}{m_e c}\right) \tilde{\vec{\mathbf{S}}} \cdot \vec{\mathbf{B}}$
  - $\vec{\boldsymbol{\mu}} = g \frac{e}{2m_e} \tilde{\vec{\mathbf{S}}}$
- B_z field only $\tilde{\mathbf{H}} = -\left(\frac{eB}{m_e c}\right) \tilde{\mathbf{S}}_z$
  - $\tilde{\mathbf{H}} \left|\pm\right\rangle = \mp \frac{\hbar}{2} \frac{eB}{m_e c} \left|\pm\right\rangle = \pm \frac{1}{2} \hbar \omega_B \left|\pm\right\rangle$
  - $\omega_B = \frac{|e|B}{m_e c}$
- Time evolution $\boldsymbol{\mathcal{U}}(t,0) = \exp\left(\frac{-i\omega_B \tilde{\mathbf{S}}_z t}{\hbar}\right)$
  - $\left|\alpha, 0; t\right\rangle = c_+ \exp\left(\frac{-i\omega_B t}{2}\right) \left|+\right\rangle + c_- \exp\left(\frac{+i\omega_B t}{2}\right) \left|-\right\rangle$

Spin Precession

- $|\alpha, 0; t\rangle = c_+ \exp\left(\frac{-i\omega_B t}{2}\right)|+\rangle + c_- \exp\left(\frac{+i\omega_B t}{2}\right)|-\rangle$
- $c_+ = 1, c_- = 0 \quad \Rightarrow \quad |\alpha, 0; t\rangle = \exp\left(\frac{-i\omega_B t}{2}\right)|+\rangle$
- $\left|\langle + \left| \exp\left(\frac{-i\omega_B t}{2}\right)|+\rangle \right\rangle\right|^2 = 1$
- $\left|\langle - \left| \exp\left(\frac{-i\omega_B t}{2}\right)|+\rangle \right\rangle\right|^2 = 0$

Spin Precession

- $|\alpha, 0; t\rangle = c_+ \exp\left(\frac{-i\omega_B t}{2}\right)|+\rangle + c_- \exp\left(\frac{+i\omega_B t}{2}\right)|-\rangle$
- $c_+ = 1/\sqrt{2},\ c_- = 1/\sqrt{2}$
  $\Rightarrow \quad |\alpha, 0; t\rangle = \frac{1}{\sqrt{2}}\left[\exp\left(\frac{-i\omega_B t}{2}\right)|+\rangle + \exp\left(\frac{i\omega_B t}{2}\right)|-\rangle\right]$
- Probability it is in $\left|\tilde{\mathbf{S}}_x; \pm\right\rangle$ state

$$|\langle\alpha; \pm|\alpha, 0; t\rangle|^2 = \frac{1}{4}\left|\left[\langle+| \pm \langle-|\right]\left[\exp\left(\frac{-i\omega_B t}{2}\right)|+\rangle \pm \exp\left(\frac{i\omega_B t}{2}\right)|-\rangle\right]\right|^2$$

$$= \frac{1}{4}\left|\exp\left(\frac{-i\omega_B t}{2}\right) \pm \exp\left(\frac{+i\omega_B t}{2}\right)\right|^2$$

$$= \begin{cases} \cos^2\frac{\omega_B t}{2} & S_x = + \\ \sin^2\frac{\omega_B t}{2} & S_x = - \end{cases}$$

Spin Precession

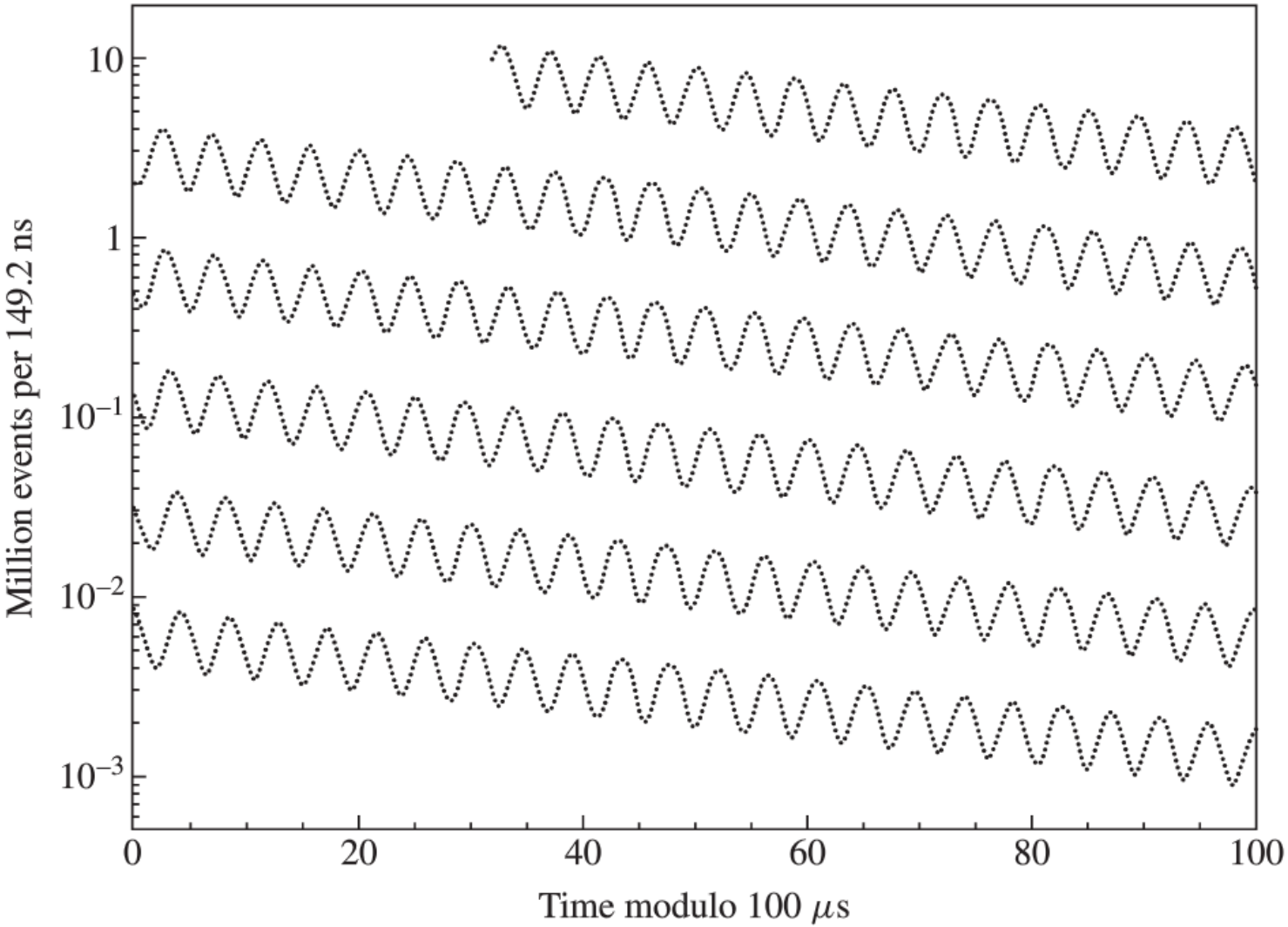
- Expectation value:

$$\left\langle \alpha;t \left| \tilde{\mathbf{S}}_x \right| \alpha;t \right\rangle = \langle \alpha;t \,|S_x,+\rangle \left\langle S_x,+ \left| \tilde{\mathbf{S}}_x \right| S_x,+ \right\rangle \langle S_x,+\,|\alpha;t\rangle$$
$$+ \langle \alpha;t \,|S_x,-\rangle \left\langle S_x,- \left| \tilde{\mathbf{S}}_x \right| S_x,- \right\rangle \langle S_x,-\,|\alpha;t\rangle$$
$$= \frac{\hbar}{2}\left[|\langle S_x,+|\alpha;t\rangle|^2 - |\langle S_x,-|\alpha;t\rangle|^2\right]$$
$$= \frac{\hbar}{2}\left[\cos^2\left(\frac{\omega_B t}{2}\right) - \sin^2\left(\frac{\omega_B t}{2}\right)\right] = \frac{\hbar}{2}\cos\omega_B t$$

- Spin operator:

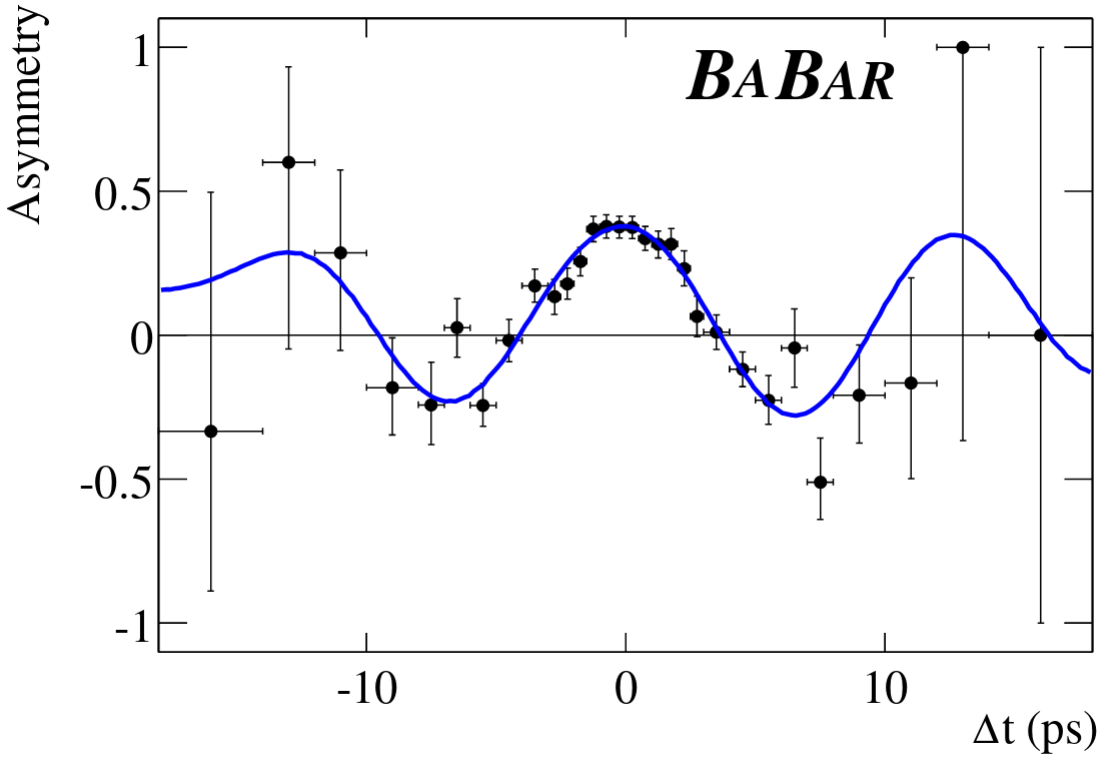
$$\tilde{\mathbf{S}}_x \doteq |S_x,+\rangle \left\langle S_x,+ \left| \tilde{\mathbf{S}}_x \right| S_x,+ \right\rangle \langle S_x,+| +$$
$$|S_x,-\rangle \left\langle S_x,- \left| \tilde{\mathbf{S}}_x \right| S_x,- \right\rangle \langle S_x,-|$$

Spin Precession



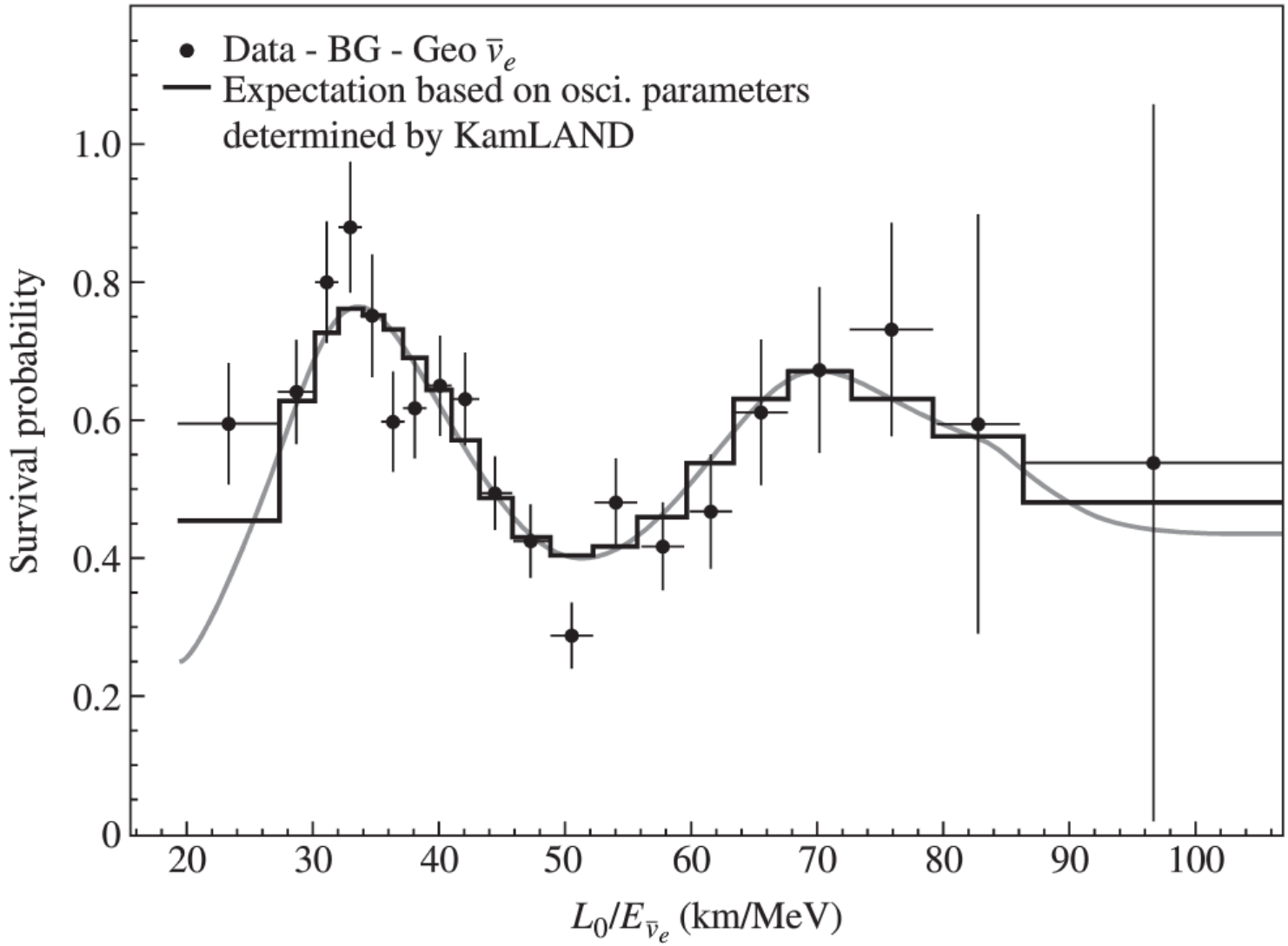
Spin Precession

B Meson Oscillations

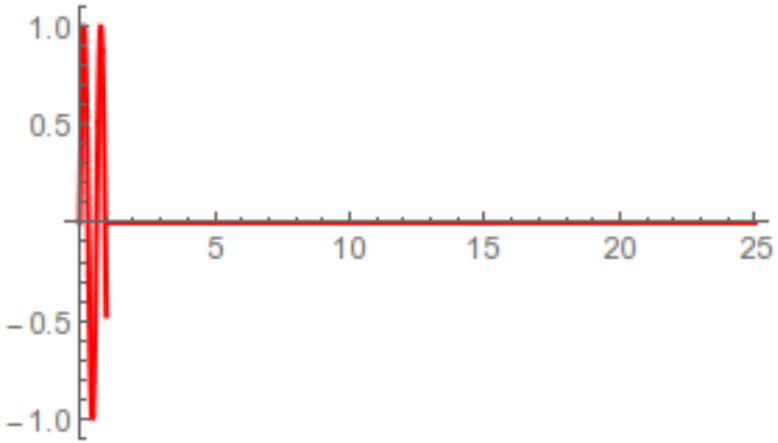


$\Delta m = 0.0003$ eV

Neutrino Oscillations



Pulse Time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0.5
−0.5
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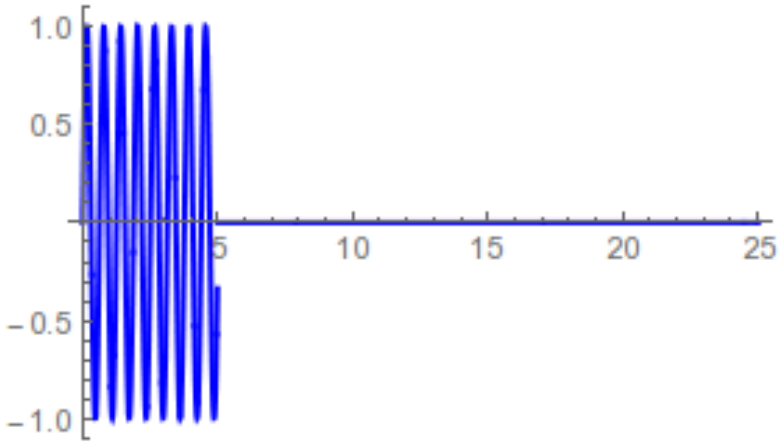
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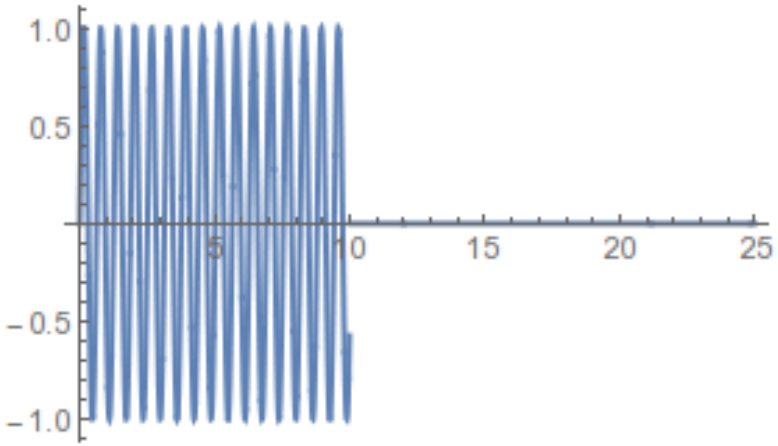
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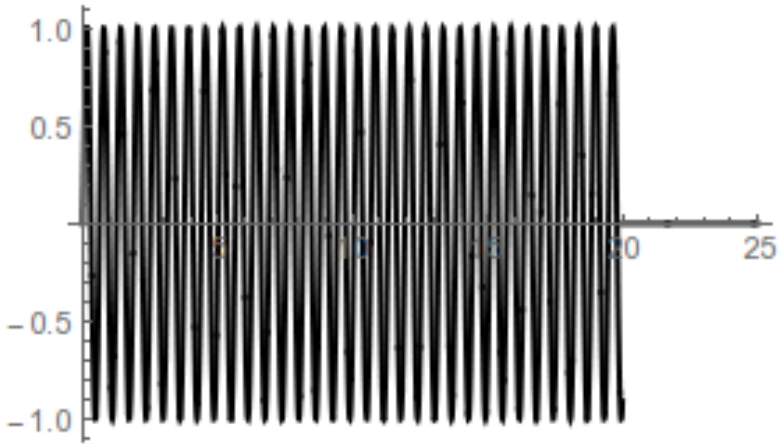
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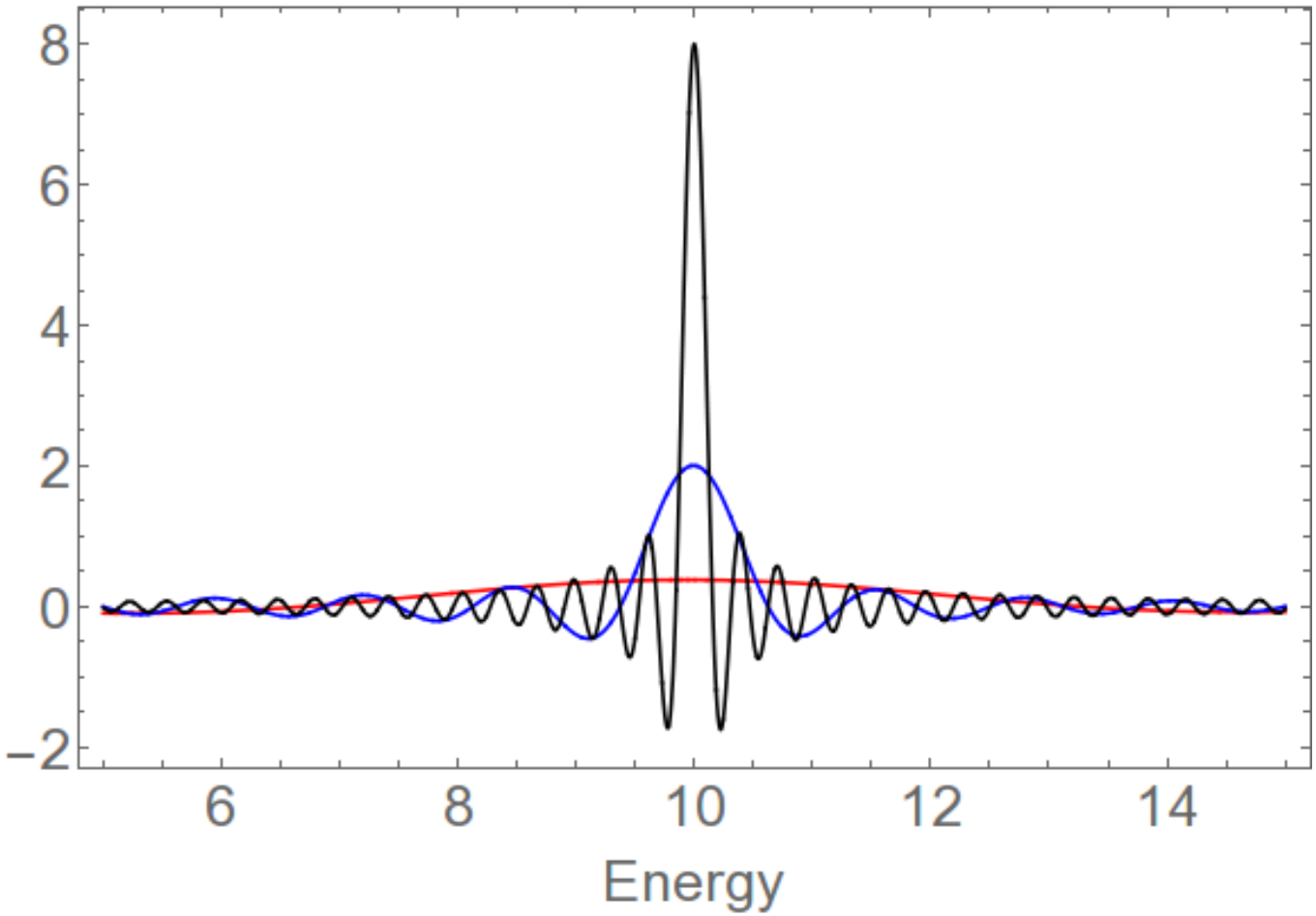
Pulse Energy



Peaked Function