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Electrodynamics 1

CH. 6 SYMMETRIES AND CONSERVATION LAWS LECTURE NOTES

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In Electro and Magneto statics we have the Maxwell Equations

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\rho(\vec{r}) = 0 \quad , \vec{J} = 0$$

$$\vec{\nabla} \cdot \vec{E} = 0 \, , \, \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad , \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla^2 \vec{E} - \frac{1}{c^2} \partial_t^2 \vec{E} = 0 \quad , \quad \nabla^2 \vec{B} - \frac{1}{c^2} \partial_t^2 \vec{B} = 0$$

we then wish to solve some of these differential equations by making guesses

$$\vec{E}(\vec{r}) = \vec{E}_0 f(x - ct) \longrightarrow \text{Moving in x-direction}$$

$$\vec{\nabla} \cdot \vec{E} = \partial_x E_{0x} f(x-ct) + \underbrace{\partial_y E_{0y} \cancel{f(x-ct)}}_{0} + \partial_z E_{0z} \cancel{f(x-ct)} = \partial_x E_{0x} f(x-ct) = 0$$

we can also take the curl of this Electric Field

$$\vec{\nabla} \times \vec{E} = \hat{x}_i \epsilon_{ijk} \partial_j E_0 f(x-ct) \hat{z}$$

This of course leads to

$$\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}$$

Where our magnetic field is $\vec{B} = \vec{B}_0 g(r - ct)$, we then have

$$E_0 f'(x-ct) \hat{y} = c \vec{B}_0 g'(r-ct)$$

The Electric and Magnetic Fields are then

$$\vec{E} = E_0 f(x-ct) \hat{z} \quad , \quad \vec{B} = -\frac{E_0}{c} f(x-ct) \hat{y}$$

If we then look at $\vec{E} \times \vec{B}$,

$$\vec{E} \times \vec{B} = \frac{E_0^2}{c} f^2 \hat{x}$$

where the above is the pointing vector. The pointing vector tells us how energy is transferred in a system.

we now look at the energy density,

$$\mathcal{E} = \frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2$$

we then have

$$\frac{\partial}{\partial t}(\mathcal{E}) = \epsilon_0 \vec{E} \cdot \frac{\partial}{\partial t} \vec{E} + \frac{1}{\mu_0} \vec{B} \cdot \frac{\partial}{\partial t} \vec{B} = \epsilon_0 \vec{E} \cdot \left(\frac{1}{\mu_0 \epsilon_0} \vec{\nabla} \times \vec{B} - \frac{1}{\epsilon_0} \vec{J} \right) + \frac{1}{\mu_0} \vec{B} (-\vec{\nabla} \times \vec{E})$$

This goes to

$$\frac{\partial}{\partial t} \mathcal{E} + \vec{E} \cdot \vec{J} = \frac{1}{\mu_0} \left(\vec{E} \cdot (\vec{\nabla} \times \vec{B}) - \vec{B} \cdot (\vec{\nabla} \times \vec{E}) \right)$$

Where we can say

$$\vec{E} \cdot (\vec{\nabla} \times \vec{B}) - \vec{B} \cdot (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \cdot (\vec{E} \times \vec{B})$$

$$\partial_i \epsilon_{ijk} E_j B_k = \epsilon_{ijk} (\partial_i E_i) B_k + \epsilon_{ijk} E_j (\partial_i B_k) = \epsilon_{kij} B_k (\partial_i E_j) - \epsilon_{ijk} E_j (\partial_i B_k)$$

Which Finally is

$$\vec{\nabla} \cdot (\vec{E} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot (\vec{\nabla} \times \vec{B})$$