Example: ODE with fixed value boundary condition

Solve:

$$\frac{\partial^2 \varphi}{\partial x^2} + \text{Cos}[x]\, \varphi(x) = 0$$

in the interval $0 \le x \le 2\pi$, with the boundary conditions that $\varphi(0) = \varphi(2\pi) = e$, Euler's constant.

Coarse version

In[1212]:=
```
ndiv = 10;
dx = 2. π / ndiv;
npts = ndiv - 1;
mat = Table[If[i == j, (-2/dx^2 + Cos[j dx]), 0] + If[Abs[i - j] == 1, 1/dx^2, 0],
   {i, 1, npts}, {j, 1, npts}];
MatrixForm[
 mat]
```

Out[1216]//MatrixForm=

$$\begin{pmatrix}
-4.25704 & 2.53303 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2.53303 & -4.75704 & 2.53303 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2.53303 & -5.37508 & 2.53303 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.53303 & -5.87508 & 2.53303 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2.53303 & -6.06606 & 2.53303 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2.53303 & -5.87508 & 2.53303 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2.53303 & -5.37508 & 2.53303 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2.53303 & -4.75704 & 2.5330 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.53303 & -4.257
\end{pmatrix}$$

In[1221]:=
```
bvec = Table[0, {j, 1, npts}];
```
$$\text{bvec}[[1]] = \frac{-E}{dx^2};$$
$$\text{bvec}[[\text{npts}]] = \frac{-E}{dx^2};$$
```
MatrixForm[bvec]
```

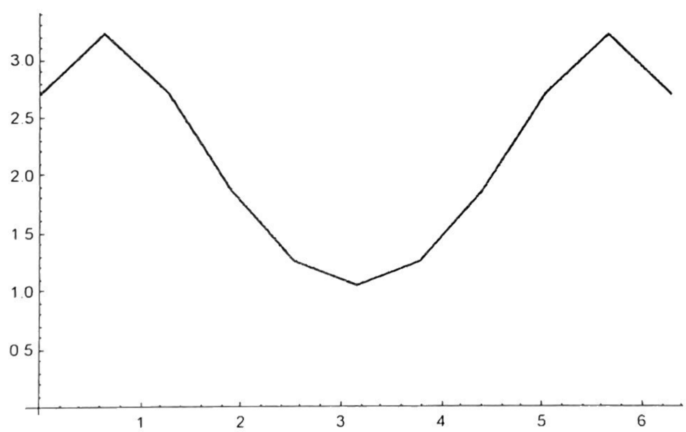
Out[1224]//MatrixForm=
$$\begin{pmatrix} -6.88549 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -6.88549 \end{pmatrix}$$

In[1225]:=
```
phi = LinearSolve[mat, bvec]
```
Out[1225]=
```
{3.24177, 2.72987, 1.88495, 1.26997, 1.06062, 1.26997, 1.88495, 2.72987, 3.24177}
```

In[1228]:=
```
phiTot = Join[{E}, phi, {E}];
```

In[1230]:=
```
phidat = Table[{(j - 1) * dx, phiTot[[j]]}, {j, 1, ndiv + 1}]
```
Out[1230]=
```
{{0., e}, {0.628319, 3.24177}, {1.25664, 2.72987}, {1.88496, 1.88495},
 {2.51327, 1.26997}, {3.14159, 1.06062}, {3.76991, 1.26997},
 {4.39823, 1.88495}, {5.02655, 2.72987}, {5.65487, 3.24177}, {6.28319, e}}
```

In[1231]:=
```
ListPlot[phidat, Joined → True]
```
Out[1231]=



Fine version

In[1232]:=
```
ndiv = 100;
dx = 2. π / ndiv;
npts = ndiv - 1;
mat = Table[If[i == j, (-2/dx^2 + Cos[j dx]), 0] + If[Abs[i - j] == 1, 1/dx^2, 0],
    {i, 1, npts}, {j, 1, npts}];
MatrixForm[
  mat];
```

In[1237]:=
```
bvec = Table[0, {j, 1, npts}];
bvec[[1]] = -E/dx^2;
bvec[[npts]] = -E/dx^2;
MatrixForm[bvec];
```

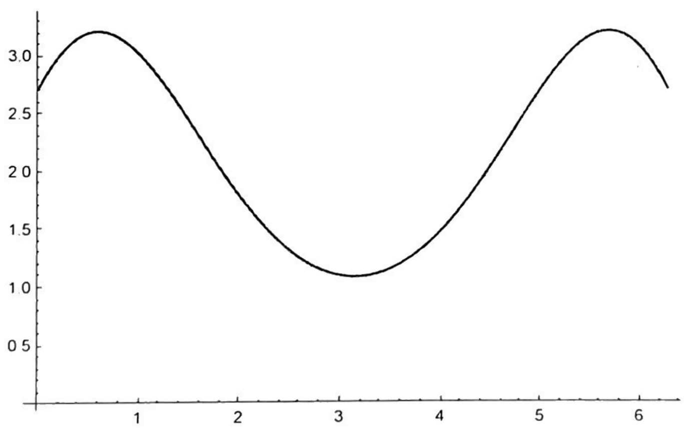
In[1246]:=
```
phi = LinearSolve[mat, bvec];
```

In[1242]:=
```
phiTot = Join[{E}, phi, {E}];
```

In[1244]:=
```
phidat = Table[{(j - 1) * dx, phiTot[[j]]}, {j, 1, ndiv + 1}];
```

In[1245]:=
```
ListPlot[phidat, Joined → True]
```

Out[1245]=



Check

In[1366]:=
```
appFxn = Interpolation[phidat, InterpolationOrder → 6]
```

Out[1366]=
```
InterpolatingFunction[ Domain: {{0., 6.28}} Output: scalar ]
```

In[1367]:=
```
Plot[appFxn''[x] + Cos[x] appFxn[x], {x, 0, 2 π}]
```

Out[1367]=