E & M I
Homework 1

1) Charged Slab with width:

In the first workshop we used Gauss' Law and symmetry considerations to calculate the electric field due to an infinite, uniform, thin charged slab lying in the x-y plane.

Consider a similar infinite slab, but with a spread-out charge distribution perpendicular to the slap.

$$\rho(\vec{r}) = \rho(z) = \rho_0 \, e^{-\alpha|z|}$$

This might be used as a model for a nano-scale metal surface where the charge distribution is related to the "skin depth" or finite extension of the surface states of the electrons in the metal.

a) Explain why and how you can use Gauss' Law to determine the electric field due to this slab. What is the general form for the electric field? Justify your answer.

b) Describe carefully, and draw a picture, showing the volume and surface you will use to solve the integrals in Gauss' Law

c) Solve the two integrals (volume integral and surface integral) in Gauss' Law to determine the electric field everywhere.

d) Determine the electrostatic potential, $\phi(\vec{r})$, that corresponds to this electric field. (Hint: you can either note that the potential is the integral of the field or guess and check a function for the potential with the property that the negative of the gradient of the function gives the field.

e) Consider your results for the field and the potential in the limits as $z \to \infty$ and $z \to 0$. Do these limits make physical sense, comparing them to the thin slab? Explain.

f) Note that your result disagrees with our symmetry arguments from Questions 1a, 1b, and 1c of Workshop 1. Which of the symmetry arguments doesn't work for this case and why?

2) Classical "Hydrogen Atom" Field:

If we consider the electron's probability density in a hydrogen atom as a static, classical charge density (I know, quite a stretch) we can make some classical predictions about the atom.

Consider a charge distribution of the form:

$$\rho(r) = \rho_0 \, e^{-\alpha \, r}$$

The charge distribution has a total charge Q and $\alpha = \frac{2}{a_0}$ where a_0 is the length scale in the problem. For a hydrogen atom, $Q = -e$ and a_0 is the Bohr radius.

a) Determine what ρ_0 needs to be in terms of Q and a_0. Be sure to explain how you determined your answer.

Hint: There is a standard trick for doing these sorts of integrals:

$$\int x^n\, e^{-\alpha x}\, dx = (-1)^n \frac{\partial^n}{\partial \alpha^n} \int e^{-\alpha x}\, dx$$

Of course, if you're not in a test (or qualifier) you can always pull up Mathematica, Wolfram Alpha, or whatever python library you might use.

b) Explain how you will use Gauss' Law to determine the electric field everywhere for the atom. Draw a picture to illustrate your approach.

c) Calculate both the volume and the surface integral in Gauss' Law, showing your work. Use these results to determine the electric field everywhere.

d) As always, we need to check our results. Show that your result for the field makes physical sense in the limit that r gets large.

NOTE: This means more than finding what happens if $r = \infty$. Check the functional behavior in the limit as $r \to \infty$.

d) Check that your result also is well behaved in the limit as $r \to 0$. Remember that this charge distribution is NOT due to a point charge, just the charged cloud.

3) Classical Hydrogen Atom potential:

The equation 16.27 in the textbook gives the classical hydrogen atom electric potential. This is the potential of both the electron cloud and the proton of the hydrogen atom, modeled as a point charge.

(Note, this is in Gaussian Units while we've been using SI units in class. You should be able to handle this difference.)

a) Consider your results to Problem 2. What is the total electric field of the atom, including both the electron cloud (Problem 2) and the proton? Explain your work.

b) Show that your total electric field agrees with the potential given in 16.27.

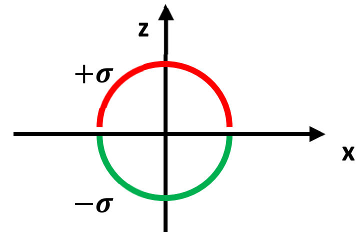
c) Determine the electrostatic potential energy of the electron cloud of the hydrogen atom. Use the total electrostatic potential but don't include the energy due to the proton.

4) The "Dipole Sphere":

The electric potential and electric field of a sphere with a uniform charge on its surface is a common, simple example done in introductory physics classes. Let's change this up a bit for some more useful practice.

a) As a reminder, write down the electric potential and electric field for a sphere of radius R centered at the origin, $\vec{r} = 0$. Include results for both $r > R$ and $r < R$. You don't have to actually solve this, but it would be good practice just to be sure you can do it.

Next, consider the same sphere of radius R but now the surface charge is split between the upper and lower hemispheres.



The upper hemisphere ($z > 0$ in Cartesian Coordinates, $0 \leq \theta \leq \frac{\pi}{2}$ in Spherical Coordinates) has a constant positive surface charge density, $+\sigma$. The lower hemisphere ($z < 0, \frac{\pi}{2} \leq \theta \leq \pi$) has a constant negative surface charge density, $-\sigma$. The magnitudes of the charge densities are the same.

b) Sketch what you think the electric field will look like in the x-z plane shown.

c) Explain why you can't directly use Gauss' Law to solve for the field of this charge distribution.

d) Using a direct integration, solve for the electric potential everywhere on the z-axis. This includes $z > R, z < -R,$ and $-R < z < R$. Of course, show your work.

e) Calculate the electric field along the z-axis, $\vec{E}(z) = E(z)\,\hat{e}_z$ using your result for the potential.

f) The properties of the potential and field of this "Dipole Sphere" are quite different from those of the uniformly charge sphere. What are some of the differences? Explain the physics of these differences.

For example, you might consider what happens for $|z| \to \infty, |z| \to R$, and the field and potential in the interior of the spheres.