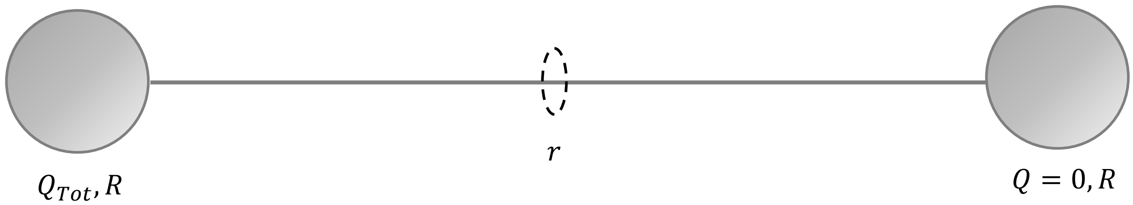
E & M I

Workshop 7 – Maxwell & Faraday, 3/23/2022

In today's workshop, we'll consider an argument for Maxwell's addition to Ampere's Law. There will be some hand-waving, neglecting some issues of time-dependence, but it gives interesting results and also gives some practice with electrostatics and Ampere's Law.



Consider two conducting spheres, both with radius a, that are very far apart> we can treat them as basically isolated spheres. Initially the left sphere has a charge Q_{Tot} and the right sphere is uncharged.

A) Using the results from electrostatics, explain/justify briefly why (for electro-STATICS) that:

i) The electric field for a charged sphere will be zero inside and perpendicular to the sphere at the surface (and outside).

ii) All the charge will be on the surface of the sphere.

iii) The electric potential is a constant inside and on the surface and equal to

$$\phi(R) = k\frac{Q}{a}, \qquad k = \frac{1}{4\pi\epsilon_0}$$

$$\left(\vec{\nabla}\cdot\vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\epsilon_0}, \qquad \vec{E}(\vec{r}) = -\vec{\nabla}\phi(\vec{r}), \qquad F_q = q\,\vec{E}(\vec{r})\right)$$

Consider what happens if we connect the two spheres with a conducting wire. Assume that the wire is thin enough that the net charge on the wire will be much smaller than Q_{Tot}.

B) Explain what will happen to the charge in the system when the spheres are connected. What will be the charge distribution a very long time after the spheres are connected? Why?

The wire has a resistance R that is large enough so that the charge remains uniformly distributed on the spheres (the current is not too large). Remember Ohm's Law:

$$I(t) = \frac{\Delta V(t)}{R} = \frac{\phi_L(t) - \phi_R(t)}{R}$$

$\phi_L(t)$ and $\phi_R(t)$ are the potentials of the left and right spheres.

C) Write down a differential equation for the time-dependent charge on the right sphere, $Q_R(t)$.

D) Solve the differential equation from (C) for the charge $Q_R(t)$ and the current in the wire $I(t)$. Show that these have the correct values at $t = 0$ (when the spheres are connected) and $t \to \infty$.

Hint: What is the general solution to:

$$\frac{d}{dt} f(t) + \gamma\, f(t) = C, \qquad C = \text{Constant}$$

(Don't spend too much time on this. If you get stuck, ask.)

Next consider Ampere's Law using a small "Amperian" loop of radius r around the center of the wire as shown.

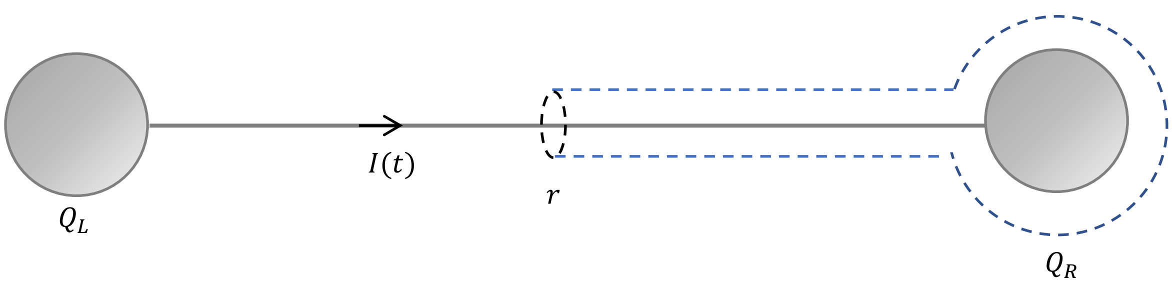
E) Consider Ampere's Law in the magneto-static approximation:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \quad \Rightarrow \quad \oint \vec{B} \cdot d\vec{l} = \mu_0 \iint \vec{J} \cdot \hat{n}\, dS = \mu_0\, I_{enclosed}$$

Using this result and the simplest surface for the loop, a flat disk bounded by the loop, calculate the "quasi-static" approximation to the magnetic field, $\vec{B}(t)$ at the center of the wire.

Stoke's Theorem, and Ampere's Law, should work for ANY surface with the boundary being the loop given. You might see where this is going…

Consider a surface that is in the shape of a "flask", with the open end being the amperian loop, a long cylindrical "neck" around the wire, and a spherical bulb centered on, the right sphere:



F) What is the current through this new surface? This is a "demonstration" that the magnetostatic Ampere's Law is insufficient.

G) Determine the electric field due to the right sphere (don't over-think this) and show that you will get the same result as part E using:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \vec{E} \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 \left(\iint \vec{J} \cdot \hat{n}\, dS + \epsilon_0 \frac{\partial}{\partial t} \iint \vec{E} \cdot \hat{n}\, dS \right)$$

Bonus Question: What is the problem with how we did part E, considering Maxwell?