PHYS 5013 October 4, 2020
Exam Sample One Mullen

Name: ______________________________

Each of the following problems is designed to test you understanding and integration of the material we have studied. Please note that:

- You may use your notes or anything in your own handwriting.
- The problems are broken up into small parts; in no case do you need the result of earlier questions to proceed, so if you get stuck, just move on.
- Explanations are as important as having the right answer. You must not only state the right answer, but make it clear how you derived it.
- If you don't know how to do the entire problem, try to communicate what you do understand.
- Make sure your writing is readable!

You will have up to two hours for this exam. Cheating will be punished by the most gruesome method I can devise. And I can be pretty inventive. Don't cheat. This exam is broken into three parts.

Part I The first part consists of several true/false questions, for a total of 40 points. If you mark a statement as false you must explain why it is false or give a counter example in the space provided.

Part II The second part involves a short essay. Please write enough to convince me that you understand the material. It is worth 20 points.

Part III The third part involves detailed problem solving, and is worth 60 points. Please read the questions carefully, and show all your work.

Part I

______ 1. The magnetic field is an axial vector.

a. True b. False

______ 2. The Levi-Civita tensor $\epsilon_{123321} = -1$.

a. True b. False

______ 3. The metric tensor for an orthogonal co-ordinates system is diagonal.

a. True b. False

______ 4. Requiring that a particle travel on the path taking the least time is an example of a global constraint.

a. True b. False

______ 5. A set of linearly independent vectors is orthonormal.

a. True b. False

______ 6. Noether's theorem states that every symmetry of a system described by a Lagrangian has an associated conserved quantity.

a. True b. False

______ 7. There can be solutions of calculus of variations problems that are not found by the Euler-Lagrange equation even when the problem is well-posed.

a. True b. False

______ 8. Every dependent variable in the action generates a separate Euler-Lagrange equation.

a. True b. False

______ 9. Orthogonal transformations preserve angles.

a. True b. False

______ 10. Contravariant and covariant vectors transform the same way.

a. True b. False

Part II

11. We have had several examples of how mathematical physics must allow for the different *representations* of a quantity. Give three examples of how this is the case and their consequences.

Part III

12. A slender rod of lenght L subject to an external load $F(x)$ will buckle slightly to one side by a distance $w(x,t)$. The Lagrangian for the system is approximately:

$$\mathcal{L} = \int_0^L \left\{ \frac{1}{2}\mu \left(\frac{\partial w}{\partial t}\right)^2 - \frac{1}{2}EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + F(x)w \right\} dx$$

where E is the constant elastic modulus of the material and I is the (presumed) constant second moment of the cross-sectional area.

a. What is the associated Euler-Lagrange equation for the dynamics of the motion?

b. Is there a conserved Noether charge associated with the variation $t \to t + \epsilon\xi$? Explain.

c. Is there a conserved Noether charge associated with the variation $w \to w + \epsilon\eta$? Explain.

13. The parabolic coordinates u, v and w are defined in terms of the Cartesian coordinates by:

$$\begin{aligned} x &= \sqrt{uv}\cos w \\ y &= \sqrt{uv}\sin w \\ z &= \tfrac{1}{2}(u-v) \end{aligned}$$

where $u \geq 0$ and $v \geq 0$. (This system of coordinates in useful in problems of atomic physics when you calculate ionization amplitudes in an external field.)

a. Show that this is an orthogonal coordinate system.

b. Calculate the gradient in this coordinate system.

c. Calculate the divergence in this coordinate system.

d. Calculate the Laplacian in this coordinate system.

14. Consider a curve $y(x) > 0$ where $y(x_a) = y(x_b) = 0$. and the total area under the curve is A. Find the curve that has the minimum length L.