Homework Assignment #6

Math Methods

Reading Quiz Due: Friday, October 15th, 11:00am

Homework Due: Tuesday, October 19th, midnight

Instructions:

Reading: Please read Chapter 3. The reading quiz will be due Wednesday before class. Much of the chapter deals with concepts in matrices that I believe you already understand. If this is not the case, be sure to let me know by posting to the discussion board and also providing a detailed answer to the quiz question about what material I should review.

Problems: Below is a list of questions and problems from the texbook due by the time and date above. It is not sufficient to simply obtain the correct answer. You must also explain your calculation, and each step so that it is clear that you understand the material.

Homework should be written legibly, on standard size paper. Do not write your homework up on scrap paper. If your work is illegible, it will be given a zero.

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1. Submit your answer to problem (4) from the previous homework set. (This is the long homework problem dealing with $F_{\mu\nu}$.)

2. Consider the function
$$f(x) = 4x^3 - 32x^2 + 66x - 18$$
   (a) Solve $f(x) = 0$ for x, obtaining both symbolic and numeric answers.
   (b) Solve $f'(x) = 0$ for x, obtaining both symbolic and numeric answers.
   (c) Plot the function and verify that your roots and extrema are correct.

3. Plot the Taylor series expansion of the sine function over the interval $0 < x < 2\pi$ and determine how many terms you need in order to get reasonable accuracy. (*Hint:* You can use the **Table** command to generate a list of the terms and the **Total** command to sum up the list.)

4. Consider the 10×10 matrix $\mathcal{A}_{ij} = \sin(ij)$ and vector 10×1 column vector $b_i = i$. Solve the numerical problem
$$\mathcal{A}\,\vec{x} = \vec{b}$$
for the unknown vector $\vec{x}$ using the command **LinearSolve**. (*Hint:* If this takes a while to calculate on your computer, then you have gotten caught in one of the traps I warned you about in lecture.)

5. **Euler method breakdown:** Consider a simple harmonic oscillator governed by
$$\ddot{x} = -x$$
with $x(0) = 1$, and $\dot{x}(0) = 0$. (We have set the spring constant and the mass to one). Define $v \equiv \dot{x}$.
   (a) Consider the curve $(x(t), v(t))$. What should it look like for the exact solution?

(b) Solve this with a standard Euler approach:

$$\begin{aligned} v_{i+1} &= v_i - x_i \Delta t \\ x_{i+1} &= x_i + v_i \Delta t \end{aligned}$$

(Convince yourself that this is a reasonable discretization scheme and is consistent with the equation of motion.) Plot the orbit (x_i, v_i) for $0 \leq t \leq 8\pi$ for some reasonable choice of step size Δt. What type of orbit do you get?

(c) Alter the algorithm so that you instead use:

$$\begin{aligned} v_{i+1} &= v_i - x_i \Delta t \\ x_{i+1} &= x_i + v_{i+1} \Delta t \end{aligned}$$

Thus the new position depends upon the *new* velocity. Plot the orbits again. Does this give a better result? Note that this is partially an *implicit scheme*, in that we use some information at time $i+1$ to calculate the solution at time $i+1$.

(d) The energy E_i at time i is given by $(x_i^2 + v_i^2)/2$. Calculate analytically the energy at time $i+1$ in the algorithm of part (b), in terms of x_i and v_i. What do you get for $(E_{i+1} - E_i)/\Delta t \sim \dot{E}$? Is this consistent with your graph? Do the same for the algorithm of part (c). Why is it better?

6. Assume you wish to solve for the eigenstates of an electron in an infinite 1D quantum well with an electric field, $\mathcal{E}$ applied across the well. Your Hamiltonian is given by:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} - e\mathcal{E}x\,\psi = E\,\psi$$

where $0 \leq x \leq L$. You are interested in finding out how the energy of the first two eigenstates changes as a function of the strength of the electric field.

(a) What is the unit in which you will measure distance?

(b) What is the unit with which you will measure the energy? What is its physical meaning?

(c) What is the dimensionless parameter that is the "control knob" that corresponds to increasing the field? What is its meaning?

(d) At what value of this parameter would you expect to see deviations from the infinite square well solutions?

(e) Solve the problem numerically. Plot the energy of the lowest ten eigenstates for a "small" value of the applied field. How do they vary with n, the number of the eigenstate?

(f) Solve the problem numerically. Plot the energy of the lowest ten eigenstates for a "large" value of the applied field. How do they vary with n, the number of the eigenstate?

(g) Plot the energy of the lowest two eigenstates as a function of the applied field sweeping from small to large values of the applied field. Is your estimate above for the critical values of the field correct?

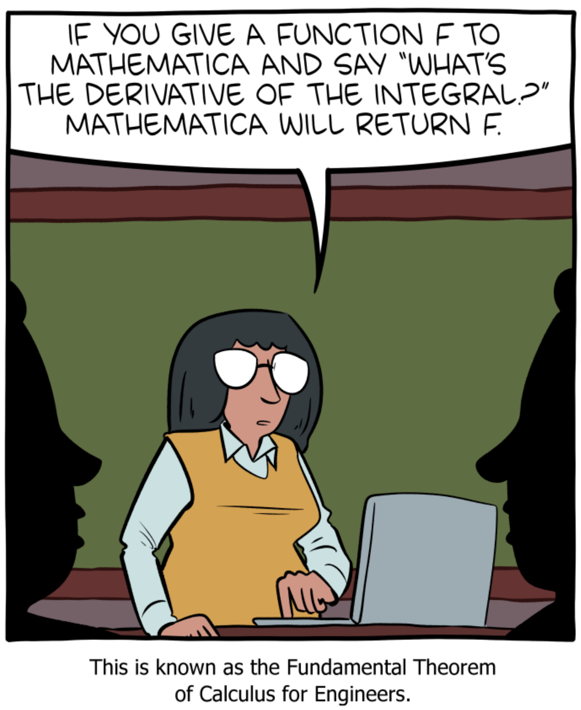

Figure 1: Cartoon by Z. Weinersmith, *"Saturday Morning Breakfast Cereal".*