Key points of 03/09 lecture

- Single quantum particle (non-relativistic) in 2D area ($A = L^2$; A area, L length) with periodic boundary conditions (no force):

$$\hat{\mathcal{H}}\psi = \mathcal{E}\psi \quad \text{with} \quad \hat{\mathcal{H}} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$$

$$\psi(x,y) \underset{\substack{\uparrow \\ \text{separable}}}{=} \varphi(x)\varphi(y)$$

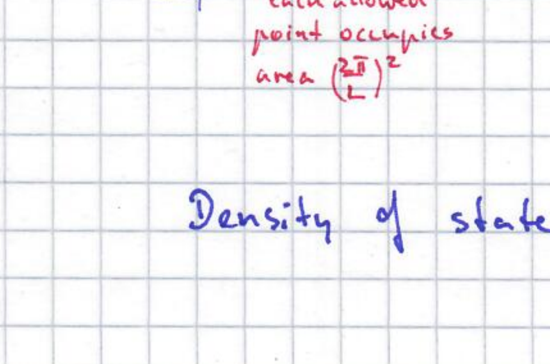
Periodic BCs implies: $\varphi(x+L) = \varphi(x)$

$$\Rightarrow e^{ik_x(x+L)} = e^{ik_x x} \Rightarrow k_x = \frac{2\pi}{L} n_x$$

$$n_x = 0, \pm 1, \pm 2, \dots$$

$$\mathcal{E} = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2\right)$$

In k space: number of states with $\mathcal{E} \leq E$: $N(E)$



area in k space

$$N(E) = \frac{\pi k^2}{\left(\frac{2\pi}{L}\right)^2} = \frac{m E L^2}{2\pi\hbar^2}$$

one state per $\left(\frac{2\pi}{L}\right)^2$

Density of states: $D(E) = \dfrac{dN(E)}{dE} = \dfrac{mL^2}{2\pi\hbar^2}$ — 2D, non-rel. particle; periodic BC

$N(E)$ counts states up to E. ~~Thin~~ onion ring: $N(E_2) - N(E_1) = \underbrace{\frac{N(E_2) - N(E_1)}{E_2 - E_1}}_{D(E)}(E_2 - E_1)$

shell