# 5163, Homework Assignment 4 due on Friday, 02/18/2022, at 6pm (to be uploaded to Canvas)

This homework set consists of four problems.

Problem 1:
A solid is composed of $N$ atoms whose nuclei have angular momentum $\hbar/2$ and thus a nuclear magnetic moment of magnitude $\mu$. When the solid is placed in a homogeneous field along a given axis, the magnetic moment of each of the atoms can have the value $\mu$ or $-\mu$. Let us assume that the interaction between the magnetic moments and the external field is much, much stronger than the interaction between neighboring magnetic moments.
(a) Use the canonical ensemble to calculate the internal energy of the solid in the external field at temperature $T$.
(b) Use the canonical ensemble to calculate the entropy of the solid in the external field at temperature $T$.

Problem 2:
(a) An experimentalist measures the specific heat $C_V$ of a gas of non-interacting one-dimensional particles of mass $m$ at temperature $T$. The potential of the $j$-th particle is given by

$$V(x_j) = \epsilon_0 \left| \frac{x_j}{a} \right|^n , \tag{1}$$

where $\epsilon_0$ and $a$ are constants with units "energy" and "length", respectively, and $n$ is an integer, $n = 1, 2, \cdots$. From the measurements, can the experimentalist determine the value of $n$?
(b) The experimentalist repeats the measurement for a gas of non-interacting two-dimensional particles of mass $m$ at temperature $T$ for which the potential of the $j$-th particle is given by

$$V(x_j, y_j) = \epsilon_0 \left( \frac{\rho_j}{a} \right)^n , \tag{2}$$

where $\rho_j = \sqrt{x_j^2 + y_j^2}$. What is $C_V$ in this case?
The following integral may be helpful:

$$\int_0^\infty \exp(-z) z^{(1-n)/n} dz = \Gamma(1/n). \tag{3}$$

Problem 3:
For zero-mass particles, the energy-momentum relation is $E = c|\vec{p}|$. This relationship can also be used to approximate "ordinary" particles when $kT \gg mc^2$. A gas described by this relationship is sometimes referred to as an extremely relativistic classical gas.
Assuming three-dimensional space, calculate the pressure and energy per particle of such a gas as functions of the density and temperature.

Problem 4:
This problem serves as a review of undergraduate quantum knowledge in preparation for quantum statistical mechanics.
(a) A one-dimensional harmonic oscillator potential is a potential of the form

$$V(x) = \frac{1}{2}kx^2. \tag{4}$$

What is the energy and degeneracy of the ground state of a system consisting of five non-interacting particles of mass $m$ that are confined by $V(x)$ in the cases that
(ai) the particles are spin-0 bosons,
(aii) the particles are spin-$\frac{1}{2}$ fermions,
(aiii) the particles are spin-$\frac{1}{2}$ bosons,
(aiv) the particles are spin-0 fermions, and
(av) the particles are spin-$\frac{5}{2}$ fermions?
(b) Repeat part (a) for the isotropic two-dimensional harmonic oscillator potential.
(c) Which of the cases (ai)-(av) is physically possible/impossible? Explain.