Diagonalizing Transformation

Out[48]//MatrixForm=

$$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

In[49] = eout = Eigensystem[mat];

Out[50]=
$$\left\{\frac{1}{2}\left(-5-\sqrt{5}\right), \frac{1}{2}\left(-3-\sqrt{5}\right), \frac{1}{2}\left(-5+\sqrt{5}\right), \frac{1}{2}\left(-3+\sqrt{5}\right)\right\}$$

In[61] = evecs = eout[[2]];

In[52] = bmat = Transpose[evecs];

bmatInv = Simplify[Inverse[bmat]];

In[54] = MatrixForm[bmat]

Out[54]//MatrixForm=

in[55] = MatrixForm[bmatInv]

Out[55]/MatrixForm=

$$\begin{pmatrix}
\frac{1}{2\theta} \left(-5 + \sqrt{5} \right) & \frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & \frac{1}{2\theta} \left(5 - \sqrt{5} \right) \\
\frac{1}{2\theta} \left(5 + \sqrt{5} \right) & -\frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & \frac{1}{2\theta} \left(5 + \sqrt{5} \right) \\
\frac{1}{2\theta} \left(-5 - \sqrt{5} \right) & -\frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{1}{2\theta} \left(5 + \sqrt{5} \right) \\
\frac{1}{2\theta} \left(5 - \sqrt{5} \right) & \frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{1}{2\theta} \left(5 - \sqrt{5} \right)
\end{pmatrix}$$

In[56] = bmatInv.mat.bmat // Simplify // MatrixForm

Out[56]/MatrixForm=