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Electrodynamics 1

CH. 5 MAGNETOSTATICS IN VACUUM LECTURE NOTES

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Faradays Law

Faradays Law mathematically is

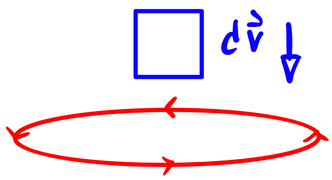
$$\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}$$

Where we have,

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint \vec{B} \cdot \hat{n}\, ds$$

Faradays Law says that an electro/magnetic Field can be created

We look at a simple scenario



Where the same happens if the hoop is moving and the block is stationary

We look at the moving block,

$$\oint \vec{E}' \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot \hat{n}\, ds + \int \vec{\nabla} \times (\vec{r} \times \vec{B}) \cdot \hat{n}\, ds$$

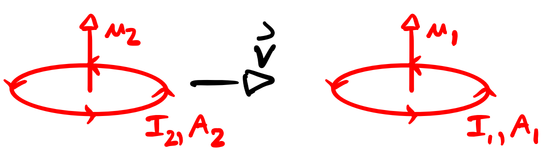
We then have

$$\oint (\vec{E}' - \vec{v} \times \vec{B}) \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot \hat{n}\, ds \implies \oint \vec{E}' \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot \hat{n}\, ds$$

We have our electric potential energy as

$$U = \frac{1}{2} \sum' q\, \phi(\vec{r}_q) = \frac{1}{2} \int \rho(\vec{r}) \varphi(\vec{r})\, d^3r = \frac{\epsilon_0}{2} \int E^2 d^3r$$

Lets look at two separate loops



We then have

$$dW_e = -I_2 \Delta t\, \mathcal{E}_1 \quad w/ \quad \mathcal{E}_1 = -A_2 \frac{\partial B_{12}}{\partial x} \frac{\Delta x}{\Delta t} = -A_2 \frac{\partial B_{12}}{\partial x} V_x$$

Where Further we see

$$dW_e = I_2 A_2 \frac{\partial B_{12}}{\partial x} V_x \Delta t = \mu_2 \frac{\partial B_{12}}{\partial x} V_x \Delta t$$

The force is then

$$\vec{F} = -\vec{\nabla}(\vec{m}_2, \vec{B}_1) = -\vec{\nabla}(m_2 B_{1z})$$

We then Finally have

$$dW_M = \vec{F} \cdot \Delta\vec{x} = -m_2 \frac{\partial B_{1z}}{\partial x} V \Delta t$$

For the hoop that is static,

$$dw_T = dw_{e2} + dw_{m2} + dw_{e1} = -dw_{m1}$$

Looking at the math we see

$$dw = \vec{m}_1 \cdot \vec{B}_2 \quad , \quad u_m = I_1 \int \vec{B}_2 \cdot \hat{n}\, ds = I_1 \int (\vec{\nabla} \times \vec{A}_2) \cdot \hat{n}\, ds = I_1 \oint \vec{A}_2 \cdot d\vec{l}$$

If we examine the current along the loop,

$$I_1 d\vec{l} = \vec{J} \cdot \hat{n}\, ds\, dl$$

The electric potential energy is

$$U_m = \int \vec{J} \cdot \vec{A}\, d^3r \quad w/ \quad \vec{J} = \frac{1}{\mu_0}(\vec{\nabla} \times \vec{B}) \quad \therefore \quad U_m = \frac{1}{2\mu_0} \int (\vec{\nabla} \times \vec{B}) \cdot \vec{A}\, d^3r$$

we then proceed to calculate

$$(\vec{\nabla} \times \vec{B}) \cdot \vec{A} = \epsilon_{ijk} (\partial_i B_i) A_i = \epsilon_{ijk} (\partial_j B_k A_i) - B_K (\partial_j A_i)$$

$$= \partial_j \epsilon_{jki} (B_k A_i) - B_K \epsilon_{kij} (\partial_j A_i) = \vec{\nabla} \cdot (\vec{B} \times \vec{A}) + \vec{B} \cdot (\vec{\nabla} \times \vec{A})$$

This means our potential energy will be

$$U_m = \frac{1}{2\mu_0} \int \underbrace{\vec{\nabla} \cdot (\vec{B} \times \hat{A})\, d^3r}_{=0 \ \rightarrow \ \text{Gauss' Law}} + \int \vec{B} \cdot (\vec{\nabla} \times \hat{A})\, d^3r = \frac{1}{2\mu_0} \int B^2\, d^3\vec{r}$$

So, Finally we have

$$U_m = \frac{1}{2} \int \vec{J} \cdot \vec{A}\, d^3r = \frac{1}{2\mu_0} \int \frac{J(\vec{r}) \cdot J(\vec{r}')}{|\vec{r} - \vec{r}'|}\, d^3\vec{r}\, d^3\vec{r}'$$

we now move on to looking at an example

Example: Solenoid

We can say

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \;,\quad \vec{B}\cdot\vec{L} = \mu_0 N I \quad \therefore \quad B = \mu_0 \frac{N}{L} I$$

The potential energy is then

$$u = \frac{1}{2\mu_0}\int B^2 d^3r = \frac{1}{2\mu_0}\left(\mu_0 \frac{N}{L} I\right)^2 A\cdot L = \frac{\mu_0}{2}\frac{N^2}{L^2} I^2 A$$

We can then say

$$du = I\,\Delta t \cdot N \frac{dB}{\Delta t} = N I \,\Delta t\, A\left(\frac{\mu_0 N}{L}\right)\frac{\Delta I}{\Delta t} = \frac{\mu_0}{L} A N^2 \, I\, dI$$

We then say

$$u = \frac{\mu_0}{L} A N^2 \int_0^{I_1} I\, dI = \frac{\mu_0}{2}\frac{A N^2 I^2}{L}$$