PHYS5153 Assignment 8

Due: 6:00pm on 11/5/2021 (prior to class commencing).
Marking: Total of 10 marks (weighting of each question is indicated).
Fine print: Solutions should be presented legibly (handwritten or LaTeX is equally acceptable) so that the grader can follow your line of thinking and any mathematical working should be appropriately explained/described. If you provide only equations you will be marked zero. If you provide equations that are completely wrong but can demonstrate some accompanying logical reasoning then you increase your chances of receiving more than zero. If any of your solution has relied on a reference or material other than the textbook or lectures, please note this and provide details.

Question 1 (3 marks)

Consider a half-cylinder of radius r_0 and mass m that is allowed to roll on a horizontal plane, as in Fig. []. Take the angle φ to correspond to the deviation of the upper flat plane from horizontal.

(a) Compute the moment of inertia tensor with respect to the half-cylinder's center-of-mass.

(b) Adapt your result from (a) to show that the Lagrangian for the half-cylinder can be written as,

$$L = \frac{1}{2}\left[\frac{3}{2} - \frac{8}{3\pi}\cos(\varphi)\right] m r_0^2 \dot{\varphi}^2 + \frac{4mgr_0}{3\pi}\cos(\varphi). \tag{1}$$

Hint: You might find it easiest to use the contact point of the half-cylinder with the horizontal plane as your point of reference.

(c) Show that when the half-cylinder is near equilibrium its motion is equivalent to that of a pendulum of effective length

$$l = \left(\frac{9\pi}{8} - 2\right) r_0. \tag{2}$$

Question 2 (3 marks)

Consider a system of 3 masses, two with mass m and one with mass $2m$ constrained to slide (frictionlessly) about a ring of radius r_0. All three masses are linked by 3 identical springs with associated spring constant k.

(a) Compute the normal modes of the system and comment briefly on the nature of each mode.

(b) Assuming the three masses are initially prepared near their equilibrium condition. The heavier mass is perturbed slightly away from this configuration while the others are held in place. Assuming the masses are simultaneously released, solve for the ensuing motion. Which of the normal modes participate?

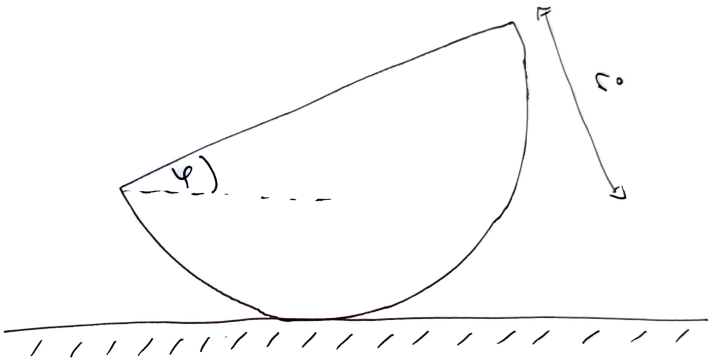

Figure 1: The semi-cylinder of Q1.

Question 3 (4 marks)

The remainder of this assignment will be focused on the paper Appl. Phys. B **66**, 181 (1998) by Daniel James. This work discusses the formation of crystals of trapped ions, which is a platform that is being pursued to realize a quantum computer by many prominent academic research groups (Maryland, Innsbruck), national labs (NIST) and companies (Honeywell, IonQ).

(a) Read up to the end of Sec. 2 of the manuscript. In just a few sentences, summarize the key points of the work.

(b) Explain what the terms in Eq. (1) are describing.

(c) Derive Eq. (5).

(d) Look at the result of Table 1 for the equilibrium positions of the ions. Focusing only on N even, can you explain the trends in:

  (i) the spacing between the two central ions,

  (ii) the position of the outermost ions.