## Course Review

#### P. Gutierrez

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- State vector  $|\alpha\rangle$ .
- Dual correspondence  $a^* \langle \alpha | \stackrel{DC}{\Longleftrightarrow} a | \alpha \rangle$
- Normalization and positivity  $\langle \alpha | \alpha \rangle \geq 0$ .
- Observable  $\tilde{\mathbf{A}} = \tilde{\mathbf{A}}^{\dagger}$ .
  - Eigenvalues  $\tilde{\mathbf{A}} |a_i\rangle = a_i |a_i\rangle$ .
  - Eigenvalues real  $a_i = a_i^*$ .
  - Eigenvectors orthogonal  $\langle a_i | a_j \rangle = \delta_{ij}$ .
  - Completeness  $\sum_i |a_i\rangle\langle\,a_i| = ilde{\mathbf{1}}.$
  - Expansion  $|\alpha\rangle = \sum_i |a_i\rangle \langle a_i | \alpha \rangle$
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  - $\tilde{\mathbf{U}}\tilde{\mathbf{U}}^{\dagger} = \tilde{\mathbf{1}}$ .
  - $e^{-ipx'/\hbar} |x\rangle = |x+x'\rangle$ .
  - Infinitesimal operation  $e^{-ip\delta x'/\hbar} = \tilde{1} ip\delta x'/\hbar$
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