E & M I

Workshop 8 – Plane Waves, 3/28/2022

We won't spend much time considering time dependence and waves in E&M I, but we'll consider the basic derivations and properties of waves to build some mental pictures.

Today, in particular, you should be working in groups of 3.

1) Picturing waves:

(This is an exercise developed at Oregon State University. It has been given to groups of physics and astronomy faculty at the AAPT/APS/AAS New Faculty Workshop with some very interesting results and discussions.)

Each group will be given sheets of paper with a set of Coordinate Axes like the one attached below, and a vector $\vec{k}$. The vector $\vec{k}$ has the units of inverse length.

A) On your paper, draw lines for the positions $\vec{r}$ that satisfy:

$$\vec{k} \cdot \vec{r} = n, \qquad n = \text{integers}$$

There will be one line for each integer n. Of course, you don't have to do EVERY integer, but you should describe how you drew each of these lines.

B) On the same paper, draw a representation of the function:

$$f_{\vec{k}}(\vec{r}) = A \sin\left(\vec{k} \cdot \vec{r}\right)$$

Of course, doing this exactly would require you to make a 3D plot, which is not really feasible. Figure out a way to illustrate this function, perhaps by shading or a representation along various lines in $\vec{r}$. Give a brief description of this function.

C) Finally, consider the behavior of the function:

$$f_{\vec{k},\alpha} = A \sin(\vec{k} \cdot \vec{r} - \alpha\, t)$$

t is, of course, time. You can either represent this on your picture by indicating what happens to the function with time and/or describe this behavior in your answers.

2) Exploring Plane Waves:

In class last week, we kind of waved our hands a bit and came up with a description of wave properties for E & M fields in a vacuum. Here we'll go a little further.

A) We did this in class, but I want to make sure that you can do this. Starting with the vacuum Maxwell's equations:

$$\vec{\nabla} \cdot \vec{E} = 0, \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\partial_t \vec{B}, \qquad \vec{\nabla} \times \vec{B} = \frac{1}{c^2}\, \partial_t \vec{E}, \qquad \frac{1}{c^2} = \mu_0\, \epsilon_0$$

Derive the wave equations for $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$. You don't have to go into great detail, as it's in the notes and any textbook, but you should be sure that you can recreate this quickly and efficiently.

B) Consider solutions to the wave equations of the form:

$$\vec{E}(\vec{r},t) = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega\, t)}, \qquad \vec{B}(\vec{r},t) = \vec{B}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega\, t)}$$

Where $\vec{E}_0$ and $\vec{B}_0$ are complex, constant vectors, and $\vec{k}$ and ω are real constants.

Show that these are solutions to the wave equations. Use the wave equations and Maxwell's equations to determine the relations between $\vec{E}_0, \vec{B}_0, \vec{k},$ and ω.

C) Let's consider a more specific example of these solutions for this part. Let:

$$\vec{k} = k\, \hat{z}, \qquad \vec{E}_0 = E_0\, \hat{x}, \qquad E_0^* = E_0$$

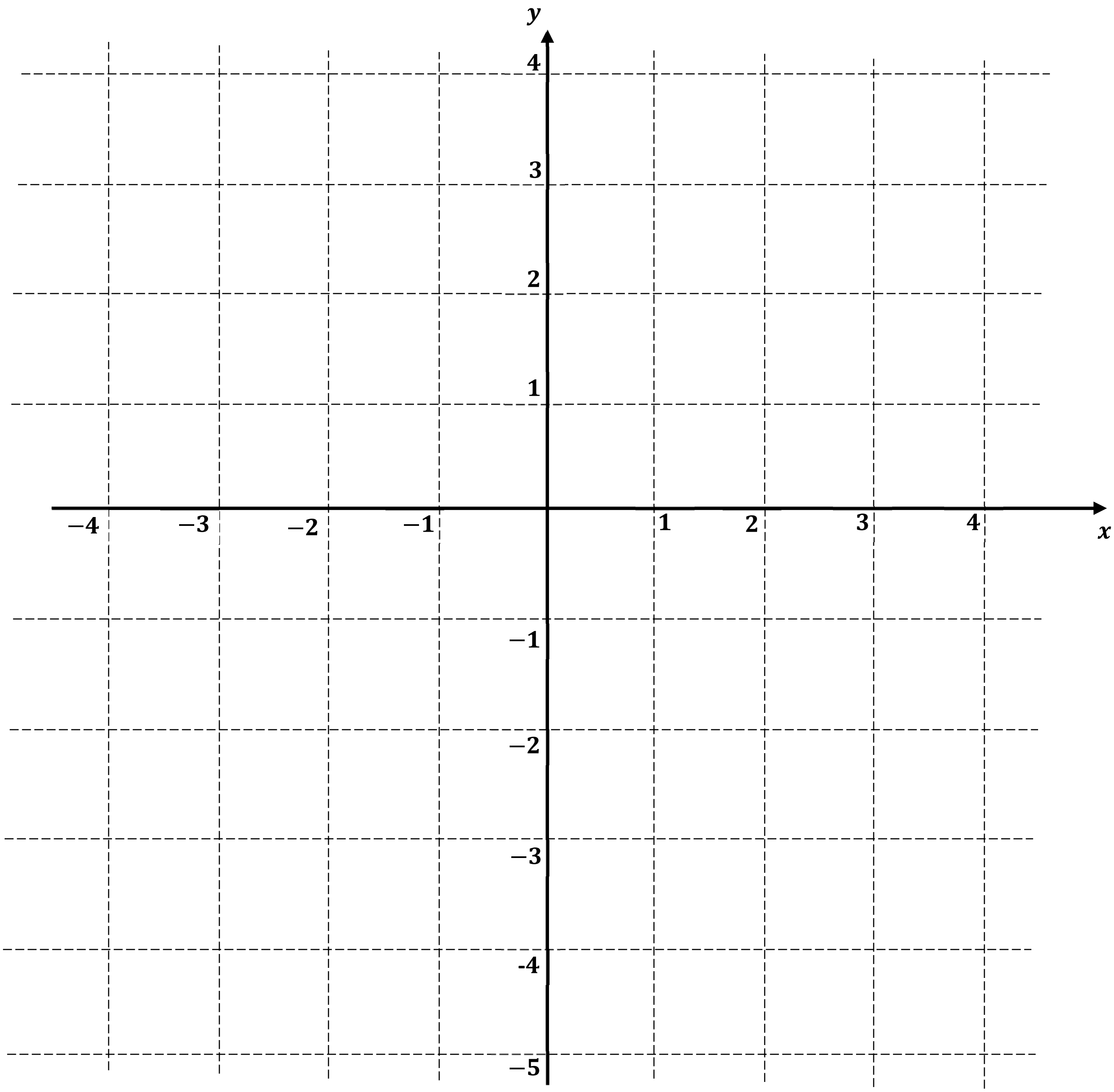
Solve for the energy density of the wave, averaged over one period of the oscillation. Remember that:

$$u(t) = \frac{\epsilon_0}{2}\left(\left|\vec{E}(t)\right|^2 + c^2 \left|\vec{B}(t)\right|^2 \right), \qquad \langle\, u\,\rangle = \frac{1}{T}\int_0^T u(t)\, dt, \qquad T = \frac{2\pi}{\omega}$$

D) Using the same example as in (C), calculate the Poynting vector for the propagating waves:

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$$

Relate this to the result above for $u(t)$ to justify the argument made in class that the Poynting vector is related to the energy flux in electromagnetic waves.



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