PHYS5153 Assignment 2

Due: 1:30pm on 09/08/2021 (prior to class commencing).
Marking: Total of 10 marks (weighting of each question is indicated).
Fine print: Solutions should be presented legibly (handwritten or LaTeX is equally acceptable) so that the grader can follow your line of thinking and any mathematical working should be appropriately explained/described. If you provide only equations you will be marked zero. If you provide equations that are completely wrong but can demonstrate some accompanying logical reasoning then you increase your chances of receiving more than zero. If any of your solution has relied on a reference or material other than the textbook or lectures, please note this and provide details.

Question 1 (2.5 marks)

Consider a classical particle subject to the effective 1D potential,

$$V(x) = \frac{x^2}{2} - \frac{x_0\sqrt{1+gx^2}}{\sqrt{1+gx_0^2}}, \tag{1}$$

where $x_0, g > 0$ are constants.

(a) Determine the extrema of $V(x)$ and comment on their nature as a function of g and x_0.

(b) Using your answer to (a) as a guide, sketch/plot the potential for various values of g and assuming $x_0 = 1$. You may use any numerical tools at your disposal if that is useful.

(c) Imagine that the particle is placed at the point $x(0) = x_0 > 0$ and is initially at rest. Without explicitly solving the dynamics, comment on the expected behaviour of $x(t)$ as a function of g and x_0.

Question 2 (4 marks)

Consider the nonlinear iterative map,

$$x_{n+1} = \mu \sin(\pi x_n) \tag{2}$$

where $x \in [0, 1]$ and $0 \leq \mu \leq 1$.

(a) Construct a bifurcation diagram of the map with $x_0 = 0.4$. To do this you will need to write a short code (in whatever language you choose) and evaluate the iterative map for a range of $0 < \mu < 1$. An example of what to expect is shown in Fig. 1. Note: While you should compute values $x_0, x_1, x_2, ..., x_N$, in the bifurcation diagram you should only plot the last handful of values of x_n you evaluate. For example, for $N = 10^3$ one might only plot $x_{900}, ..., x_{1000}$ in the bifurcation diagram.

(b) This map can feature up to two fixed points depending on the value of μ. Determine the fixed points (you may leave one of them as a transcendental equation – it will be obvious which this is!) and thus the special value μ_0 that delineates maps featuring two fixed points ($\mu > \mu_0$ from those with only one ($\mu < \mu_0$).

(c) Assume $\mu > \mu_0$. Perform a stability analysis of the fixed points of the map by studying the derivative of the function that defines the map, e.g., $x_{n+1} = f(x_n)$ with $f(x) = \mu \sin(\pi x)$. For the critical point defined by a transcendental equation you will find the stability depends on the value of μ. You may use numerical methods to determine for what values of μ this point is stable/unstable.

(d) Explain how your results for (b) and (c) relate to the features of the bifurcation diagram in Fig. 1.

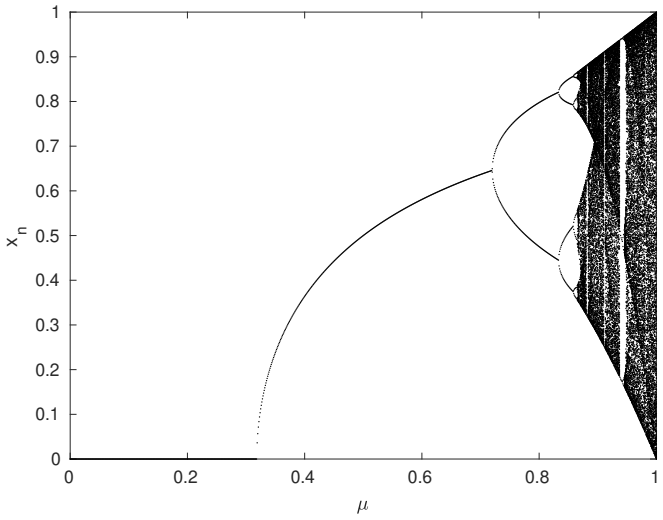

Figure 1: Bifurcation diagram described in Q2 part (a).

Question 3 (2 marks)

Consider the map defined by:

$$x_{n+1} = \begin{cases} 2\alpha x_n, & \text{if } x_n < 1/2 \\ 2\alpha(1 - x_n), & \text{if } x_n \geq 1/2 \end{cases}$$

where $0 \leq \alpha \leq 1$. For what values of α is the map chaotic?

Question 4 (2 marks)

For this question, you need to read the paper "Deterministic chaos in the elastic pendulum: A simple laboratory for nonlinear dynamics", R. Cuerno, A. F. Ranada, and J. J. Ruiz-Lorenzo, Am. J. Phys. 60, 73 (1992). You can access it at https://aapt.scitation.org/doi/10.1119/1.17047. To download the pdf you will need to be on campus, use the University VPN or access the journal via the library website.

(a) Give a one paragraph summary of the manuscript including their methods and main results.

(b) Outline the procedure used to generate Fig. 3 (e.g., what "recipe" did the authors follow to obtain this figure). Discuss and comment on what is plotted in each panel of Fig. 3 (e.g., what information is being conveyed about the system and what are the authors trying to report).