Key points of 03/07 lecture

- Canonical ensemble, quantum:

$$\hat{\rho}_{can} = \frac{e^{-\beta\hat{\mathcal{H}}}}{Tr(e^{-\beta\hat{\mathcal{H}}})} = \frac{e^{-\beta\hat{\mathcal{H}}}}{Q(N,T,V)}$$

$Q = Tr(e^{-\beta\hat{\mathcal{H}}})$

$$\langle\hat{A}\rangle = Tr(\hat{\rho}_{can}\,\hat{A})$$

the order here does not matter

- $$Tr(e^{-\beta\hat{\mathcal{H}}}) = \sum_m e^{-\beta E_m} = \int_{-\infty}^{\infty}\langle x|e^{-\beta\hat{\mathcal{H}}}|x\rangle dx$$

E_m eigenenergies of $\hat{\mathcal{H}}$

here: single particle in one spatial dimension

- Reminders from quantum: $\langle x|y\rangle = \delta(x-y)$

$$\int |x\rangle\langle x|\,dx = \hat{1} \quad \text{(closure)}$$

$$|y\rangle = \int |x\rangle\,\delta(x-y)\,dx$$

- Example: $$\langle\hat{\mathcal{H}}\rangle = Tr(\hat{\rho}\,\hat{\mathcal{H}}) = Tr\left(\frac{\hat{\mathcal{H}}e^{-\beta\hat{\mathcal{H}}}}{Tr(e^{-\beta\hat{\mathcal{H}}})}\right) = -\frac{\partial}{\partial\beta}\log\big(\underbrace{Tr(e^{-\beta\hat{\mathcal{H}}})}_{Q(N,T,V)}\big)$$

Same as in classical SM