Legendre polynomial expansion

Check the basis:

In[1]:= `basis = Table[LegendreP[n, x], {n, 0, 4}]`

Out[1]= $\left\{1, x, \frac{1}{2}\left(-1+3x^2\right), \frac{1}{2}\left(-3x+5x^3\right), \frac{1}{8}\left(3-30x^2+35x^4\right)\right\}$

In[2]:= `Integrate[basis^2, {x, -1, 1}]`

Out[2]= $\left\{2, \frac{2}{3}, \frac{2}{5}, \frac{2}{7}, \frac{2}{9}\right\}$

In[3]:= $\texttt{basis = Table}\left[\sqrt{\frac{2n+1}{2}}\ \texttt{LegendreP[n, x], \{n, 0, 4\}}\right]$

Out[3]= $\left\{\frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}}\,x, \frac{1}{2}\sqrt{\frac{5}{2}}\left(-1+3x^2\right), \frac{1}{2}\sqrt{\frac{7}{2}}\left(-3x+5x^3\right), \frac{3\left(3-30x^2+35x^4\right)}{8\sqrt{2}}\right\}$

In[4]:= `Integrate[basis^2, {x, -1, 1}]`

Out[4]= {1, 1, 1, 1, 1}

Expand the Gaussian with 5 terms

In[33]:= `fxn = Exp[-2 x^2];`

In[12]:= `coeffs = Integrate[fxn basis, {x, -1, 1}]`

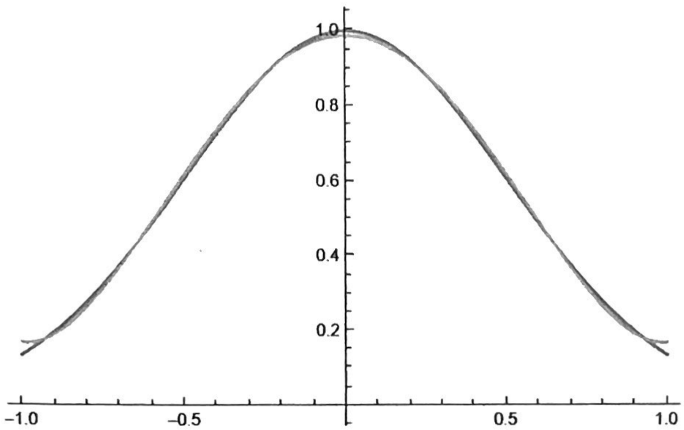
Out[12]= $\left\{\frac{1}{2}\sqrt{\pi}\,\mathrm{Erf}\left[\sqrt{2}\right], 0, -\frac{6\sqrt{10}+e^2\sqrt{5\pi}\,\mathrm{Erf}\left[\sqrt{2}\right]}{16\,e^2}, 0, \frac{3}{256}\left(-\frac{250\sqrt{2}}{e^2}+33\sqrt{\pi}\,\mathrm{Erf}\left[\sqrt{2}\right]\right)\right\}$

In[13]:= `appFxn = coeffs.basis`

Out[13]= $\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left[\sqrt{2}\right] + \frac{1}{2048\sqrt{2}}9\left(3-30x^2+35x^4\right)\left(-\frac{250\sqrt{2}}{e^2}+33\sqrt{\pi}\,\mathrm{Erf}\left[\sqrt{2}\right]\right) - \frac{1}{32\,e^2}\sqrt{\frac{5}{2}}\left(-1+3x^2\right)\left(6\sqrt{10}+e^2\sqrt{5\pi}\,\mathrm{Erf}\left[\sqrt{2}\right]\right)$

In[14] = `Plot[{fxn, appFxn}, {x, -1, 1}]`

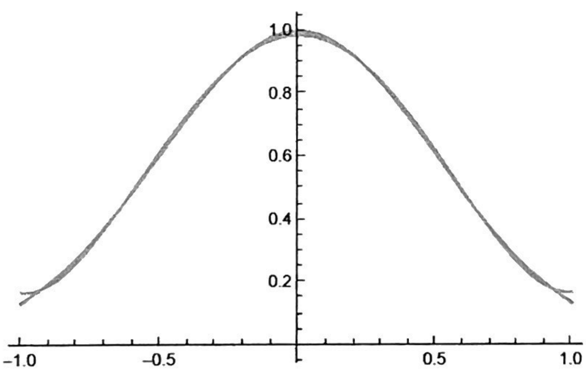
Out[14]=



Expand the Gaussian

```
basis = Table[Sqrt[(2 n + 1)/2] LegendreP[n, x], {n, 0, 7}];
coeffs = Integrate[fxn basis, {x, -1, 1}];
appFxn2 = coeffs.basis;
Plot[{fxn, appFxn, appFxn2}, {x, -1, 1}]
```

Out[31]=



In[32] = `Plot[{appFxn - fxn, appFxn2 - fxn}, {x, -1, 1}]`

Out[32]=