PHYS5153 Assignment 7

Due: 6:00pm on 10/28/2021. *To be confirmed.*
Marking: Total of 10 marks (weighting of each question is indicated).
Fine print: Solutions should be presented legibly (handwritten or LaTeX is equally acceptable) so that the grader can follow your line of thinking and any mathematical working should be appropriately explained/described. If you provide only equations you will be marked zero. If you provide equations that are completely wrong but can demonstrate some accompanying logical reasoning then you increase your chances of receiving more than zero. If any of your solution has relied on a reference or material other than the textbook or lectures, please note this and provide details.

Question 1 (3 marks)

In class we expressed the components of the angular velocity $\boldsymbol{\omega}$ along the body-fixed axes in terms of Euler angles as

$$\boldsymbol{\omega}_{bf} = \begin{pmatrix} \dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi \\ \dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi \\ \dot{\phi}\cos\theta + \dot{\psi} \end{pmatrix}. \tag{1}$$

Show that the angular velocity along the space-fixed axes in terms of Euler angles is instead give by,

$$\boldsymbol{\omega}_{sf} = \begin{pmatrix} \dot{\psi}\sin\theta\sin\phi + \dot{\theta}\cos\phi \\ -\dot{\psi}\sin\theta\cos\phi + \dot{\theta}\sin\psi \\ \dot{\psi}\cos\theta + \dot{\phi} \end{pmatrix}. \tag{2}$$

Question 2 (2 marks)

Consider a 3D cone with uniform mass density (and total mass M).

(a) Find the moment of inertia tensor assuming the body-fixed co-ordinate system is such that the origin is placed at:

i) the center-of-mass (COM) of the cone and the z-axis passes through the sharp tip of the cone
ii) the sharp tip (apex) of the cone and the z-axis passes through the COM

(b) Check your results to (a) by verifying that they are consistent with Steiner's parallel axes theorem:

$$I_{jk} = I'_{jk} - M(|\mathbf{R}|^2\delta_{jk} - R_j R_k), \tag{3}$$

where I is the moment of inertia tensor in the COM basis, I' relative to the cone tip and $\mathbf{R}$ is the vector from the origin O of the COM basis to the cone tip.

Question 3 (3 marks)

Consider a toy model of a diatomic molecule where the atoms are taken to be point particles with masses m_1 and m_2 connected by a massless rigid rod of length $2b$. Assume the molecule rotates in such a fashion that the rigid rod makes a constant angle θ_0 with respect to the z-axis of the space-fixed co-ordinate system and the atoms trace out circular orbits in respective xy-planes (i.e., their space-fixed z co-ordinates are fixed). What is the angular momentum of the molecule and what is the magnitude of the torque that must be applied (in the space-fixed frame) for the molecule to continue precessing about a fixed axis? Solve this question by:

(a) computing the momentum of inertia using a body-fixed co-ordinate system set by the principle axes

(b) computing the rate of change of the angular momentum using a space-fixed co-ordinate system

Question 4 (2 marks)

Consider a cone of mass M, height h and half-angle α rolling on a plane without slipping. The cone is taken to be orientated such that it is rolling on its slanted side (i.e., both the apex of the cone and edge of the base make contact with the surface upon which the cone is rolling). Compute the kinetic energy.