Physics 5403 Homework #1
Spring 2022

Instructor Bruno Uchoa
Due date: February 02, 2022

January 25, 2022

1 Addition of angular momentum

Consider two particles with spin 3/2 and spin 1/2.

a) Compute the total angular momentum states $|j_1, j_2; J, M\rangle$ in terms of the single particle product states $|j_1, m_1\rangle|j_2, m_2\rangle$.

b) Suppose the Hamiltonian of the two particles has the form

$$\mathcal{H} = \alpha \mathbf{S}_1 \cdot \mathbf{S}_2,$$

with α a constant. If the system is initially ($t = 0$) in the following eigenstate of $\mathbf{S}_1^2$, $\mathbf{S}_2^2$, S_{1z}, S_{2z},

$$|j_1 j_2; m_1 m_2\rangle = \left|\frac{3}{2}\frac{1}{2}; \frac{1}{2}\frac{1}{2}\right\rangle,$$

what is the probability of finding the system in state $\left|\frac{3}{2}\frac{1}{2}; \frac{3}{2} - \frac{1}{2}\right\rangle$ at time $t > 0$?

2 Clebsh-Gordan Coefficients

Using recursion relations, verify the special case of the Clebsh-Gordan coefficient

$$\langle j1; j0|j1; jj\rangle = \sqrt{\frac{j}{j+1}}.$$

Hint: write down the relevant Clebsh-Gordan completeness relation that includes this coefficient. That will give you one equation with two unknown coefficients. Find then a convenient recursion relation to obtain the second equation for those two same coefficients and solve them.

3 Wigner-Eckart theorem

Consider a system formed formed by two spinless particles with angular momentum $j_1 = 1$ and $j_2 = 1$.

a) Assuming that the system is subjected to a spherically symmetric potential, using the Wigner-Eckart theorem, find the selection rules for the matrix elements of the momentum operator components P_x, P_y and P_z

$$\langle \alpha, j', m'|P_i|\alpha j m\rangle,$$

where α is a quantum number which is independent of the of the magnetic quantum numbers m and m'. Compute the matrix elements explicitly for $j' = 2$ and $j = 1$ (use the Clebsh-Gordan table for that).

b) Using the definition for the product of spherical tensors,

$$T_q^{(k)} = \sum_{q_1 q_2} T_{q_1}^{(k_1)} T_{q_2}^{(k_2)} \langle k_1 k_2; q_1 q_2 | k_1 k_2; kq \rangle,$$

compute the generic form for $T_{\mathbf{q}}^{(2)}$ in terms of the operator components of $\mathbf{P}$.