Lectures 01

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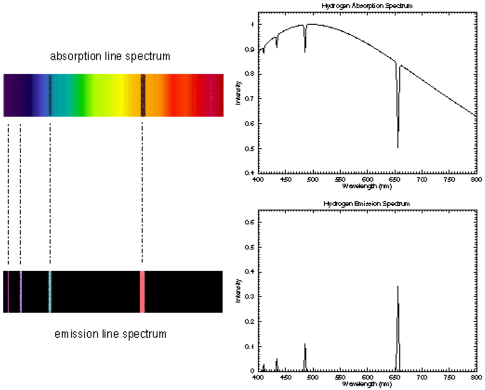
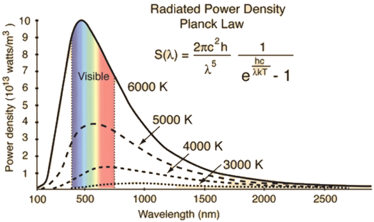
University of Oklahoma

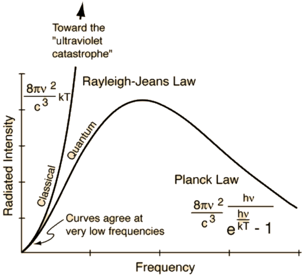
Absorption Emission Spectra



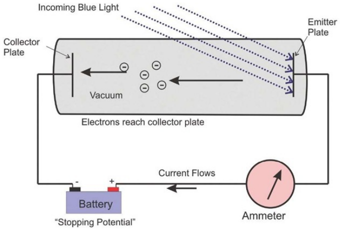
Two ways of showing the same spectra: on the **left** are pictures of the dispersed light and on the **right** are plots of the intensity vs. wavelength. Notice that the pattern of spectral lines in the absorption and emission line spectra are the **same** since the gas is the same.

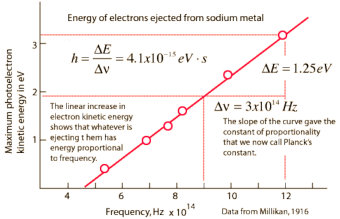
Black-Body Radiation



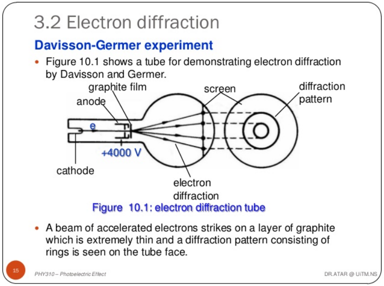


Photoelectric Effect





Electron Diffraction



3.2 Electron diffraction

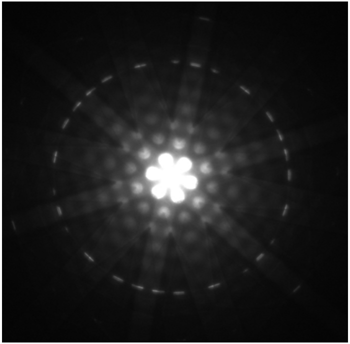
Davisson-Germer experiment

- Figure 10.1 shows a tube for demonstrating electron diffraction by Davisson and Germer.

Figure 10.1: electron diffraction tube

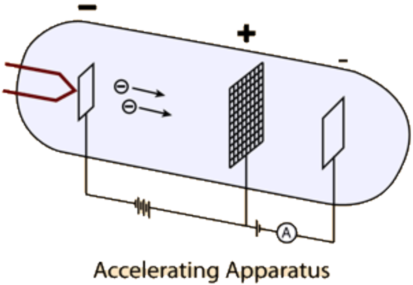
- A beam of accelerated electrons strikes on a layer of graphite which is extremely thin and a diffraction pattern consisting of rings is seen on the tube face.

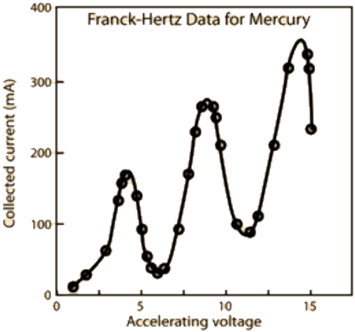
15 PHY310 – Photoelectric Effect DR.ATAR @ UiTM.NS



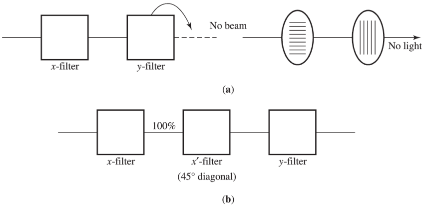
Davisson and Germer

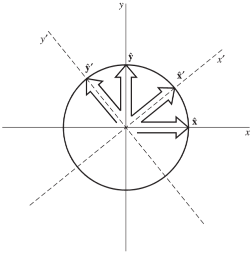
Franck Hertz Experiment

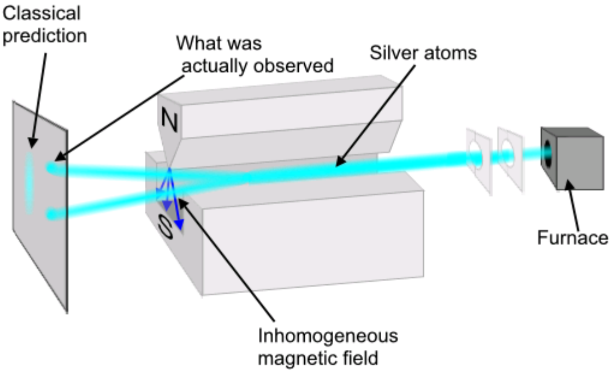




Light Polarization

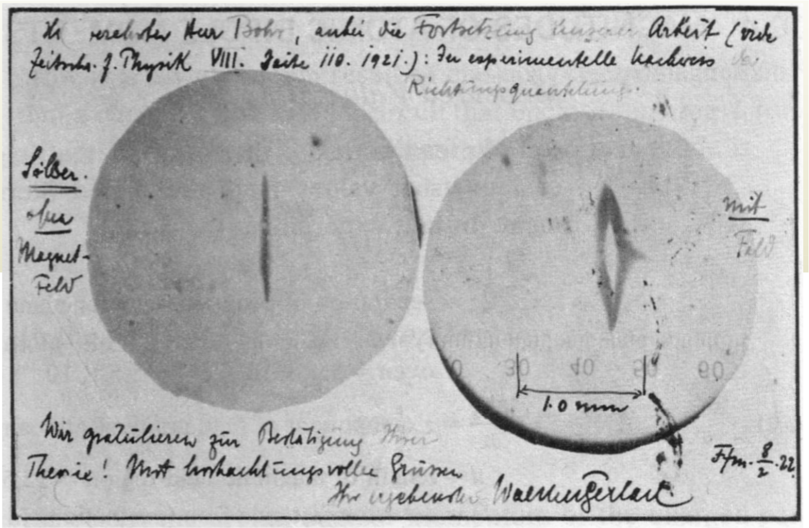






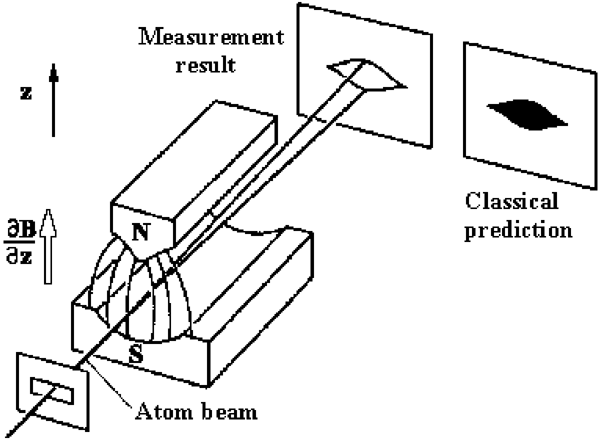
Classical prediction
What was actually observed
Silver atoms
N
S
Furnace
Inhomogeneous magnetic field

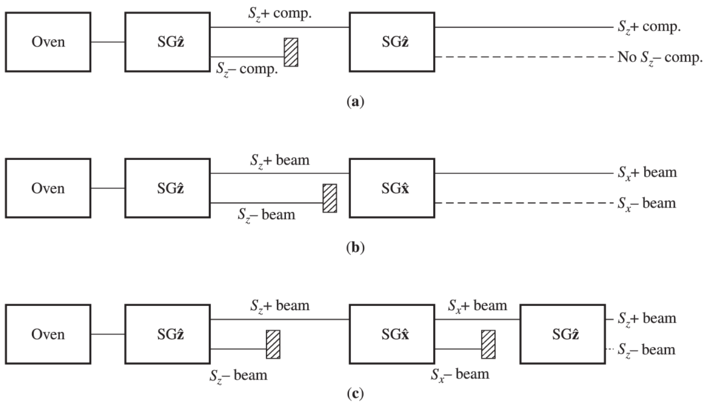
Result

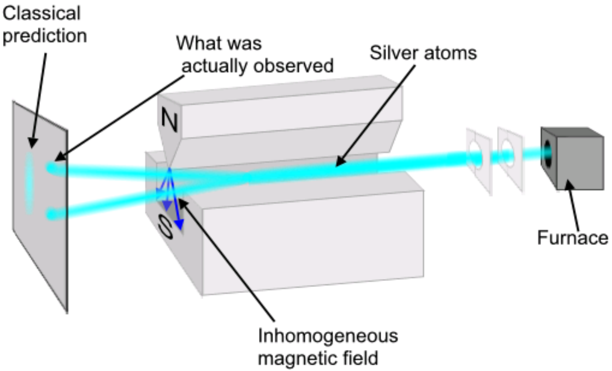


Silber
ohne Magnet-Feld
mit Feld
1.0 mm
Wir gratulieren zur Bestätigung Ihrer Theorie!

Apparatus




Oven
SGẑ
S_z+ comp.
S_z– comp.
SGẑ
S_z+ comp.
No S_z– comp.
(a)
Oven
SGẑ
S_z+ beam
S_z– beam
SGx̂
S_x+ beam
S_x– beam
(b)
Oven
SGẑ
S_z+ beam
S_z– beam
SGx̂
S_x+ beam
S_x– beam
SGẑ
S_z+ beam
S_z– beam
(c)



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Stern-Gerlach States

$$|S_z;+\rangle$$
$$|S_z;-\rangle$$

$$|S_x;+\rangle = \frac{1}{\sqrt{2}}\left[|S_z;-\rangle + |S_z;+\rangle\right]$$
$$|S_x;-\rangle = \frac{1}{\sqrt{2}}\left[|S_z;-\rangle - |S_z;+\rangle\right]$$

$$|S_y;+\rangle = \frac{1}{\sqrt{2}}\left[|S_z;+\rangle + i\,|S_z;-\rangle\right]$$
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Linear Vector Space

Definition 1: A linear vector space $\mathbb{V}$ is a collection of objects of the form $|V\rangle$, called vectors, for which there exists:

1. A definite rule for forming the vector sum, denoted $|V\rangle + |W\rangle$
2. A definite rule for multiplication by scalars a denoted $a\,|V\rangle$ with the following features:
   - The results of these operations is another element of the space, a feature called closure: $|V\rangle + |W\rangle \in \mathbb{V}$ and $a\,|V\rangle \in \mathbb{V}$.
   - Scalar multiplication is distributive in the vectors: $a\,(|V\rangle + |W\rangle) = a\,|V\rangle + a\,|W\rangle$.
   - Addition is commutative: $|V\rangle + |W\rangle = |W\rangle + |V\rangle$.
   - Addition is associative: $|V\rangle + (|W\rangle + |Z\rangle) = (|V\rangle + |W\rangle) + |Z\rangle$
   - There exits a null vector $|0\rangle$ obeying $|V\rangle + |0\rangle = |V\rangle$
   - For every vector $|V\rangle$ there exists an inverse under addition, $|-V\rangle$, such that $|V\rangle + |-V\rangle = |0\rangle$

Definition 2: The numbers a are called the field over which the vector space is defined.

If the field consists of all real numbers, we have a real vector space, if they are complex, we have a complex vector space. Note, the vectors are neither real nor complex, the adjective applies only to the scalars.

Dual Vector Space

- $\langle\alpha| \overset{\text{DC}}{\Longleftrightarrow} |\alpha\rangle$
- $c^* \langle\alpha| \overset{\text{DC}}{\Longleftrightarrow} c\,|\alpha\rangle$
- Inner product $\langle\alpha\,|\beta\rangle$ = Complex Number
- Orthognal $\langle\alpha\,|\beta\rangle = 0$
- Normalization $\langle\alpha\,|\alpha\rangle = 1$
- Postulates:
  - $\langle\alpha\,|\beta\rangle = \langle\beta\,|\alpha\rangle^*$
  - $\langle\alpha\,|\alpha\rangle \geq 0$
- Operators (*in general*) $\tilde{\mathrm{X}}\,|\alpha\rangle \neq \langle\alpha|\,\tilde{\mathrm{X}}$
  - $\tilde{\mathrm{X}}\,|\alpha\rangle = |\beta\rangle$
  - $\langle\alpha|\,\tilde{\mathrm{X}} = \langle\gamma|$
  - $\langle\alpha|\,\tilde{\mathrm{X}}^\dagger \overset{\text{DC}}{\Longleftrightarrow} \tilde{\mathrm{X}}\,|\alpha\rangle$
  - In general $\tilde{\mathrm{X}}\tilde{\mathrm{Y}} \neq \tilde{\mathrm{Y}}\tilde{\mathrm{X}}$
  - $\tilde{\mathrm{X}} = |\alpha\rangle\langle\beta|$

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Matrix Representation

- $\tilde{\mathbf{A}} = \sum_{ij} |a_i\rangle\langle a_i|\, \tilde{\mathbf{A}}\, |a_j\rangle\langle a_j| = \sum_{ij} \left\langle a_i \middle| \tilde{\mathbf{A}} \middle| a_j \right\rangle |a_i\rangle\langle a_j|$
- Matrix elements $\left\langle a_i \middle| \tilde{\mathbf{A}} \middle| a_j \right\rangle = \left\langle \text{row} \middle| \tilde{\mathbf{A}} \middle| \text{column} \right\rangle$
  - In it's own basis matrix is diagonal $\left\langle a_i \middle| \tilde{\mathbf{A}} \middle| a_j \right\rangle = a_j \delta_{ij}$
- State can be expressed as a column vector;
  $|\alpha\rangle = \sum_i |a_i\rangle\langle a_i|\, |\alpha\rangle$
- Matrix elements $\langle a_i |\alpha\rangle$
- Matrix multiplication

$$\tilde{\mathbf{A}}\, |\alpha\rangle = \sum_{ij} |a_i\rangle\langle a_i|\, \tilde{\mathbf{A}}\, |a_j\rangle\langle a_j|\, |\alpha\rangle$$

$$= \sum_{ij} |a_i\rangle \left\langle a_i \middle| \tilde{\mathbf{A}} \middle| a_j \right\rangle \langle a_j |\alpha\rangle$$

Uncertainty Principle

- Incompatible observable $\left[\tilde{\mathbf{A}}, \tilde{\mathbf{B}}\right] \neq 0$
  - Measure $\tilde{\mathrm{A}}$ in the state $|a_2\rangle$
  - Measure $\tilde{\mathrm{B}}$ in the state $|a_2\rangle = \sum_j |b_j\rangle \langle b_j |a_2\rangle$; only the probability of the value is known.
- Uncertainty principle
  - Schwartz inequality $\langle \alpha|\alpha\rangle \langle \beta|\beta\rangle \geq |\langle \alpha|\beta\rangle|^2$
  - Dispersion $\Delta\tilde{\mathbf{A}} = \tilde{\mathbf{A}} - \left\langle \tilde{\mathbf{A}} \right\rangle \tilde{\mathbf{1}}$
  - $\left\langle \left(\Delta\tilde{\mathbf{A}}\right)^2 \right\rangle = \left\langle \tilde{\mathbf{A}}^2 - 2\tilde{\mathbf{A}} \left\langle \tilde{\mathbf{A}} \right\rangle + \left\langle \tilde{\mathbf{A}} \right\rangle^2 \right\rangle = \left\langle \tilde{\mathbf{A}}^2 \right\rangle - \left\langle \tilde{\mathbf{A}} \right\rangle^2$
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  - Anti-Hermitian operator $\left[\tilde{\mathrm{A}}, \tilde{\mathrm{B}}\right]$ has imaginary eigenvalues.

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Uncertainty Principle – *continued*

- Schwartz inequality
  - $\left\langle \left(\Delta\tilde{\mathbf{A}}\right)^2 \right\rangle \left\langle \left(\Delta\tilde{\mathbf{B}}\right)^2 \right\rangle \geq \left| \left\langle \left(\Delta\tilde{\mathbf{A}}\Delta\tilde{\mathbf{B}}\right) \right\rangle \right|^2$
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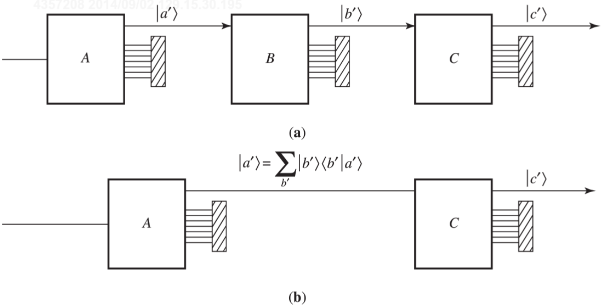
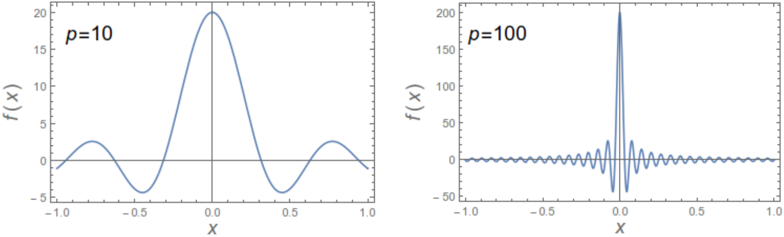


A
$|a'\rangle$
B
$|b'\rangle$
C
$|c'\rangle$
(a)
$|a'\rangle = \sum_{b'} |b'\rangle\langle b'|a'\rangle$
A
C
$|c'\rangle$
(b)

Lectures 02

P. Gutierrez

Department of Physics & Astronomy
University of Oklahoma



$$f(x) = \frac{2\sin(px)}{x} \quad \Rightarrow \quad \begin{cases} \lim\limits_{x\to\infty} f(x) \to 0 \\ \lim\limits_{x\to 0} f(x) \to 2p \end{cases}$$

Translation Operator

- $\mathcal{J}(dx)\,|x\rangle = |x+dx\rangle$
- Propose $\mathcal{J}(d\vec{\mathbf{x}}) = 1 - i\tilde{\mathbf{K}}\cdot d\vec{\mathbf{x}}$
- Unitary operator (*maintain normalization*):

$$\langle\alpha\,|\alpha\rangle = \left\langle\alpha\left|\mathcal{J}^{\dagger}(d\vec{\mathbf{x}}')\mathcal{J}(d\vec{\mathbf{x}}')\right|\alpha\right\rangle \quad\Rightarrow\quad \mathcal{J}^{\dagger}(d\vec{\mathbf{x}}')\mathcal{J}(d\vec{\mathbf{x}}') = 1$$
$$\Rightarrow\quad \tilde{\mathbf{K}} = \tilde{\mathbf{K}}^{\dagger}$$

- Successive translations:

$$\mathcal{J}(d\vec{\mathbf{x}}'')\mathcal{J}(d\vec{\mathbf{x}}') = \mathcal{J}(dx''+dx').$$

- Inverse translation:

$$\mathcal{J}(-d\vec{\mathbf{x}}') = \mathcal{J}^{-1}(d\vec{\mathbf{x}}').$$

- No translation:

$$\lim_{d\vec{\mathbf{x}}'\to 0}\mathcal{J}(d\vec{\mathbf{x}}') = \mathcal{J}(0) = 1.$$

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