

---

---

---

---

---



$$H = \sum_e h_e O_z^{(e)} + \sum_{e,m} J_{em} O_z^{(e)} O_z^{(m)}$$

Binary variables  $\rightarrow \{0, 1\}$   
 $\rightarrow \{+1, -1\}$

bits  $\in \{0, 1\}$  denoted  $x$        $s = 1 - 2x = (-1)^x$

bits  $\in \{+1, -1\}$  denoted  $s$

Hamiltonian in this picture  
becomes a function of binary  
variables. Called pseudo Boolean forms.

$$(-1)^0 \rightarrow 1$$

$$(-1)^1 \rightarrow -1$$

$f : \{0, 1\}^n \rightarrow \mathbb{R}$   
 $\underbrace{\quad}_{n\text{-long bit string}} \quad \underbrace{\quad}_{\text{real numbers}}$

$\tilde{f} : \{+1, -1\}^{n \times n} \rightarrow \mathbb{R}$

$$\tilde{f} = \sum_e h_e S_e + \sum_{e,m} J_{em} S_e \cdot S_m$$

Matrix embedding

$$S = 1 - 2X \Rightarrow X = \frac{1-S}{2}$$

Polarization versus standard bits.

$$S \rightarrow \sigma_z$$

$$X \rightarrow |X\rangle\langle X|$$

$$= \frac{1 + (-1)^X \sigma_z}{2}$$

Ex.  $f(x, y, z) = 1 + x + 2xy + 3xyz$  easy to program

How can I use penalty functions to embed universal boolean gates?

→ Can I use a cost function to emulate the NAND gate?

$$\begin{array}{cc} x \\ y \end{array} \rightarrow \boxed{D} \oplus z = x \bar{\wedge} y$$

$$f(x, y) = \bar{x}\bar{y} + \bar{x}y + x\bar{y}$$

simplify  $\Rightarrow$

x	y	$\bar{z} = x \bar{\wedge} y$
0	0	1
0	1	1
1	0	1
1	1	0

$$1 - xy = f'(x, y)$$

$$\text{using } \bar{g} = 1 - g$$

$$x^2 = x$$

idempotent property

$$x \wedge y \rightarrow xy$$

$$\bar{x} = 1 - x$$

$$\overline{(xy)} = 1 - xy$$

$$S = 1 - 2x$$

$$(1-x)(1-y) + \bar{x}y + x\bar{y}$$

$$= 1 - x - y + xy + y - xy + x - xy$$

$$= 1 - xy$$

$$x = |x\rangle\langle x| =$$

let  $f = 1 - xy$ . find real  $h, j$  such that

$$\frac{1 + (-1)^x \partial_z}{2}$$

$$\tilde{f} = \sum_{e=1}^2 h_e S_e + \frac{1}{2} \sum_{e,m=1}^2 J_{em} S_e S_m$$

and evaluate  $f, \tilde{f}$  on all inputs. + constant

$f(x, y) = \tilde{f}((-1)^x, (-1)^y)$  solve this equation by finding  $h_1, h_2, J_{12}$

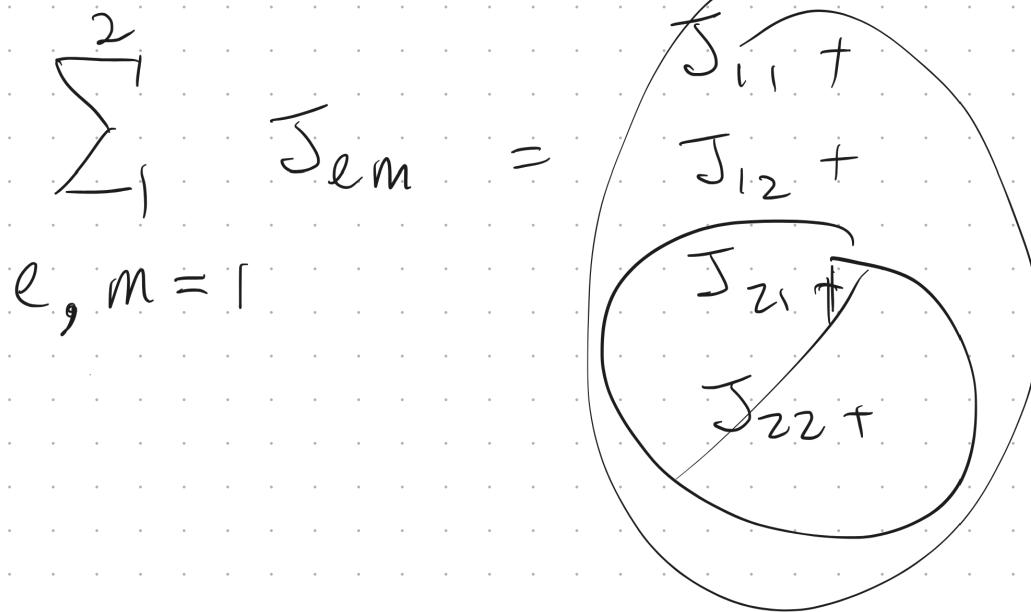
example.  $\tilde{f} = 1 - S_1 S_2 \Rightarrow J_{12} ? = -1$

$S_1$	$S_2$	$\tilde{f}$
+1	+1	0
-1	+1	2
+1	-1	2
-1	-1	0

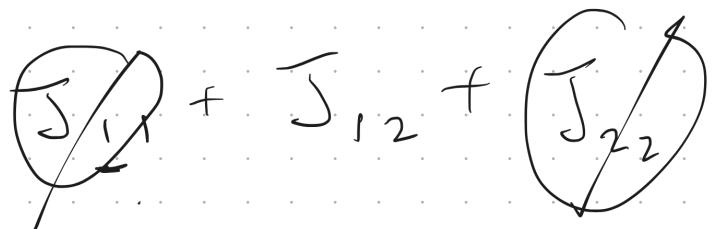
$$\rightarrow 1 - (-1) \cdot 1 = 2$$

constant = 1

$H^+ = H \Rightarrow$  eigenvalues hermitian, are real.



$$\sum_{1 \leq l \leq m \leq 2} J_{em} =$$



$$f(\vec{x}) = \vec{h}^T \vec{x} + \vec{x}^T \vec{J} \vec{x}$$

alternative notation  
\* optimisation theory

$$\vec{f}_2 = 1 - S_1 S_2 + \underline{S_2 S_3} - 3 \underline{S_1 S_3}$$

find constant,  $\vec{h}, \vec{J}$

$$\frac{1}{2} \sum_{e,m=1}^3 J_{em} S_e S_m + \sum_{e=1}^3 h_e S_e + C$$

Translate  
from polarity ( $\pm 1$ )  
to Boolean

$$f(x_1, x_2, x_3) =$$

$$f((-1)^{x_1}, (-1)^{x_2}, (-1)^{x_3})$$

$$S_k = 1 - 2 X_k$$

$$J_{12} = J_{21} = ?$$

$$J_{13} = J_{31} = ?$$

$$J_{23} = J_{32} = ?$$

$$\bar{J}_{11} = 0$$

$$\bar{J}_{22} = 0$$

$$\bar{J}_{33} = 0$$

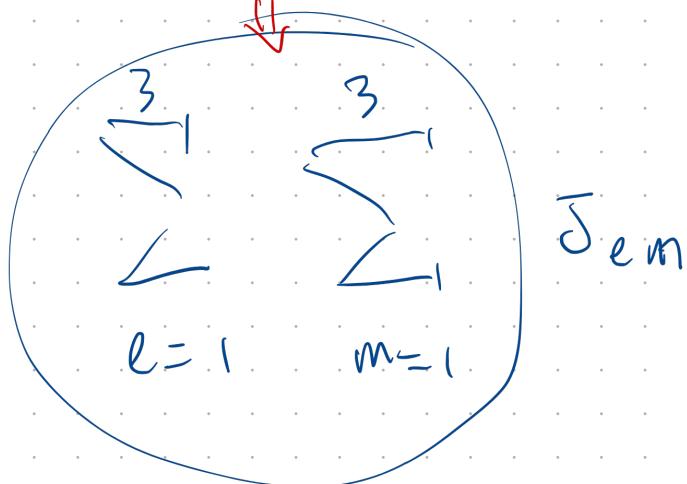
$$C = ?$$

$$\frac{1+(-1)^k}{2}$$

binary  
 $\downarrow$   
 polarisation  
 $\downarrow$   
 $S \rightarrow \sigma_z$   
 $Z$  (hamiltonian)

$$\sum_{l=1}^3 J_{em} = J_{12} + J_{21} + J_{13} + J_{31} + J_{23} + J_{32}$$

$\underbrace{e_1, m=1}_{\downarrow}$



$$f = 1 - x_1 x_2$$

$$1 - \left[ \frac{1-s_1}{2} \right] \left[ \frac{1-s_2}{2} \right]$$

$$= 1 - \frac{1}{4} (1-s_1)(1-s_2) =$$

$$\tilde{f} = 1 - \frac{1}{4} (1-s_1 - s_2 + s_1 s_2)$$

$$\tilde{f}((-1)^{x_1}, (-1)^{x_2}) = f(x_1, x_2)$$

How to turn  $f, \tilde{f}$  into a hamiltonian in terms of  $\sigma_z$ ?

$$f = 1 - x_1 x_2 \Rightarrow \sum_{l,m=1}^2 (1-x_1 x_2) |x_1 x_2\rangle \langle x_1 x_2|$$

$$\sum_{e=1}^3 h_e S_e = h_1 S_1 + h_2 S_2 + h_3 S_3$$

↑ constants      ↑ variables

$f(x, y, z) = 0$  when  $z = x \bar{\wedge} y$

$x, y, z \in \{0, 1\}$

otherwise  $f \geq \Delta \geq 1$

find  $f$ .

You can use  $X Y Z$  (3-body terms)

$x$	$y$	$z$	$f$
0	0	0	$\Delta$
0	0	1	0
0	1	0	$\Delta$
0	1	1	0
1	0	0	$\Delta$
1	0	1	0
1	1	0	0
1	1	1	$\Delta$

$\rightarrow \bar{x} \bar{y} \bar{z} \cdot \Delta$

$\leftrightarrow \Delta(1 - \bar{x} \bar{y} \bar{z})$

$\rightarrow \Delta(1 - \bar{x} y z)$

$\rightarrow \Delta(1 - x \bar{y} z)$

$\rightarrow \Delta(1 - x y \bar{z})$

NAND

$x$	$y$	$x \bar{\wedge} y$
0	0	1
0	1	1
1	0	1
1	1	0

$$\begin{array}{r}
 \begin{array}{c}
 \overline{x} \overline{y} \\
 \overline{0} \overline{0} \\
 \overline{0} \overline{1} \\
 \overline{0} \overline{0} \\
 \overline{1} \overline{0} \\
 \overline{1} \overline{1}
 \end{array}
 \end{array}
 \quad
 \begin{array}{l}
 \overline{x} \overline{y} \\
 + 2 \overline{x} \overline{y} \\
 - 2 \overline{x} \overline{y} \\
 \hline
 3 \overline{x} \overline{y}
 \end{array}$$

$$\overline{x} \overline{y} + 2 \overline{x} \overline{y} + 3 \overline{x} \overline{y} = 2 \overline{x} \overline{y}$$

$$f(x, y, z) + \Delta z$$

#1. code f and validate.

#2. The method I gave you results in a 3-body term:

e.g.  $\overline{x} \overline{y} \overline{z}$ . Understand and solve the constraint problem.

#3. Know how to embed into gates.

$$\begin{aligned}
 \tilde{f}(s_1, s_2) &= 1 - s_1 \cdot s_2 \\
 &\quad + 2 s_1 s_2
 \end{aligned}$$

$$\begin{aligned}
 \tilde{f}(-1, 1) &= 1 + (-1) \\
 &\quad + 2(-1)(1) = \\
 &\quad 1 - 2 = -1
 \end{aligned}$$

$$c=1$$

$$h_1 = -1$$

$$h_2 = -1$$

$$S_{12} = 2$$

$$\tilde{f} = \sum_{\ell=1}^2 h_\ell S_\ell + \frac{1}{2} \sum_{\ell, m=1}^2 S_\ell m S_m$$

$$\begin{aligned}
 &\sum_{\ell, m=1}^2 S_\ell m S_m \\
 &1 \leq \ell \leq m \leq 2
 \end{aligned}$$

