Electrodynamics
Computational Lab

Neutrino Mixing (1)

December 3, 2025

1. A Warm Up

In class we used the fundamentals of the Rabi Equation to construct the probability of a neutrino switching between an electron neutrino ν_e into a muon neutrino ν_μ given by:

$$P_{\nu_e \to \nu_\mu} = \sin^2\theta \sin^2\left(\frac{(m_1^2 - m_2^2)Lc^3}{4E\hbar}\right)$$

Where L is the distance from the source in meters, c is the speed of light, and E is the total energy of the neutrino in eV (electron volts), and .$(m_1^2 - m_2^2)$ is the difference in the squares of the mass eigenstates.

Some useful constants for this problem:

$h \cdot c = 1.23984193 \times 10^{-6} eV \cdot m$

$(m_1^2 - m_2^2) = 8 \times 10^{-5} eV^2/c^4$

$1\text{MeV} = 1 \times 10^6$ eV

$\theta \approx 69^o \approx 1.20428$ Rad

a If we consider an electron neutrino (produced in the sun) with total energy of 8MeV. How far must this neutrino travel before it oscillates to the maximum probability of being measured as muon neutrino?

b How many oscillations ($\nu_e \to \nu_\mu \to \nu_e$) will take place if this neutrino travels from the sun to the earth? (sun to earth distance $d = 1.5 \times 10^{11} m$)

c How many oscillations as it travels through the earth? (radius of earth $r = 6.37 \times 10^6 m$)

2. A More Realistic Model

a Using the same values from the previous question, produce a plot of the probability of an **electron neutrino** produced in the sun to be measured as an **electron neutrino** at a distance L from the sun if it is a *monenergenic* (only 8MeV) neutrino. Plot a large range of L ($10^3 < L < 10^7 m$) using a computer (Mathematica, Python, etc). I recommend using a log scale on the x-axis for this large a range of L. After you are successful plotting the probability for an 8MeV neutrino, on the same axis, add in the probability for a 4MeV neutrino.

b Many weak decay processes produce neutrinos with a spectrum of energies rather than one specific energy (like the previous question). For this question, lets assume we have a decay process that produces a uniform distribution of neutrino energies from 4MeV to 8MeV as in Figure 1 below.

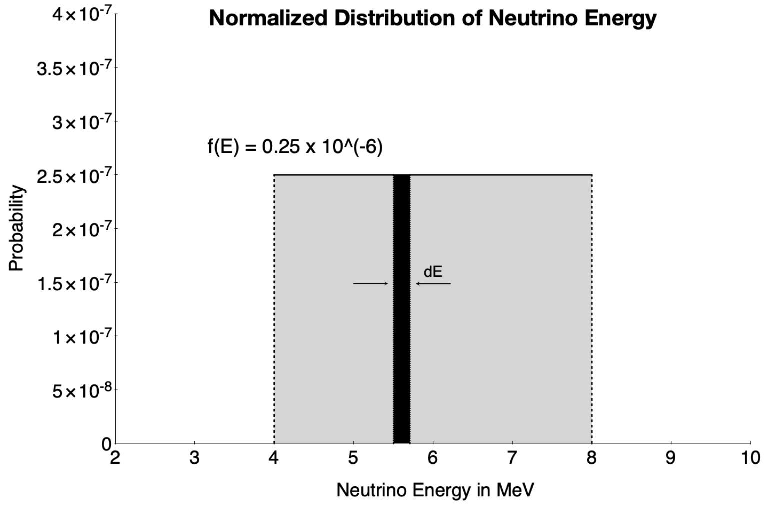

Figure 1: Uniform Neutrino Energy Distribution

To calculate the probability at a specific distance from a source (L) given a spectrum of neutrino energies we must average over the all the independent probabilities at every distance L. To calculate this average over a continuous distribution we will have to integrate using an integral of the form:

$$P_{\nu_e \to \nu_e} = \int_{E_{lower}}^{E_{upper}} P(L, E) f(E) dE$$

with $P(L, E)$ the probability you used in part (a), $f(E)$ is the normalized energy distribution, which in this case is $f(E) = 1/(4MeV)$ Using Python, etc (See sample code below), produce a plot similar to part (a) but for a spectrum of neutrino energies. This means you are going to have to evaluate the integral above for every L in the range $10^3 < L < 10^7 m$.

c If you set up a neutrino detector on Earth to measure the flavor of neutrinos produced in the sun what considerations would you have to take into account based on what you discovered in your plots in parts (a) and (b)?

d Pick another distribution of neutrino energies (Normal, Binomial, Poisson, or something asymmetric) and see if you produce a similar result as part (b).

3. Numerical Integration with Scipy.integrate.simps example

```
import numpy
from scipy import integrate

def f(x):
return x ** 4

x = numpy.linspace(0, 1, 50)
integral = integrate.simps(f(x), x)

print("The integral of f(x) = x^2 from 0 to 1 is:", integral)
```