

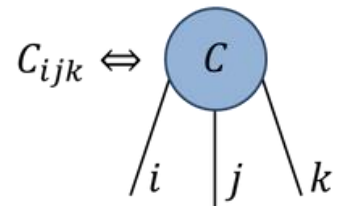
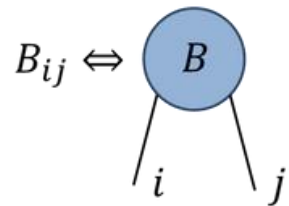
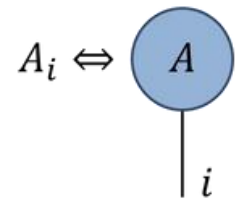
2021.9.13

徐直前

单精度导致的rho误差

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2,2717 底端 debug.double/p0 /rho			

$$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & \cdots & B_{1n} \\ \vdots & \ddots & \vdots \\ B_{m1} & \cdots & B_{mn} \end{bmatrix} \quad C = \left[\begin{bmatrix} \begin{bmatrix} c_{111} & \cdots & c_{1n1} \end{bmatrix}^1 \\ \vdots \\ \begin{bmatrix} c_{m11} & \cdots & c_{mn1} \end{bmatrix}^2 \end{bmatrix}^3 \right]^l$$



$$\begin{array}{c} \text{---} \text{ } \text{ } \text{ } \text{---} \\ | \quad | \\ i \quad k \end{array} \text{ } \text{ } \text{ } \text{---} \text{ } \text{ } \text{---} = \text{---} \text{ } \text{ } \text{---} \text{ } \text{ } \text{---} \text{ } \text{ } \text{---} \text{ } \text{ } \text{---} \\ | \quad | \quad | \quad | \\ i \quad \quad j \quad \quad k$$

$$\Updownarrow$$

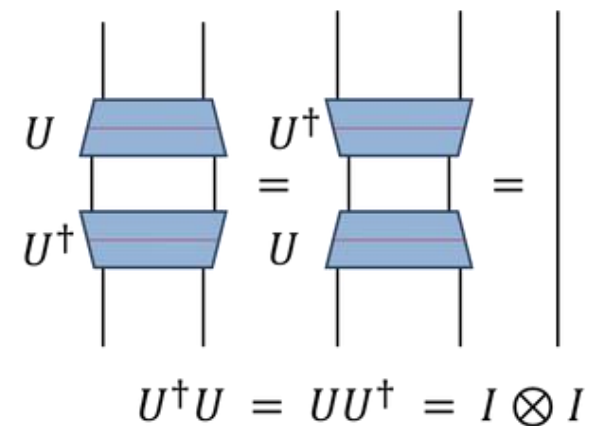
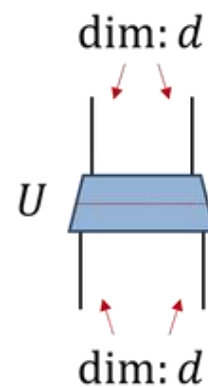
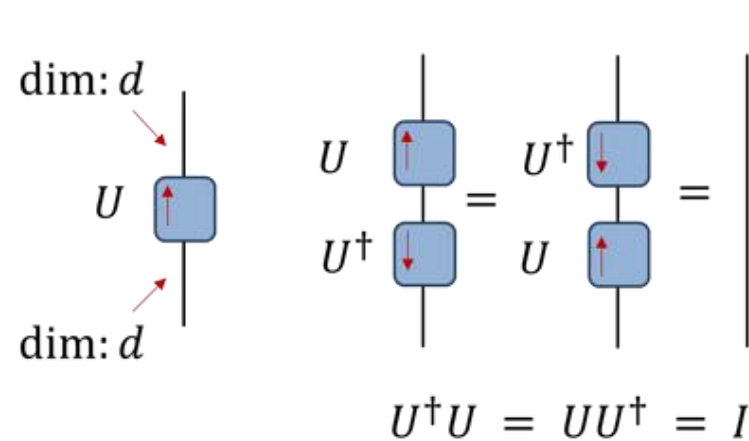
$$C_{ik} = \sum_j A_{ij} B_{jk}$$

$$\begin{array}{c} \text{---} \text{ } \text{ } \text{---} \\ | \quad | \\ i \quad k \end{array} \text{ } \text{ } \text{ } \text{---} \text{ } \text{ } \text{---} = \text{---} \text{ } \text{ } \text{---} \text{ } \text{ } \text{---} \text{ } \text{ } \text{---} \text{ } \text{ } \text{---} \\ | \quad | \quad | \quad | \quad | \quad | \\ i \quad \quad j \quad \quad n \quad \quad k$$

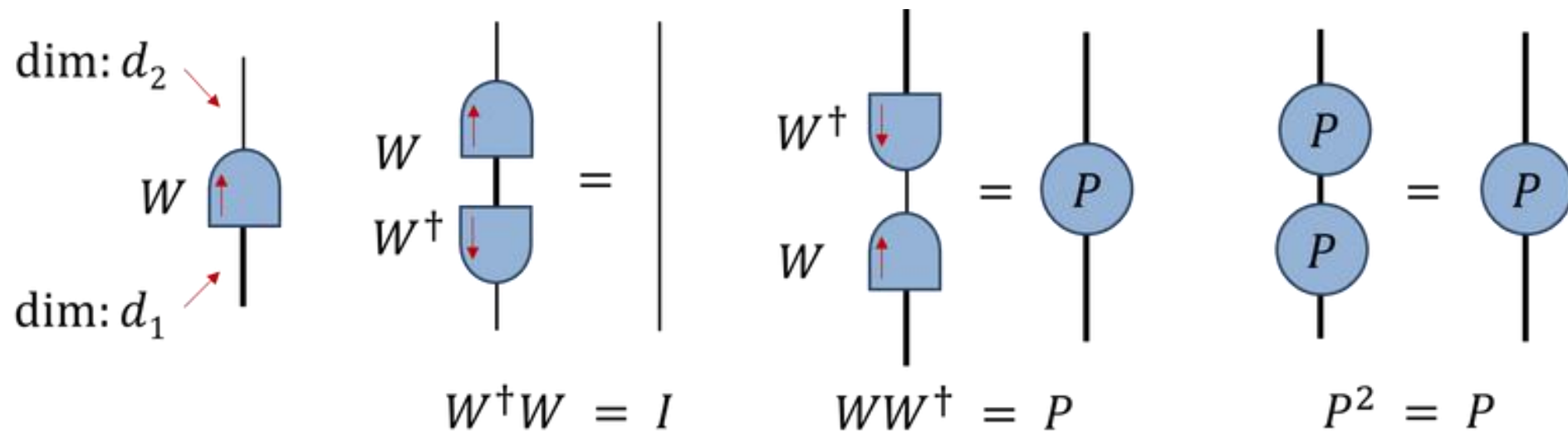
$$\Updownarrow$$

$$D_{ijk} = \sum_{lmn} A_{ljm} B_{iln} C_{nmk}$$

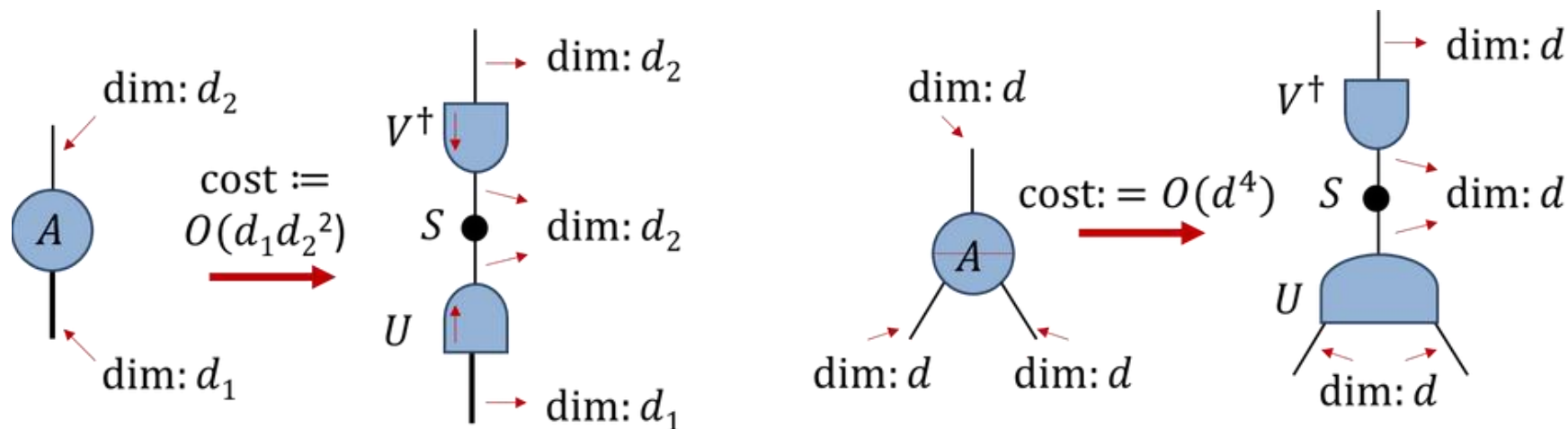
酉矩阵(Unitary matrix)



等距矩阵(Isometric matrix)



奇异值分解(Singular value decomposition)

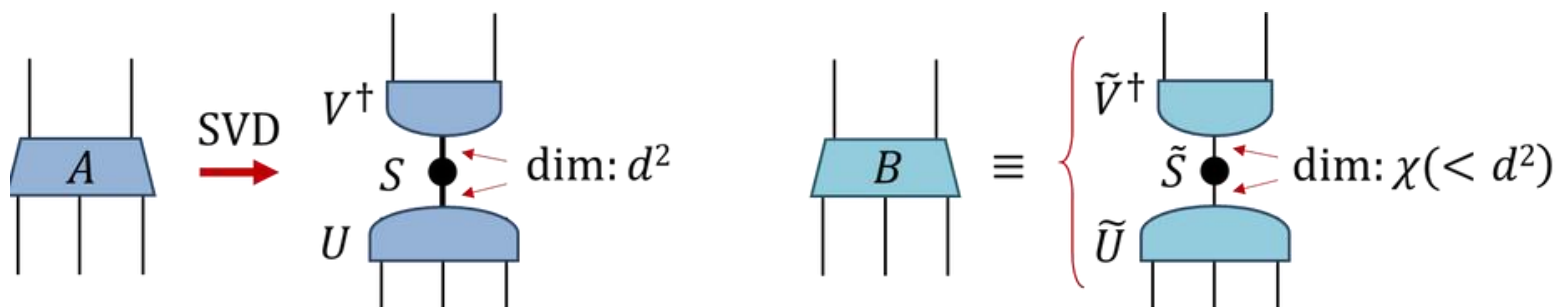


$$A = USV^\dagger$$

U 是等距矩阵, V 是酉矩阵, S 是对角矩阵, 对角元素也叫奇异值, 分解开销 $O(d_1 d_2^2)$

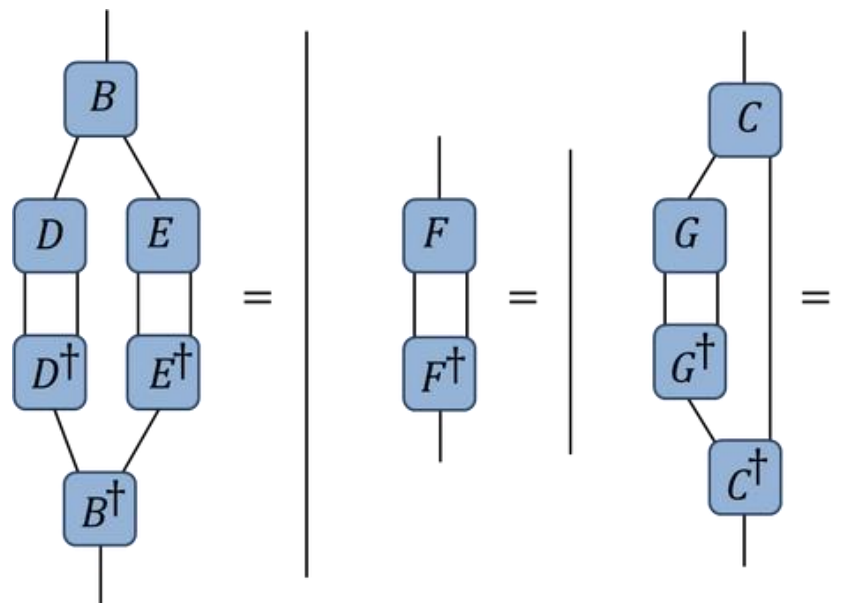
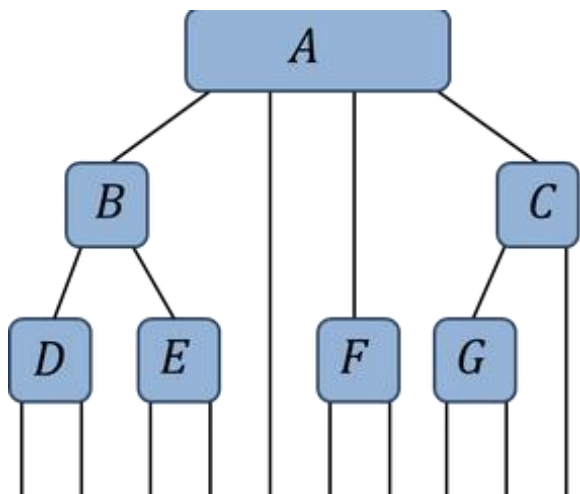
分解的阶(rank)与张量近似

- 张量A的分解阶等于A做SVD之后非零奇异值的个数
- 为了近似A，构造一个维度相同的B，但B的阶小于A（通过直接舍弃A的奇异值中最小的那几个），使得误差最小
- 达到削减维度减少计算量的目的

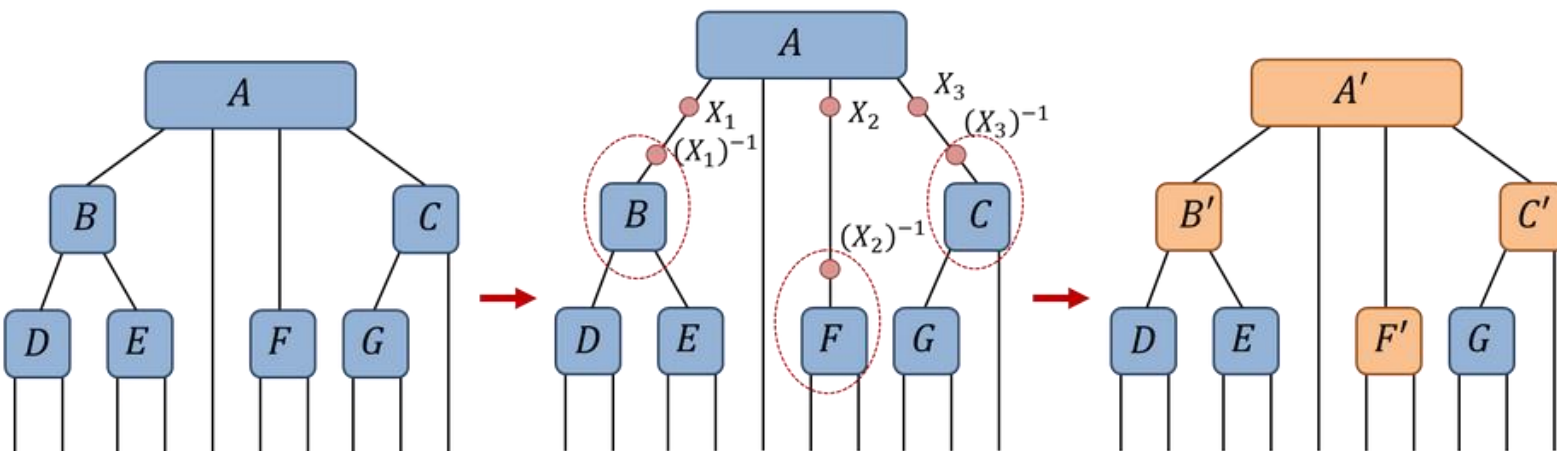
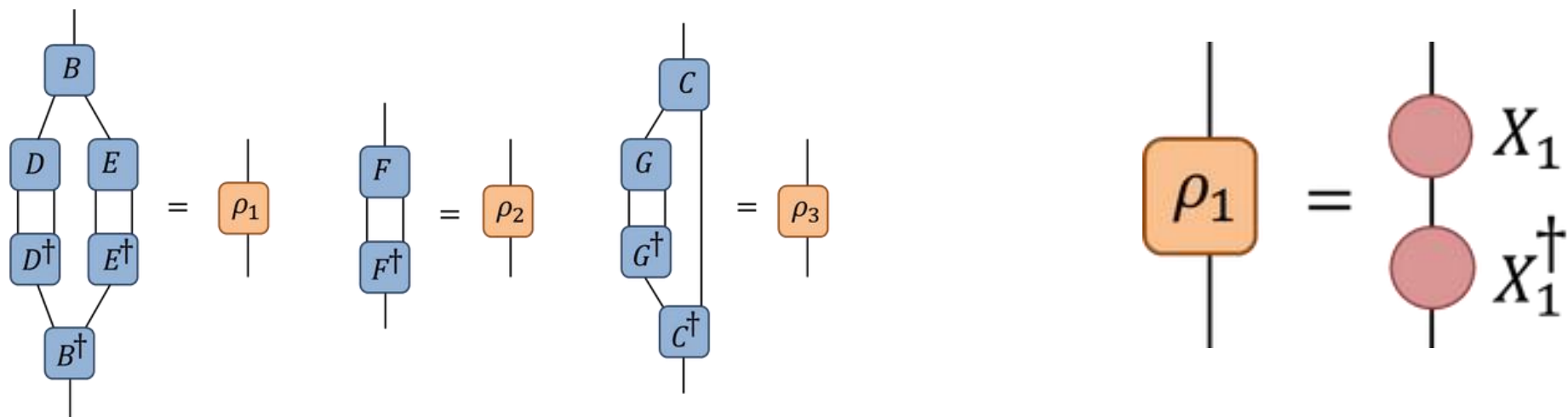


(中心正交性?) Center of Orthogonality

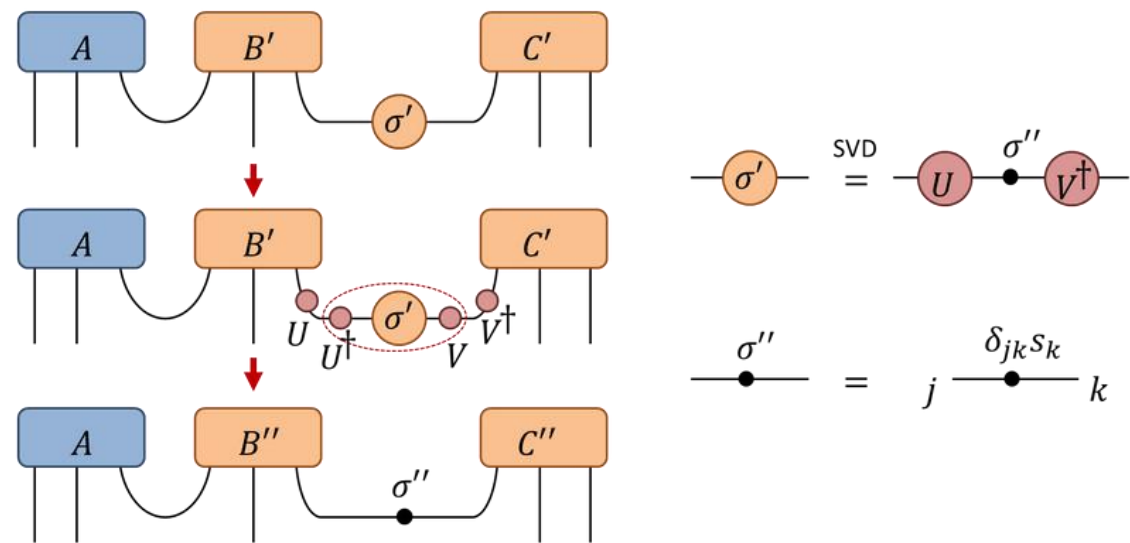
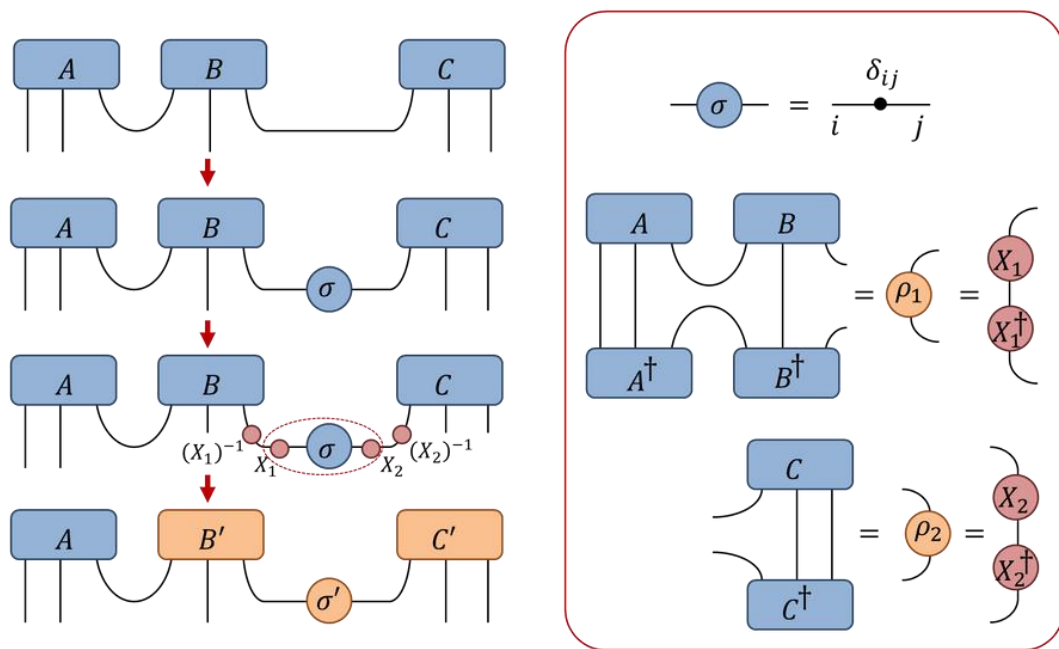
- 张量A的中心正交性, 当且仅当每个维度都是等距的(Isometric)



在张量网络中构造中心正交性



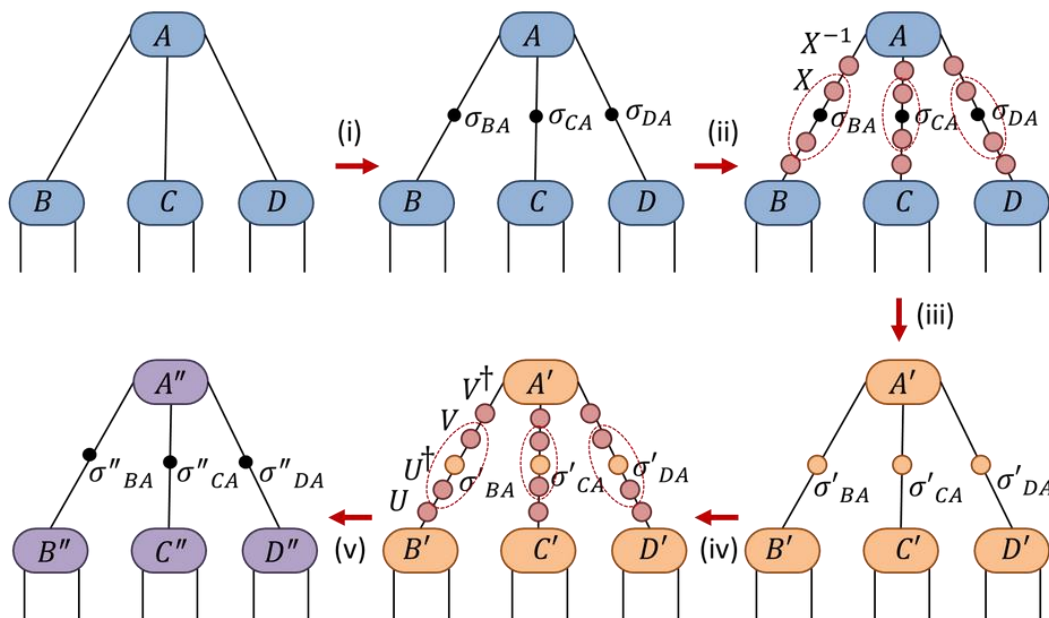
链接的中心正交性



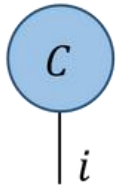
张量网络中我们希望削减张量之间的链接，就可以通过把链接中心正交化，这个链接就成了包含一系列从小到大（？）奇异值的对角矩阵，这时候直接舍弃掉最小奇异值就可以了，误差就是舍掉的奇异值的平方

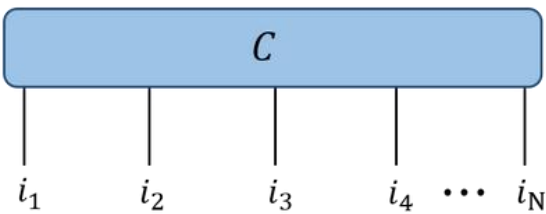
张量网络的正规形式(Canonical forms)

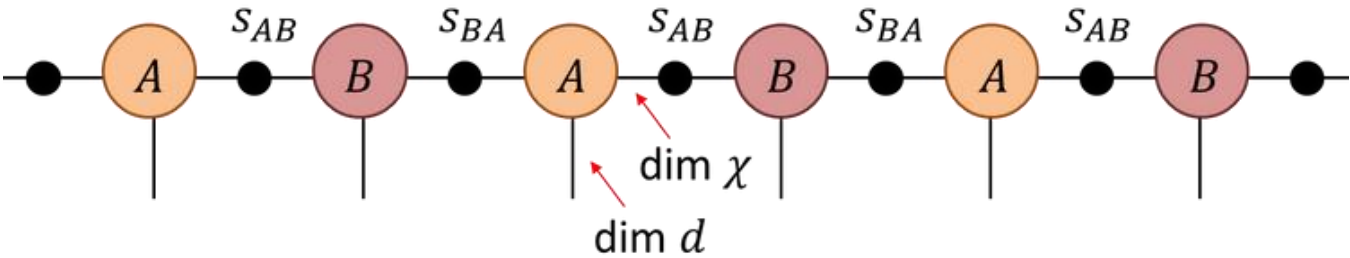
- 当且仅当所有链接都是中心正交的
- 可以非常方便的削减张量之间维度大小，同时误差也很好计算
- 可以从奇异值中提取信息



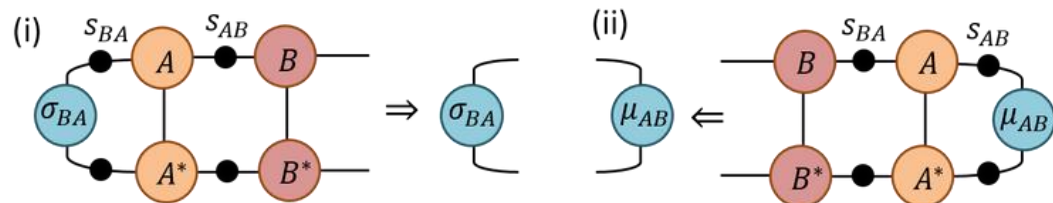
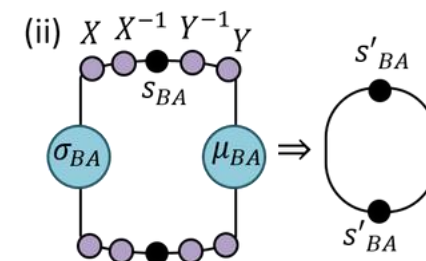
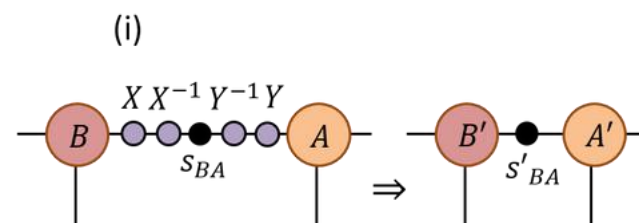
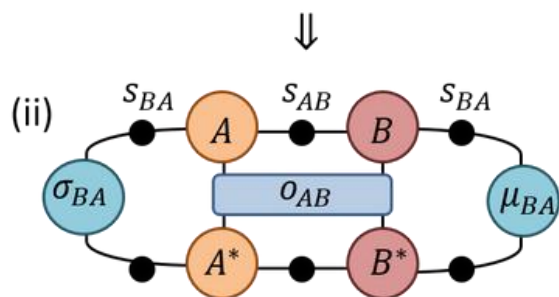
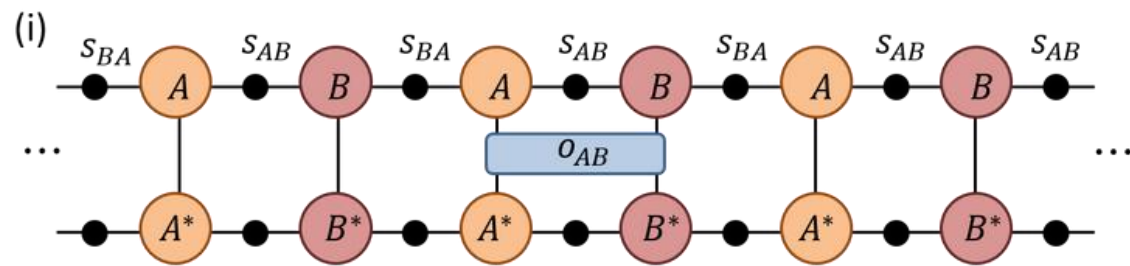
矩阵乘积态MPS

$$|\phi\rangle = C_1|1\rangle + C_2|2\rangle + \cdots + C_d|d\rangle = \sum_{i=1}^d C_i|i\rangle$$
A blue circle labeled C with a single vertical line extending downwards from its center, labeled i .

$$|\psi\rangle = \sum_{i_1, i_2, \dots, i_N} C_{i_1, i_2, \dots, i_N} |i_1\rangle |i_2\rangle \cdots |i_N\rangle$$
A blue rectangle labeled C with multiple vertical lines extending downwards from its bottom edge. The lines are labeled from left to right as $i_1, i_2, i_3, i_4, \dots, i_N$.



收敛法和正规化



MPS时间演化

