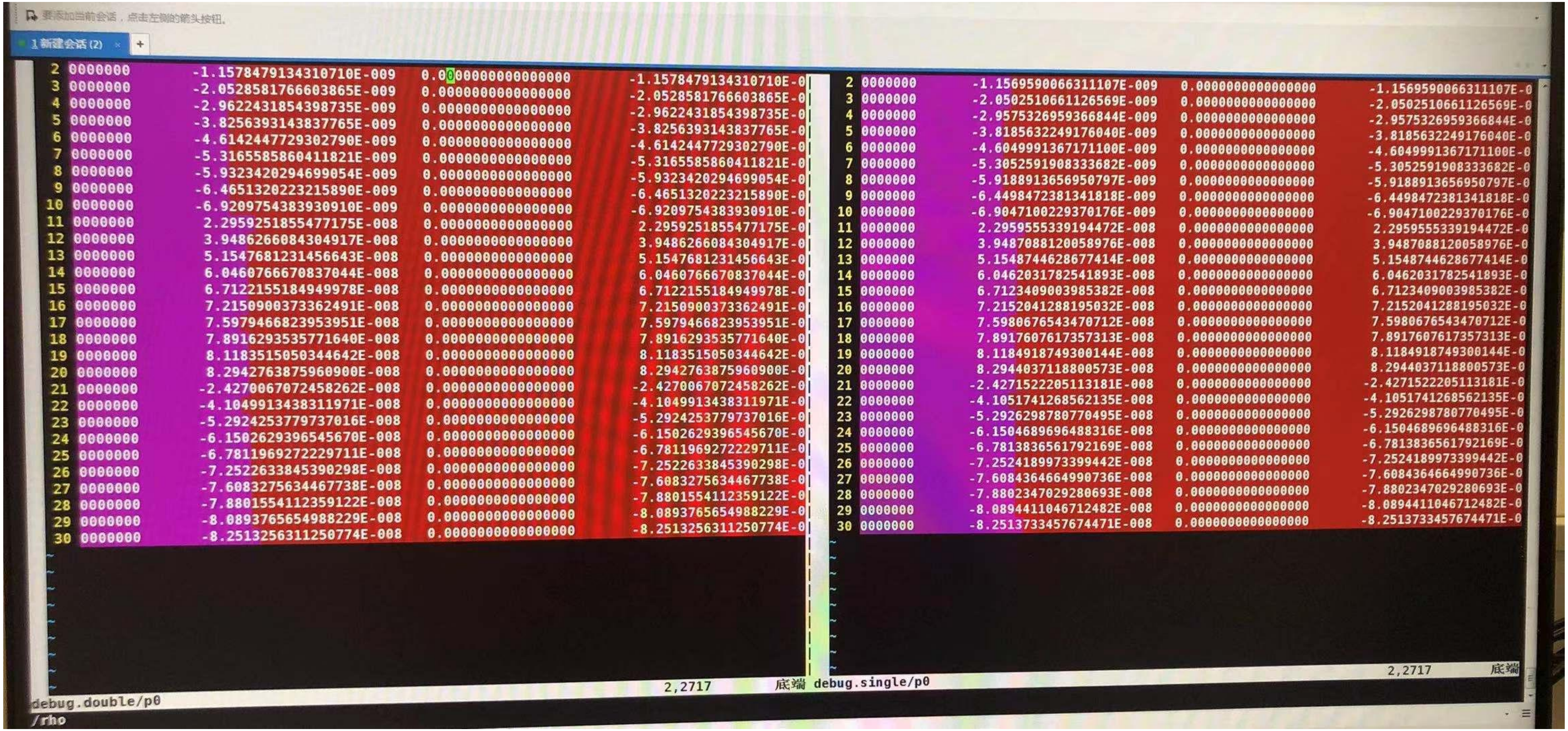
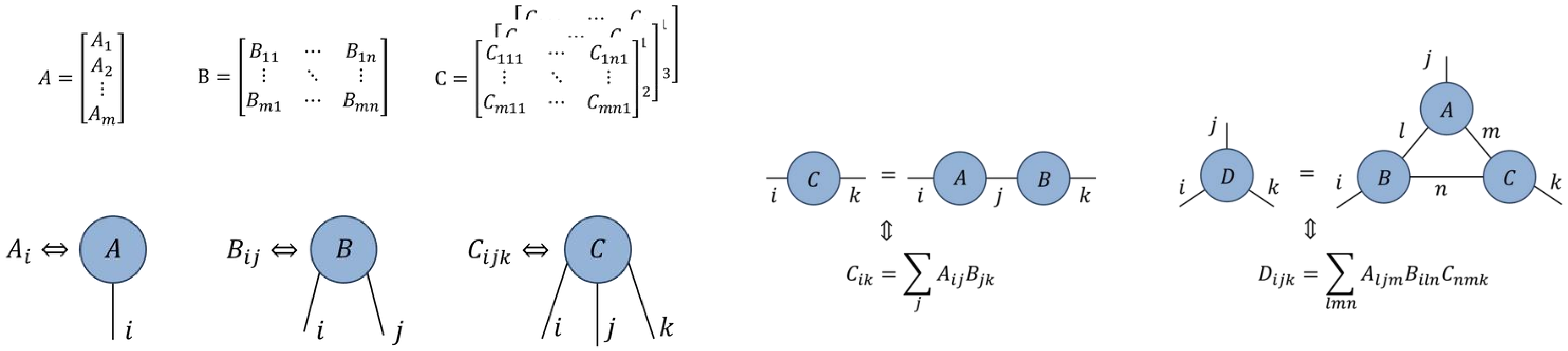
2021.9.13

徐直前

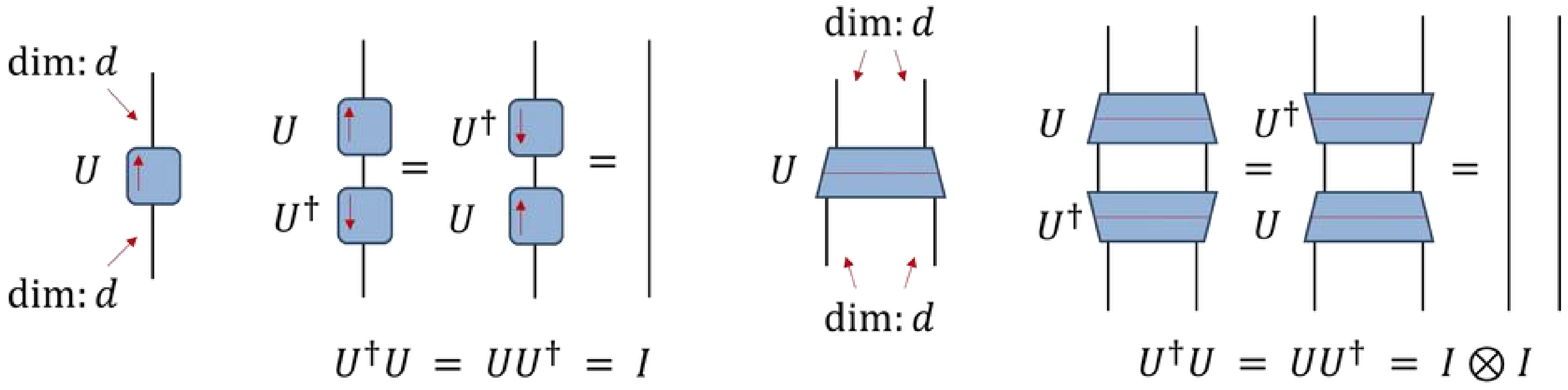
单精度导致的rho误差



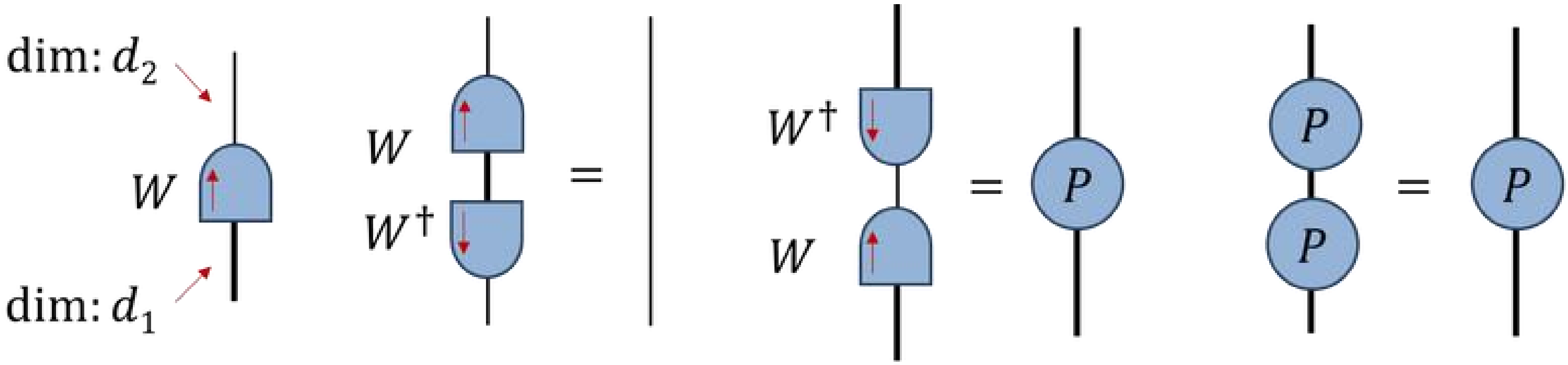


$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{bmatrix}$
$B = \begin{bmatrix} B_{11} & \cdots & B_{1n} \\ \vdots & \ddots & \vdots \\ B_{m1} & \cdots & B_{mn} \end{bmatrix}$
$C = \begin{bmatrix} C_{111} & \cdots & C_{1n1} \\ \vdots & \ddots & \vdots \\ C_{m11} & \cdots & C_{mn1} \end{bmatrix}$
$A_i \Leftrightarrow$
$B_{ij} \Leftrightarrow$
$C_{ijk} \Leftrightarrow$
$C_{ik} = \sum_j A_{ij} B_{jk}$
$D_{ijk} = \sum_{lmn} A_{ljm} B_{iln} C_{nmk}$

酉矩阵(Unitary matrix)

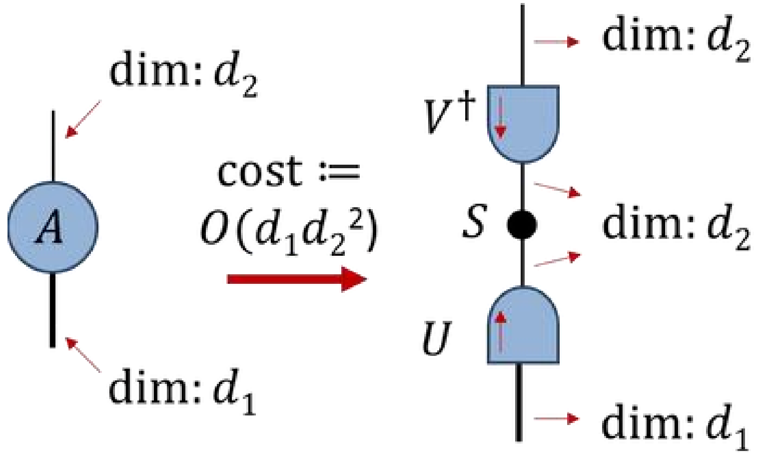


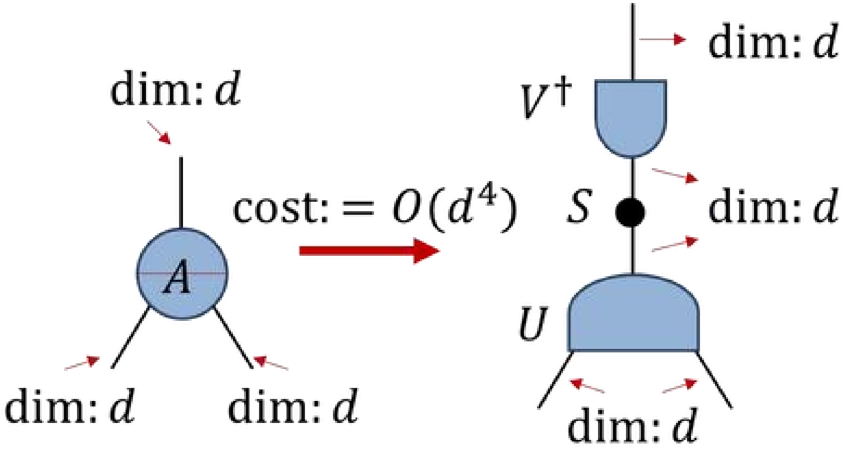
等距矩阵(Isometric matrix)



$W^\dagger W = I$ $WW^\dagger = P$ $P^2 = P$

奇异值分解(Singular value decomposition)



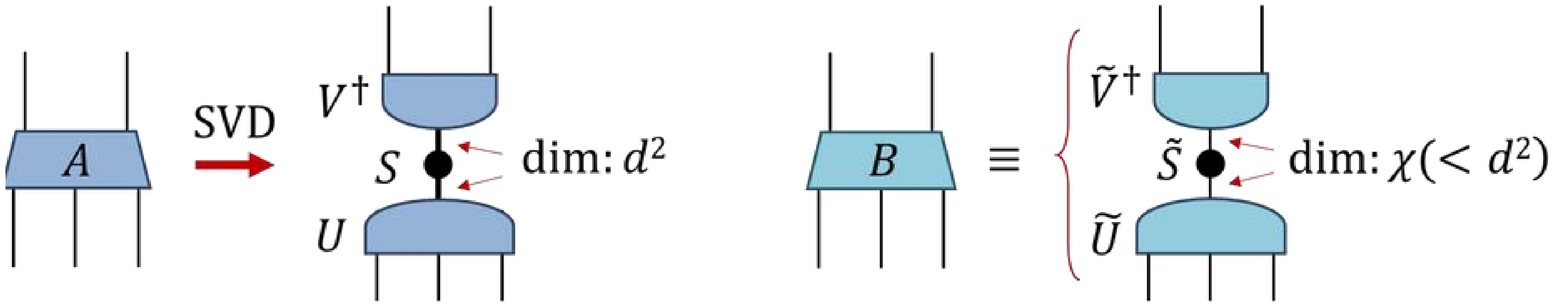


$$A = USV^\dagger$$

U是等距矩阵，V是酉矩阵，S是对角矩阵，对角元素也叫奇异值，分解开销O(d1d2^2)

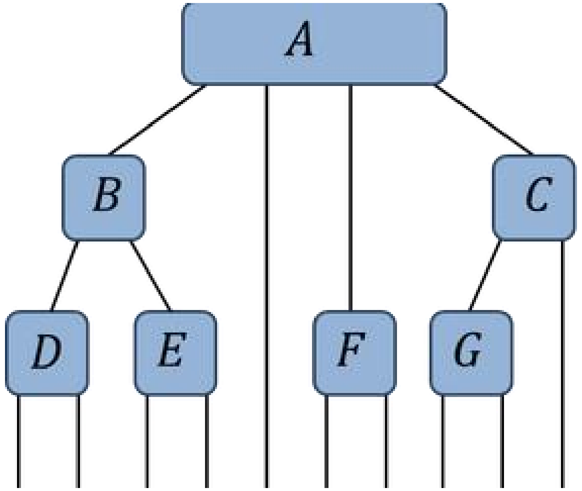
分解的阶(rank)与张量近似

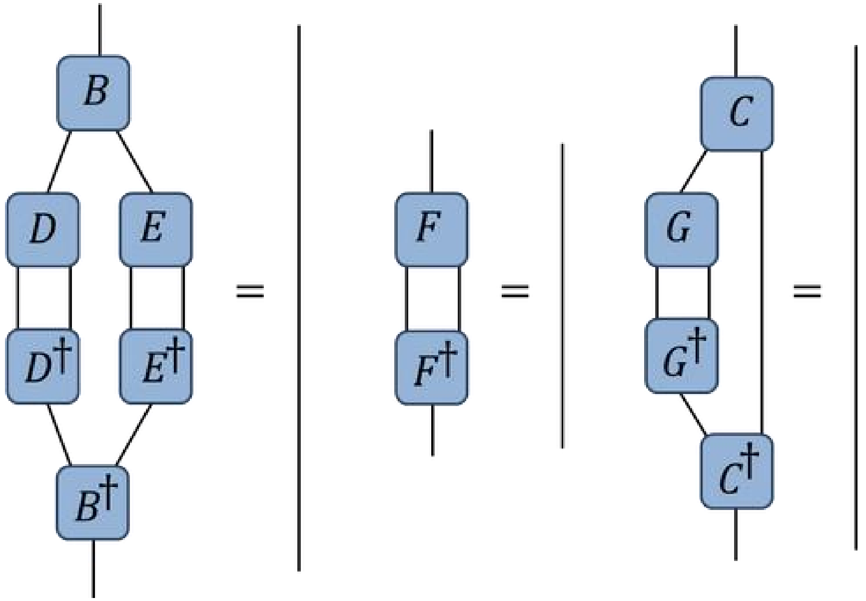
- 张量A的分解阶等于A做SVD之后非零奇异值的个数
- 为了近似A，构造一个维度相同的B，但B的阶小于A（通过直接舍弃A的奇异值中最小的那几个），使得误差最小
- 达到削减维度减少计算量的目的



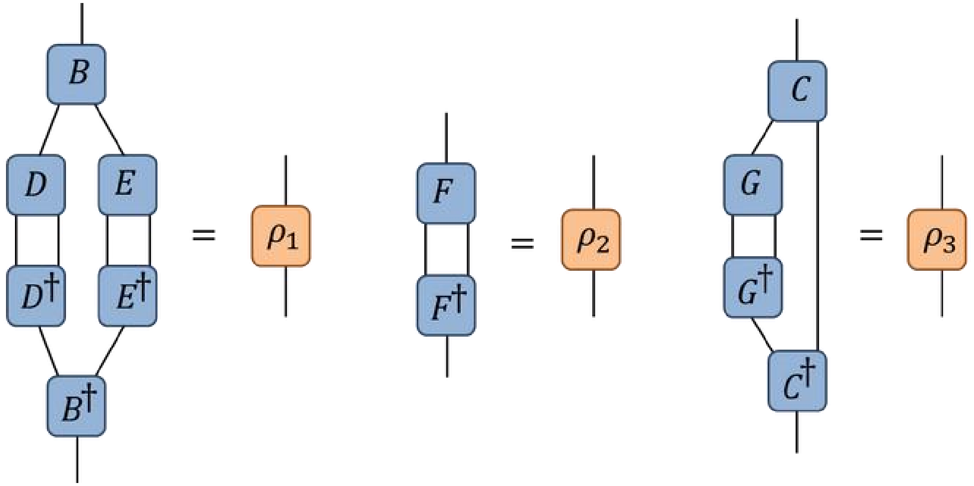
（中心正交性？） Center of Orthogonality

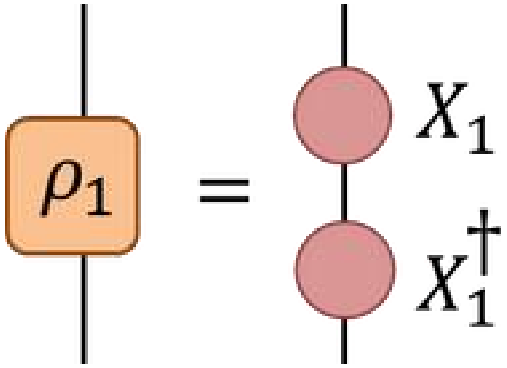
- 张量A的中心正交性，当且仅当每个维度都是等距的(Isometric)

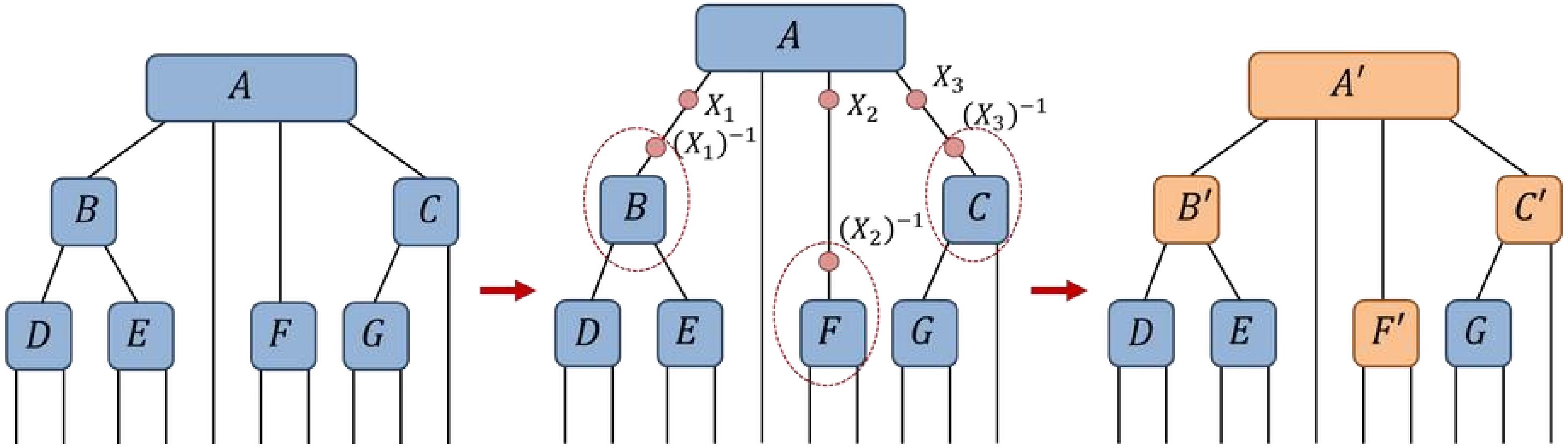




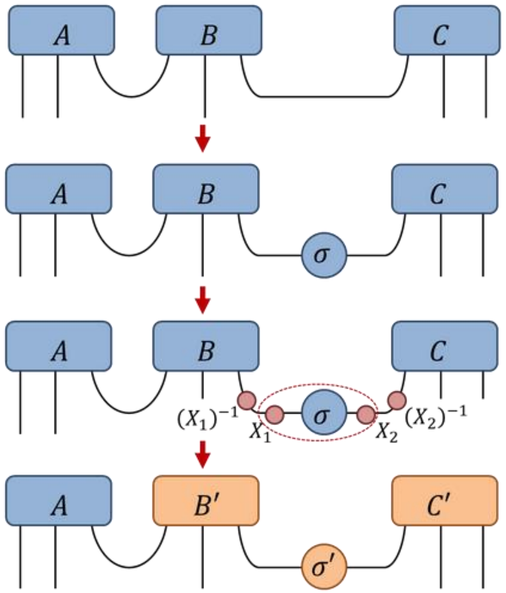
在张量网络中构造中心正交性

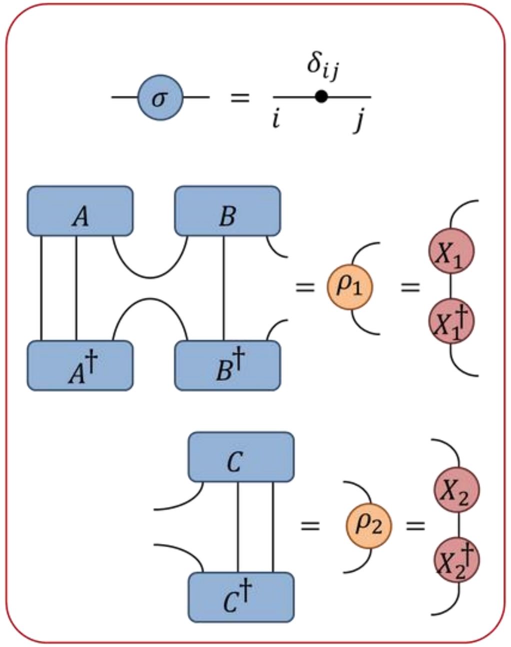


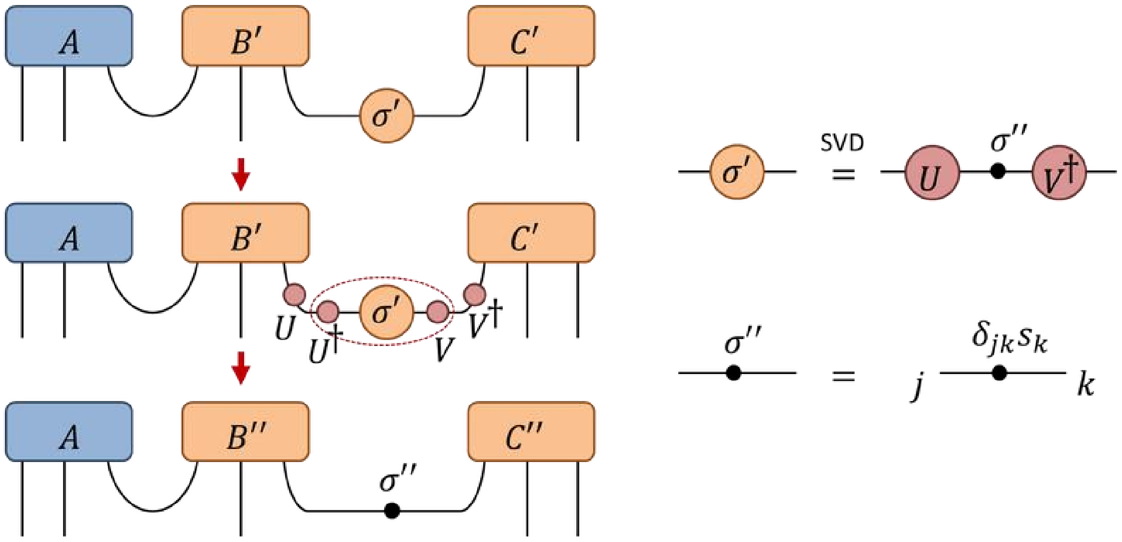




链接的中心正交性



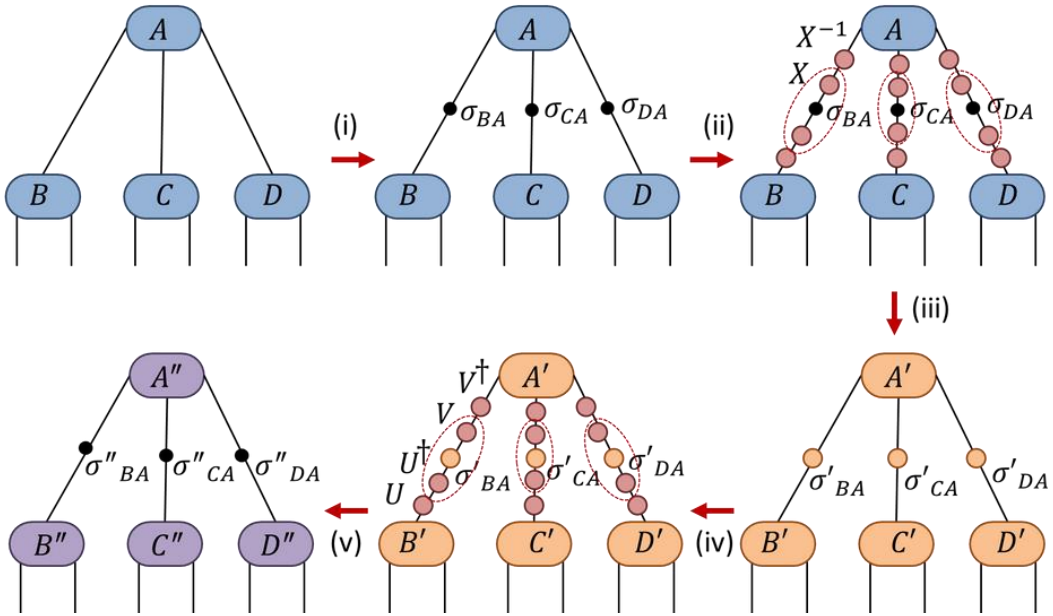




张量网络中我们希望削减张量之间的链接，就可以通过把链接中心正交化，这个链接就成了包含一系列从小到大（？）奇异值的对角矩阵，这时候直接舍弃掉最小奇异值就可以了，误差就是舍掉的奇异值的平方

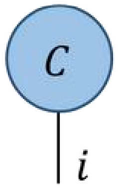
张量网络的正规形式(Canonical forms)

- 当且仅当所有链接都是中心正交的
- 可以非常方便的削减张量之间维度大小，同时误差也很好计算
- 可以从奇异值中提取信息

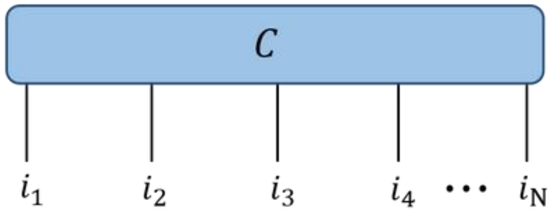


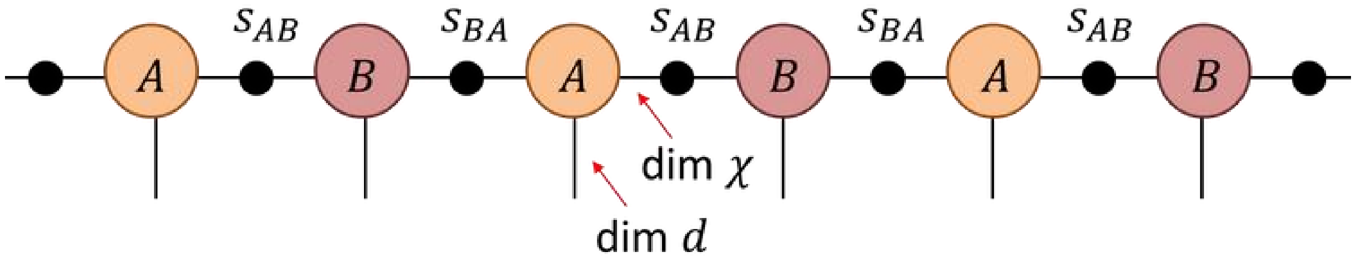
矩阵乘积态MPS

$$|\phi\rangle = C_1|1\rangle + C_2|2\rangle + \cdots + C_d|d\rangle = \sum_{i=1}^{d} C_i|i\rangle$$

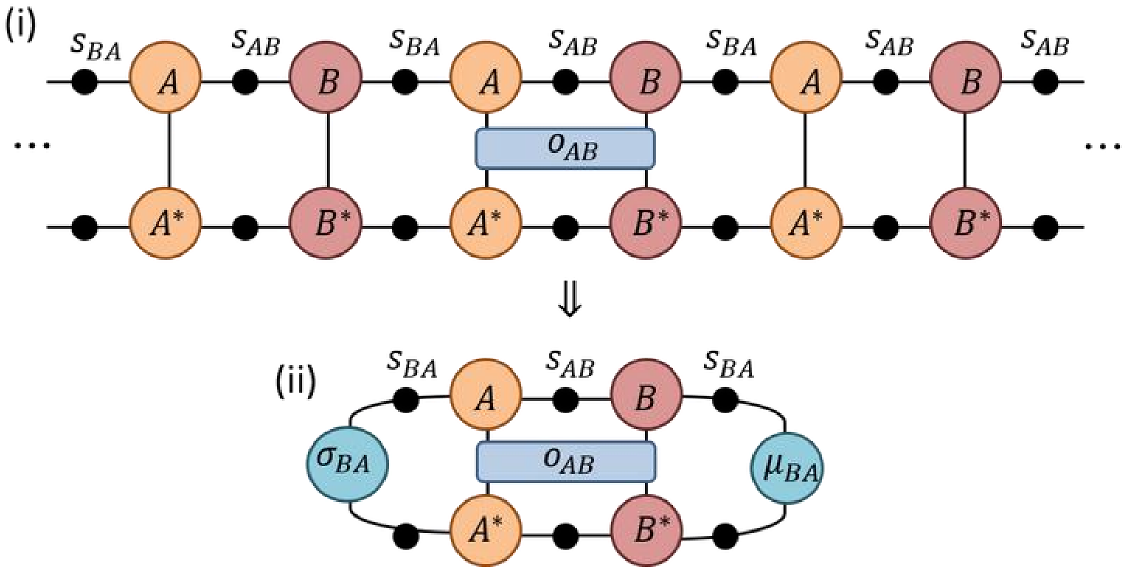


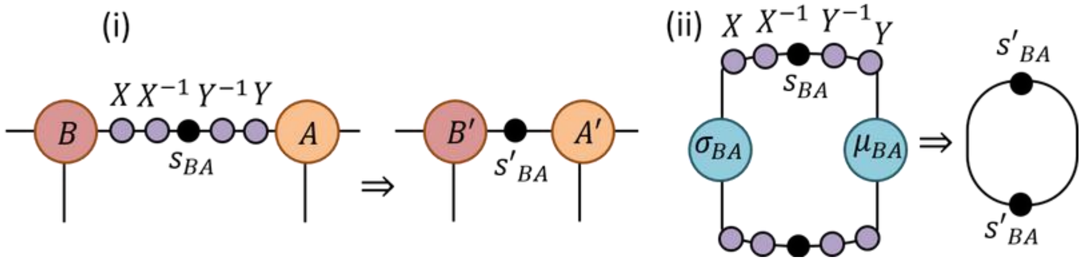
$$|\psi\rangle = \sum_{i_1,i_2,\dots,i_N} C_{i_1,i_2,\dots,i_N}|i_1\rangle|i_2\rangle\cdots|i_N\rangle$$

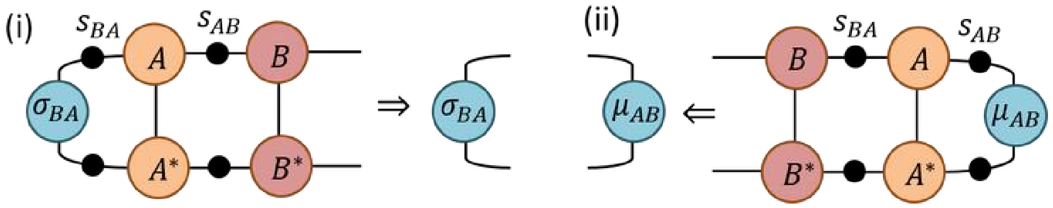




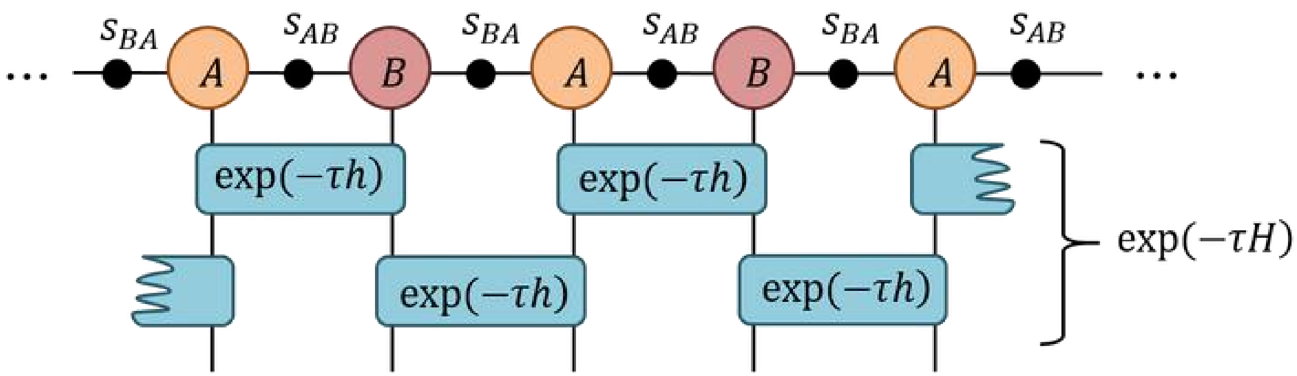
收敛法和正规化


(i)
s_{BA} s_{AB} s_{BA} s_{AB} s_{BA} s_{AB} s_{AB}
A B A B A B
o_{AB}
A^* B^* A^* B^* A^* B^*
$\Downarrow$
(ii)
s_{BA} s_{AB} s_{BA}
σ_{BA} o_{AB} μ_{BA}

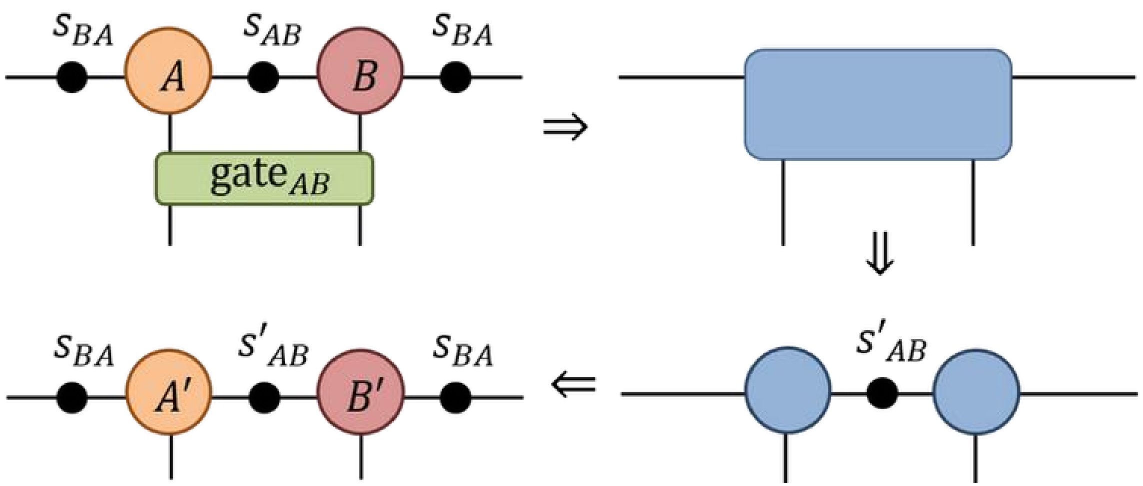

(i)
$X\,X^{-1}\,Y^{-1}\,Y$
B s_{BA} A $\Rightarrow$ B' s'_{BA} A'
(ii) X X^{-1} Y^{-1} Y
s_{BA}
σ_{BA} μ_{BA} $\Rightarrow$
s'_{BA}
s'_{BA}


(i)
s_{BA} s_{AB}
σ_{BA} A B A^* B^* $\Rightarrow$ σ_{BA}
(ii)
μ_{AB} $\Leftarrow$
s_{BA} s_{AB}
B A μ_{AB} B^* A^*

MPS时间演化



s_{BA} s_{AB} s_{BA} s_{AB} s_{BA} s_{AB}
A B A B A
exp(−τh)
exp(−τh)
exp(−τh)
exp(−τh)
exp(−τH)



s_{BA} s_{AB} s_{BA}
A B
gate_{AB}
$\Rightarrow$
$\Downarrow$
s'_{AB}
$\Leftarrow$
s_{BA} s'_{AB} s_{BA}
A′ B′