TSP using Genetic Algorithms

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***Abstract*—This document shows the effect that crossover type, crossover rate, mutation rate, and elitism strategy have on a Genetic Algorithm solving a TSP problem.**

***Index Terms*—Genetic Algorithms, Traveling salesman problem, Computer Science**

I. Objective and Problem definition

The objective of the assignment is to implement a GA(genetic algorithm) system for the TSP(Traveling Salesman Problem) using the two data sets provided at:
http://elib.zib.de/pub/mp-testdata/tsp/tsplib/tsp/ulysses22.tsp
http://elib.zib.de/pub/mp-testdata/tsp/tsplib/tsp/eil51.tsp
The Traveling salesman problem begins with a graph of n cities at various coordinates. The objective is to find an efficient path from city x to all other cities, and then back back to city x. An algorithm is effective in solving this if its results are better than a brute force or random strategy.

II. Parameters

For this paper we will be using the following instance parameters in all cases:

- Maximum population : 1000[1]
- Maximum Generation Span : 1000
- Tournament Size : 5
- Rounding Digits : 5
- Termination period of non improvement: 20% of maximum population (tour will stop crossover/mutation if it does not improve for 200 generations. 190 of these 'stalled' generations are then removed from the graph)

In the following tests, the following parameters are used:

- Elitism Mode
  - top10P (Top 10% of tours are automatically moved to next generation).
  - top1 (Best tour in generation is automatically moved to next generation).
- Crossover Type
  - UOX (Uniform Order Crossover)
  - PMX (Partially Mapped Crossover)
- Crossover Rate
  - 100%
  - 90%
- Mutation Rate
  The chance that each tour x will be altered slightly
  - 0%
  - 10%
  - 100%

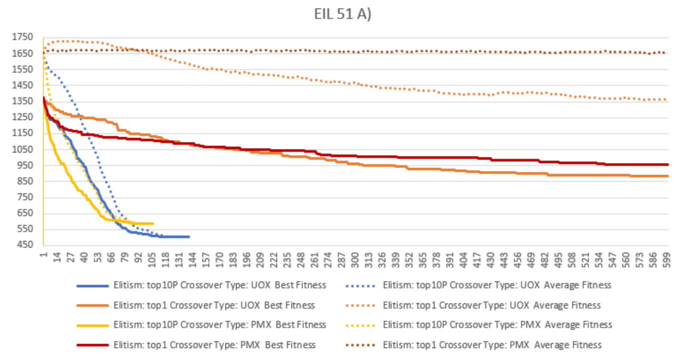
III. Results

On the following graphs, a solid line represents a set of parameters best fitness, and a dotted line represents said parameters average fitness.
Line Colors:

- Blue: Elitism Strategy: top10P. Crossover Type: UOX
- Orange: Elitism Strategy: top1. Crossover Type: UOX
- Yellow: Elitism Strategy: top10P. Crossover Type: PMX
- Red: Elitism Strategy: top1. Crossover Type: PMX

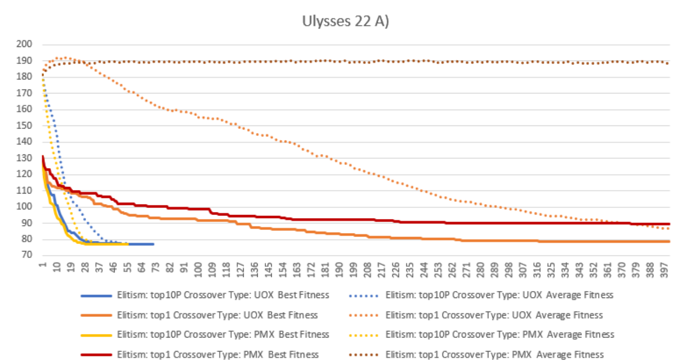
[1]*In the included graphics, graphs were cropped to 400 generations for Ulysses22 and 600 generations for EIL51

A. Crossover rate 100% Mutation rate 0%



PARAMETERS	BEST	AVG
Elitism: top10P Crossover Type: UOX	503.8856	503.8856
Elitism: top1 Crossover Type: UOX	884.421	1360.573
Elitism: top10P Crossover Type: PMX	586.3626	586.3626
Elitism: top1 Crossover Type: PMX	956.4292	1650.732

Fig. 1. EIL with Crossover 1.0 and Mutation 0.0

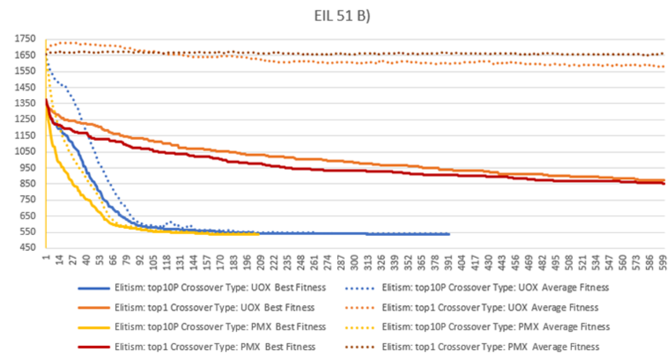


PARAMETERS	BEST	AVG
Elitism: top10P Crossover Type: UOX	77.21008	77.21008
Elitism: top1 Crossover Type: UOX	78.56307	86.57986
Elitism: top10P Crossover Type: PMX	76.99683	76.99683
Elitism: top1 Crossover Type: PMX	89.58875	181.7409

Fig. 2. Ulysses with Crossover 1.0 and Mutation 0.0

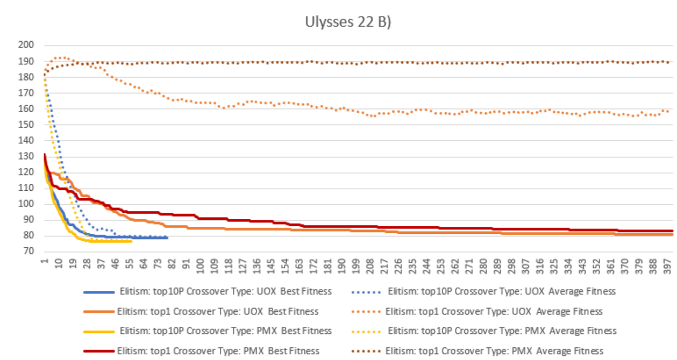
On both datasets it can be seen that the elitism strategy of automatically moving the top 10% of tours to the next generation causes very rapid convergence, however, this also significantly improves the results of each generation. In (1) it can be seen that the fitness achieved by Elitism: top10P after 14 generations is the same as Elitism: top1 with the same crossover type achieves after 352 generations.
Another notable detail is that the PMX crossover type improves the fitness faster than OUX, however, it seems to converge quicker (1) , or with worse results (2)

B. Crossover rate 100% Mutation rate 10%



PARAMETERS	BEST	AVG
Elitism: top10P Crossover Type: UOX	538.6172	541.7831
Elitism: top1 Crossover Type: UOX	873.1162	1581.548
Elitism: top10P Crossover Type: PMX	537.0897	540.0003
Elitism: top1 Crossover Type: PMX	853.5541	1652.498

Fig. 3. EIL with Crossover 1.0 and Mutation 0.1

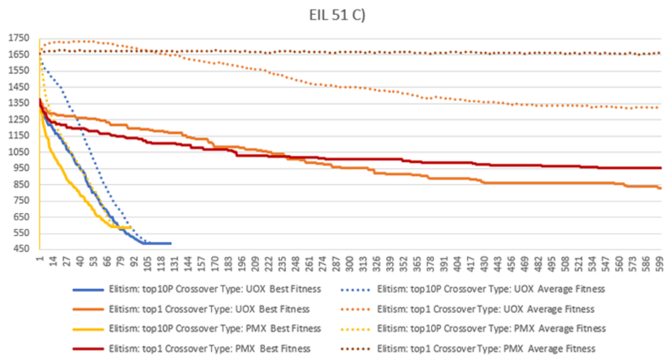


PARAMETERS	BEST	AVG
Elitism: top10P Crossover Type: UOX	78.95679	79.52036
Elitism: top1 Crossover Type: UOX	81.26408	155.4937
Elitism: top10P Crossover Type: PMX	76.58661	77.21971
Elitism: top1 Crossover Type: PMX	83.33588	181.7409

Fig. 4. Ulysses with Crossover 1.0 and Mutation 0.1

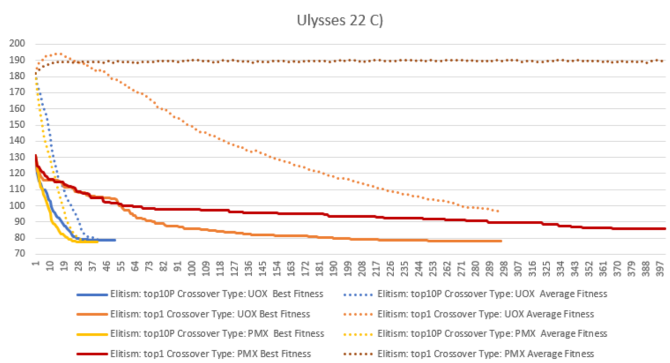
After adding in the slight mutation rate if 10%, it can be said that convergence is delayed for both elitism strategy's. This does not seem to have significantly changed the final results, as can be seen that between both (1),(3) and (2),(4). In fact, the results are slightly *worse* than they were without the mutation.

C. *Crossover rate 90% Mutation rate 0%*



PARAMETERS	BEST	AVG
Elitism: top10P Crossover Type: UOX	485.9578	485.9578
Elitism: top1 Crossover Type: UOX	830.3958	1320.639
Elitism: top10P Crossover Type: PMX	587.5629	587.5629
Elitism: top1 Crossover Type: PMX	951.5277	1651.468

Fig. 5. EIL with Crossover 0.9 and Mutation 0.0

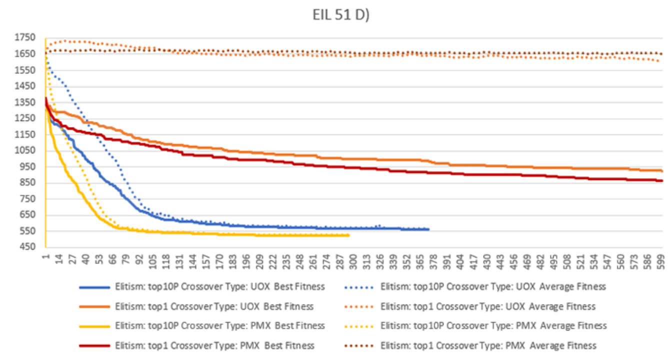


PARAMETERS	BEST	AVG
Elitism: top10P Crossover Type: UOX	78.81823	78.81823
Elitism: top1 Crossover Type: UOX	78.11784	96.72657
Elitism: top10P Crossover Type: PMX	77.76866	77.76866
Elitism: top1 Crossover Type: PMX	85.90859	181.7409

Fig. 6. Ulysses with Crossover 0.9 and Mutation 0.0

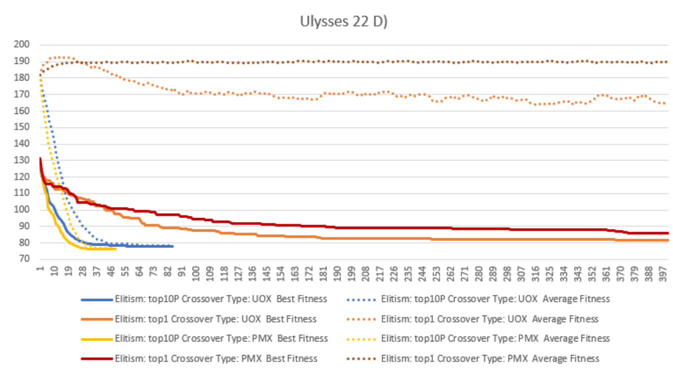
Lowering the crossover rate produces very similar patterns to (1) and (2). However, the resulting fitness does increase slightly across the board, as can be seen between (1),(5) and (2),(6). This could be a fluke result of not using a large enough sample size for averaging.

D. *Crossover rate 90% Mutation rate 10%*



PARAMETERS	BEST	AVG
Elitism: top10P Crossover Type: UOX	565.2316	568.1254
Elitism: top1 Crossover Type: UOX	926.1977	1609.733
Elitism: top10P Crossover Type: PMX	526.3307	529.1418
Elitism: top1 Crossover Type: PMX	862.8355	1652.198

Fig. 7. EIL with Crossover 0.9 and Mutation 0.1

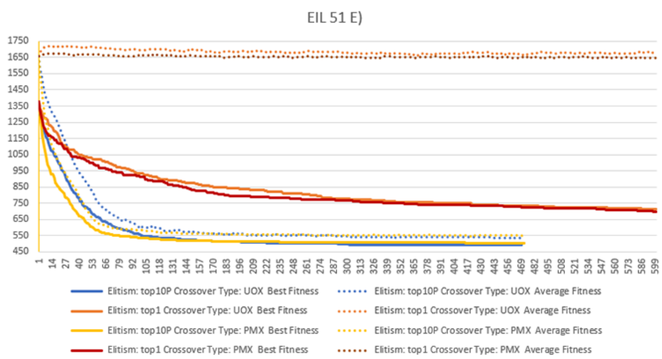


PARAMETERS	BEST	AVG
Elitism: top10P Crossover Type: UOX	77.7301	78.38323
Elitism: top1 Crossover Type: UOX	81.86005	163.7863
Elitism: top10P Crossover Type: PMX	76.3776	76.9352
Elitism: top1 Crossover Type: PMX	86.12767	181.7409

Fig. 8. Ulysses with Crossover 0.9 and Mutation 0.1

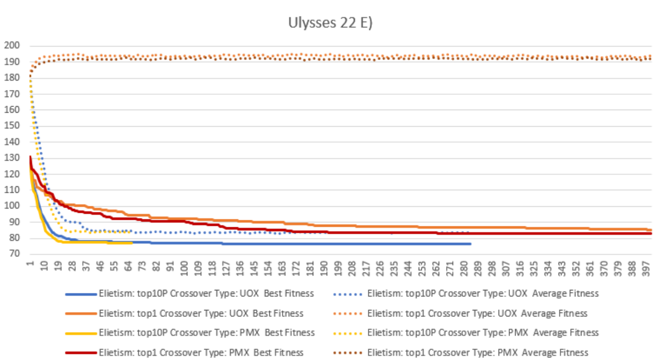
These parameters produce a strange effect compared to the previous experiments. In the case of comparing (1) and (7), the tour utilizing the top10P elitism strategy resulted in a 11.5% less efficient path, however, the tour using the top1 elitism strategy actually improved by approximately 4.6%.

E. Crossover rate 100% Mutation rate 100%



PARAMETERS	BEST	AVG
Elitism: top10P Crossover Type: UOX	490.783	535.6787
Elitism: top1 Crossover Type: UOX	715.2315	1655.855
Elitism: top10P Crossover Type: PMX	503.4053	547.4482
Elitism: top1 Crossover Type: PMX	698.5237	1644.579

Fig. 9. EIL with Crossover 1.0 and Mutation 1.0

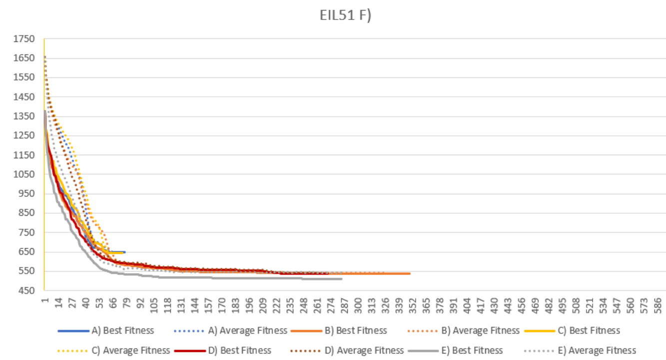


PARAMETERS	BEST	AVG
Elietism: top10P Crossover Type: UOX	76.74086	83.07785
Elietism: top1 Crossover Type: UOX	85.44687	181.7409
Elietism: top10P Crossover Type: PMX	77.20251	83.47124
Elietism: top1 Crossover Type: PMX	83.01536	181.7409

Fig. 10. Ulysses with Crossover 1.0 and Mutation 1.0

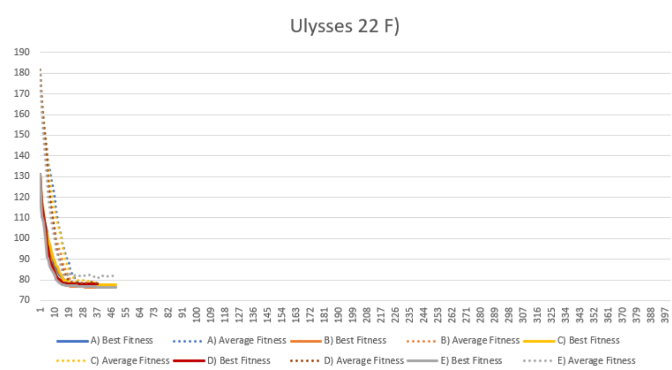
This experiment provided the best results of those tested here, but took approximately 40% longer to compute. Averaging the improvements between (9) and (10) and (1) and (2), each tour was improved by between 2% and 30%. This is understandable, as these parameters allowed the most room for crossbreeding and mutating, thus delaying convergence, as shown by the longer tails on the top10P elitism strategy tours.

F. Crossover rate 100% Mutation rate 100% PMXUOX top10P



PARAMETERS	BEST	AVG
A)	649.034	649.034
B)	536.301	539.417
C)	646.147	646.147
D)	538.19	541.372
E)	510.224	541.34

Fig. 11. EIL with Crossover rate 1.0 Mutation rate 1.0 PMXUOX top10P

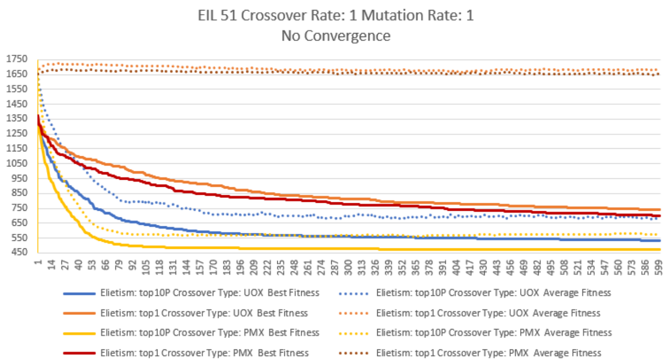


PARAMETERS	BEST	AVG
A)	77.62713	77.63074
B)	76.46537	77.04422
C)	77.7389	77.7389
D)	77.94885	78.5973
E)	76.46905	81.32277

Fig. 12. Ulysses with Crossover rate 1.0 Mutation rate 1.0 PMXUOX top10P

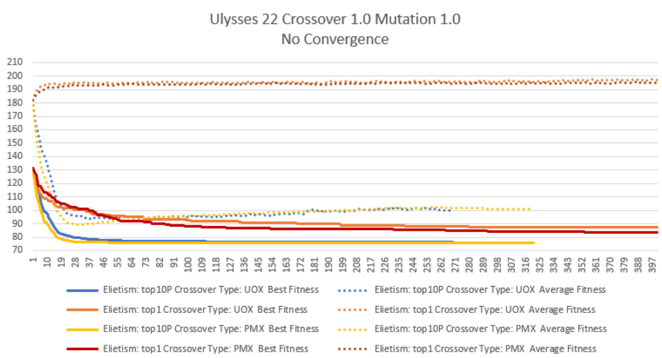
This experiment combined UOX and PMX together, running both on each tour that is generated and allowed by the crossover rate. For this, only tours using top10P were used. This did not produce any significant change from the values generated in previous sections, however, the steepness of the tours indicates that it found a quality solution in comparatively fewer generations.

G. Convergence Protection



PARAMETERS	BEST	AVG
Elietism: top10P Crossover Type: UOX	535.4196	674.6354
Elietism: top1 Crossover Type: UOX	744.521	1655.855
Elietism: top10P Crossover Type: PMX	471.4128	559.8684
Elietism: top1 Crossover Type: PMX	701.188	1649.41

Fig. 13. EIL with Crossover rate 1.0 Mutation rate 1.0 PMXUOX top10P



PARAMETERS	BEST	AVG
Elietism: top10P Crossover Type: UOX	76.3474	93.36968
Elietism: top1 Crossover Type: UOX	87.53728	181.7409
Elietism: top10P Crossover Type: PMX	75.90285	89.4048
Elietism: top1 Crossover Type: PMX	83.43486	181.7409

Fig. 14. Ulysses with Crossover rate 1.0 Mutation rate 1.0 PMXUOX top10P

By using a registry to keep track of what tours have already been tested, convergence is prevented, thus allowing the GA to run for longer and produce better results. Through this is much more expensive and time consuming, (13) and (14) produced the lowest values out of any experiment, with (13) reaching 471.4 and (14) reaching 75.9. This increases are marginal at best, and in any formal application would likely not be worth their associated cost. Additionally, the slope of the lines is much shallower, meaning it took more generations for these tests to reach an acceptable solution, with only minor improvements after that.

IV. Conclusions

A. Effect of crossover type

In general, it seems that PMX outpaces UOX in terms of improvement per generation, however, PMX seems to stop improving earlier than UOX (Yellow and Blue on 5). The result of which of them. When comparing all of the selected experiments, it seems that the question of which is better often depends on the data set and elitism strategy. In terms of final fitness result, in Ulysses 22 PMX wins over UOX in all experiments except E when top10P is used, but loses over UOX in all experiments except E when using top1. Given that the biggest difference between experiments A-D and E is that E has a high mutation rate, UOX seems to work inversely better than PMX with a high mutation rate.
In experiment F (11),(12) it can be seen that using both crossovers one after the other results mostly *worse* fitness's than using them independently. This can be interpreted to mean that PMX and UOX work against each other, resulting in worse results. Notably, it can be seen that the other parameters used have a much less dramatic effect on the results, as PMXUOX seemes to merge the lines closer to eachother than any other crossover type.

B. Effect of crossover rate

When the crossover rate is at 90%, the GA seems to perform better slightly better when comparing (1) and (1), and slightly worse when comparing (2) and (6). This indicates that it could once again be a matter of dataset, or, it could be a fluke, as the differences are minimal. A much larger gap difference in results could likely be obtained by lowering the crossover rate significantly, but that would be counteractive to the goal of this paper.

C. Effect of mutation rate

Mutation rate has a modest effect on final fitness values, but its main draw benefit pushing back convergence and randomly finding new, more efficient, tours. This can be seen between (1) and (3) with the length of the yellow and blue tails (UOX and PMX top10P best), as well as the how the dotted orange and red (UOX and PMX top1 avg) costs stay far away from their best tours. (10) also shows an example of a dramatically larger number of generations before convergence.

D. Effect of Elitism strategy

Elitism strategy has, based on these experiments, by far the most dramatic effect on best and average costs. All graphs show that top10P converges incredibly quickly compared to top1. This is because good tours, when bred, often produce likewise good tours. Since the top 10% of tours are protected in top10P, the result is the new generation having many similar tours. This repeats until every tour is essentially the same, convergence is identified, and the tour stops crossover/mutation. With top1, there is a very slim (1:200) chance that the top tour will be crossovered/mutated. This results in 1) less duplication and chance of convergence, and 2) less overall improvement. Combined, this leads to top1 being far less efficient.

E. Effect of convergence protection

Convergence protection nearly doubled the time the program took to run, and increased its memory use by a factor of

10, but, in the end it did generate superior results in most cases. By preventing already analyzed paths from being re-analyzed, it increased the odds of tours being crossbred with tours they had not bred with before. This resulted in slow but sure improvement.