Lets have a subspace spanned by states

$$
\begin{aligned}
|1\rangle &= \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |i,i\rangle \\
|2\rangle &= \frac{1}{\sqrt{N(N-1)}} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} |i,j\rangle
\end{aligned}
\tag{1}
$$

where $N = 2^n$ in our case, this subspace is invariant under Grover and oracle operator and state $|1\rangle$ is state which amplitude will be amplified by Grover search. Our oracle acts on these states as follows

$$
\begin{aligned}
\hat{O}|1\rangle &= -|1\rangle \\
\hat{O}|2\rangle &= |2\rangle
\end{aligned}
.
\tag{2}
$$

The Grover operator on the whole system reads

$$
\hat{G} = -\hat{I} + 2|\psi\rangle\langle\psi|,
\tag{3}
$$

where

$$
|\psi\rangle = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} |i,j\rangle
\tag{4}
$$

Complete Grover operator acting on invariant subspace give

$$
\begin{aligned}
\hat{G}|1\rangle &= \frac{2-N}{N}|1\rangle + \frac{2\sqrt{(N-1)}}{N}|2\rangle \\
\hat{G}|2\rangle &= \frac{2\sqrt{(N-1)}}{N}|1\rangle + \frac{N-2}{N}|2\rangle
\end{aligned}
.
\tag{5}
$$

If I have Grover that acts only one of our subsystems it has a form

$$
\hat{G}_N = -\hat{I} + 2|\psi_N\rangle\langle\psi_N|,
\tag{6}
$$

To have a operator on the whole system I use tensor product and get

$$
\hat{G}_B = \hat{I} \otimes \hat{G}_N,
\tag{7}
$$

which is the operator on Bobs subsystem. Fortunately it has the same effect on our invariant subspace

$$
\begin{aligned}
\hat{G}_B|1\rangle &= \frac{2-N}{N}|1\rangle + \frac{2\sqrt{(N-1)}}{N}|2\rangle \\
\hat{G}_B|2\rangle &= \frac{2\sqrt{(N-1)}}{N}|1\rangle + \frac{N-2}{N}|2\rangle
\end{aligned}
.
\tag{8}
$$

as the Grover on the whole space. Hence, we do not get any speed up over the original Grover operator, but it easier to implement smaller Grover operator. And it does matter if we apply the smaller operator to Alice or Bob. If I introduce operator swap $\hat{S}$ which is doing this

$$
\hat{S}|i,j\rangle = |j,i\rangle,
\tag{9}
$$

The following holds true

$$
\hat{G}_A = \hat{S}\hat{G}_B\hat{S}
\tag{10}
$$

but the basis states from invariant subspace does not change under $\hat{S}$

$$
\begin{aligned}
\hat{S}|1\rangle &= |1\rangle \\
\hat{S}|2\rangle &= |2\rangle
\end{aligned}
\tag{11}
$$

Hence it does not matter if we apply it to Bob or Alice.