Analyzing the Relationship Between the Acidity and the Conductivity of a Solution

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1 Introduction

1.1 Purpose

Understand how pH of a solution affects conductivity measured in µS.

1.2 Hypothesis

Conductivity will decrease as pH approaches 7, as the ions that would otherwise facilitate the conduction of electricity (H^+/OH^-) would be neutralized.

1.3 Variables

Independent Variable

- pH

Dependent Variable

- Conductivity [µS]

Controlled Variables

- Molarity of HCl

- Molarity of NaOH

- Volume of HCl (50 mL)

- Drop rate of NaOH ($\sim 1\,s^{-1}$)

- Spinning speed of stir bar

2 Materials

- 150 mL 0.10 M HCl

- 150 mL 0.10 M NaOH

- 1 · 100 mL buret+stand

- $1 \cdot 250\,\text{mL}$ beaker
- $1 \cdot 50\,\text{mL}$ graduated cylinder
- $1 \cdot$ stir plate+bar
- $1 \cdot$ logging device
- $1 \cdot$ pH probe
- $1 \cdot$ conductivity probe
- $1 \cdot$ USB flash disk
- Phenolphthalein

3   Procedure

1. Turn on logger
2. Plug in pH/conductivity probes and USB disk
3. Set up buret with stand
4. Put stir plate below buret
5. Fill graduated cylinder with $50\,\text{mL}$ HCl
6. Fill buret with $50\,\text{mL}$ NaOH
7. Transfer HCl to beaker
8. Put 2-3 drops of phenolphthalein in the beaker
9. Calibrate probes
10. Put beaker on top of stir plate
11. Put the stir bar in the beaker
12. Set the stir bar spinning at $\frac{1}{4}$ speed
13. Open the buret so there is ~ 1 drop s^{-1}
14. Start logger
15. Let experiment run for 50 seconds
16. Stop logger
17. Export data to USB disk
18. Release any remaining NaOH into HCl solution
19. Turn off stir plate
20. Remove stir bar
21. Clean beaker
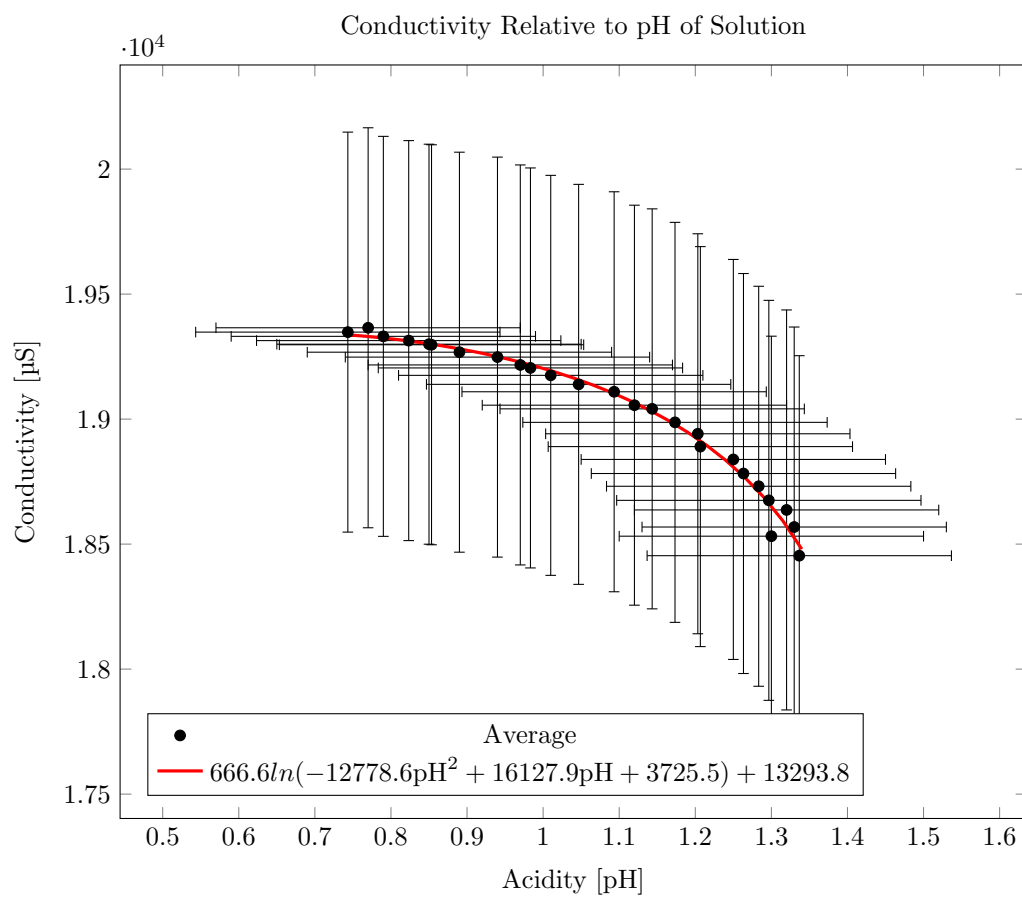22. Repeat $5 - 21$ as necessary for data collection

4 Data

Conductivity Relative to pH of Solution

Legend:
- Average
- $666.6 ln(-12778.6\text{pH}^2 + 16127.9\text{pH} + 3725.5) + 13293.8$

x-axis: Acidity [pH]
y-axis: Conductivity [µS]

Time	pH	Conductivity	
s		µS	
0.00	1.01±0.20	19 352.00	±800.
2.00	1.02±0.20	19 374.00	±800.
4.00	1.04±0.20	19 323.00	±800.
6.00	1.07±0.20	19 293.00	±800.
8.00	1.11±0.20	19 272.00	±800.
10.00	1.08±0.20	19 301.00	±800.
12.00	1.14±0.20	19 264.00	±800.
14.00	1.17±0.20	19 293.00	±800.
16.00	1.17±0.20	19 257.00	±800.
18.00	1.19±0.20	19 257.00	±800.
20.00	1.21±0.20	19 257.00	±800.
22.00	1.24±0.20	19 228.00	±800.
24.00	1.34±0.20	19 220.00	±800.
26.00	1.27±0.20	19 176.00	±800.
28.00	1.35±0.20	19 191.00	±800.
30.00	1.35±0.20	19 139.00	±800.
32.00	1.42±0.20	19 125.00	±800.
34.00	1.39±0.20	19 088.00	±800.
36.00	1.37±0.20	19 030.00	±800.
38.00	1.42±0.20	18 985.00	±800.
40.00	1.47±0.20	18 949.00	±800.
42.00	1.45±0.20	18 934.00	±800.
44.00	1.50±0.20	18 905.00	±800.
46.00	1.49±0.20	18 846.00	±800.
48.00	1.49±0.20	18 839.00	±800.
50.00	1.57±0.20	18 780.00	±800.

Table 1: Trial 1

Time	pH	Conductivity	
s		µS	
0.00	0.82±0.20	19 646.00	±800.
2.00	0.79±0.20	19 669.00	±800.
4.00	0.83±0.20	19 646.00	±800.
6.00	0.84±0.20	19 661.00	±800.
8.00	0.86±0.20	19 639.00	±800.
10.00	0.87±0.20	19 632.00	±800.
12.00	0.90±0.20	19 609.00	±800.
14.00	0.94±0.20	19 579.00	±800.
16.00	0.98±0.20	19 579.00	±800.
18.00	0.95±0.20	19 579.00	±800.
20.00	0.97±0.20	19 541.00	±800.
22.00	1.01±0.20	19 527.00	±800.
24.00	0.98±0.20	19 504.00	±800.
26.00	1.04±0.20	19 474.00	±800.
28.00	1.03±0.20	19 451.00	±800.
30.00	1.08±0.20	19 421.00	±800.
32.00	1.05±0.20	19 414.00	±800.
34.00	1.06±0.20	19 369.00	±800.
36.00	1.13±0.20	19 331.00	±800.
38.00	1.11±0.20	19 286.00	±800.
40.00	1.11±0.20	19 264.00	±800.
42.00	1.13±0.20	19 226.00	±800.
44.00	1.17±0.20	19 226.00	±800.
46.00	1.19±0.20	19 174.00	±800.
48.00	1.21±0.20	19 144.00	±800.
50.00	1.22±0.20	19 099.00	±800.

Table 2: Trial 2

Time	pH	Conductivity	
s		μS	
0.00	0.40±0.20	19 046.00	±800.
2.00	0.50±0.20	19 053.00	±800.
4.00	0.50±0.20	19 024.00	±800.
6.00	0.56±0.20	18 988.00	±800.
8.00	0.58±0.20	18 988.00	±800.
10.00	0.61±0.20	18 959.00	±800.
12.00	0.63±0.20	18 930.00	±800.
14.00	0.71±0.20	18 872.00	±800.
16.00	0.76±0.20	18 814.00	±800.
18.00	0.81±0.20	18 778.00	±800.
20.00	0.85±0.20	18 727.00	±800.
22.00	0.89±0.20	18 662.00	±800.
24.00	0.96±0.20	18 604.00	±800.
26.00	1.05±0.20	18 517.00	±800.
28.00	1.05±0.20	18 481.00	±800.
30.00	1.09±0.20	18 401.00	±800.
32.00	1.14±0.20	18 285.00	±800.
34.00	1.17±0.20	18 213.00	±800.
36.00	1.25±0.20	18 155.00	±800.
38.00	1.26±0.20	18 076.00	±800.
40.00	1.27±0.20	17 981.00	±800.
42.00	1.31±0.20	17 865.00	±800.
44.00	1.29±0.20	17 779.00	±800.
46.00	1.31±0.20	17 685.00	±800.
48.00	1.20±0.20	17 612.00	±800.
50.00	1.22±0.20	17 482.00	±800.

Table 3: Trial 3

Table 4: Average

Time	pH	Conductivity	
s		µS	
0.00	0.74±0.20	19 348.00	±800.
2.00	0.77±0.20	19 365.33	±800.
4.00	0.79±0.20	19 331.00	±800.
6.00	0.82±0.20	19 314.00	±800.
8.00	0.85±0.20	19 299.67	±800.
10.00	0.85±0.20	19 297.33	±800.
12.00	0.89±0.20	19 267.67	±800.
14.00	0.94±0.20	19 248.00	±800.
16.00	0.97±0.20	19 216.67	±800.
18.00	0.98±0.20	19 204.67	±800.
20.00	1.01±0.20	19 175.00	±800.
22.00	1.05±0.20	19 139.00	±800.
24.00	1.09±0.20	19 109.33	±800.
26.00	1.12±0.20	19 055.67	±800.
28.00	1.14±0.20	19 041.00	±800.
30.00	1.17±0.20	18 987.00	±800.
32.00	1.20±0.20	18 941.33	±800.
34.00	1.21±0.20	18 890.00	±800.
36.00	1.25±0.20	18 838.67	±800.
38.00	1.26±0.20	18 782.33	±800.
40.00	1.28±0.20	18 731.33	±800.
42.00	1.30±0.20	18 675.00	±800.
44.00	1.32±0.20	18 636.67	±800.
46.00	1.33±0.20	18 568.33	±800.
48.00	1.30±0.20	18 531.67	±800.
50.00	1.34±0.20	18 453.67	±800.

5 Calculations

5.1 Average

$$\langle \mu S(pH_i) \rangle \tag{1}$$

$$\frac{\sum\limits_{i} \mu S(pH_i)}{i} \tag{2}$$

$$\frac{\mu S(pH_1) + \mu S(pH_2) + ... + \mu S(pH_{i-1}) + \mu S(pH_i)}{i} \tag{3}$$

5.2 Pearson Correlation Coefficient

The Pearson Correlation Coefficient $r_{x,y}$ is defined as

$$r_{x,y} = \frac{\sum\limits_{i=1}^{n} cov(x,y)}{\sigma_x \sigma_y} \tag{4}$$

where $cov(x,y)$ is the covariance and σ_x/σ_y are the standard deviations of sets x and y. This can be expanded to

$$r_{x,y} = \frac{\sum\limits_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{\sum\limits_{i=1}^{n} (x_i - \overline{x})^2 \sum\limits_{i=1}^{n} (y_i - \overline{y})^2} \tag{5}$$

where $\overline{x}/\overline{y}$ are the averages of sets x and y. $r_{x,y}$ was then calculated by substituting sets x and y with the pH and conductivity sets, respectively.

5.3 Error and Minimization

Error was calculated using sum squared error, which is defined as follows:

$$E = \sum\limits_{i=1}^{n} (y_i - f(x_i))^2 \tag{6}$$

While a program carried out the minimization of the function, the basic premise of minimization is to "follow the gradient," so to speak. The program follows the gradient

$$E' = \frac{\partial}{\partial x}(\sum\limits_{i=1}^{n} (y_i - f(x_i))^2) \tag{7}$$

similar to Euler's method for solving differential equations. Minimizing the function is simply following the "negative" slope to the local minima of the function.

5.4   Line of Best Fit

While creating my best fit line, I took into account the axis units. pH is a logarithmic scale, and µS is a linear scale. From there, I hypothesized that my line of best fit would be logarithmic in nature.

$$f(x) = kln(px) + h \tag{8}$$

However, this did not produce lower error than a second order polynomial,

$$g(x) = ax^2 + bx + c \tag{9}$$

which was surprising. The root cause seemed to come from the fact that the logarithmic curve, by nature, "flattens out" much more rapidly than a polynomial curve. To compensate, I concatenated the two functions:

$$(f \circ g)(x) = kln(p(ax^2 + bx + c)) + h \tag{10}$$

$$(f \circ g)(x) = kln(ax^2 + bx + c) + h \tag{11}$$

This approach led to a much lower error than either function alone, as shown below:

Error		
$(f \circ g)(x)$	$f(x)$	$g(x)$
27423.187126	286923.387218	46056.2186406

6   Conclusion

My hypothesis, that the conductivity would decrease as pH approached 7, was partially correct. I was only able to test with an acidic starting point, meaning that I only observed the pH approaching 7 from $0 \leq pH < 7$. However, I can conclude that as the pH approaches 7 from $0 \leq pH < 7$, the conductivity decreases. Simply from a mathematical standpoint, the Pearson correlation coefficient—the measure of the "correlatedness" of any two datasets—of the pHand conductivity returns -0.95, which is an almost exact (negative) linear relationship. This is evident by looking at the graph, which shows that as the pH increased, the conductivity decreased. When the pH was ~ 0.74 (starting pH), the conductivity was $19\,348.00\,\mu S$. When the pH increased to ~ 1.34, however, the conductivity decreased to $18\,453.70\,\mu S$. Of course, there are many more values in between, but the general trend is similar to the one described.

The biggest difficulty I had in my lab was keeping the probes calibrated. In my lab, I was using $0.10\,M$ HCl and NaOH, which have pH values of 1 and 13, respectively. However, the pH probe in particular would not hold its calibration, and in my second and third trials, the starting pH of the HCl is incorrectly reported as 0.82 and 0.40. Even so, this does not invalidate my conclusion, because there is still an obvious downward trend in the conductivity-pH data.

In the future, I could compensate for probe calibration issues by running more trials, so that I could average away systematic error. In this lab, I only had enough time to run 3 trials, but, in the future, I could run more trials to compensate for sensor drift.