Investigating the Effect of String Tension on Fundamental Frequency

Tarik Onalan

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1 Introduction

1.1 Purpose

Understand how string tension affects the fundamental frequency of the string.

1.2 Hypothesis

As tension increases, the fundamental frequency of the string will also increase.

1.3 Variables

Independent Variable

- Tension [N]

Dependent Variable

- Fundamental frequency [Hz]

Controlled Variables

- String length [0.5 m]
- String mass [0.1 kg]
- Type of string

2 Materials

- 1 · 0.5 m string
- 1 · pitch sensor
- 1 · spring scale

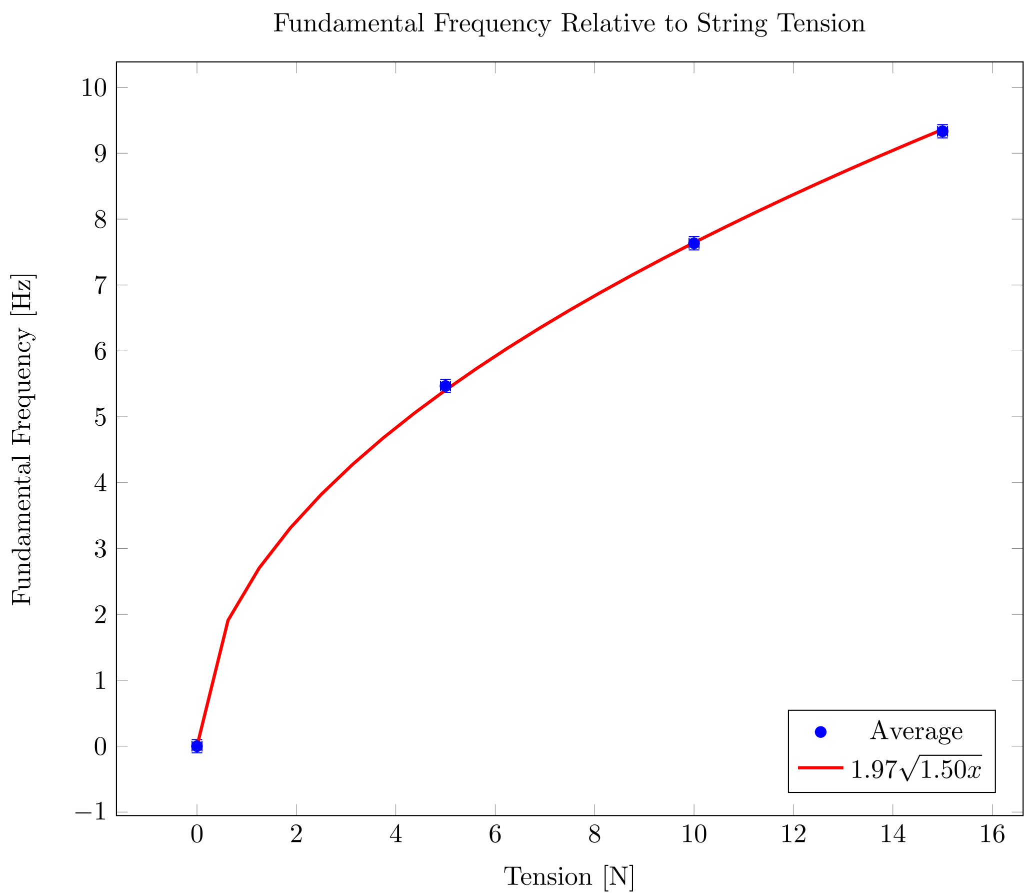
3 Procedure

1. Turn on pitch sensor
2. Tighten string to 0 N
3. "Pluck" string
4. Record frequency
5. Repeat steps 2-4 with 5 N, 10 N, and 15 N
6. Repeat steps 2-5 as necessary for data collection

4 Data

Table 1: Raw Data:

Tension	Trial 1 s	Trial 2 s	Trial 3 s	Average s
0.0±0.1	0.0±0.1	0.0±0.1	0.0±0.1	0.0±0.1
5.0±0.1	5.5±0.1	5.2±0.1	5.7±0.1	5.5±0.1
10.0 ±0.1	7.4±0.1	7.8±0.1	7.7±0.1	7.6±0.1
15.0 ±0.1	9.4±0.1	9.1±0.1	9.5±0.1	9.3±0.1



5 Calculations

5.1 Average

$$\langle \mathrm{Hz(N)} \rangle \tag{1}$$

$$\frac{\sum_i \mathrm{Hz(N)}}{i} \tag{2}$$

$$\frac{\mathrm{Hz(N_1)} + \mathrm{Hz(N_2)} + \ldots + \mathrm{Hz(N_{i-1})} + \mathrm{Hz(N_i)}}{i} \tag{3}$$

5.2 Error and Minimization

Error was calculated using sum squared error, which is defined as follows:

$$E = \sum_{i=1}^{n} (y_i - f(x_i))^2 \tag{4}$$

A program carried out the minimization of the above function, which entails tracing the derivative of the function with respect to a given parameter

$k \in \{a, b, c, d, ...\}$ to the local minima of the function

$$E' = \frac{\partial}{\partial k}(\sum_{i=1}^{n}(y_i - f(x_i))^2) \tag{5}$$

5.3 Line of Best Fit

When choosing a model for my data, I had to keep in mind the basic features. I knew that the tension and fundamental frequency were related by the function

$$?(x) = \frac{\sqrt{\frac{T}{m/L}}}{2L} \tag{6}$$

where T is tension, m is mass, and L is length. Because of this, I assumed my data would also follow a square-root function.
I constructed my function like so:

$$f(x) = a\sqrt{bx + c} + d \tag{7}$$

As a control, I also constructed a linear function:

$$g(x) = a(bx + c) + d \tag{8}$$

As I had predicted, the square-root function yielded much less error when modeling the data, which is why I chose it as my model. The errors for the square-root and linear functions are shown below:

Error

$f(x)$	$g(x)$
0.004 72	3.948 33

6 Conclusion

My hypothesis, that the fundamental frequency of the string would increase as the tension of the string increased, was correct. The graph of the data shows a marked upward trend. When there was 0.0 N of force on the string, the fundamental frequency was 0.0 Hz. However, when the tension was increased to 15.0 N, the fundamental frequency increased to 9.3 Hz. Also, given that the data very closely matches the square-root used to model it, and the square root function is a monotonically increasing function, it would be safe to assume that the relationship shown by the data, that the fundamental frequency increases as string tension increases, is true.
A difficulty I experienced with this lab was keeping the spring scale still while conducting a trial. While I did have it attached to an immobile object, the scale was prone to slippage, and would occasionally slacken the string. This is slightly shown in the data, where there are dips in the fundamental frequency (see 5 N trial 2, 10 N trial 1, 15 N trial 2). However, these errors are not large enough to nullify my conclusion.
In the future, I would like to be able to conduct more trials, so any errors like the one described above could be "averaged away", per se. While the errors in my data were not particularly large, it would have been better to conduct more trials to see exactly where the fundamental frequency was.