1-Q: $|0\rangle =$  , $|1\rangle =$ 

$|\tilde{0}\rangle =$  , $|\tilde{1}\rangle =$ 

$$|\tilde{x}\rangle = \frac{1}{\sqrt{2}} \sum_{y=0}^{2-1} e^{\frac{i\pi}{2} 2x\cdot y} |y\rangle \Rightarrow \frac{1}{\sqrt{2}} \left[\cancel{e^{\frac{2\pi i}{2} x \cdot 0}} |0\rangle + e^{\frac{2\pi i}{2} x \cdot 1} |1\rangle \right]$$

$$\Rightarrow \boxed{\frac{1}{\sqrt{2}} \left[|0\rangle + e^{i\pi x} |1\rangle \right]}$$

$$e^{i\theta} = \cos\theta + i \,\mathrm{sen}\,\theta$$

$$\theta = \pi$$

$$e^{i\pi} = -1 + i \cdot 0$$

$$e^{i\pi} + 1 = 0$$

$$QFT|0\rangle = |+\rangle$$

$$= \frac{1}{\sqrt{2}} \left[|0\rangle + e^{i\pi 0} |1\rangle \right] = \frac{1}{\sqrt{2}} \left[|0\rangle + |1\rangle \right] = |+\rangle$$

$$QFT|1\rangle = |-\rangle = ¿?$$

2 ingredientes:

1) $H|x_k\rangle \Rightarrow$ $|\tilde{0}\rangle : (|0\rangle + |1\rangle)/\sqrt{2}$

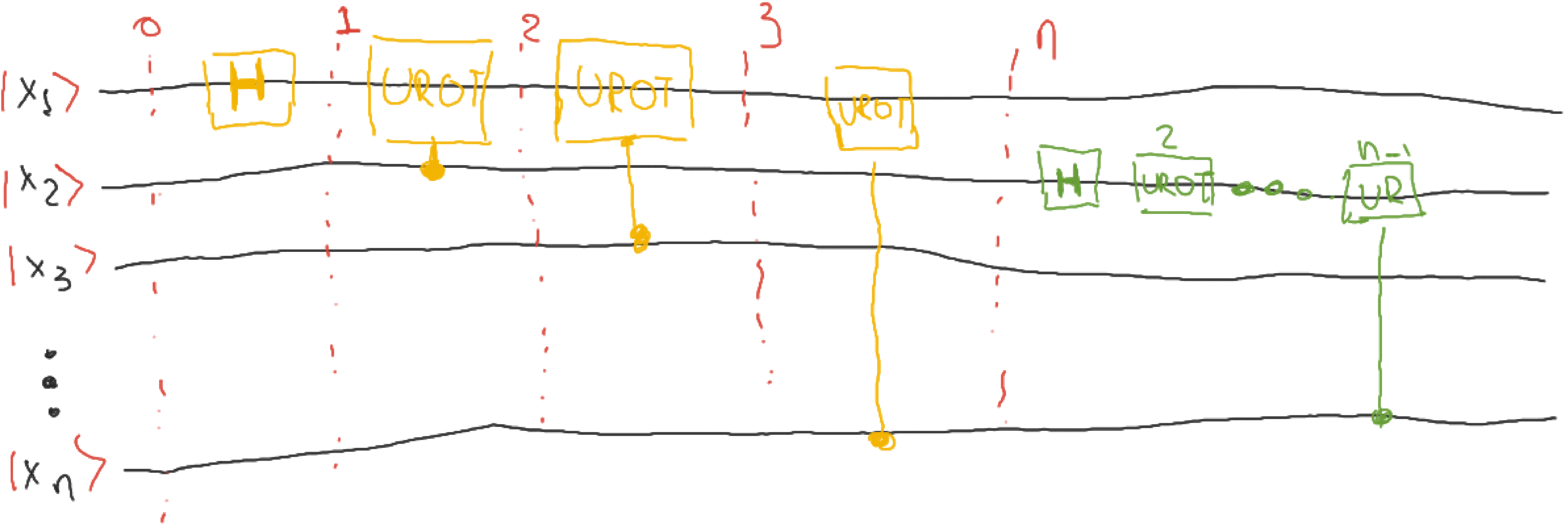
$\tilde{0}\ \tilde{1}$ $\quad |\tilde{1}\rangle : (|0\rangle - |1\rangle)/\sqrt{2}$

$$= \left(|0\rangle + e^{\frac{2\pi i x_k}{2}} |1\rangle\right)/\sqrt{2}$$

2) $UROT_k |x_j\rangle = e^{\frac{2\pi i}{2^k} x_j} |x_j\rangle$; $x_j : e^{\frac{2\pi i \cdot 0}{2^k}} |0\rangle = |0\rangle$

$|1\rangle : e^{\frac{2\pi i \cdot 1}{2^k}} |1\rangle = e^{\frac{2\pi i}{2^k}} |1\rangle$

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^k} \end{pmatrix}$$



Step 0: $|x_1, x_2, x_3, \dots, x_n\rangle$

Step 1: $\left[|0\rangle + e^{\frac{2\pi i x_1}{3}}|1\rangle\right] \otimes |x_2, x_3, \dots, x_n\rangle$

Step 2: $\left[|0\rangle + e^{\frac{2\pi i x_2}{2^2}} e^{\frac{2\pi i}{2^1} x_1} |1\rangle\right] \otimes |x_2, x_3, \dots, x_n\rangle$

Step n: $\left[|0\rangle + e^{\frac{2\pi i}{2^n} x_n} \cdot e^{\frac{2\pi i}{2^{n-1}} x_{n-1}} \cdots e^{\frac{2\pi i}{2^0} x_1} |1\rangle\right] \otimes |x_2\, x_3 \dots, x_n\rangle$

QFT → circuit

$$\boxed{|\tilde{x}\rangle = \frac{1}{\sqrt{N}} \sum_{y=0}^{y=N-1} e^{\frac{i2\pi}{N} x \cdot y} |y\rangle} \rightsquigarrow \text{QFT}$$