

$$1-Q: |0\rangle = \begin{array}{c} \uparrow \\ \text{---} \\ \downarrow \end{array} \quad , \quad |1\rangle = \begin{array}{c} \text{---} \\ \downarrow \end{array}$$

$$|\tilde{0}\rangle = \begin{array}{c} \text{---} \\ \swarrow \end{array} \quad , \quad |\tilde{1}\rangle = \begin{array}{c} \text{---} \\ \searrow \end{array}$$

$$|\tilde{x}\rangle = \frac{1}{\sqrt{2}} \sum_{y=0}^{2^x-1} e^{\frac{i\pi}{2} 2^x y} |y\rangle \Rightarrow \frac{1}{\sqrt{2}} [e^{\frac{2\pi i}{2} x \cdot 0} |0\rangle + e^{\frac{2\pi i}{2} x \cdot 1} |1\rangle]$$

$$\Rightarrow \frac{1}{\sqrt{2}} [ |0\rangle + e^{i\pi x} |1\rangle ]$$

$$\begin{aligned} \text{QFT } |0\rangle &= |+\rangle \\ &= \frac{1}{\sqrt{2}} [ |0\rangle + e^{i\pi \cdot 0} |1\rangle ] = \frac{1}{\sqrt{2}} [ |0\rangle + |1\rangle ] = \underline{|+\rangle} \end{aligned}$$

$$\text{QFT } |1\rangle = |-\rangle = ?$$

$$\begin{aligned} e^{i\theta} &= \cos\theta + i\sin\theta \\ \theta &= \pi \\ e^{i\pi} &= -1 + i \cdot 0 \\ e^{i\pi} + 1 &= 0 \end{aligned}$$

2 ingredients:

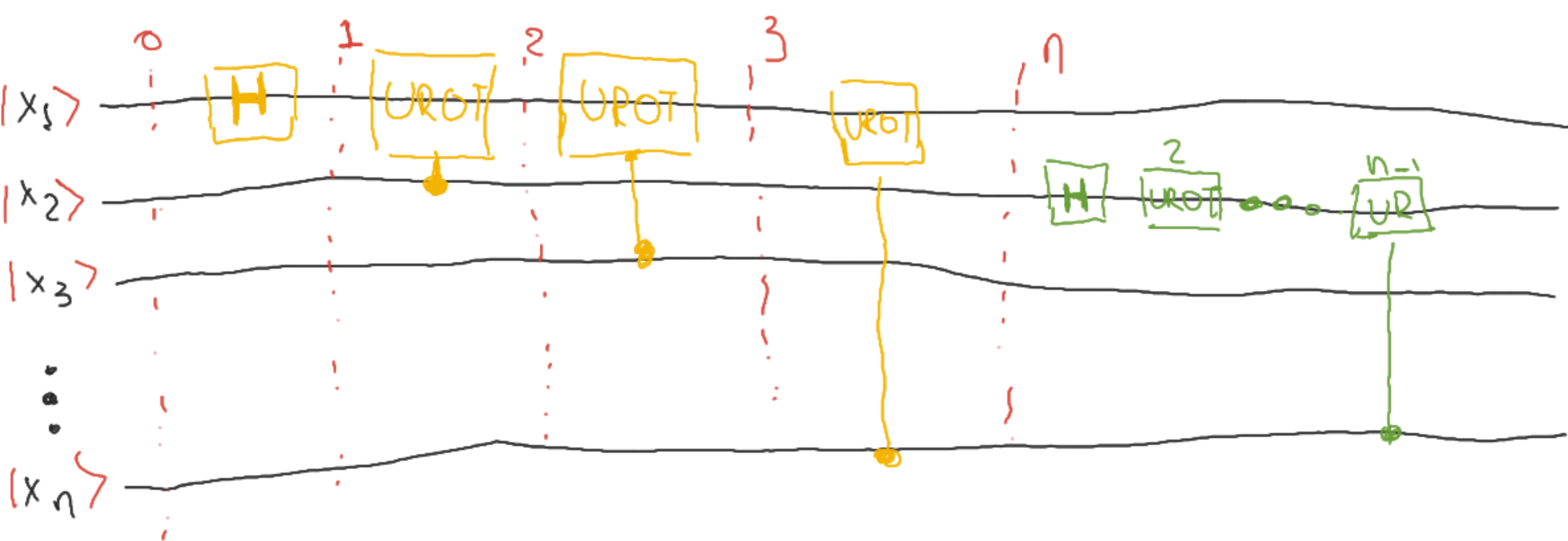
$$1) H|x_k\rangle \Rightarrow \begin{matrix} |0\rangle: (\underline{10} + \underline{11})/\sqrt{2} \\ |\tilde{1}\rangle: (\underline{10} - \underline{11})/\sqrt{2} \end{matrix}$$

$$= (|0\rangle + e^{\frac{2\pi i}{2} x_k} |1\rangle) / \sqrt{2}$$

$$2) U_{ROT_k} |x_j\rangle = e^{\frac{2\pi i}{2^k} x_j} |x_j\rangle; \quad \begin{matrix} |0\rangle: e^{\frac{2\pi i}{2^k} \cdot 0} |0\rangle = |0\rangle \\ |1\rangle: e^{\frac{2\pi i}{2^k} \cdot 1} |1\rangle = e^{\frac{2\pi i}{2^k}} |1\rangle \end{matrix}$$

$$\uparrow$$

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^k} \end{pmatrix}$$



Step 0:  $|x_1, x_2, x_3, \dots, x_n\rangle$

Step 1:  $[|0\rangle + e^{\frac{2\pi i}{3} x_1} |1\rangle] \otimes |x_2, x_3, \dots, x_n\rangle$

Step 2:  $[|0\rangle + e^{\frac{\pi i}{2^2} x_2} e^{\frac{2\pi i}{2^1} x_1} |1\rangle] \otimes |x_2, x_3, \dots, x_n\rangle$

Step n:  $[|0\rangle + e^{\frac{2\pi i}{2^n} x_n} e^{\frac{2\pi i}{2^{n-1}} x_{n-1}} \dots e^{\frac{2\pi i}{2^1} x_1} |1\rangle] \otimes |x_2, x_3, \dots, x_n\rangle$

QFT  $\rightarrow$  circuit

$$|\tilde{x}\rangle = \frac{1}{\sqrt{N}} \sum_{y=0}^{y=N-1} e^{i2\pi \frac{x \cdot y}{N}} |y\rangle$$

$\leadsto$  QFT