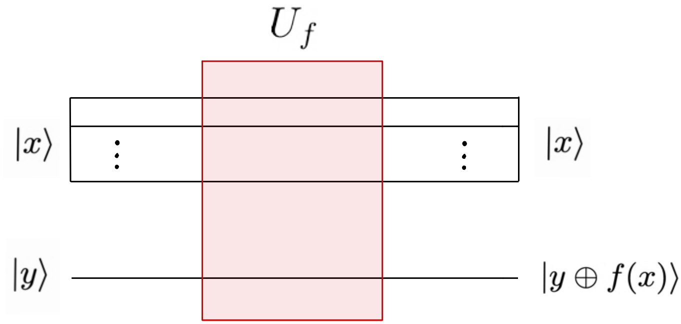
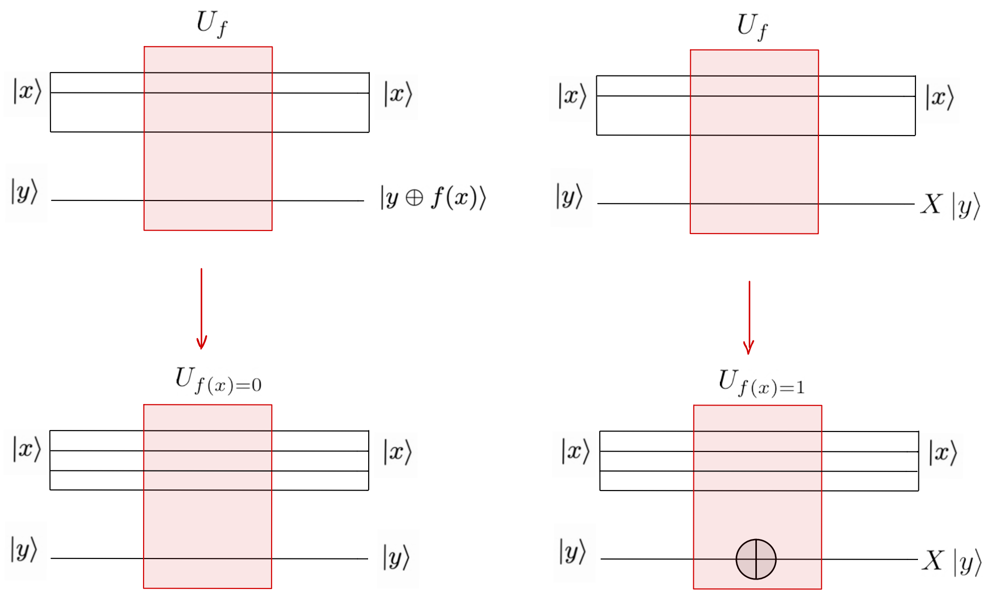
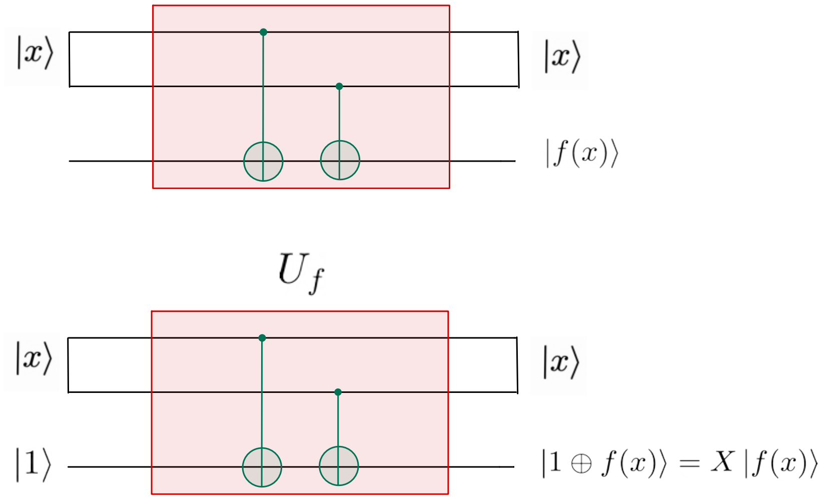


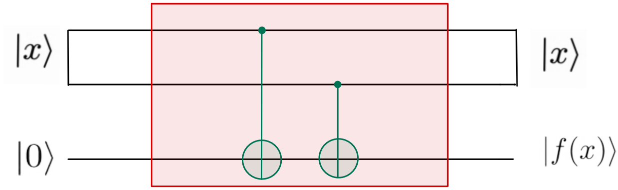
$x \in \{0,1\}^n$

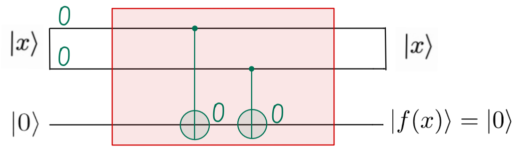
$y \in \{0,1\}$

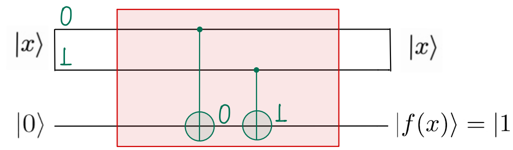


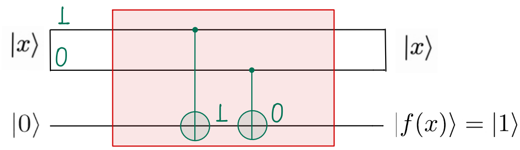


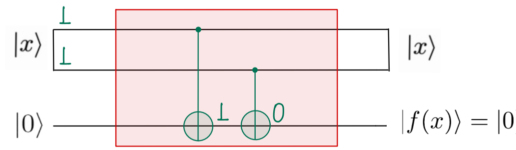
$|x\rangle$
$|x\rangle$
$|f(x)\rangle$
U_f
$|x\rangle$
$|x\rangle$
$|1\rangle$
$|1 \oplus f(x)\rangle = X\,|f(x)\rangle$

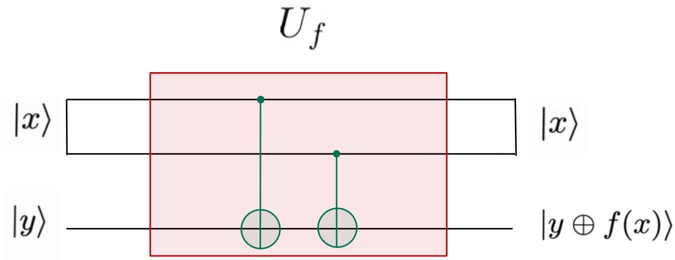

$|x\rangle$
$|0\rangle$
$|x\rangle$
$|f(x)\rangle$


$|x\rangle$
0
0
$|0\rangle$
0
0
$|x\rangle$
$|f(x)\rangle = |0\rangle$

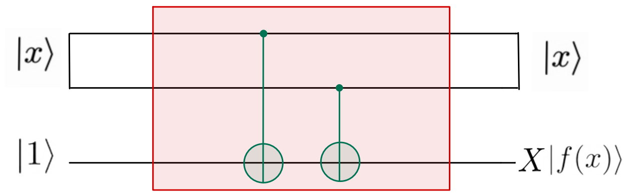

$|x\rangle$
0
1
$|0\rangle$
0
1
$|x\rangle$
$|f(x)\rangle = |1\rangle$

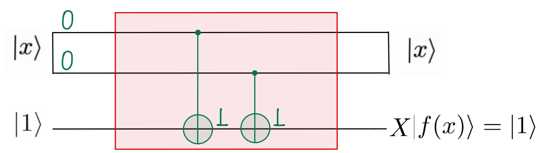

$|x\rangle$
1
0
$|0\rangle$
1
0
$|x\rangle$
$|f(x)\rangle = |1\rangle$

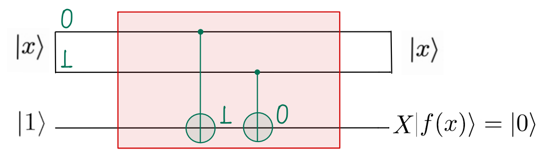

$|x\rangle$
1
1
$|0\rangle$
1
0
$|x\rangle$
$|f(x)\rangle = |0\rangle$

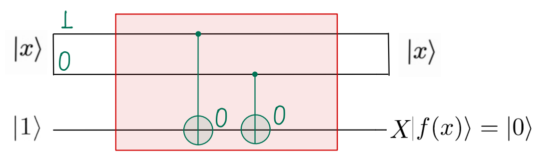


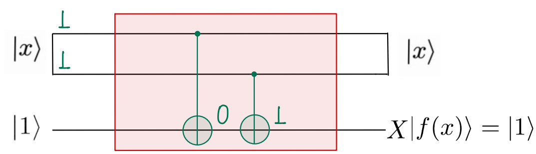
$x \in \{0,1\}^2$
$y \in \{0,1\}$

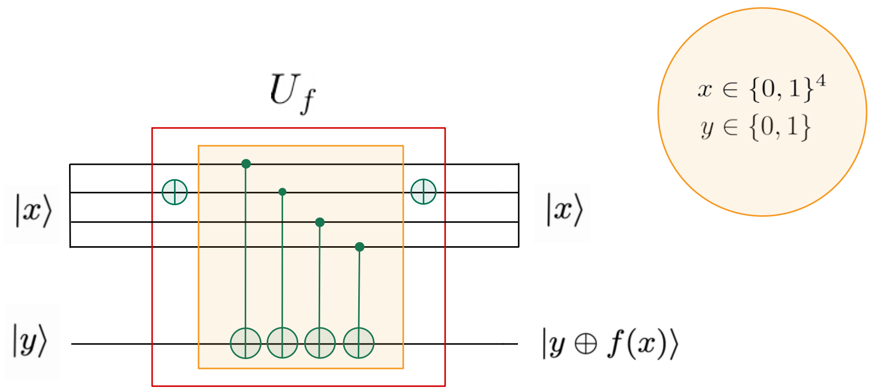




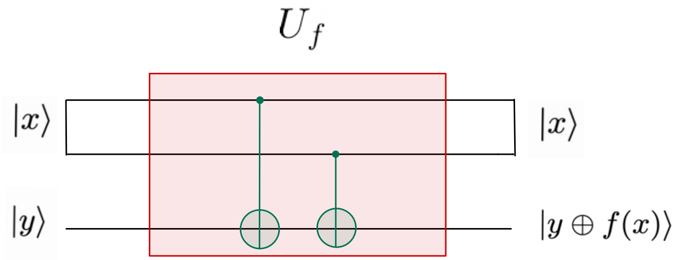








U_f
$|x\rangle$
$|x\rangle$
$|y\rangle$
$|y \oplus f(x)\rangle$
$x \in \{0,1\}^4$
$y \in \{0,1\}$



$$x \in \{0,1\}^2$$
$$y \in \{0,1\}$$

$$U_f |x\rangle|-\rangle = \frac{1}{\sqrt{2}}|x\rangle|\underbrace{0\oplus \underbrace{f(x)}_{0}}_{0}\rangle - \frac{1}{\sqrt{2}}|x\rangle|\underbrace{1\oplus \underbrace{f(x)}_{0}}_{1}\rangle$$

$$= \frac{1}{\sqrt{2}}|x\rangle|0\rangle - \frac{1}{\sqrt{2}}|x\rangle|1\rangle$$

$$= |x\rangle \underbrace{\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)}_{|-\rangle}$$

$$= |x\rangle|-\rangle$$

$$U_f |x\rangle|-\rangle = \frac{1}{\sqrt{2}}|x\rangle|\underbrace{0\oplus \underbrace{f(x)}_{1}}_{1}\rangle - \frac{1}{\sqrt{2}}|x\rangle|\underbrace{1\oplus \underbrace{f(x)}_{1}}_{0}\rangle$$

$$= \frac{1}{\sqrt{2}}|x\rangle|1\rangle - \frac{1}{\sqrt{2}}|x\rangle|0\rangle$$

$$= -|x\rangle \underbrace{\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)}_{|-\rangle}$$

$$= -|x\rangle|-\rangle$$

$$\boxed{U_f |x\rangle|-\rangle = (-1)^{f(x)}|x\rangle|-\rangle}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$|\psi_{4,0}\rangle = \frac{1}{\sqrt{2}}(-1)^{f(0)}\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) + \frac{1}{\sqrt{2}}(-1)^{f(1)}\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$= \frac{1}{2}\left((-1)^{f(0)} + (-1)^{f(1)}\right)|0\rangle + \frac{1}{2}\left((-1)^{f(0)} - (-1)^{f(1)}\right)|1\rangle$$

$$H|x\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^x |1\rangle\right)$$

$(-1)^0 = 1$

$x=0$: $H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$

$= |+\rangle$

$(-1)^1 = -1$

$x=1$: $H|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$

$= |-\rangle$

$|0\rangle \qquad |1\rangle$

$$\boxed{|\chi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle}$$

$$\vec{r} = \alpha\,\hat{x} + \beta\,\hat{y}$$

$n = 2$

$\to |00\rangle$
$\to |01\rangle$
$\to |10\rangle$
$\to |11\rangle$

$2^n = 4$

$\langle 00|01\rangle = 0$

$\langle 00|10\rangle = 0$

$\vdots$

$\langle 00|00\rangle = 1 \implies$

$\langle 00| = \underset{q_0}{\langle 0|} \otimes \underset{q_1}{\langle 0|}$

$|00\rangle = \underset{q_0}{|0\rangle} \otimes \underset{q_1}{|0\rangle}$

$$\langle 00|00\rangle = \langle q_0, 0\,;\, q_1, 0 \,|\, q_0, 0\,;\, q_1, 0\rangle$$

$$= \left(\underset{q_0}{\langle 0|} \otimes \underset{q_1}{\langle 0|}\right)\left(\underset{q_0}{|0\rangle} \otimes \underset{q_1}{|0\rangle}\right)$$

$$= \underbrace{\langle 0|0\rangle}_{1}\,\underbrace{\langle 0|0\rangle}_{1}$$

$$\overset{q_0\, q_1\, q_2\, q_3}{\langle 0100 \,|\, 0010\rangle} = \underbrace{\langle 0|0\rangle}_{1}\,\underbrace{\cancel{\langle 1|0\rangle}}_{0}\,\underbrace{\cancel{\langle 0|1\rangle}}_{0}\,\underbrace{\langle 0|0\rangle}_{1}$$

$$= 0$$

$$|\chi\rangle = |x_1, x_2, x_3, \ldots, x_n\rangle$$

$$|0\rangle = (0, 0, 0, \ldots, 0)$$

$$\langle \hat{0}^n | x \rangle = \langle 0 | x_1 \rangle \langle 0 | x_2 \rangle \cdots \langle 0 | x_n \rangle$$

| Si $x_1 = x_2 = \cdots = x_n = 0$ / De lo contrario

$= 1$ $\qquad\qquad$ $\langle \hat{0} | x \rangle = 0$

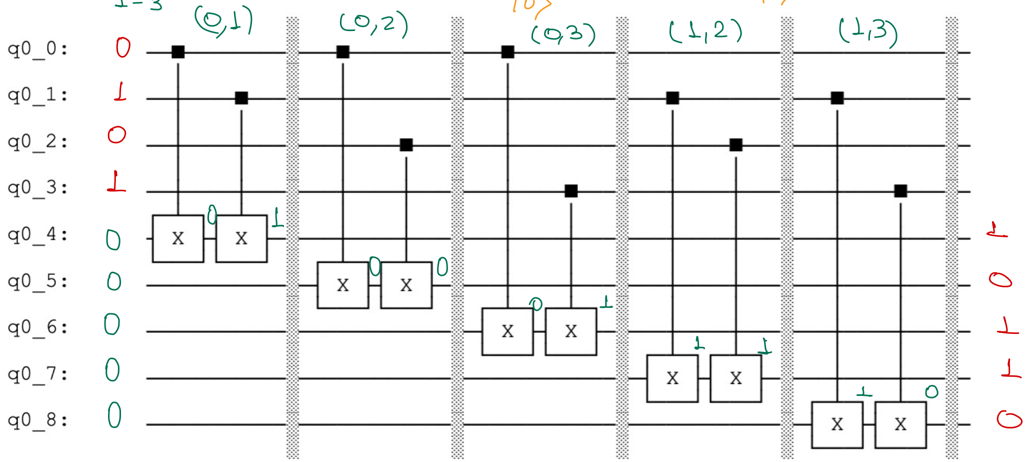
Sesión 5 – Grover

Aristas → 0-1
0-2
0-3
1-2
1-3

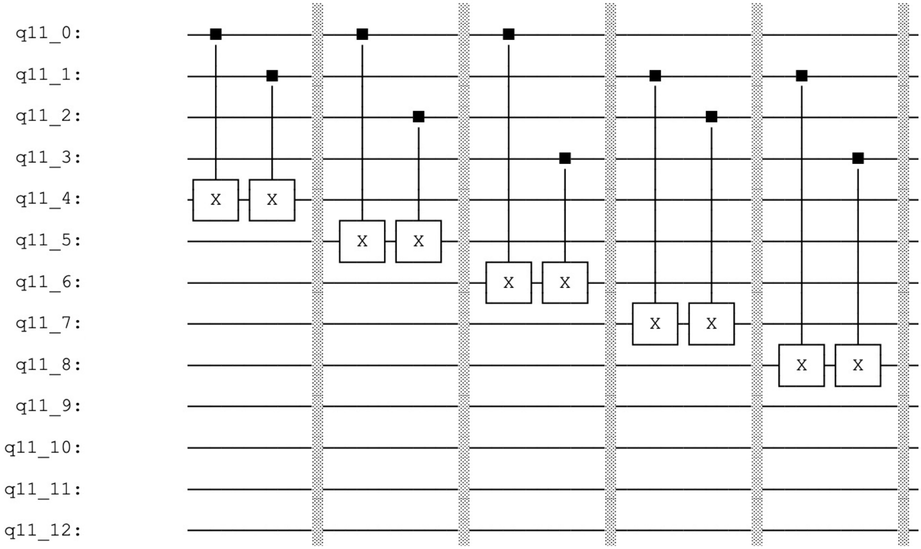
Ejemplo

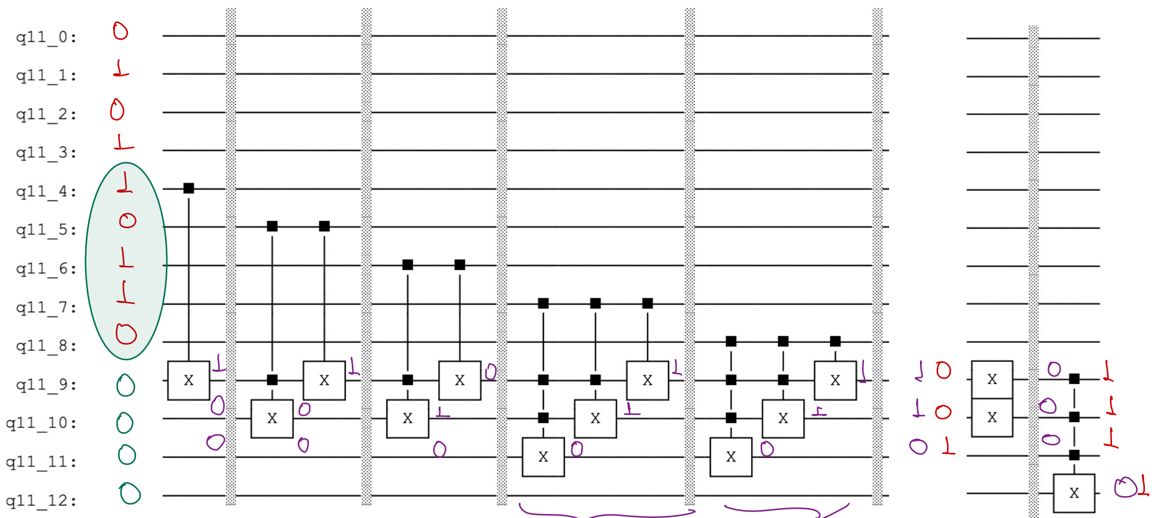
$\{q_0; q_2\}$ Azul → $|0\rangle$

$\{q_1; q_3\}$ Naranjo → $|1\rangle$



Correctas: (0,1) ; (0,3) y (1;2)





$011_{(2)} = 11_{(2)} = 3$

$4 \rightarrow 100_{(2)} \rightarrow$

$q_{11} = 1$

$q_{10} = 0$

$q_9 = 0$