

# Collection of Integration Problems

With Rarest Collections

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28th October, 2023



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# Introduction

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# Chapter 1

## Indefinite Integrals

### 1.1 Simplification and Substitution Based

Here we need to simplify first and then substitute with  $z = f(x)$  and integrate them. There are several types of simplification, such as

1. Partial Fractions
2.  $\frac{1}{\sqrt{f(x)} - \sqrt{g(x)}}$  or  $\frac{1}{f(x)g(x)}$ , where  $f(x) - g(x) = \text{constant}$
3. Trigonometric based simplification
4. Many more...

#### Problems

- |   |   |
|---|---|
| 1. $\int \frac{x-1}{(x+1)(x-2)} dx$                       | 9. $\int \frac{1}{(1+x^2)^{\frac{3}{2}}} dx$    |
| 2. $\int \frac{(x-1)^2}{(x^2+1)^2} dx$                    | 10. $\int \frac{1}{(2ax+x^2)^{\frac{3}{2}}} dx$ |
| 3. $\int \frac{x^2+1}{x^4-1} dx$                          | 11. $\int \frac{1}{(a^2-b^2x^2)^{3/2}} dx$      |
| 4. $\int \frac{x^2-1}{x^4-1} dx$                          | 12. $\int \frac{1}{(x+2)(x^2+1)} dx$            |
| 5. $\int \frac{1}{x\sqrt{x^2-a^2}} dx$ , where $a \geq 0$ | 13. $\int \frac{\sqrt{x-1}}{x\sqrt{x+1}} dx$    |
| 6. $\int \frac{x^2}{(1+x^2)(1+\sqrt{1+x^2})} dx$          | 14. $\int \frac{x^9}{(4x^2+1)^6} dx$            |
| 7. $\int \sqrt{x} (1+x^{1/3})^4 dx$                       | 15. $\int \frac{1}{x\sqrt{1-x^3}} dx$           |
| 8. $\int \frac{1}{x(x^n+1)} dx$                           |   |

$$16. \int \frac{1}{x^2 (x^n + 1)^{(n-1)/n}} dx$$

$$17. \int (x^2 + x)(x^{-8} + 2x^{-9})^{1/10} dx$$

$$18. \int x^5(1 + x^3)^{2/3} dx$$

$$19. \int \sqrt{x^2 + 1} \frac{\ln(x^2 + 1) - 2 \ln x}{x^4} dx$$

$$20. \int \sqrt{\frac{1 - \sqrt{x}}{1 + \sqrt{x}}} dx$$

$$21. \int \frac{1}{x} \sqrt{\frac{1 - \sqrt{x}}{1 + \sqrt{x}}} dx$$

$$22. \int \frac{x^2}{\sqrt{1-x}} dx$$

$$23. \int \frac{\sqrt{a} - \sqrt{x}}{1 - \sqrt{ax}} dx$$

$$24. \int \frac{(\sqrt{x})^5}{\sqrt{x}^7 + x^6} dx$$

$$25. \int \frac{x^4 + 1}{x(x^2 + 1)^2} dx$$

$$26. \int \frac{1}{\sqrt[3]{x} + \sqrt[4]{x}} dx$$

$$27. \int \frac{x^2 - 1}{x^3 \sqrt{2x^4 - 2x^2 + 1}} dx$$

$$28. \int \frac{1}{(7x - 10 - x^2)^{3/2}} dx$$

$$29. \int \frac{1}{(2x - 5 - x^2)^{3/2}} dx$$

$$30. \int (e^x + e^{-x})^2 (e^x - e^{-x}) dx$$

$$31. \int \sqrt{e^x - 1} dx$$

$$32. \int \sqrt{\frac{e^x - 1}{e^x + 1}} dx$$

$$33. \int \frac{1 + x}{x + e^{-x}} dx$$

$$34. \int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx$$

$$35. \int \sqrt{1 - \sin x} dx$$

$$36. \int \sqrt{1 - \cos x} dx$$

$$37. \int \sqrt{1 + \sin x} dx$$

$$38. \int \sqrt{1 + \cos x} dx$$

$$39. \int \sqrt{\sec x - 1} dx$$

$$40. \int \sqrt{\operatorname{cosec} x - 1} dx$$

$$41. \int (\sqrt{\cot x} - \sqrt{\tan x}) dx$$

$$42. \int \sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^3 x}} dx$$

$$43. \int \frac{1}{\sqrt{\sin^3 x \cos^5 x}} dx$$

$$44. \int \frac{1}{\sin x - \sin 2x} dx$$

$$45. \int \frac{1 - \cos x}{\cos x(1 + \cos x)} dx$$

$$46. \int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$$

$$47. \int \frac{\sec^2 x}{(\sec x + \tan x)^n} dx$$

$$48. \int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx$$



## 1.2 U-V Rule or By Parts

There are several types of these integration.

### 1.2.1 Type I: $\int e^{g(x)}(g'(x) f(x) + f'(x)) dx$

When  $g(x) = x$ , it becomes  $\int e^x(f(x) + f'(x)) dx$ , which is most common problem. Here you need to first convert into above forms, and then just identify  $f(x)$  and then balance  $f'(x)$  and continue. If you use this trick, you will solve any problem very quickly.

#### Problems

1.  $\int \frac{\ln x}{(1 + \ln x)^2} dx$
2.  $\int e^{x^2 - z^2} \{2x \cos(2xz) - 2z \sin(2xz)\} dx$
3.  $\int e^{x^x} (\ln x + 1) x^{2x} dx$
4.  $\int e^{(e^x + e^{-x})} (e^{2x} + 2e^x - e^{-x} - 1) dx$
5.  $\int \frac{2+x}{x^3} e^{-x} dx$

### 1.2.2 Type II: $\int f(x)g'(x) + f'(x)g(x) dx$ or Cancellation Based

#### Problems

1.  $\int e^x (\sin x + \cos x) dx$
2.  $\int (1 + \ln x) \ln(\ln x) dx$
3.  $\int \frac{\sec^2(1 + \ln x) - \tan(1 + \ln x)}{x^2} dx$
4.  $\int dx$

### 1.2.3 Type III: More ILATE Problems

#### Problems

- |  |       |  |
|--|-------|--|
| 1. $\int \frac{\ln(1 + \sqrt[6]{x})}{\sqrt[3]{x} + \sqrt{x}} dx$ | [IIT] | 7. $\int \sec^{-1} x dx$                 |
| 2. $\int \sin \sqrt{x} dx$                                       |       | 8. $\int \cot^{-1} x dx$                 |
| 3. $\int \cos \sqrt{x} dx$                                       |       | 9. $\int \operatorname{cosec}^{-1} x dx$ |
| 4. $\int \sin^{-1} x dx$   |       | 10. $\int \sin^{-1} \sqrt{x} dx$         |
| 5. $\int \cos^{-1} x dx$   |       | 11. $\int \cos^{-1} \sqrt{x} dx$         |
| 6. $\int \tan^{-1} x dx$   |       | 12. $\int \tan^{-1} \sqrt{x} dx$         |

13.  $\int \sec^{-1} \sqrt{x} \, dx$

16.  $\int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} \, dx$

14.  $\int \cot^{-1} \sqrt{x} \, dx$

17.  $\int \cot^{-1} (x^2 + x + 1) \, dx$

15.  $\int \operatorname{cosec}^{-1} \sqrt{x} \, dx$

18.  $\int \frac{\tan^{-1} x}{x^4} \, dx$

### 1.3 $\lambda - \mu$ Based Problems

There are several forms.

#### Problems

1.  $\int \frac{a \sin x + b \cos x}{b \sin x + a \cos x} \, dx$

3.  $\int \frac{x}{(7x - 10 - x^2)^{3/2}} \, dx$

2.  $\int \frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} \, dx$

4.  $\int \frac{x}{(2x - 5 - x^2)^{3/2}} \, dx$

### 1.4 Using Differentiation

Usually, form of this type is  $\int \frac{f(x)}{g^2(x)} \, dx$ . We need to extract into below form.

$$\frac{d}{dx} \left( \frac{\psi(x)}{g(x)} \right) = \frac{f(x)}{g^2(x)} + h(x)$$

Just we need to find  $\psi(x)$  such that  $h(x)$  can be integrated easily.

#### Problems

1.  $\int \frac{x^2 + n(n-1)}{(x \sin x + n \cos x)^2} \, dx$

2.  $\int \frac{a + b \sin x}{(b + a \sin x)^2} \, dx$

### 1.5 Bi-quadratic Forms

It is one of beautiful section of interesting problems. What you need is observation and practice!

Table 1.1: Rules of substitution

Forms	Substitution
$\int \frac{1}{(ax+b)\sqrt{px+q}} dx$	$px+q = z^2$
$\int \frac{1}{(ax^2+bx+c)\sqrt{px+q}} dx$	$px+q = z^2$
$\int \frac{1}{(ax+b)\sqrt{px^2+qx+r}} dx$	$ax+b = \frac{1}{z}$
$\int \frac{1}{(ax^2+b)\sqrt{px^2+q}} dx$	$\sqrt{px^2+q} = xz$
$\int \frac{1}{(x-a)^m(x-b)^n} dx$	$x-a = z(x-b)$
$\int R(x, \sqrt{ax^2+bx+c}) dx$	$\sqrt{ax^2+bx+c} = \begin{cases} t \pm x\sqrt{a}, & \text{if } a > 0 \\ tx \pm \sqrt{c}, & \text{if } c > 0 \\ (x-\alpha)t, & \text{if } \alpha \text{ is real root of } ax^2+bx+c=0 \end{cases}$

Last one is called **Euler's Substitution**. There could be many forms like these.

### Problems

1.  $\int \frac{x^2+1}{x^4+1} dx$

2.  $\int \frac{x^2-1}{x^4+1} dx$

3.  $\int \frac{x^2}{x^4+1} dx$

4.  $\int \frac{1}{x^4+1} dx$

5.  $\int \frac{x^2-1}{x^4+x^2+1} dx$

6.  $\int \frac{x^2+1}{x^4-x^2+1} dx$

7.  $\int \frac{1}{x^4+x^2+1} dx$

8.  $\int \frac{1}{x^4-x^2+1} dx$

9.  $\int \frac{1}{x^4+ax^2+1} dx$

10.  $\int \frac{(x-1)^2}{x^4+x^2+1} dx$

11.  $\int \frac{x^2+a^2}{x^4+a^4} dx$

12.  $\int \frac{x^4+1}{x^6+1} dx$

13.  $\int \frac{x^2-1}{(x^2+1)\sqrt{1+x^4}} dx$

14.  $\int \frac{\sqrt{x^2+1}}{x^4} dx$

15.  $\int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}} dx$

16.  $\int \frac{x^4+1}{x(x^2+1)^2} dx$

17.  $\int \frac{1}{(x+1)\sqrt{x^2-1}} dx$

$$18. \int \frac{1}{(x-1)\sqrt{x^2+4}} dx$$

$$19. \int \frac{1}{(x^2-4)\sqrt{x+1}} dx$$

$$20. \int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}} dx$$

$$21. \int \frac{1}{(1-x^2)\sqrt{1+x^2}} dx$$

$$22. \int \frac{1}{x\sqrt{ax-x^2}} dx$$

$$23. \int \frac{x^2}{(x-1)^3(x+1)} dx$$

$$24. \int \frac{x}{x^3-1} dx$$

$$25. \int \frac{x^2-2}{x^5-x} dx$$

$$26. \int \frac{x^2+x+1}{(x+1)^2(x+2)} dx$$

$$27. \int x \sqrt{\frac{1-x}{1+x}} dx$$

$$28. \int \frac{1}{x\sqrt{1+x^3}} dx$$

$$29. \int \frac{1+x^2}{\sqrt{1-x^2}} dx$$

$$30. \int \frac{1}{1+\sqrt{x^2+2x+2}} dx$$

$$31. \int \frac{1}{x+\sqrt{x^2-x+1}} dx$$

$$32. \int \frac{1}{x-\sqrt{x^2+2x+4}} dx$$

$$33. \int \frac{x}{(7x-10-x^2)^{3/2}} dx$$
<sup>1</sup>

### Trigonometric

$$34. \int \frac{\sin x + \cos x}{\sin^4 x + \cos^4 x} dx$$

$$35. \int \frac{1}{\sin^4 x + \cos^4 x} dx$$

$$36. \int \frac{1}{\sin^6 x + \cos^6 x} dx$$

$$37. \int \frac{\cot x + \cot^3 x}{1 + \cot^3 x} dx$$

$$38. \int \sqrt{\tan x} + \sqrt{\cot x} dx$$

$$39. \int \sqrt{\tan x} dx$$

$$40. \int \sqrt{\cot x} dx$$

---

<sup>1</sup>It can be solved other ways too!

## 1.6 Miscellaneous Problems

1.  $\int \frac{x^6 + x^3 + 1}{(2x^6 + 3x^3 + 6)^{2/3}} dx = ?$

3.  $\int \frac{\sec x}{\sqrt{1 + 2 \sec x}} \sqrt{\frac{\operatorname{cosec} x - \cot x}{\operatorname{cosec} x + \cot x}} dx$

2.  $\int \frac{x^2 + x}{(e^x + x + 1)^2} dx = ?$

4.  $\int \frac{f'(x)x - f(x)}{(f(x) + 1)\sqrt{f(x)x - x^2}} dx = ?$

5. Let  $4x^4 - 24x^3 + 31x^2 + 6x - 8 = 0$  has roots  $\alpha, \beta, \gamma$  and  $\delta$ , where  $\alpha < \beta < \gamma < \delta$ , then

$$\int \left( \frac{x - \delta}{x - \gamma} \right)^{\alpha + \beta + \delta} dx = ?$$

6.  $\int \frac{\cos 5x + \cos 4x}{1 - 2 \sin 3x} dx = -g(x) \sin x + C$

(a)  $\int g(x) dx = ?$

(b)  $\int \frac{(1 - g(x))^2}{1 + \tan x} dx = ?$

7.  $\int \frac{1}{\sqrt{1 + \cos x} \sqrt{\sin x + \cos x}} dx$

8.  $\int (\cos 3x \cos 5x \cos 6x \cos 7x - \cos x \cos 2x \cos x \cos 8x) dx$

9.  $\int \frac{\ln(1 + x)}{x^2} dx$

10. Evaluate

$$\int \frac{\tan\left(\frac{\pi}{4} - x\right)}{\cos^2 x \sqrt{\tan^3 x + \tan^2 x + \tan x}} dx$$

11.  $\int \frac{(x - 1)\sqrt{x^4 + 2x^3 - x^2 + 2x + 1}}{x^2(x + 1)} dx$

12.  $\int \frac{1}{2 \sin x + \sec x} dx$

**Answers** 

---

1.

2.

3.

4.

5.

6.

7. Use  $t = \tan \frac{x}{2}$ .

8.  $\frac{1}{8} \left( \frac{\sin 21x}{21} - \frac{\sin 13x}{13} \right) + C$

9.

10.  $-2 \tan^{-1} \sqrt{\tan x + \cot x + 1} + C$

11.  $\sqrt{t^2 + 2t - 3} - \ln(t + 1 + \sqrt{t^2 + 2t - 3}) - \sqrt{3} \sin^{-1} \left( \frac{t + 5}{t + 2} \right) + C$ , where  $t = x + \frac{1}{x}$

12.

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# Chapter 2

## Definite Integrals

### 2.1 Basic Rules

There are very common basic rules and common mistakes. The basic rules are

1.  $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$
2.  $\int_a^b f(x) dx = \int_0^{\frac{b-a}{2}} [f(\frac{a+b}{2}-x) + f(\frac{a+b}{2}+x)] dx$ 
  - (a)  $\int_0^{2a} f(x) dx = \int_0^a [f(a-x) + f(a+x)] dx$
  - (b)  $\int_{-a}^a f(x) dx = \int_0^a [f(-x) + f(x)] dx$
3.  $\int_{a+mT}^{a+nT} f(x) dx = (n-m) \int_0^T f(x) dx$ , where  $f(x)$  is periodic with periodicity  $T$ .

The common mistakes are

1. **Limit based problem:** You need to check that *there should not be any asymptotic between upper limit and lower limit*. In that case you need to split up your limit.
2. **Substitution based problem:** While substituting  $z = f(x)$ , we need to check  $f(x)$  is asymptotic in domain  $\mathbb{D} \in (\text{lower limit}, \text{upper limit})$ . If there is, just need to split up!

### Problems

1.  $\int_{-1}^3 \tan^{-1} \frac{x}{x^2+1} + \tan^{-1} \frac{x^2+1}{x} dx$
2.  $\int_0^{\pi/2} \ln \sin x dx$
3.  $\int_0^\theta \ln(1 + \tan \theta \tan x) dx$
4.  $\int_{-\infty}^\infty \frac{2023^{-|x|}}{1 + 5^{\sin^{-1}(\sin^5 x)}} dx$
5.  $\int_0^{\pi/2} \sqrt{1 + \sin x} dx$
6.  $\int_0^\pi \sqrt{1 + \sin x} dx$

$$7. \int_0^{\infty} \frac{1}{(x + \frac{1}{x})^2} dx$$

$$9. \int_0^{\pi} \cos(x + \cos x) dx$$

$$8. \int_1^{\infty} \frac{\ln x}{x^2} dx$$

$$10. \int_{-\infty}^0 \frac{1}{x^3 - 1} dx$$

## 2.2 Feynman's Trick

We need to consider some  $I(a) = \int_a^b f(x, a) dx$  such that  $I'(a)$  can be integrated easily. After that we need to integrate that and the constant is evaluated by putting  $I(0)$  or some constant for which  $\int_a^b f(x, 0) dx$  is integrated easily.

### Problems

$$1. \int_0^1 \frac{x^2 - 1}{\ln x} dx$$

$$3. \int \frac{\tan^{-1}(2 \sin x)}{\sin x} dx$$

$$2. \int_0^1 \frac{\ln(1+x)}{x^2 + 1} dx$$

$$4. \int dx$$

## 2.3 Frullani's Integral

$$\int_0^{\infty} \frac{f(ax) - f(bx)}{x} dx = (f(\infty) - f(0)) \ln \frac{a}{b} \quad [\text{Proof}]$$

### Problems

1.

## 2.4 Gamma and Beta Integral

## 2.5 Laplace Transformation

## 2.6 Miscellaneous problems

### Problems

$$1. \int_0^{2\pi} \frac{\min(\sin x, \cos x)}{\max(e^{\sin x}, e^{\cos x})} dx$$

$$2. \text{Min value of } f(x) = \int_0^2 e^{|t-x|} dt$$

$$3. \int_0^{2024} x^2 - \lfloor x \rfloor \lceil x \rceil dx$$



4.  $\int_0^{\pi/2} \frac{x}{\tan x} dx$

5. For all real numbers  $x$ , let  $f(x) = |x^2 + x|$ ,  $I_1 = \int_{-2020}^0 f(x) dx$ ,  $I_2 = \int_0^{2019} f(x) dx$ , calculate  $|I_1 - I_2|$ .

6.  $\int_{1/e}^e \frac{\tan^{-1} x}{x} dx$

## 2.7 Advanced Problems

### Problems

1.  $\lim_{\epsilon \rightarrow 0^+} \epsilon^4 \int_0^{\pi} 2^{-\epsilon} \tan^5 x dx = ?$

2.  $\int_0^1 \left\lfloor \sqrt{1 + \frac{1}{x}} \right\rfloor dx$

3.  $\int_0^\infty \sin ax dx$

4.  $\int_0^\infty \frac{\cos ax - \cos bx}{x} dx$

5.  $\int_0^\infty \frac{b \sin ax - a \sin bx}{x^2} dx$

6.  $\int_0^{\pi/2} \frac{\sqrt[3]{\tan x}}{(\sin x + \cos x)^2} dx$

7.  $\int_0^\pi \left( \frac{\sin 2x \sin 3x \sin 5x \sin 30x}{\sin x \sin 6x \sin 10x \sin 15x} \right)^2 dx$

8.  $\int_{-1/2}^{1/2} \sqrt{x^2 + 1 + \sqrt{x^4 + x^2 + 1}} dx$

9.  $\left\lfloor 10^{20} \int_2^\infty \frac{x^9}{x^{20} - 48x^{10} + 575} dx \right\rfloor$

10.  $\int_0^1 \left( \sum_{n=1}^\infty \frac{\lfloor 2^n x \rfloor}{3^n} \right)^2 dx$

11.  $\int \sqrt{(\sin 20x + 3 \sin 21x + \sin 22x)^2 + (\cos 20x + 3 \cos 21x + \cos 22x)^2} dx$

12.  $\int_0^\infty \frac{e^{-2x} \sin 3x}{x} dx$

13.  $\int_0^{2\pi} \cos 2022x \frac{\sin 10050x}{\sin 50x} \frac{\sin 10251x}{\sin 51x} dx$

14.  $\int_0^1 x^{\frac{1}{3}}(1-x)^{\frac{2}{3}} dx$

15.  $\left\lfloor \log_{10} \int_{2022}^{\infty} 10^{-x^3} dx \right\rfloor$

16.  $\int_{-2}^2 |(x-2)(x-1)x(x+1)(x+2)| dx$

17.  $\int_1^2 \left( e^{\frac{1}{1-(x-1)^2}} + 1 \right) + \left( 1 + \frac{1}{\sqrt{1-\ln(x-1)}} \right) dx$

18.  $\int_0^1 \ln x \sin(\ln x) dx$

19.  $\int_0^1 \ln^{2020} x dx$

20.  $\int_{-\infty}^{\infty} \frac{\sin\left(2x - \frac{1}{x}\right)}{2x^3 - x} dx$

21.  $\int_p^q \left( e^{\frac{1}{1-(x-1)^2}} + 1 \right) + \left( 1 + \frac{1}{\sqrt{1-\ln(x-1)}} \right) dx$ , where  $1 \leq p < q$

22.  $\int_0^{100} \lceil \sqrt{x} \rceil dx$

23.  $\int_0^1 \sqrt{(x-1)^3 + 1} + x^{2/3} + (1-x)^{3/2} - \sqrt[3]{1-x^2} dx$

24. Find  $\alpha$  such that

$$\lim_{x \rightarrow 0^+} x^\alpha I(x) = a \text{ given } I(x) = \int_0^\infty \sqrt{t+1} e^{-xt} dt$$

where  $a$  is a non-zero real number.

25. Define  $H_n = \sum_{k=1}^n \frac{1}{k}$ . Evaluate

$$\sum_{n=1}^{2017} \binom{2017}{n} H_n (-1)^n$$

26. The numerical value of the following integral

$$\int_0^1 (-x^2 + x)^{2017} \lfloor 2017x \rfloor dx$$

can be expressed as  $a \frac{(m!)^2}{n!}$ , where  $a$  is minimized. Find  $a + m + n$ .

27. Let  $T$  be defined by the recurrence relation  $T_{n+1} = 2xT_n - T_{n-1}$  with  $T_0 = 1$  and  $T_1 = x$ . What is the value of

$$\sum_{n=2}^{\infty} \int_0^1 T_n dx$$

28. Compute the following limit

$$\lim_{n \rightarrow \infty} \int_0^1 \frac{nx^n}{\sqrt{4x^3 - x^2 + 1}} dx$$

29.  $\lim_{n \rightarrow \infty} n^2 \int_0^{1/n} x^{x+1} dx = ?$

30.  $\sum_{a=1}^{\infty} \sum_{b=1}^{\infty} \frac{1}{a^2b + 2ab + ab^2} = ?$

31. Let  $f : \mathbb{R}_{>0} \rightarrow \mathbb{R}$  (where  $\mathbb{R}_{>0}$  is the set of all positive real numbers) be differentiable and satisfy the equation

$$f(y) - f(x) = \frac{x^x f(y^y)}{y^y f(x^x)}$$

for all real  $x, y > 0$ . Furthermore,  $f'(1) = 1$ . Find  $f(x)$ .

32. Suppose the following equality holds, where  $a, b, c$  are integers and  $K$  is the constant of integration:

$$\int \frac{\sin^a x - \cos^a x}{\sin^b x \cos^b x} dx = \frac{\operatorname{cosec}^c x}{c} + \frac{\sec^c x}{c} + K$$

If  $a = 2021$ , compute  $a + b + c$ .

33. Let  $c(x) = \frac{e^x + e^{-2x}}{2}$ , defined in interval  $1 \leq x \leq 2$ , Let  $c^{-1}(x)$  be the inverse of  $c(x)$ . Compute

$$\int_{c(1)}^{c(2)} c^{-1}(x) dx$$

34. Compute the infinite sum

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\binom{n+1}{2}}$$

35.  $\int_0^{\pi/3} \sec x \sqrt{\tan x \sqrt{\tan x \sqrt{\tan x \sin x}}} dx$

36. Compute:  $\lim_{x \rightarrow 0} \left( 1 + \int_0^x \frac{\cos t - 1}{t^2} dt \right)^{\frac{1}{x}}$

37.  $\int_0^{\pi/2} \cot x \ln(\cos x) dx$

38.  $\sum_{n=2}^{\infty} \ln \left( \frac{n^3 + 1}{n^3 - 1} \right)$

39.  $\sum_{n=2}^{\infty} \sqrt{n^2 + 3n + 2} - \sqrt{n^2 + n} - 1$

40. A unit cube is rotated around an axis containing its longest diagonal. Compute the volume swept out by the rotating.

**Answers** \_\_\_\_\_

- |   |                              |                                      |
|---|------------------------------|--------------------------------------|
| 1. $\frac{1}{4}$  | 13. $6\pi$                   | 28. $\frac{1}{2}$                    |
| 2. $\frac{7}{4}$  | 14. $\frac{2\pi}{9\sqrt{3}}$ | 29. $\frac{1}{2}$                    |
| 3. $\frac{1}{a}$  | 15. $-2022^3 - 8$            | 30. $\frac{7}{4}$                    |
| 4. $\ln \frac{b}{a}$  | 16. $\frac{19}{3}$           | 31. $x \ln x$                        |
| 5. $ab \ln \frac{b}{a}$   | 17. $3$                      | 32. $6060$                           |
| 6. $\frac{2\sqrt{3}\pi}{9}$   | 18. $\frac{1}{2}$            | 33. $.5e^2 + 1.25e^{-4} - .75e^{-2}$ |
| 7. $7\pi$   | 19. $2020!$                  | 34. $4 \ln 2 - 2$                    |
| 8. $\frac{\sqrt{7}}{2\sqrt{2}} + \frac{3}{4\sqrt{2}} \ln \frac{\sqrt{7} + 2}{\sqrt{3}}$ | 20. $\pi$                    | 35. $\frac{8}{7}(2^{7/8} - 1)$       |
| 9. $10^{16} + \frac{10^{10} - 1}{3} + \frac{10^4}{5}$                                   | 21. $q^2 - p^2$              | 36. $\frac{1}{\sqrt{e}}$             |
| 10. $\frac{27}{32}$   | 22. $715$                    | 37. $-\frac{\pi^2}{24}$              |
| 11. $3x + 2 \sin x + C$   | 23. $1$                      | 38. $\ln 3 - \ln 2$                  |
| 12. $\tan^{-1} \frac{3}{2}$   | 24. $\frac{3}{2}$            | 39. $\frac{1}{2}$                    |
|   | 25. $-\frac{1}{2017}$        | 40. $\frac{\pi}{\sqrt{3}}$           |
|   | 26. $7060$                   |                                      |
|   | 27. $-1$                     |                                      |
-

## Problems

1.

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### My Box



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