GUIDELY MATHEMATICS CHALLENGE

Class VIII

Topics: Rational Numbers, Square Numbers, Cube Numbers

Time: 1 Hour 30 Minutes **Maximum Marks: 50**

Section A - Conceptual Understanding (1 mark each)

1. If $x = \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$ and $y = \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$, then $x + y$ is:
   A. 6
   B. 12
   C. 24
   D. $2\sqrt{35}$

2. The cube of $\left(\frac{2}{3}\right)^{-2}$ is:
   A. $\frac{8}{27}$
   B. $\frac{27}{8}$
   C. $\frac{729}{64}$
   D. $\frac{64}{729}$

3. Which of these is always a perfect square for any integer n?
   A. $n^3 + n^2$
   B. $(n + 1)^3 - n^3$
   C. $n^4 + 2n^2 + 1$
   D. $n^5 - n$

4. The rational number between $\frac{1}{3}$ and $\frac{1}{2}$ is:
   A. $\frac{7}{12}$
   B. $\frac{5}{12}$
   C. $\frac{9}{16}$
   D. $\frac{11}{24}$

5. If n^3 ends with 8, then n^2 must end with:
   A. 2
   B. 4
   C. 6
   D. 8

6. The smallest number by which 2592 must be multiplied to make it a perfect cube is:
   A. 2
   B. 3
   C. 6
   D. 12

Section B - Problem Solving (2 marks each)

1. Find three rational numbers between $\frac{2}{7}$ and $\frac{3}{8}$ without taking average
2. Prove that the cube of any odd number is always odd
3. Simplify: $\left(\frac{125}{64}\right)^{-2/3} + \left(\frac{256}{625}\right)^{-1/4} + \left(\frac{\sqrt[3]{216}}{49}\right)^{-1/2}$
4. If $\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} = 3$, find $x + \frac{1}{x}$

Section C - Advanced Applications (3 marks each)

1. Show that $0.3\overline{7}$ can be expressed as $\frac{17}{45}$, then generalize the method for any decimal of form $0.a\overline{b}$
2. Find all perfect cubes between 1000 and 2000 that are also perfect squares
3. Prove that $\sqrt[3]{2} + \sqrt[3]{4}$ is irrational using the fact that $\sqrt[3]{2}$ is irrational
4. If a and b are positive rational numbers with $a \neq b$, prove that $\frac{a+b}{2} > \sqrt{ab}$ (AM-GM inequality for two numbers)
5. Find all integer solutions to $x^3 - y^3 = 19$

Section D - Proofs and Extended Problems (5 marks each)

1. **(a)** Prove that the sum of a rational and irrational number is irrational
   (b) Hence prove that $\sqrt[3]{9} + \sqrt{3}$ is irrational
2. **(a)** Find the smallest perfect square divisible by 12, 18, and 27
   (b) Generalize the method for finding the smallest perfect square divisible by given numbers
3. **(a)** If $x + \frac{1}{x} = 5$, find $x^3 + \frac{1}{x^3}$
   (b) Develop a general formula for $x^n + \frac{1}{x^n}$ in terms of $x + \frac{1}{x}$

— End of Advanced Challenge Paper —