Collection of Integration Problems

With Rarest Collections

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Introduction

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Chapter 1

Indefinite Integrals

1.1 Simplification and Substitution Based

Here we need to simplify first and then substitute with $z = f(x)$ and integrate them. There are several types of simplification, such as

1. Partial Fractions
2. $\frac{1}{\sqrt{f(x)} - \sqrt{g(x)}}$ or $\frac{1}{f(x)\ g(x)}$, where $f(x) - g(x) = constant$
3. Trigonometric based simplification
4. Many more...

Problems

1. $\int \frac{x-1}{(x+1)(x-2)}\ dx$
2. $\int \frac{(x-1)^2}{(x^2+1)^2}\ dx$
3. $\int \frac{x^2+1}{x^4-1}\ dx$
4. $\int \frac{x^2-1}{x^4-1}\ dx$
5. $\int \frac{1}{x\sqrt{x^2-a^2}}\ dx$, where $a \geq 0$
6. $\int \frac{x^2}{(1+x^2)(1+\sqrt{1+x^2})}\ dx$
7. $\int \sqrt{x}\ (1+x^{1/3})^4\ dx$
8. $\int \frac{1}{x(x^n+1)}\ dx$
9. $\int \frac{1}{(1+x^2)^{\frac{3}{2}}}\ dx$
10. $\int \frac{1}{(2ax+x^2)^{\frac{3}{2}}}\ dx$
11. $\int \frac{1}{(a^2-b^2x^2)^{3/2}}\ dx$
12. $\int \frac{1}{(x+2)(x^2+1)}\ dx$
13. $\int \frac{\sqrt{x-1}}{x\ \sqrt{x+1}}\ dx$
14. $\int \frac{x^9}{(4x^2+1)^6}\ dx$
15. $\int \frac{1}{x\sqrt{1-x^3}}\ dx$

16. $\int \frac{1}{x^2\ (x^n+1)^{(n-1)/n}}\ dx$

17. $\int (x^2+x)(x^{-8}+2x^{-9})^{1/10}\ dx$

18. $\int x^5(1+x^3)^{2/3}\ dx$

19. $\int \sqrt{x^2+1}\ \frac{\ln(x^2+1)-2\ln x}{x^4}\ dx$

20. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\ dx$

21. $\int \frac{1}{x}\ \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\ dx$

22. $\int \frac{x^2}{\sqrt{1-x}}\ dx$

23. $\int \frac{\sqrt{a}-\sqrt{x}}{1-\sqrt{ax}}\ dx$

24. $\int \frac{(\sqrt{x})^5}{\sqrt{x})^7+x^6}\ dx$

25. $\int \frac{x^4+1}{x(x^2+1)^2}\ dx$

26. $\int \frac{1}{\sqrt[3]{x}+\sqrt[4]{x}}\ dx$

27. $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}\ dx$

28. $\int \frac{1}{(7x-10-x^2)^{3/2}}\ dx$

29. $\int \frac{1}{(2x-5-x^2)^{3/2}}\ dx$

30. $\int \left(e^x+e^{-x}\right)^2\ \left(e^x-e^{-x}\right)\ dx$

31. $\int \sqrt{e^x-1}\ dx$

32. $\int \sqrt{\frac{e^x-1}{e^x+1}}\ dx$

33. $\int \frac{1+x}{x+e^{-x}}\ dx$

34. $\int \frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}\ dx$

35. $\int \sqrt{1-\sin x}\ dx$

36. $\int \sqrt{1-\cos x}\ dx$

37. $\int \sqrt{1+\sin x}\ dx$

38. $\int \sqrt{1+\cos x}\ dx$

39. $\int \sqrt{\sec x-1}\ dx$

40. $\int \sqrt{\text{cosec } x-1}\ dx$

41. $\int (\sqrt{\cot x}-\sqrt{\tan x})\ dx$

42. $\int \sqrt{\frac{\cos x-\cos^3 x}{1-\cos^3 x}}\ dx$

43. $\int \frac{1}{\sqrt{\sin^3 x\ \cos^5 x}}\ dx$

44. $\int \frac{1}{\sin x-\sin 2x}\ dx$

45. $\int \frac{1-\cos x}{\cos x(1+\cos x)}\ dx$

46. $\int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}\ dx$

47. $\int \frac{\sec^2 x}{(\sec x+\tan x)^n}\ dx$

48. $\int \frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}\ dx$

1.2 U-V Rule or By Parts

There are several types of these integration.

1.2.1 Type I: $\int e^{g(x)}(g'(x)\ f(x)+f'(x))\ dx$

When $g(x)=x$, it becomes $\int e^x(f(x)+f'(x))\ dx$, which is most common problem. Here you need to first convert into above forms, and then just identify $f(x)$ and then balance $f'(x)$ and continue. If you use this trick, you will solve any problem very quickly.

Problems

1. $\int \dfrac{\ln x}{(1+\ln x)^2}\ dx$
2. $\int e^{x^2-z^2}\left\{2x\cos(2xz)-2z\sin(2xz)\right\}\ dx$
3. $\int e^{x^x}(\ln x+1)x^{2x}\ dx$
4. $\int e^{(e^x+e^{-x})}(e^{2x}+2e^x-e^{-x}-1)\ dx$
5. $\int \dfrac{2+x}{x^3}e^{-x}\ dx$

1.2.2 Type II: $\int f(x)g'(x)+f'(x)g(x)\ dx$ or Cancellation Based

Problems

1. $\int e^x(\sin x+\cos x)\ dx$
2. $\int (1+\ln x)\ln(\ln x)\ dx$
3. $\int \dfrac{\sec^2(1+\ln x)-\tan(1+\ln x)}{x^2}\ dx$
4. $\int\ dx$

1.2.3 Type III: More ILATE Problems

Problems

1. $\int \dfrac{\ln(1+\sqrt[6]{x})}{\sqrt[3]{x}+\sqrt{x}}\ dx$ **[IIT]**
2. $\int \sin\sqrt{x}\ dx$
3. $\int \cos\sqrt{x}\ dx$
4. $\int \sin^{-1}x\ dx$
5. $\int \cos^{-1}x\ dx$
6. $\int \tan^{-1}x\ dx$
7. $\int \sec^{-1}x\ dx$
8. $\int \cot^{-1}x\ dx$
9. $\int \operatorname{cosec}^{-1}x\ dx$
10. $\int \sin^{-1}\sqrt{x}\ dx$
11. $\int \cos^{-1}\sqrt{x}\ dx$
12. $\int \tan^{-1}\sqrt{x}\ dx$

13. $\int \sec^{-1}\sqrt{x}\ dx$
14. $\int \cot^{-1}\sqrt{x}\ dx$
15. $\int \operatorname{cosec}^{-1}\sqrt{x}\ dx$
16. $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}\ dx$
17. $\int \cot^{-1}(x^2+x+1)\ dx$
18. $\int \frac{\tan^{-1}x}{x^4}\ dx$

1.3 $\lambda-\mu$ Based Problems

There are several forms.

Problems

1. $\int \frac{a\sin x+b\cos x}{b\sin x+a\cos x}\ dx$
2. $\int \frac{3e^x-5e^{-x}}{4e^x+5e^{-x}}\ dx$
3. $\int \frac{x}{(7x-10-x^2)^{3/2}}\ dx$
4. $\int \frac{x}{(2x-5-x^2)^{3/2}}\ dx$

1.4 Using Differentiation

Usually, form of this type is $\int \frac{f(x)}{g^2(x)}\ dx$. We need to extract into below form.

$$\frac{d}{dx}\left(\frac{\psi(x)}{g(x)}\right)=\frac{f(x)}{g^2(x)}+h(x)$$

Just we need to find $\psi(x)$ such that $h(x)$ can be integrated easily.

Problems

1. $\int \frac{x^2+n(n-1)}{(x\sin x+n\cos x)^2}\ dx$
2. $\int \frac{a+b\sin x}{(b+a\sin x)^2}\ dx$

1.5 Bi-quadratic Forms

It is one of beautiful section of interesting problems. What you need is observation and practice!

Table 1.1: Rules of substitution

Forms	Substitution
$\int \frac{1}{(ax+b)\sqrt{px+q}}\,dx$	$px+q=z^2$
$\int \frac{1}{(ax^2+bx+c)\sqrt{px+q}}\,dx$	$px+q=z^2$
$\int \frac{1}{(ax+b)\sqrt{px^2+qx+r}}\,dx$	$ax+b=\frac{1}{z}$
$\int \frac{1}{(ax^2+b)\sqrt{px^2+q}}\,dx$	$\sqrt{px^2+q}=xz$
$\int \frac{1}{(x-a)^m(x-b)^n}\,dx$	$x-a=z(x-b)$
$\int R(x,\sqrt{ax^2+bx+c})\,dx$	$\sqrt{ax^2+bx+c}=\begin{cases} t\pm x\sqrt{a}, & \text{if } a>0 \\ tx\pm\sqrt{c}, & \text{if } c>0 \\ (x-\alpha)\,t, & \text{if } \alpha \text{ is real root of } ax^2+bx+c=0 \end{cases}$

Last one is called **Euler's Substitution**.There could be many forms like these.

Problems

1. $\int \frac{x^2+1}{x^4+1}\,dx$
2. $\int \frac{x^2-1}{x^4+1}\,dx$
3. $\int \frac{x^2}{x^4+1}\,dx$
4. $\int \frac{1}{x^4+1}\,dx$
5. $\int \frac{x^2-1}{x^4+x^2+1}\,dx$
6. $\int \frac{x^2+1}{x^4-x^2+1}\,dx$
7. $\int \frac{1}{x^4+x^2+1}\,dx$
8. $\int \frac{1}{x^4-x^2+1}\,dx$
9. $\int \frac{1}{x^4+ax^2+1}\,dx$
10. $\int \frac{(x-1)^2}{x^4+x^2+1}\,dx$
11. $\int \frac{x^2+a^2}{x^4+a^4}\,dx$
12. $\int \frac{x^4+1}{x^6+1}\,dx$
13. $\int \frac{x^2-1}{(x^2+1)\sqrt{1+x^4}}\,dx$
14. $\int \frac{\sqrt{x^2+1}}{x^4}\,dx$
15. $\int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}\,dx$
16. $\int \frac{x^4+1}{x(x^2+1)^2}\,dx$
17. $\int \frac{1}{(x+1)\sqrt{x^2-1}}\,dx$

18. $\int \frac{1}{(x-1)\sqrt{x^2+4}}\,dx$

19. $\int \frac{1}{(x^2-4)\sqrt{x+1}}\,dx$

20. $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}}\,dx$

21. $\int \frac{1}{(1-x^2)\sqrt{1+x^2}}\,dx$

22. $\int \frac{1}{x\sqrt{ax-x^2}}\,dx$

23. $\int \frac{x^2}{(x-1)^3(x+1)}\,dx$

24. $\int \frac{x}{x^3-1}\,dx$

25. $\int \frac{x^2-2}{x^5-x}\,dx$

26. $\int \frac{x^2+x+1}{(x+1)^2(x+2)}\,dx$

27. $\int x\sqrt{\frac{1-x}{1+x}}\,dx$

28. $\int \frac{1}{x\sqrt{1+x^3}}\,dx$

29. $\int \frac{1+x^2}{\sqrt{1-x^2}}\,dx$

30. $\int \frac{1}{1+\sqrt{x^2+2x+2}}\,dx$

31. $\int \frac{1}{x+\sqrt{x^2-x+1}}\,dx$

32. $\int \frac{1}{x-\sqrt{x^2+2x+4}}\,dx$

33. $\int \frac{x}{(7x-10-x^2)^{3/2}}\,dx$ [1]

Trigonometric

34. $\int \frac{\sin x+\cos x}{\sin^4 x+\cos^4 x}\,dx$

35. $\int \frac{1}{\sin^4 x+\cos^4 x}\,dx$

36. $\int \frac{1}{\sin^6 x+\cos^6 x}\,dx$

37. $\int \frac{\cot x+\cot^3 x}{1+\cot^3 x}\,dx$

38. $\int \sqrt{\tan x}+\sqrt{\cot x}\,dx$

39. $\int \sqrt{\tan x}\,dx$

40. $\int \sqrt{\cot x}\,dx$

[1] It can be solved other ways too!

1.6 Miscellaneous Problems

1. $\displaystyle\int \frac{x^6+x^3+1}{(2x^6+3x^3+6)^{2/3}}\,dx = ?$

2. $\displaystyle\int \frac{x^2+x}{(e^x+x+1)^2}\,dx = ?$

3. $\displaystyle\int \frac{\sec x}{\sqrt{1+2\sec x}}\sqrt{\frac{\operatorname{cosec}\,x-\cot x}{\operatorname{cosec}\,x+\cot x}}\,dx$

4. $\displaystyle\int \frac{f'(x)x-f(x)}{(f(x)+1)\sqrt{f(x)x-x^2}}\,dx = ?$

5. Let $4x^4-24x^3+31x^2+6x-8=0$ has roots α, β, γ and δ, where $\alpha<\beta<\gamma<\delta$, then $\displaystyle\int\left(\frac{x-\delta}{x-\gamma}\right)^{\alpha+\beta+\delta} dx = ?$

6. $\displaystyle\int \frac{\cos 5x+\cos 4x}{1-2\sin 3x}\,dx = -g(x)\sin x + C$

   (a) $\displaystyle\int g(x)\,dx = ?$

   (b) $\displaystyle\int \frac{(1-g(x))^2}{1+\tan x}\,dx = ?$

7. $\displaystyle\int \frac{1}{\sqrt{1+\cos x}\,\sqrt{\sin x+\cos x}}\,dx$

8. $\displaystyle\int(\cos 3x\cos 5x\cos 6x\cos 7x-\cos x\cos 2x\cos x\cos 8x)\,dx$

9. $\displaystyle\int \frac{\ln(1+x)}{x^2}\,dx$

10. Evaluate
$$\int \frac{\tan\left(\frac{\pi}{4}-x\right)}{\cos^2 x\sqrt{\tan^3 x+\tan^2 x+\tan x}}\,dx$$

11. $\displaystyle\int \frac{(x-1)\sqrt{x^4+2x^3-x^2+2x+1}}{x^2(x+1)}\,dx$

12. $\displaystyle\int \frac{1}{2\sin x+\sec x}\,dx$

Answers

1.

2.

3.

4.

5.

6.

7. Use $t = \tan\dfrac{x}{2}$.

8. $\dfrac{1}{8}\left(\dfrac{\sin 21x}{21} - \dfrac{\sin 13x}{13}\right) + C$

9.

10. $-2\tan^{-1}\sqrt{\tan x + \cot x + 1} + C$

11. $\sqrt{t^2+2t-3} - \ln\left(t+1+\sqrt{t^2+2t-3}\right) - \sqrt{3}\sin^{-1}\left(\dfrac{t+5}{t+2}\right) + C$, where $t = x + \dfrac{1}{x}$

12.

Chapter 2

Definite Integrals

2.1 Basic Rules

There are very common basic rules and common mistakes. The basic rules are

1. $\int_a^b f(x)\ dx = \int_a^b f(a+b-x)\ dx$
2. $\int_a^b f(x)\ dx = \int_0^{\frac{b-a}{2}} [f(\frac{a+b}{2}-x) + f(\frac{a+b}{2}+x)]\ dx$
   (a) $\int_0^{2a} f(x)\ dx = \int_0^a [f(a-x)+f(a+x)]\ dx$
   (b) $\int_{-a}^a f(x)\ dx = \int_0^a [f(-x)+f(x)]\ dx$
3. $\int_{a+mT}^{a+nT} f(x)\ dx = (n-m)\int_0^T f(x)\ dx$, where $f(x)$ is periodic with periodicity T.

The common mistakes are

1. **Limit based problem**: You need to check that ***there should not be any asymptotic between upper limit and lower limit***. In that case you need to split up your limit.
2. **Substitution based problem**: While substituting $z = f(x)$, we need to check $f(x)$ is asymptotic in domain $\mathbb{D} \in$ (lower limit, upper limit). If there is, just need to split up!

Problems

1. $\int_{-1}^3 \tan^{-1}\frac{x}{x^2+1} + \tan^{-1}\frac{x^2+1}{x}\ dx$
2. $\int_0^{\pi/2} \ln\sin x\ dx$
3. $\int_0^{\theta} \ln(1+\tan\theta\tan x)\ dx$
4. $\int_{-\infty}^{\infty} \frac{2023^{-|x|}}{1+5^{\sin^{-1}(\sin^5 x)}}\ dx$
5. $\int_0^{\pi/2} \sqrt{1+\sin x}\ dx$
6. $\int_0^{\pi} \sqrt{1+\sin x}\ dx$

7. $\int_0^\infty \frac{1}{(x+\frac{1}{x})^2}\ dx$
8. $\int_1^\infty \frac{\ln x}{x^2}\ dx$
9. $\int_0^\pi \cos\left(x+\cos x\right)\ dx$
10. $\int_{-\infty}^0 \frac{1}{x^3-1}\ dx$

2.2 Feynman's Trick

We need to consider some $I(a)=\int_a^b f(x,a)\ dx$ such that $I'(a)$ can be integrated easily. After that we need to integrate that and the constant is evaluated by putting $I(0)$ or some constant for which $\int_a^b f(x,0)\ dx$ is integrated easily.

Problems

1. $\int_0^1 \frac{x^2-1}{\ln x}\ dx$
2. $\int_0^1 \frac{\ln(1+x)}{x^2+1}\ dx$
3. $\int \frac{\tan^{-1}(2\sin x)}{\sin x}\ dx$
4. $\int\ dx$

2.3 Frullani's Integral

$$\int_0^\infty \frac{f(ax)-f(bx)}{x}\ dx=(f(\infty)-f(0))\ln\frac{a}{b}$$ **[Proof]**

Problems

1.

2.4 Gamma and Beta Integral

2.5 Laplace Transformation

2.6 Miscellaneous problems

Problems

1. $\int_0^{2\pi} \frac{\min(\sin x,\cos x)}{\max(e^{\sin x},e^{\cos x})}\ dx$
2. Min value of $f(x)=\int_0^2 e^{|t-x|}\ dt$
3. $\int_0^{2024} x^2-\lfloor x\rfloor\lceil x\rceil\ dx$

4. $\displaystyle\int_0^{\pi/2} \frac{x}{\tan x}\, dx$

5. For all real numbers x, let $f(x) = |x^2 + x|$, $I_1 = \displaystyle\int_{-2020}^{0} f(x)\, dx$, $I_2 = \displaystyle\int_0^{2019} f(x)\, dx$, calculate $|I_1 - I_2|$.

6. $\displaystyle\int_{1/e}^{e} \frac{\tan^{-1} x}{x}\, dx$

2.7 Advanced Problems

Problems

1. $\displaystyle\lim_{\epsilon \to 0^+} \epsilon^4 \int_0^{\frac{\pi}{2}-\epsilon} \tan^5 x\, dx = ?$

2. $\displaystyle\int_0^1 \left\lfloor \sqrt{1 + \frac{1}{x}} \right\rfloor\, dx$

3. $\displaystyle\int_0^{\infty} \sin ax\, dx$

4. $\displaystyle\int_0^{\infty} \frac{\cos ax - \cos bx}{x}\, dx$

5. $\displaystyle\int_0^{\infty} \frac{b \sin ax - a \sin bx}{x^2}\, dx$

6. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sqrt[3]{\tan x}}{(\sin x + \cos x)^2}\, dx$

7. $\displaystyle\int_0^{\pi} \left(\frac{\sin 2x \sin 3x \sin 5x \sin 30x}{\sin x \sin 6x \sin 10x \sin 15x}\right)^2\, dx$

8. $\displaystyle\int_{-1/2}^{1/2} \sqrt{x^2 + 1 + \sqrt{x^4 + x^2 + 1}}\, dx$

9. $\displaystyle\left\lfloor 10^{20} \int_2^{\infty} \frac{x^9}{x^{20} - 48x^{10} + 575}\, dx \right\rfloor$

10. $\displaystyle\int_0^1 \left(\sum_{n=1}^{\infty} \frac{\lfloor 2^n x \rfloor}{3^n}\right)^2 dx$

11. $\displaystyle\int \sqrt{(\sin 20x + 3 \sin 21x + \sin 22x)^2 + (\cos 20x + 3 \cos 21x + \cos 22x)^2}\, dx$

12. $\displaystyle\int_0^{\infty} \frac{e^{-2x} \sin 3x}{x}\, dx$

13. $\displaystyle\int_0^{2\pi} \cos 2022x\, \frac{\sin 10050x}{\sin 50x}\, \frac{\sin 10251x}{\sin 51x}\, dx$

14. $\int_0^1 x^{\frac{1}{3}}(1-x)^{\frac{2}{3}}\ dx$

15. $\left\lfloor \log_{10} \int_{2022}^{\infty} 10^{-x^3}\ dx \right\rfloor$

16. $\int_{-2}^{2} |(x-2)(x-1)x(x+1)(x+2)|\ dx$

17. $\int_1^2 \left(e^{\frac{1}{\sqrt{1-(x-1)^2}}} + 1 \right) + \left(1 + \frac{1}{\sqrt{1-\ln(x-1)}} \right)\ dx$

18. $\int_0^1 \ln x\ \sin(\ln x)\ dx$

19. $\int_0^1 \ln^{2020} x\ dx$

20. $\int_{-\infty}^{\infty} \frac{\sin\left(2x - \frac{1}{x}\right)}{2x^3 - x}\ dx$

21. $\int_p^q \left(e^{\frac{1}{\sqrt{1-(x-1)^2}}} + 1 \right) + \left(1 + \frac{1}{\sqrt{1-\ln(x-1)}} \right)\ dx$, where $1 \leq p < q$

22. $\int_0^{100} \left\lceil \sqrt{x} \right\rceil\ dx$

23. $\int_0^1 \sqrt{(x-1)^3+1} + x^{2/3} + (1-x)^{3/2} - \sqrt[3]{1-x^2}\ dx$

24. Find α such that

$$\lim_{x \to 0^+} x^\alpha I(x) = a \text{ given } I(x) = \int_0^\infty \sqrt{t+1}\ e^{-xt}\ dt$$

where a is a non-zero real number.

25. Define $H_n = \sum_{k=1}^{n} \frac{1}{k}$. Evaluate

$$\sum_{n=1}^{2017} \binom{2017}{n} H_n (-1)^n$$

26. The numerical value of the following integral

$$\int_0^1 (-x^2+x)^{2017} \lfloor 2017x \rfloor\ dx$$

can be expressed as $a\frac{(m!)^2}{n!}$, where a is minimized. Find $a+m+n$.

27. Let T be defined by the recurrence relation $T_{n+1} = 2xT_n - T_{n-1}$ with $T_0 = 1$ and $T_1 = x$. What is the value of

$$\sum_{n=2}^{\infty} \int_0^1 T_n dx$$

28. Compute the following limit

$$\lim_{n\to\infty} \int_0^1 \frac{nx^n}{\sqrt{4x^3 - x^2 + 1}}\, dx$$

29. $\lim\limits_{n\to\infty} n^2 \displaystyle\int_0^{1/n} x^{x+1}\, dx = ?$

30. $\displaystyle\sum_{a=1}^{\infty}\sum_{b=1}^{\infty} \frac{1}{a^2b + 2ab + ab^2} = ?$

31. Let $f : \mathbb{R}_{>0} \to \mathbb{R}$ (where $\mathbb{R}_{>0}$ is the set of all positive real numbers) be differentiable and satisfy the equation

$$f(y) - f(x) = \frac{x^x}{y^y}\frac{f(y^y)}{f(x^x)}$$

for all real $x, y > 0$. Furthermore, $f'(1) = 1$. Find f(x).

32. Suppose the following equality holds, where a, b, c are integers and K is the constant of integration:

$$\int \frac{\sin^a x - \cos^a x}{\sin^b x\ \cos^b x}\, dx = \frac{\operatorname{cosec}^c x}{c} + \frac{\sec^c x}{c} + K$$

If $a = 2021$, compute $a + b + c$.

33. Let $c(x) = \dfrac{e^x + e^{-2x}}{2}$, defined in interval $1 \le x \le 2$, Let $c^{-1}(x)$ be the inverse of $c(x)$. Compute

$$\int_{c(1)}^{c(2)} c^{-1}(x)\, dx$$

34. Compute the infinite sum

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\binom{n+1}{2}}$$

35. $\displaystyle\int_0^{\pi/3} \sec x \sqrt{\tan x \sqrt{\tan x \sqrt{\tan x\ \sin x}}}\, dx$

36. Compute: $\lim\limits_{x\to 0} \left(1 + \displaystyle\int_0^x \frac{\cos t - 1}{t^2}\, dt\right)^{\frac{1}{x}}$

37. $\displaystyle\int_0^{\pi/2} \cot x \ln(\cos x)\, dx$

38. $\displaystyle\sum_{n=2}^{\infty} \ln\left(\frac{n^3 + 1}{n^3 - 1}\right)$

39. $\displaystyle\sum_{n=2}^{\infty} \sqrt{n^2 + 3n + 2} - \sqrt{n^2 + n} - 1$

40. A unit cube is rotated around an axis containing its longest diagonal. Compute the volume swept out by the rotating.

Answers

1. $\frac{1}{4}$
2. $\frac{7}{4}$
3. $\frac{1}{a}$
4. $\ln \frac{b}{a}$
5. $ab \ln \frac{b}{a}$
6. $\frac{2\sqrt{3}\pi}{9}$
7. 7π
8. $\frac{\sqrt{7}}{2\sqrt{2}} + \frac{3}{4\sqrt{2}} \ln \frac{\sqrt{7}+2}{\sqrt{3}}$
9. $10^{16} + \frac{10^{10}-1}{3} + \frac{10^4}{5}$
10. $\frac{27}{32}$
11. $3x + 2\sin x + C$
12. $\tan^{-1} \frac{3}{2}$
13. 6π
14. $\frac{2\pi}{9\sqrt{3}}$
15. $-2022^3 - 8$
16. $\frac{19}{3}$
17. 3
18. $\frac{1}{2}$
19. 2020!
20. π
21. $q^2 - p^2$
22. 715
23. 1
24. $\frac{3}{2}$
25. $-\frac{1}{2017}$
26. 7060
27. -1
28. $\frac{1}{2}$
29. $\frac{1}{2}$
30. $\frac{7}{4}$
31. $x \ln x$
32. 6060
33. $.5e^2 + 1.25e^{-4} - .75e^{-2}$
34. $4 \ln 2 - 2$
35. $\frac{8}{7}(2^{7/8} - 1)$
36. $\frac{1}{\sqrt{e}}$
37. $-\frac{\pi^2}{24}$
38. $\ln 3 - \ln 2$
39. $\frac{1}{2}$
40. $\frac{\pi}{\sqrt{3}}$

Problems

1.

hello

hello hello

hello

My Box

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