

# Advanced Mathematics Question Paper

Class: VIII

Time: 1 Hour 30 Minutes  
Max Marks: 40

## Topic: Rational Numbers, Powers & Exponents, Square Numbers

### Section A: Challenging Multiple Choice Questions (1 Mark Each)

Total:  $5 \times 1 = 5$  Marks

1. If  $\left(\frac{a}{b}\right)^{x-2} = \left(\frac{b}{a}\right)^{x-4}$ , then the value of  $x$  is:
  - a) 0
  - b) 1
  - c) 2
  - d) 3
2. The value of  $\left(\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}}\right)$  is:
  - a) 0
  - b) 1
  - c)  $\sqrt{4} - 1$
  - d)  $1 - \sqrt{4}$
3. If  $n$  is a positive integer, which of the following cannot be a perfect square?
  - a)  $5n^2$
  - b)  $9n^2$
  - c)  $16n^2$
  - d)  $20n^2$
4. The rationalizing factor of  $\frac{1}{\sqrt[3]{5}-\sqrt[3]{2}}$  is:
  - a)  $\sqrt[3]{5} + \sqrt[3]{2}$
  - b)  $\sqrt[3]{25} + \sqrt[3]{10} + \sqrt[3]{4}$
  - c)  $\sqrt[3]{5} - \sqrt[3]{2}$

- d)  $\sqrt[3]{25} - \sqrt[3]{10} + \sqrt[3]{4}$
5. If  $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$  and  $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ , then  $x^2 + y^2$  is:
- a) 10  
b) 49  
c) 98  
d) 100

## Section B: Short Answer Questions (2 Marks Each)

**Total:  $5 \times 2 = 10$  Marks**

6. Find the value of  $\left(\frac{64}{125}\right)^{-2/3} + \left(\frac{256}{625}\right)^{-1/4}$ .
7. If  $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$ , find the values of  $a$  and  $b$ .
8. Prove that  $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = 1$ .
9. Find the smallest rationalizing factor of  $\sqrt[4]{8}$ .
10. If  $a = 2 + \sqrt{3}$ , find  $a^2 + \frac{1}{a^2}$ .

## Section C: Long Answer Questions (3 Marks Each)

**Total:  $5 \times 3 = 15$  Marks**

11. Solve for  $x$ :
- $$2^{2x} - 6 \times 2^x + 8 = 0$$
12. If  $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$  and  $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ , find  $x^2 + xy + y^2$ .
13. Prove that  $\sqrt{5}$  is irrational by contradiction.
14. Find the value of  $\sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}$ .
15. If  $a = 9 + 4\sqrt{5}$ , find  $\sqrt{a} - \frac{1}{\sqrt{a}}$ .

## Section D: Application-Based Problem (5 Marks)

**Total:  $1 \times 5 = 5$  Marks**

16. A right-angled triangle has legs of lengths  $\sqrt{2}$  and  $\sqrt{3}$ .
- (a) Find the exact length of the hypotenuse.
- (b) If a square has the same area as this triangle, find the side length of the square.
- (c) Compare the perimeter of the triangle and the square.

# Marking Scheme (Sample Answers)

## Section A

1. d) 3 (Hint: Equate exponents after expressing both sides with the same base.)
2. c)  $\sqrt{4} - 1$  (Rationalize each term and observe telescoping cancellation.)
3. d)  $20n^2$  (20 is not a perfect square, so  $20n^2$  can't be a perfect square.)
4. b)  $\sqrt[3]{25} + \sqrt[3]{10} + \sqrt[3]{4}$  (Use  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ .)
5. c) 98 (Compute  $x + y = 10$  and  $xy = 1$ , then use  $x^2 + y^2 = (x + y)^2 - 2xy$ .)

## Section B

6.  $\frac{25}{16} + \frac{5}{4} = \frac{45}{16}$  (Simplify exponents and evaluate.)
7.  $a = 11, b = -6$  (Rationalize the denominator and compare terms.)
8. **Proof:** Combine exponents and simplify to 1.
9.  $\sqrt[4]{2}$  (Multiply by  $\sqrt[4]{8} \times \sqrt[4]{2} = \sqrt[4]{16} = 2$ .)
10. 14 (Compute  $a^2 = 7 + 4\sqrt{3}$  and  $\frac{1}{a^2} = 7 - 4\sqrt{3}$ .)

## Section C

11.  $x = 1, 2$  (Let  $2^x = t$ , solve quadratic  $t^2 - 6t + 8 = 0$ .)
12. 49 (Compute  $x + y = 8$  and  $xy = 1$ , then  $x^2 + xy + y^2 = (x + y)^2 - xy$ .)
13. **Proof by contradiction:** Assume  $\sqrt{5} = \frac{p}{q}$  in lowest terms, derive contradiction.
14. 3 (Let  $x = \sqrt{6 + x}$ , solve  $x^2 - x - 6 = 0$ .)
15. 4 (Let  $\sqrt{a} = \sqrt{5} + 2$ , then compute  $\sqrt{a} - \frac{1}{\sqrt{a}}$ .)

## Section D

16. (a) Hypotenuse  $= \sqrt{(\sqrt{2})^2 + (\sqrt{3})^2} = \sqrt{5}$ .  
(b) Area of triangle  $= \frac{1}{2} \times \sqrt{2} \times \sqrt{3} = \frac{\sqrt{6}}{2}$ . Side of square  $= \sqrt{\frac{\sqrt{6}}{2}}$ .  
(c) Perimeter of triangle  $= \sqrt{2} + \sqrt{3} + \sqrt{5}$ . Perimeter of square  $= 4 \times \sqrt{\frac{\sqrt{6}}{2}}$ .