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# SELF-ORGANIZING, TWO-TEMPERATURE ISING MODEL DESCRIBING HUMAN SEGREGATION

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A two-temperature Ising-Schelling model is introduced and studied for describing human segregation. The self-organized Ising model with Glauber kinetics simulated by Müller *et al.* exhibits a phase transition between segregated and mixed phases mimicking the change of tolerance (local temperature) of individuals. The effect of external noise is considered here as a second temperature added to the decision of individuals who consider a change of accommodation. A numerical evidence is presented for a discontinuous phase transition of the magnetization.

*Keywords*: Segregation; socio-economic models; complex systems; lattice theory; nonequilibrium phase transition.

## 1. Introduction

The problem of human segregation is an important problem of society and politics even in the 21st century.[1] Social sciences have been investigating the reasons and nature of segregation for a long time. Sociologist have introduced several models, one of them is the Schelling model.[2] From a physicists point of view that model is a 3-state voter-type non-equilibrium model (groups A, B and empty), with spin-exchange dynamics at zero temperature ($T = 0$) on a 2-dimensional square lattice. Although the model describes segregation by a quench without external reasons, unwanted frozen states may also occur.

Recently it was shown by computer simulations[3] that there exists a simpler model, namely the Glauber-Ising model,[4,5] which captures the essence of human segregation. Besides that, the usage of the simple $T > 0$ spin-flip dynamics makes it computationally easier and one can avoid the frozen states of the Schelling model. In this model the temperature plays the role of a global tolerance; by varying it, the model may or may not evolve into the ordered (segregated) state.

However a constant, global tolerance is rather artificial in a society, it can vary from individual to individual and can change in time as well. By introducing a local, time-dependent temperature (tolerance), with a feedback mechanism from the local neighborhood the model becomes more realistic. The results do not change too much,[6] a self-organization of the average temperature occurs. The parameters of this model are the global rate of forgetting (of tolerance) and the response of local tolerances on the neighborhood. The sum of these local changes determine the local temperature. Hereafter this self-organized segregation model will be called SO-Seg model.

Stepping further towards more realistic models, one can pose the question: what happens to this model if the decision of individuals are affected by an independent external noise as well. The external noise can be an artifact of a random environment, housing, moving situation, presence of shopping centers ... etc. The external noise introduced here as a second temperature, i.e., individuals are connected to a second heat bath.

Two-temperature two-state voter-type models have been intensively investigated recently and have become the prototypes of non-equilibrium models (for a review investigated see Ref. 7). An important finding of these studies was the discovery of relevant factors affecting the phase transitions of models exhibiting $Z_2$ (up-down) symmetry.[8] In particular models with general, isotropic spin-flip dynamics maintaining the $Z_2$ symmetry can be classified as two temperature models, where one temperature controls the bulk, the other the interface fluctuations. The Ising model is a special case of these models, where both temperatures are non-zero and the time-reversal symmetry drives the system into an equilibrium state. The transition of these $Z_2$ symmetric, two-temperature models has been found to be continuous, Ising type unless the bulk temperature is zero. In the latter case it is first order, voter model class type.[7] In this work we investigate the effect of a second temperature applied as an external, independent heat bath to the spins of the SO-Seg model.

## 2. The Model

The model is defined on 2-dimensional square lattice, with periodic boundary conditions and Ising spins ($s_i = (1, -1)$) distributed initially randomly (zero initial magnetization = no segregation). The kinetics follows a Glauber spin-flip (sequential) update with heath bath acceptance rate (see Ref. 9), depending on the local temperatures. The spin is flipped if a random number between 0 and 1 is smaller than the probability $\exp(-\Delta E/k_B T)/[1 - \exp(-\Delta E/k_B T)]$, where the change of energy is $\Delta E = \Delta \sum_{i,j} s_i s_j$. In the SO-Seg model each individual has four interacting nearest neighbors and a randomized initial local temperature, with an average value $\langle T_1(0)\rangle = 1.5$. This local temperature is lowered by $\delta T_F$ at each update for modeling the loss of tolerance. This alone would just make a quench to $T_1 \to 0$ with domain coarsening.

To model people's awareness of the dangers of segregation they can increase their own temperature (tolerance) by $\delta T_a$, if all four neighbors of an individual belong to the same group as $s_i$. If all four neighbors belong to different groups the local temperature is decreased by the same amount.

Our external noise is described by a heat bath of a second temperature $T_2$ applied to the decisions. The actual spin-flip will be the logical OR of internal and external flip decisions. The magnetization $M(t) = \langle s_i(i) \rangle$, the average number of like neighbors minus unlike neighbors $\langle N(t) \rangle$, and the average self-organizing internal temperature (tolerance) $\langle T_1(t) \rangle$ is followed up to $t_{\max} = 2 \times 10^5$ Monte Carlo sweeps (MCS) of the lattice.

## 3. Simulation Results

The simulations were performed on $L = 400$, 2000 and 4000 sized square lattices, up to $t_{\max} = 2 \times 10^5$ MCS, with cooling rates: $\delta T_F = 0.01, 0.02$ and tolerance steps: $\delta T_a = 0.003, 0.002$. The phase transition of the SO-Seg model is at $\delta T_a = 0.0029$,[5] so $\delta T_a = 0.003$ corresponds to super-critical, $\delta T_a = 0.002$ to sub-critical situations.

As Fig. 1 shows the inclusion of a small second temperature $T_2 <\sim 1$ does not change the composition of neighbors and $T_1$ in the steady state (as $t \to \infty$). The same can be seen by plotting $N(t)$ (see Fig. 2) and the average internal temperature of individuals (Fig. 3). For stronger external noise the domains are destroyed, but the average tolerance goes to zero too. This means that the unsegregated state can be maintained with the help of strong external noise, without worrying about people's local tolerance. The same analysis for $\delta T_a = 0.002$ resulted in similar trends in $N(t = 30\,000)$ (see Fig. 4) and in the average tolerance $T_1(t = 30\,000)$. Note that one observes even a weak increase in the asymptotic values for $T_2 < 1$. Running the simulations on larger sizes there were no changes in this, excluding the possibility of finite size errors.

By increasing the second temperature $T_2$ the transitions of $N(\infty)$ (see Fig. 4) and $T_1(\infty)$ are very smeared. The magnetization density $m = M(\infty)/L^2$ on the other hand shows a sharp fall, indicating a first order phase transition at $T_2^* = 0.135(2)$. A fitting attempt using the form $m = A(T_2^* - T_2)^\beta$ did not result in agreement with the 2d Ising class continuous phase transition behavior,

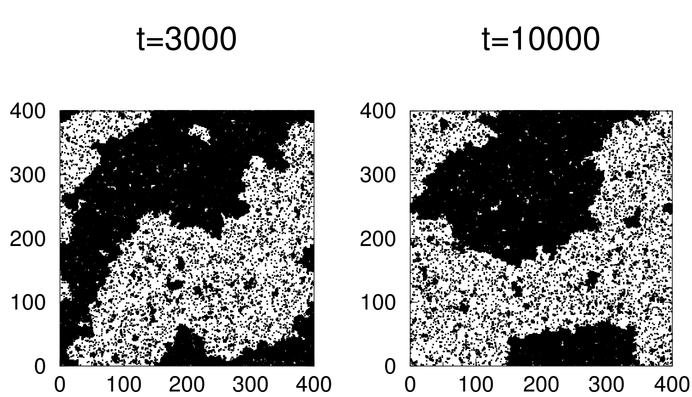


Fig. 1. Clusters survive small external noise: $T_2 = 0.5$.

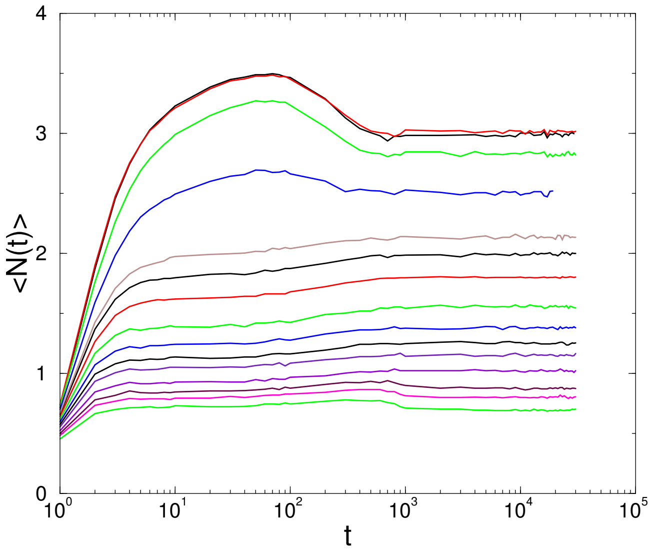


Fig. 2. Average composition as a function of time and external temperature $T_2 = 0, 1, 2, \ldots, 10$ (top to bottom).

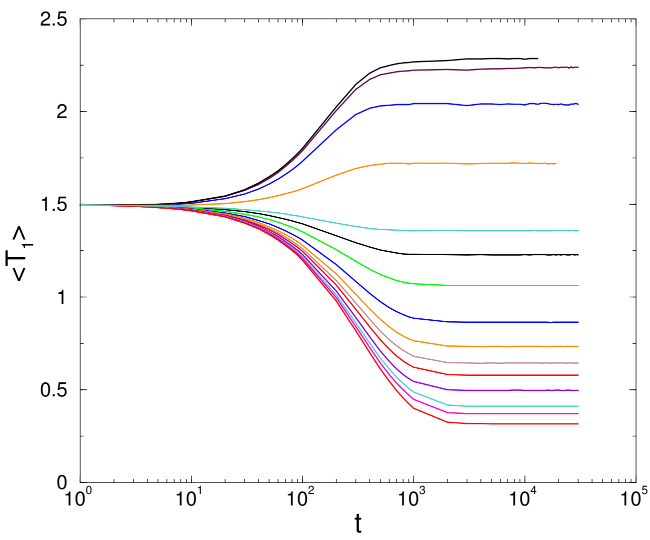


Fig. 3. The average tolerance of individuals as a function of time and external temperature $T_2 = 0, 1, 2, \ldots, 10$ (top to bottom).

characterized by $\beta = 1/8$.[7] Furthermore a hysteresis cycle can also be found by starting the simulations with different initial conditions (ordered vs. disordered), which is a clear hallmark of a first order phase transition. This is in contrast with the results for two-temperature $Z_2$ symmetric models exhibiting Ising transition in 2d (the bulk noise in nonzero of course). One may understand the discontinuous

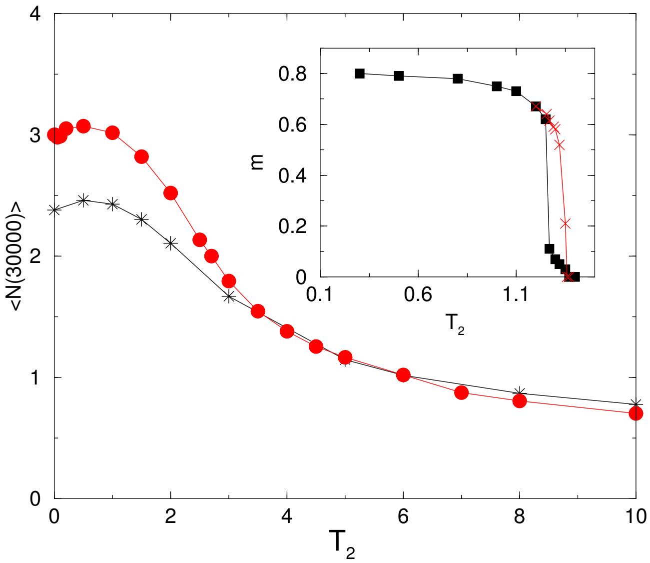


Fig. 4. Average composition as a function of external temperature ($T_2$). Bullets (higher data on the left) correspond to $\delta T_a = 0.003$ forgetting rate, crosses to $\delta T_a = 0.002$ forgetting rate. Inset: The magnetization density for $\delta T_a = 0.003$ shows a first order transition.

transition here by realizing that this model is effectively a coupled system: $T_2$ temperature Ising and a $T_l$ local temperature (tolerance) field. The temperatures can increase if at least four neighbors are in the same state and decrease without condition. In the language of reaction diffusion systems it is a quadruple model, where at least four neighbors are needed for creation but the removal is spontaneous. It is well known that in 2d such a quadruple model exhibits a first order transition.[10]

Simulations with different initial temperatures ($T_1(0) = 2.5$), cooling rates ($T_F = 0.2$) and $\delta T_a$ have not resulted in changes in the transition point $T_2^*$.

## 4. Conclusion

A two-temperature, self-organized Ising-Schelling model has been introduced and investigated by numerical simulations. A low second temperature, which represents the external noise to the decision of individuals for moving does not change the segregation behavior of the model. A temperature higher than $T_2 = 0.135(2)$ randomizes the segregated, ordered domains and results in low average tolerance of individuals. While the self-organized tolerance and the average composition of the steady state show continuous variation on $T_2$ the magnetization exhibits a first order phase transition. The threshold did not show considerable dependence on the model parameters.

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