SEQUENCES AND SERIES

Introduction

Sequences and series are fundamental concepts in mathematics that appear in various disciplines, including algebra, calculus, and real analysis. A sequence is an ordered list of numbers following a particular rule, while a series is the sum of the terms in a sequence.

1. Sequences

A sequence is a function whose domain is the set of natural numbers. It is represented as:

$a_1, a_2, a_3, \ldots$

where each term follows a specific pattern or rule.

Types of Sequences

1. Arithmetic Sequence

- A sequence where each term is obtained by adding a fixed number (common difference) to the previous term.
  - General formula:

    $a_n = a_1 + (n - 1) d$

  - Example:
    - Given $a_1 = 3$ and $d = 5$, the sequence is:

      3, 8, 13, 18, 23, ...

2. Geometric Sequence

- A sequence where each term is obtained by multiplying the previous term by a fixed number (common ratio).
  - General formula:

    $a_n = a_1 * r^{(n-1)}$

  - Example:
    - Given $a_1 = 2$ and $r = 3$, the sequence is:

      2, 6, 18, 54, 162, ...

3. Fibonacci Sequence

- A sequence where each term is the sum of the two preceding ones.

- Formula:

  Fn = Fn-1 + Fn-2

- Example:

  - Starting with 0 and 1, the sequence is:

    0, 1, 1, 2, 3, 5, 8, 13, ...

2. Series

A series is the sum of the terms of a sequence. It is represented as:

$S_n = a_1 + a_2 + a_3 + ... + a_n$

Types of Series

1. Arithmetic Series

  - The sum of the terms in an arithmetic sequence.

  - Sum formula:

    $S_n = (n/2) * (2a_1 + (n-1) d)$

  - Example:

    - Find the sum of the first 10 terms of the sequence 3, 8, 13, 18, 23, ...:

      $S_{10} = (10/2) * (2 * 3 + (10 - 1) * 5)$

      $S_{10} = 5 * (6 + 45) = 5 * 51 = 255$

2. Geometric Series

  - The sum of the terms in a geometric sequence.

  - Sum formula:

    $S_n = a_1 * (1 - r^n) / (1 - r)$, if $|r| < 1$

  - Example:

    - Find the sum of the first 4 terms of the sequence 2, 6, 18, 54, ... with r = 3:

      $S_4 = 2 * (1 - 3^4) / (1 - 3)$

      $S_4 = 2 * (1 - 81) / (-2)$

      $S_4 = 2 * 40.5 = 81$

Conclusion

Sequences and series play a crucial role in mathematics, appearing in various applications such as finance, computer science, and physics. Understanding their properties and formulas allows for

solving complex mathematical problems efficiently.