

Polynomials

Definition of Polynomials

A polynomial is an algebraic expression consisting of variables, coefficients, and exponents combined using addition, subtraction, and multiplication.

A general polynomial of degree n is written as:

$$P(x) = a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + \dots + a_1 \cdot x + a_0$$

where:

- n is a non-negative integer (degree of the polynomial),
- a_n, a_{n-1}, \dots, a_0 are constants (coefficients).

Types of Polynomials

1. **Constant Polynomial:** A polynomial with no variables (e.g., $P(x) = 5$).
2. **Linear Polynomial:** A polynomial of degree 1 (e.g., $P(x) = 2x + 3$).
3. **Quadratic Polynomial:** A polynomial of degree 2 (e.g., $P(x) = x^2 - 4x + 5$).
4. **Cubic Polynomial:** A polynomial of degree 3 (e.g., $P(x) = x^3 - 2x^2 + 3x - 1$).

Operations on Polynomials

1. **Addition:** $(2x^2 + 3x + 1) + (x^2 - x + 2) = 3x^2 + 2x + 3$
2. **Multiplication:** $(x+2)(x-3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$
3. **Division:** Using long division or synthetic division.

Example Problem

Find the value of the polynomial $P(x) = 3x^3 - 2x^2 + 4x - 5$ at $x = 2$.

$$P(2) = 3(2)^3 - 2(2)^2 + 4(2) - 5 = 24 - 8 + 8 - 5 = 19$$