Introduction to Algebra

What is Algebra?

Algebra is a branch of mathematics that uses symbols, letters, and numbers to represent relationships and solve problems.
It is a fundamental aspect of mathematics used in various fields such as science, engineering, economics, and everyday life.

Basic Terminology

- Variables: Symbols (usually letters) that represent unknown values (e.g., x, y, z).
- Constants: Fixed values that do not change (e.g., 5, -3, 2.7).
- Expressions: A combination of variables, constants, and mathematical operations (e.g., 3x + 5).
- Equations: A mathematical statement showing two expressions are equal (e.g., 2x + 3 = 7).
- Coefficients: Numbers that multiply a variable (e.g., in 4x, the coefficient is 4).

Fundamental Operations

Addition and Subtraction

- Like terms can be added or subtracted. Example:
  - 3x + 5x = 8x
  - 7y - 2y = 5y

Multiplication

- Variables and constants follow the distributive property:
  - 3(x + 2) = 3x + 6
  - x * x = x^2

Division

- Division follows the inverse property:
  - (6x) / 3 = 2x
  - x^2 / x = x

Order of Operations

Use PEMDAS/BODMAS (Parentheses, Exponents, Multiplication/Division, Addition/Subtraction) to evaluate expressions.
Example:
- 3 + 4 * 2 = 3 + 8 = 11 (Multiplication first)

Solving Simple Equations
One-Step Equations
- x + 5 = 12 -> Subtract 5 -> x = 7
- 3x = 12 -> Divide by 3 -> x = 4

Two-Step Equations
- 2x + 3 = 7
  - Subtract 3 from both sides -> 2x = 4
  - Divide by 2 -> x = 2

Polynomials and Expressions
Like Terms & Unlike Terms
- Like terms: 3x and 5x (both have x)
- Unlike terms: 3x and 7y (different variables)

Operations with Polynomials
- (3x + 2) + (4x - 5) = 7x - 3
- (2x)(3x) = 6x^2

Factoring in Algebra
Common Factorization
- 6x + 9 = 3(2x + 3)

Factoring Quadratics
- x^2 + 5x + 6 = (x + 2)(x + 3)

Linear Equations & Graphing
Slope-Intercept Form
- y = mx + b (m = slope, b = y-intercept)

Graphing a Line
- Find two points using the equation.
- Plot and draw a straight line through them.

Practice Exercises

1. Simplify: 5x + 3x - 2

2. Solve: 2x + 3 = 11

3. Factor: x^2 + 7x + 10

4. Graph the equation: y = 2x + 1

Answer Key

1. 8x - 2

2. x = 4

3. (x + 5)(x + 2)

4. (Graph solution)

Algebra is a foundational subject that strengthens problem-solving skills. Keep practicing and exploring its applications!