LOGARITHMIC FUNCTIONS

Introduction to Logarithms

A logarithm is the inverse operation of exponentiation. It helps us solve equations where the unknown variable is an exponent.

If:

$a^x = b$

Then, the logarithmic form is:

$\log_a b = x$

where:

- a is the base,
- b is the argument,
- x is the exponent.

For example:

$2^3 = 8$ can be written as $\log_2 8 = 3$.

Common Types of Logarithms

1. Common Logarithm: Base 10, written as log x.
2. Natural Logarithm: Base e (Euler's number, e approximately 2.718), written as ln x.
3. Binary Logarithm: Base 2, written as $\log_2 x$.

Properties of Logarithms

1. Product Rule: $\log_a (MN) = \log_a M + \log_a N$
2. Quotient Rule: $\log_a (M/N) = \log_a M - \log_a N$
3. Power Rule: $\log_a (M^p) = p \log_a M$
4. Change of Base Formula: $\log_a b = \log_c b / \log_c a$

Solving Logarithmic Equations

Example 1: Solve $\log_2 x = 5$

Solution:

Convert to exponential form:

$2^5 = x$

$x = 32$

Example 2: Solve $\log_3 (x - 2) + \log_3 (x + 4) = 2$

Solution:

Using product rule:

$$\log_3 [(x - 2)(x + 4)] = 2$$

Convert to exponential form:

$$(x - 2)(x + 4) = 3^2$$

$$x^2 + 2x - 8 = 9$$

$$x^2 + 2x - 17 = 0$$

Solving quadratic:

$$x = (-2 \pm \sqrt{72}) / 2$$

$$x = -1 \pm 3\sqrt{2}$$

Only valid solutions are taken.

Applications of Logarithms

1. Compound Interest: $A = P e^{(rt)}$ solved using logarithms.
2. pH Scale: $pH = -\log [H+]$.
3. Earthquake Magnitude: Richter scale.
4. Information Theory: Measures data entropy.

Conclusion

Logarithmic functions are essential in mathematics and real-world applications. Understanding properties and rules simplifies complex calculations.