Vectors and Spaces

Introduction to Vectors

A vector is an object that has both **magnitude** and **direction**. Vectors are often represented as **ordered tuples** of numbers.

Example of a vector in 3D space:

v = (x, y, z)

Vector Operations

1. **Vector Addition**:

   If a = (a1, a2, a3) and b = (b1, b2, b3), then:

   a + b = (a1 + b1, a2 + b2, a3 + b3)

2. **Scalar Multiplication**:

   If c is a scalar, then:

   c * a = (c * a1, c * a2, c * a3)

3. **Dot Product**:

   a . b = a1 * b1 + a2 * b2 + a3 * b3

Example

Find the dot product of a = (2,3,4) and b = (1,0,-1).

a . b = (2 * 1) + (3 * 0) + (4 * -1) = 2 + 0 - 4 = -2

Vector Spaces

A vector space is a collection of vectors that can be added together and multiplied by scalars while satisfying certain rules.

Common examples include:

- The set of real numbers R

- The set of n-dimensional space R^n