Doing an angular fit without doing an angular fit: the moments technique

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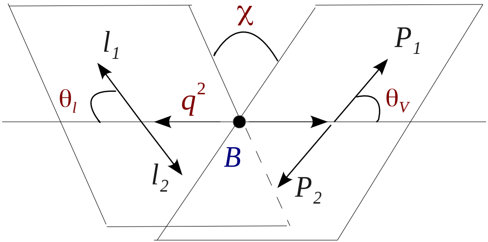
Generalized $P \to VV$

- Two massless leptons ($\ell_{1,2}$) and two pseudo-scalars ($P_{1,2}$):



- B2CC: $B_d \to \psi^{(\prime)} K\pi$, $B_s \to \psi^{(\prime)} KK$
- EWP: $B_d \to K\pi\mu\mu$, $B_s \to KK\mu\mu$
- SL: $B_d \to \{\rho, D^*\}\ell^-\overline{\nu}_\ell$ (tagged @ *BABAR*)

- Exactly same formalism, but different $N_{\rm sig}$, S/B, acceptance, ...

The moments idea

- De facto reference: arXiv:1505.02873

- Re-write the full differential rate in an orthonormal basis:

$$\frac{d\Gamma}{dq^2 d\Omega} = \sum_i \Gamma_i(q^2)\, f_i(\Omega)$$

- Orthonormality implies:

$$\int f_i(\Omega) f_j(\Omega) d\Omega = \delta_{ij}$$

- The observables/moments are extracted by simple summation:

$$\Gamma_i(q^2) = \sum_{k=1}^{N_{\mathrm{data}}} f_i(\Omega_k)$$

Example: $\bar{B} \to J/\psi K^- \pi^+$

- The orthonormal basis is constructed out of $Y_l^m \equiv Y_l^m(\theta_\ell, \chi)$ and $P_l^m \equiv \sqrt{2\pi}\, Y_l^m(\theta_V, 0)$.
- 7 complex amplitudes, $\{S, H_{0,\parallel,\perp}, D_{0,\parallel,\perp}\}$, or 13 real numbers.
- 28 Γ_i moments/observables. A few examples:

i	$f_i(\Omega)$	Γ_i^{tr}
1	$P_0^0 Y_0^0$	$\left[\lvert H_0\rvert^2 + \lvert H_\parallel\rvert^2 + \lvert H_\perp\rvert^2 + \lvert S\rvert^2 + \lvert D_0\rvert^2 + \lvert D_\parallel\rvert^2 + \lvert D_\perp\rvert^2\right]$
2	$P_1^0 Y_0^0$	$2\left[\frac{2}{\sqrt{5}} Re(H_0 D_0^*) + Re(S H_0^*) + \sqrt{\frac{3}{5}}\, Re(H_\parallel D_\parallel^* + H_\perp D_\perp^*)\right]$
5	$P_4^0 Y_0^0$	$\frac{2}{7}\left[-2(\lvert D_\parallel\rvert^2 + \lvert D_\perp\rvert^2) + 3\lvert D_0\rvert^2\right]$
28	$P_4^0 \sqrt{2}\, Im(Y_2^2)$	$-\frac{4}{7}\sqrt{\frac{3}{5}}\, Im(D_\perp D_\parallel^*)$

The moments idea (cntd.)

- Interesting things are now solvable for (no fitting):

$$
\begin{aligned}
|D_0|^2 &= \frac{7}{9}\left(\frac{\Gamma_5}{2} - \sqrt{5}\Gamma_{10}\right) \\
|D_\parallel|^2 &= \frac{7}{4}\left(\sqrt{\frac{5}{3}}\Gamma_{23} - \frac{1}{3}\left(\sqrt{5}\Gamma_{10} + \Gamma_5\right)\right) \\
|D_\perp|^2 &= \frac{7}{4}\left(-\sqrt{\frac{5}{3}}\Gamma_{23} - \frac{1}{3}\left(\sqrt{5}\Gamma_{10} + \Gamma_5\right)\right)
\end{aligned}
$$

The three "legs"

- Background subtraction
- Acceptance correction
- Evaluation of the covariance matrix

Background subtraction

- Let $\tilde{n}_{\rm b}$ and $N_{\rm b}$ be the projected background events in the signal- and side-bands, respectively.

- Define the scale-factor $x = \tilde{n}_{\rm b}/N_{\rm b}$.

- The background-subtracted "measured" moments and covariance matrix:

$$\tilde{\Gamma}_i^{\rm b} = \sum_{k=1}^{N^{\rm data}} f_i(\Omega_k) - x \sum_{k=1}^{N^{\rm b}} f_i(\Omega_k)$$

$$\tilde{C}_{ij}^{\rm b} = \sum_{k=1}^{N^{\rm data}} f_i(\Omega_k) f_j(\Omega_k) + x^2 \sum_{k=1}^{N^{\rm b}} f_i(\Omega_k) f_j(\Omega_k)$$

Acceptance correction

- Throw MC flat in $d\Omega$ and calculate the efficiency matrix:

$$E_{ij} = \int \epsilon(\Omega)\left[f_i(\Omega)f_j(\Omega)\right]d\Omega = \frac{\int d\Omega}{N_{\rm gen}^{\rm MC}}\left[\sum_{k=1}^{N_{\rm acc}^{\rm MC}} f_i(\Omega_k)f_j(\Omega_k)\right]$$

- The "true" moments and covariance matrix can now be extracted as:

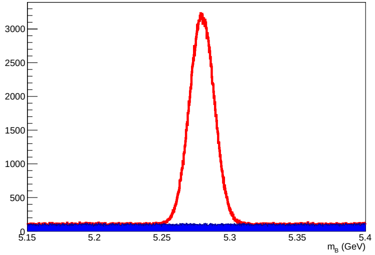
$$\Gamma_i = (E^{-1})_{ij}\tilde{\Gamma}_j$$
$$C_{ij} = (E^{-1})_{ik}\tilde{C}_{kl}(E^{-1})_{jl}$$

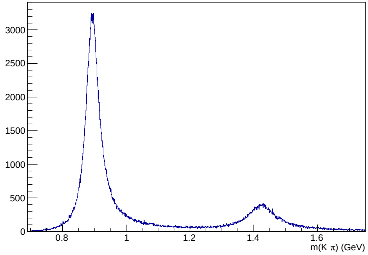
- Robust under local "holes" in acceptance. Simple counting.

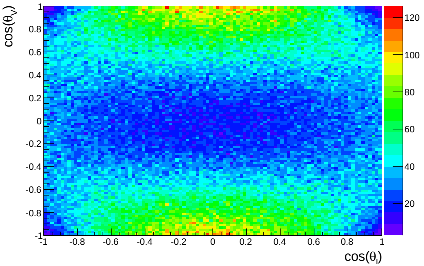
Toy set-up

- Full $m(K\pi)$ spectrum generated with LASS S-wave, $K_0(1410)$, $K^*(892)$, and $K_2(1430)$.
- 3-D acceptance function from `B2KstMuMu` analysis.
- $N_{\rm sig}$ per toy is sampled from a Poissonian with $\lambda = N_{\rm sig}^{\rm data}$
- In all, toys should be a reasonably realistic representation of full Run I dataset.

Toys (cntd.)


3000
2500
2000
1500
1000
500
0
5.15
5.2
5.25
5.3
5.35
5.4
m_B (GeV)

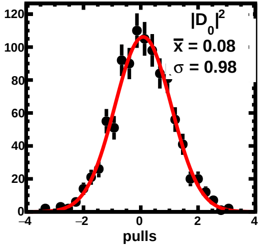

3000
2500
2000
1500
1000
500
0
0.8
1
1.2
1.4
1.6
m(K π) (GeV)

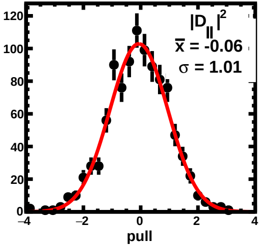

cos(θ_V)
cos(θ_l)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
1
120
100
80
60
40
20

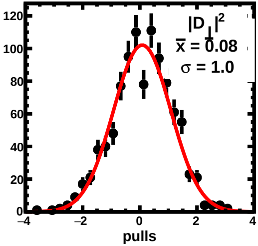
Toys results

- I'll show results here for the $|m(K\pi) - 895| < 10$ MeV bin.
- Checks done at each individual step of complication. I'll show results for the full case.
- See also toys for $\bar{B} \to K^-\pi^+\mu^-\mu^+$ in yesterday's RD meeting.

Sample pulls for $|D_{0,\parallel,\perp}|^2$







Ongoing work

- 4-D parameterization code of $\epsilon(m(K\pi), \cos\theta_\ell, \cos\theta_V, \chi)$ adopted from `B2KstMuMu` analysis. Each event is further weighted by $1/\epsilon$.

$$\tilde{\Gamma}_i^{\mathrm{b}} = \sum_{k=1}^{N^{\mathrm{data}}} \frac{1}{\epsilon} f_i(\Omega_k) - \sum_{k=1}^{N^{\mathrm{b}}} \frac{x}{\epsilon} f_i(\Omega_k)$$

$$\tilde{C}_{ij}^{\mathrm{b}} = \sum_{k=1}^{N^{\mathrm{data}}} \frac{1}{\epsilon^2} f_i(\Omega_k) f_j(\Omega_k) + \sum_{k=1}^{N^{\mathrm{b}}} \frac{x^2}{\epsilon^2} f_i(\Omega_k) f_j(\Omega_k)$$

- Validation and compatibility with the efficiency matrix formalism ongoing.

- NB: pretty statistics intensive, so we will need *quite* a bit of PHSP signal MC.

Publication strategy (for $\bar{B} \to \psi^{(\prime)} K^- \pi^+$)

- Will provide 27 normalized moments + cov. matrix in small $m(K\pi)$ bins.

- Clearly the world's most detailed description of the $m(K\pi)$ system.

- Re-visit the $Z(4430)$: are SPD waves enough to describe the system? F_D under the $K^*(892)$ should be interesting.

Relations between the Γ_i's

- The 28 Γ_i's are not all independent. 5 relations found so far. Ex:

$$\Gamma_{25} = -\sqrt{\frac{7}{3}}\Gamma_{27}$$

- Unless the complete set of relations are found, "direct" fit simply not doable.

- Also $(|H_0|^2 + |S|^2)$ seem to go together. So F_S of F_P might not be extractable. $|H_{\|,\perp}|^2$ available though.

- Unlike $S + P$, the different components don't decouple for $S + P + D$.

Summary and outlook

- The moments approach simplifies the analysis by bypassing an actual fit. Entire analysis is a counting experiment.
- Robust against low statistics and fit biases. You get what you see.
- Even allows for angular analysis when a direct fit not possible.
- Validations in parallel for B2CC/EWP/SL. Method performs very well for all modes.
- May be a JINST paper summarizing all the toy studies. The data papers can refer to this.
- Next step: what optimal physics can we extract from the moments?