

Asymptotic Notations

Session No.: 2

Course Name: Analysis and Design of Algorithms

Course Code: R1UC407B

Instructor Name: Dr. Mili Dhar

Review of the key concepts of session no

1. An algorithm is a sequence of computational steps that transform input into output.
2. Step by step instructions or flowchart or pseudocode are the ways to represent an algorithm for a program.
3. pseudocode does not have a specific syntax like any of the programming languages and thus cannot be executed on a computer.

How to predict an algorithm's performance for large inputs without actually running

3/5/2025

Dr. Mili Dhar

At the end of this session students will be able to

1. Explain Asymptotic Notations (O , Ω , Θ) based on Orders of Growth
2. Apply asymptotic notations to represent best case, worst case and average case complexity.

Session Outline

3/5/2025

1. Introduction of Asymptotic Notations
2. Calculate time and space complexity
3. A hands-on approach to apply Asymptotic Notations

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Asymptotic Notation

(Algorithm's Growth Rate or Growth of Function)

- Different types of asymptotic notations are used to measure the complexity of an algorithm.

O – Big Oh (Tightly Upper Bound)

Ω – Big omega (Tightly Lower Bound)

θ – Big theta (both Lower and Upper Bound)

o – Little Oh (Strictly Upper Bound)

ω – Little omega (Strictly Lower Bound)



Example of Asymptotic Analysis

(Best suitable for very large value of n or data sets)



Price: 35 Lakh

+



Price: 10,000/-

- The simplest example is a function $f(n) = n^2 + 3n$, the term $3n$ compared to n^2 when n is very large.
- The function " $f(n)$ " is said to be asymptotically equivalent to n^2 is written symbolically as $f(n) \sim n^2$.

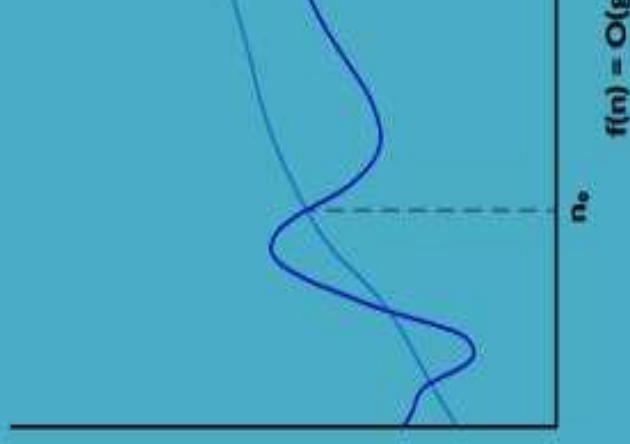
O – Big Oh Notation

- Tightly Upper Bound of an Algorithm
- Given a particular problem of size n . The time required by any algorithm for solving this problem is denoted by a function such as $f(n)$.

Lets assume, $f(n)$ and $g(n)$ are two functions

$f(n) = O(g(n))$ if there exists a positive integer n_0 and a positive constant c , such that

$$f(n) \leq c \cdot g(n) \quad \forall n \geq n_0$$



Ω – Big omega Notation

- Tightly Lower bound of the running time of an algorithm.

For a given function $f(n)$ and $g(n)$,

$f(n) \geq c \cdot g(n)$, for $n \geq n_0$; $c > 0$; then

$$f(n) = \Omega(g(n))$$



Θ – Big theta Notation

- Represents the upper and the lower bound of the running time of an algorithm
- Used for analyzing the average-case complexity of an algorithm.

For a given function $f(n)$ and $g(n)$,

$$c_1g(n) \leq f(n) \leq c_2g(n), \text{ for } n \geq n_0; c > 0; n_0 \geq 1$$

The Theta Notation is more precise than both the big-oh and Omega notation. The function $f(n) = \theta(g(n))$ if $g(n)$ is both an upper and lower bound.



o – Little Oh & ω – Little omega

Little-o Notation (Strictly Upper Bound)

$$\mathbf{f(n) = o\ g(n) \text{ if, } f(n) < c.g(n)}$$

for some constant n_0 and c .

$$f(n) = n, g(n) = n^2$$

$$\mathbf{f(n) < c.g(n) \text{ for } c=2, n \geq}$$

$$f(n) = o(g(n))$$

Little omega Notation (Strictly Lower Bound)

$$\mathbf{f(n) = \omega\ g(n) \text{ if, } f(n) > c.g(n)}$$

for some constant n_0 and c .

$$f(n) = n^2, g(n) = n$$

$$f(n) > c.g(n)$$

$$f(n) = \omega(g(n))$$



Assessment

Question -1

$F(n) = 1000n^2 + 5n^3 + 6000000n$, What is the Order of Function?

Answer : $f(n) = O(n^3)$

Question -2

$F(n) = n^5 + 500n^3 + 5000n^2 + 99999999*9999$ What is the Order of Function?

Answer : $f(n) = O(n^5)$

Question -3

$F(n) = n + 500*n + 5000*n + 50,00,000*n$ What is the Order of Function?

Answer : $f(n) = O(n)$

Write a pseudocode of swapping two numbers and find the time complexity

1. Write a pseudocode for the sum of n numbers in an array and find out time
2. Find the time complexity of linear search

Summary

Asymptotic notation describes the efficiency (time and space complexity) of algorithms as the input size grows large. It helps compare algorithms based on their growth rates.

Common Notations:

- Big-O (O): Upper bound, worst-case complexity.
- Omega (Ω): Lower bound, best-case complexity.
- Theta (Θ): Tight bound, average-case complexity.

Next Session....

We will learn about the properties of asymptotic notations