Asymptotic Properties

Session No.:3
Course Name: Analysis and Design of Algorithms
Course Code: R1UC407B
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Review of the key concepts of session no. 2

Asymptotic notation describes the efficiency (time and space complexity) of an algorithm as the input size grows large. It helps compare algorithms based on their growth rates.

Common Notations:

- Big-O (O): Upper bound, worst-case complexity.
- Omega (Ω): Lower bound, best-case complexity.
- Theta (Θ): Tight bound, average-case complexity.

How do asymptotic notations like Big-O, Omega, and Theta help in analyzing the efficiency of algorithms.?

At the end of this session students will be able to

Learning Outcome:

- Describe the properties of asymptotic notations.

Session Outline

1. Introduction to asymptotic properties

2. Different types of properties

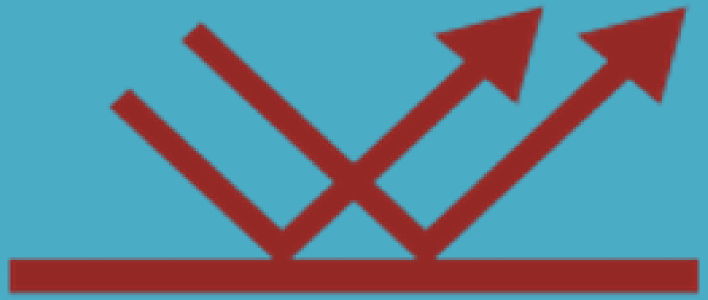
3. Solve some examples

Properties of Asymptotic Notation

1. Reflexivity
2. Symmetry
3. Transitivity
4. Transpose Symmetry
5. General Properties

Learning Activity- 1

Properties of Asymptotic Notation



1. Reflexive Properties

If f(n) is given then $\mathbf{f(n) = O(f(n))}$

Example: If $f(n) = n^3 \Rightarrow O(n^3)$

Similarly,

$$f(n) = \Omega(f(n)) \text{ or } f(n) = \Theta(f(n))$$

Learning Activity- 1

Properties of Asymptotic Notation

2. Symmetry Properties

(Valid only for Theta Notation)

If $f(n) = \Theta(g(n))$, then $g(n) = \Theta(f(n))$

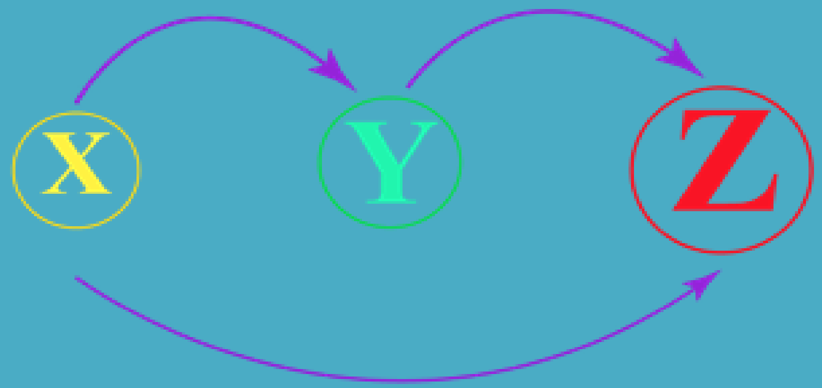
Example

If $f(n) = n^2$ and $g(n) = n^2$

then $f(n) = \Theta(n^2)$ and $g(n) = \Theta(n^2)$

Properties of Asymptotic Notation

3. Transitive Properties (for all notations)



f(n) = O(g(n)) and g(n) = O(h(n)) then

⇒ f(n) = O(h(n))

Example

If $f(n) = n$, $g(n) = n^2$ and $h(n) = n^3$

$$n<n^2<n^3$$

⇒ n is $O(n^2)$ and n^2 is $O(n^3)$ then n is $O(n^3)$

Properties of Asymptotic Notation

4. Transpose Symmetry (for Big Oh and Omega notation only)

If $f(n) = O(g(n))$ then $g(n) = \Omega(f(n))$

Example

If $f(n) = n$ and $g(n) = n^2$

then n is $O(n^2)$ and n^2 is $\Omega(n)$

Properties of Asymptotic Notation

5. General Properties

A. If f(n) = O(g(n)) or Ω (g(n) or Θ (g(n))

Then a* f(n) = O(g(n)) or Ω (g(n) or Θ (g(n))

***Example*: $f(n) = 2n^2 + 5 = O(n^2)$**

if a =5, → a* f(n) → $10n^2 + 25 = O(n^2)$

*** Valid for all notations**

Properties of Asymptotic Notation

5. General Properties

B. If $f(n) = O(g(n))$ and $f(n) = \Omega(g(n)$

Then $f(n) = \Theta(g(n))$

***Example*: $f(n) = n^2$ and $g(n) = n^2$**

$g(n) <= f(n) <= g(n)$

Properties of Asymptotic Notation

5. General Properties

C. If $f(n) = O(g(n))$ and $d(n) = O(e(n))$

Then $f(n) + d(n) = O(\max(g(n), e(n)))$

Example: $f(n) = n = O(n)$ and $d(n) = n^2 = O(n^2)$

$f(n) + d(n) = n + n^2 = O(n^2)$

Properties of Asymptotic Notation

5. General Properties

D. If $f(n) = O(g(n))$ and $d(n) = O(e(n))$

Then $f(n) * d(n) = O((g(n) * e(n)))$

Example: $f(n) = n = O(n)$ and $d(n) = n^2 = O(n^2)$

$f(n) * d(n) = n * n^2 = n^3 = O(n^3)$

Learning Activity- 2

Practice Examples

Example 1- $F(n) = n^3$ $D(n) = n^4$ $E(n) = n^6$

$T(n) = F(n) + D(n) + E(n)$

$O(T(n)) = ?$

Example 2- $F(n) = 3n^2 * n^3 + n * n^2 + 20n^2 * n^2 + 2n^2$

$O(f(n)) = ?$

Example 3- $F(n) = 3n^3 + n^{3.5} + 10n^4 + 2n^2$

$O(f(n)) = ?$

Learning Activity- 2

Practice Examples

Example 4- $F(n) = n^2 + n^4$ $D(n) = n^2$

$T(n) = F(n) * D(n)$

$O(T(n)) = ?$

Example 5- $F(n) = 3n^4 + 2n^2$

If $T(n) = 10{,}000 * F(n)$, then $O(T(n)) = ?$

Next Session....

We will learn about the Empirical analysis of algorithms