Brain-Inspired Learning Machines

From Biological Neural Networks to Artificial Neural Networks

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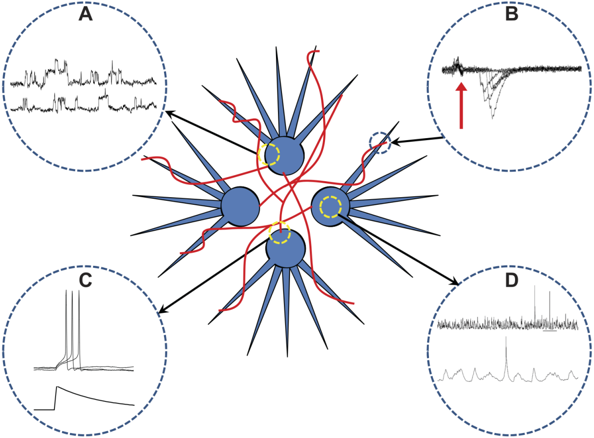
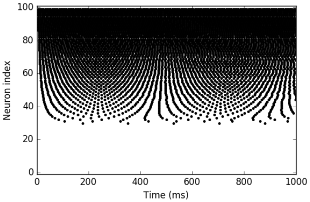


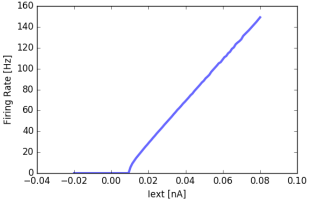
Yarom and Hounsgaard, *Physiological Reviews*, 2011

Stochastic I&F Neurons: the effect of noise on neural activations

code/brian2_activation_function.py

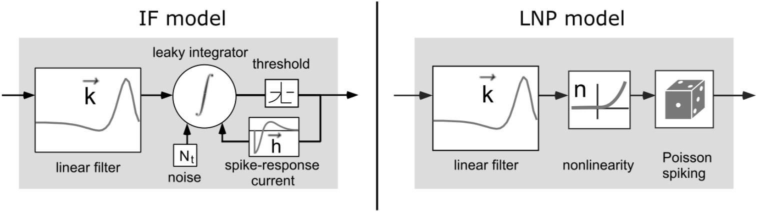
```
from brian2 import *

Cm = 50*pF; gl = 1e−9*siemens; taus = 20*ms
Vt = 10*mV; Vr = 0*mV;
sigma = 0./sqrt(ms)*mV
eqs = '''
dv/dt = − gl*v/Cm
        + sigma*xi
        + iext/Cm : volt (unless refractory)
iext : amp
'''

P = NeuronGroup(100, eqs, threshold='v>Vt', reset='v = Vr',
                refractory=0*ms, method='milstein')

P.v = Vr #Set initial V to reset voltage
P.iext = np.linspace(−.2, .8, 100)*.1*nA

s_mon = SpikeMonitor(P)

run(5.0 * second)
```





Two common approaches to modeling stochasticity in neurons:

- **Mean-field models** Solve the stochastic differential equation (SDE) of the integrate & fire neuron exactly
  - \+ Exact under some reasonable assumptions
  - \- Solution only for very special cases of the neuron dynamics
  - \- SDE's are hard!
- **Spike Response Model** Consider deterministic dynamics for the neuron and link the probability intensity of emitting a spike with a non-linear function of the state variable.
  - \+ Can be fitted to neuron (experimental) neuron models with excellent accuracy
  - \- Approximate and limited to linear differential equations
  - \+ Mathematically tractable



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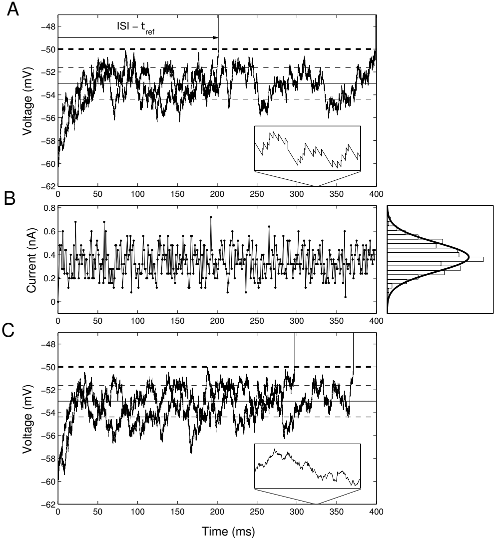
$$
\begin{aligned}
& C_m \frac{\mathrm{d}}{\mathrm{d}t} V_m = -g_L V + I_{syn} + I_{ext} + \sigma \xi(t) \\
& \text{if } V_m > V_t: \text{ elicit spike,} \\
& \qquad \text{and } V_m \leftarrow V_r \text{ during } \tau_{arp}
\end{aligned} \tag{1}
$$

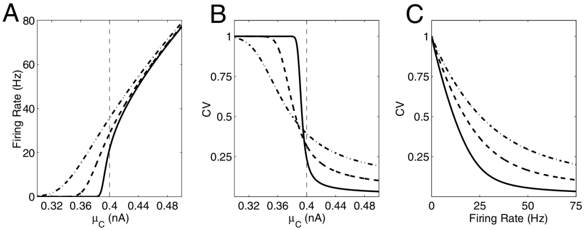
where $\xi(t) \sim N(0, 1)$ is Brownian white noise (= uncorrelated)

Diffusion Model

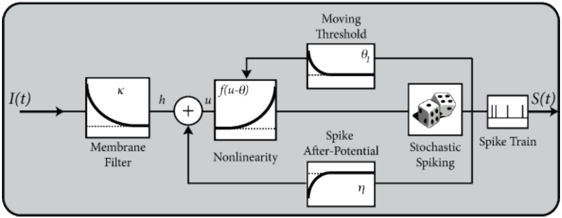


Renart, Brunel, and Wang,, 2003

$$
\begin{aligned}
\nu(I) &= \left(\tau_{ARP} + \tau_m \sqrt{\pi} \int_{\frac{V_r - V_0}{\sigma_V}}^{\frac{\theta - V_0}{\sigma_V}} \exp(x^2)(1 + \mathrm{erf}(x))\mathrm{d}x\right)^{-1} \\
V_0 &= \frac{I}{g_L} \\
\sigma_V &= \sqrt{\frac{\sigma^2}{g_L C_m}}
\end{aligned}
\tag{2}
$$



Renart, Brunel, and Wang,, 2003



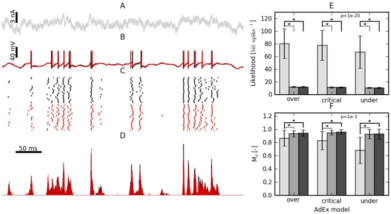
Gerstner, Kistler, Naud, and Paninski,, 2014

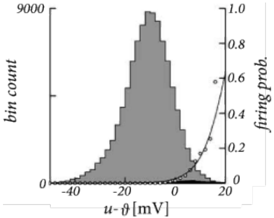
"In the Spike Response Model (SRM) the neuron model is interpreted in terms of a membrane filter (κ) as well as a function describing the shape of the spike (η)"

$$V_m(t) = V_r + \int_0^\infty \kappa(s) I(t-s)\mathrm{d}s + \underbrace{\sum_{\{t_j\}} \eta(t-t_j)}_{\text{effect of self-spiking}} \tag{3}$$

$$\nu(t) = f(V_m(t) - V_t) \quad \text{(firing rate)}$$

f is called a linking function





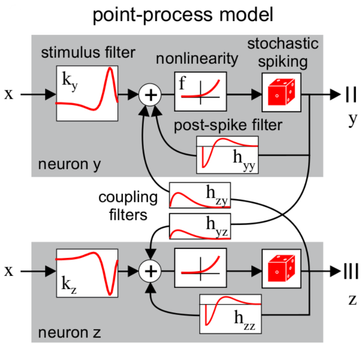
Exponential linking function:

$$f(V_m(t) - V_t) \propto \exp(\beta(V_m(t) - V_t))$$

$$P(spike \in [t, t + \Delta t]|V_m) \cong f(V_m(t) - V_t)\Delta t$$

Jolivet, Rauch, Lüscher, and Gerstner, *Journal of computational neuroscience*, 2006

Mensi, Naud, and Gerstner, *Advances in Neural Information Processing Systems*, 2011



$$P(spike \in [t, t + \Delta t] | V_m[t]) \cong \exp(\beta(V_m[t] - V_t))\Delta t$$

Simulating an LNP model (assuming α and β known)

- Compute $V_m[t]$ using SRM filters
- Compute $P[t] = \alpha \exp(\beta(V_m[t] - V_t))\Delta t$
- Spike if $u < P[t]$, where $u \in Uniform[0, 1]$ (= flip a biased coin)