

Navigation Algorithm

Get the relationship between Bezier curve and the velocity of spaceship:

$$\begin{aligned}
 B(t) &= t^3 \vec{P}_4 + 3t^2(1-t) \vec{P}_3 + 3t(1-t)^2 \vec{P}_2 + (1-t)^3 \vec{P}_1 \\
 V_X &= \frac{dX_{B(t)}}{dt} = \dots \\
 V_Y &= \frac{dY_{B(t)}}{dt} = \dots \\
 V &= \sqrt{V_X^2 + V_Y^2} = \sqrt{\left(\frac{dX_{B(t)}}{dt}\right)^2 + \left(\frac{dY_{B(t)}}{dt}\right)^2}
 \end{aligned}$$

Get the relationship between handles and the velocity of spaceships:

$$\begin{aligned}
 |P_1 - P_2| &= kV_1 \\
 |P_3 - P_4| &= kV_2
 \end{aligned}$$

Since V_2 is the velocity of spaceships when $t = 1$,

$$\begin{aligned}
 \left. \frac{dX_{B(t)}}{dt} \right|_{t=1} &= 3t^2 X_4 + [6t(t-1) + 3t^2] X_3 + [3(t-1)^2 + 3t \times 2(t-1)] X_2 + 3(t-1)^2 X_1 \\
 &= 3X_4 + 3X_3 \\
 \left. \frac{dY_{B(t)}}{dt} \right|_{t=1} &= 3Y_4 + 3Y_3
 \end{aligned}$$

So,

$$\begin{aligned}
 V_2 &= \sqrt{(3X_4 + 3X_3)^2 + (3Y_4 + 3Y_3)^2} \\
 &= \sqrt{9(X_4 + X_3)^2 + 9(Y_4 + Y_3)^2} \\
 &= 3\sqrt{(X_4 + X_3)^2 + (Y_4 + Y_3)^2} \\
 &= 3\sqrt{(X_4 - (-X_3))^2 + (Y_4 - (-Y_3))^2} \\
 &= 3|P_4 - (-P_3)| \\
 &= 3|P_4 + P_3|
 \end{aligned}$$

therefore,

$$k = \frac{|P_3 - P_4|}{3|P_4 + P_3|}$$