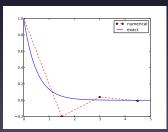
On Schemes for Exponential Decay

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Goal

The primary goal of this demo talk is to demonstrate how to write talks with Sociones and get them rendered in numerous HTML formats.

Layout.

This version utilizes beamer slides with the theme dark_gradient.

Mathematical problem

$$u'(t) = -au(t),$$
 (1)
 $u(0) = I,$ (2)

- $t \in (0, T]$
- *a*, *I*, and *T* are prescribed parameters
- u(t) is the unknown function



Numerical solution method

- Mesh in time: $0 = t_0 < t_1 \cdots < t_N = T$
- Assume constant $\Delta t = t_n t_{n-1}$
- u^n : numerical approx to the exact solution at t_n

Numerical scheme

$$u^{n+1} = \frac{1 - (1 - \theta)s\Delta t}{1 + \theta s\Delta t}u^n, \quad n = 0, 1, ..., N - 1$$

Numerical solution method

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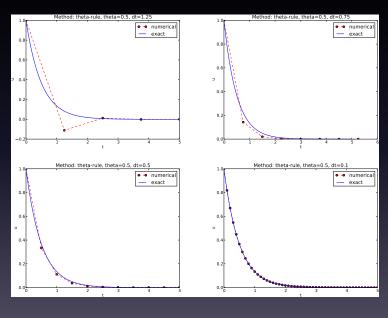
Forward Euler explained

http://youtube.com/PtJrPEIHNJw

Implementation

The numerical method is implemented in a Python function:

The Crank-Nicolson method



Exact solution of the scheme

$$u^n = A^n$$
, $A = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}$.

- Stability: |A| < 1
- No oscillations: A > 0
- Always for Backward Euler ($\theta=1$)
- $\Delta t \leq 1/a$ for normalic Euler (v=v)
- $\Delta t < 2/a$ for Crank-Nicolson ($\theta = 1/2$)

Concluding remarks:

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