

# How to improve the estimation of power curves for wind turbines

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## Abstract

We introduce a dynamical approach for the determination of power curves for wind turbines and compare it with two common methods—among them the standard procedure due to IEC 61400-12-1, i.e. the international standard prepared and published by the International Electrotechnical Commission. The main idea of the new method is to separate the dynamics of a wind turbine's power output into a deterministic and a stochastic part, corresponding to the actual behaviour of the wind turbine and external influences such as the turbulence of the wind, respectively. In particular, the governing coefficients are reconstructed from the data, and the power characteristic is extracted as the stationary states of the deterministic behaviour. Our results prove that a dynamical approach enables one to grasp the actual conversion dynamics of a wind turbine and to gain most accurate results for the power curve, independent of site-specific influences.

**Keywords:** wind energy conversion, power curve, dynamical approach, stochastic process, Langevin equation, short-time fluctuations

## 1. Introduction

With increasing importance of wind as a sustainable energy source and, consequently, the spreading of wind turbines as wind energy converter systems (WECS), wind energy research has become a new orientation. Physical findings have to be combined with technical know-how. An improved understanding of the performance of a WECS, as well as of e.g. the loads affecting certain parts of a wind turbine, can only be achieved if we comprehend its fuel, the wind, in more detail.

In the centre of our research stands the assessment of the wind turbine's power performance, i.e. the description of the wind energy conversion process. The so-called power curve, the electric power output versus the wind speed, summarizes the technical characteristics of the whole turbine and is approved to be the most important testimony for the wind turbine's performance in the view of the operator. The shape of a power curve is governed by the cubic relation between the wind speed and the corresponding power in the wind up to the so-called rated wind speed where the power takes a constant

value until the turbine cuts out. The actual energy density of the wind is reduced by the energy not usable due to a physical limit and additional losses due to several technical characteristics (see e.g. [1]). The correct and effective determination of these effects is of central interest.

A standard method for the determination of wind turbine power curves is given by [2], referred to as IEC 61400-12-1. This norm is an averaging procedure, easily applicable if enough data are available. The resulting IEC curve is the common tool to estimate a wind turbine's energy yield. However, it cannot be used to display e.g. the short-time fluctuations of the electrical power output induced by turbulent wind conditions or to explain the orographic dependences of a wind turbine's performance.

Recently, an alternative method for the estimation of wind power characteristics has been established in [3, 4]. The main idea is to reconstruct the short-time dynamics of the power conversion process and to determine the power curve as the stationary states or fixed points of this process—these stationary results represent nothing else than the ideal performance for non-fluctuating laminar wind conditions.

Therefore, we have introduced the term dynamical power curve. In detail, the dynamics of the WECS is considered as a Markovian stochastic process, and the wind speed is interpreted as a noisy driving force. The crucial point of this method is to divide the dynamics into a deterministic and a stochastic part. The stochastic part, given by dynamical noise, summarizes all the otherwise unseizable microscopic interactions and enables a macroscopic description of the considered open complex system. I.e. it also takes into account that the scalar wind speed, measured at the met mast, cannot represent the complete wind field actually acting on the wind turbine. Following Haken [5], this can be regarded as a synergetic approach to complexity. For details about the general method and other applications see [6].

The aim of this paper is to show the advantages of the recently introduced method of the dynamic power curve estimation, comparing it with two other determination methods, namely the common IEC standard and the maximum principle proposed by Rauh *et al* [7], and going back to certain examples of measured and simulated data. Therefore, we first introduce the three considered methods in more detail, perform then the comparison of methods and conclude with a short discussion.

## 2. Dynamical power curve

The dynamical power curve approach is based on highly sampled data in order to reconstruct the actual process dynamics—for more details on the conversion dynamics on small timescales see [8]. Its basic idea is to describe the electrical power output  $P$  of the wind turbine as a diffusion process, i.e. a stochastic process that satisfies the Markovian property and that can be separated into a drift and a diffusion part. Then, a typical time series can be presented as  $P(t) = P_{\text{stat}}(u) + p(t)$ , where  $P_{\text{stat}}$  denotes a stationary power value dependent on the wind speed  $u$  (or rather a non-fluctuating stationary or mean value of  $u$ , not further specified here), and  $p(t)$  refers to the corresponding short-time fluctuation around this value caused by the wind turbulences and the response of the WECS to these. The time series  $P(t)$  is assumed to be stationary with respect to a certain wind speed interval. Therefore, the dynamics of  $P(t)$  is analysed for each selected wind speed interval or bin separately. (Note that the mapping of the power values  $P(t)$  to the single wind speed bins is defined by the actual wind speed  $u(t)$ . A split-up of the wind speed into a mean value and short-time fluctuations is not considered here. Fluctuations are analysed in terms of increments. In the end, the reconstructed value of  $P_{\text{stat}}$  is mapped to the average of all values  $u(t)$  lying in the corresponding bin.)

For the evolution in time of the variable  $P$  we formulate a Langevin equation

$$\frac{d}{dt}P(t) = D^{(1)}(P; u) + \sqrt{D^{(2)}(P; u)}\Gamma(t). \quad (1)$$

$D^{(1)}$  is called drift coefficient and represents the deterministic part of the process, whereas the diffusion coefficient  $D^{(2)}$  together with the Langevin force  $\Gamma(t)$  representing

$\delta$ -correlated Gaussian white noise ( $\langle \Gamma(t) \rangle = 0$  and  $\langle \Gamma(t_1)\Gamma(t_2) \rangle = 2\delta(t_1 - t_2)$ ) describes its stochastic part. The units of  $D^{(1)}$  and  $D^{(2)}$  are  $[P] \text{ s}^{-1}$  and  $[P]^2 \text{ s}^{-1}$ , respectively, with  $[P]$  as the unit of  $P$ . The unit of  $\Gamma(t)$  arises from  $\Gamma(t) dt = dW(t)$  and  $dW(t)^2 = dt$ , where  $dW(t)$  is a Wiener process (see [9]). A simple relaxation model, as proposed in [7], would follow the equation

$$\frac{d}{dt}P(t) = -\alpha [P(t) - P_{\text{stat}}(u)] + \sqrt{\beta}\Gamma(t), \quad (2)$$

where  $\alpha$  is a constant relaxation factor that quantizes a relaxation around the stationary value  $P_{\text{stat}}$  driven by the wind speed variation and  $\beta$  the strength of additive dynamical noise.

A reconstruction of this dynamics enables the estimation of the values  $P_{\text{stat}}(u)$  and with it the determination of the power characteristic. Following [10], the coefficients  $D^{(n)}$  for  $n = 1, 2$  are given by the conditional moments  $M^{(n)}$  that can be directly calculated from the data according to

$$D^{(n)}(P; u) = \frac{1}{n!} \lim_{\tau \rightarrow 0} \frac{1}{\tau} M^{(n)}(\tau, P; u) \quad (3)$$

with

$$M^{(n)}(\tau, P; u) := \langle [P(t + \tau) - P(t)]^n \rangle |_{P(t)=P} \quad (4)$$

where  $\langle \dots \rangle$  denotes the ensemble average and  $|_{P(t)=P}$  the condition that the stochastic variable  $P(t)$  is in the state  $P$  at time  $t$ .

For small and finite  $\tau$ , we make the approximation

$$M^{(n)}(\tau, P; u) \approx n! \tau D^{(n)}(P; u) + \mathcal{O}(\tau^2), \quad (5)$$

applying an Itô–Taylor series expansion. Depending on the process it might be necessary to consider further higher-order terms. An exact derivation is given in [11].

If additional measurement noise is present,  $D^{(n)}$  is best estimated by the extrapolation

$$D^{(n)}(P; u) = \frac{1}{n!} \frac{M^{(n)}(\tau_2, P; u) - M^{(n)}(\tau_1, P; u)}{\tau_2 - \tau_1} \quad (6)$$

for suitable  $\tau_1$  and  $\tau_2$  or a corresponding linear fit [12].

Corresponding errors are calculated according to [13] with

$$\sigma [D^{(1)}(P, \tau; u)] = \sqrt{\frac{2}{\tau} \frac{D^{(2)}(P, \tau; u)}{N} - \frac{[D^{(1)}(P, \tau; u)]^2}{N}} \quad (7)$$

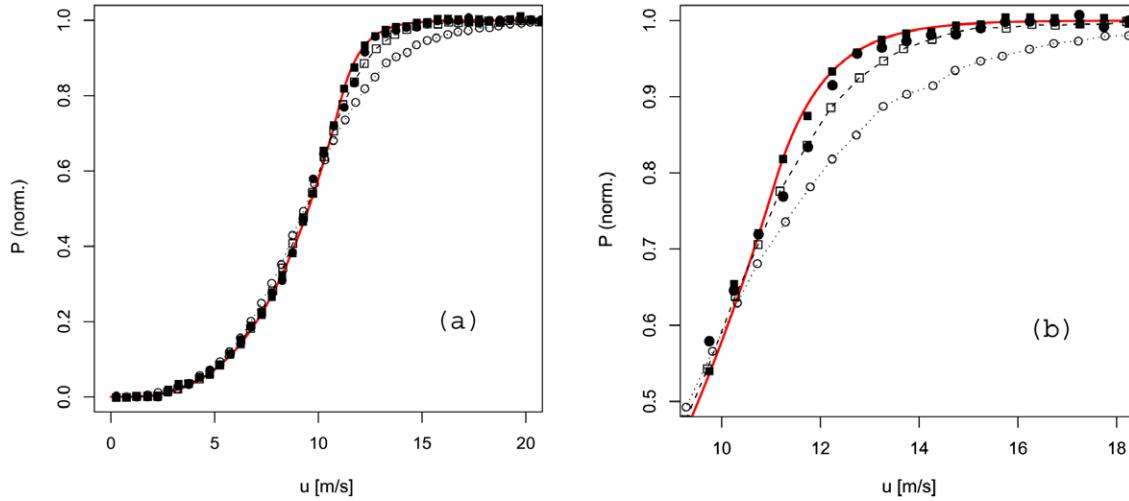
and for  $D^{(2)}$  similar, considering a finite time increment  $\tau > 0$  and a finite number of data points  $N$  in the bin.

Finally, a deterministic fixed-point analysis for each speed bin yields the stationary points  $P_{\text{stat}}(u)$ , defined by  $D^{(1)}(P; u) \equiv 0$ , and with it the dynamic power curve.

## 3. Comparison with other determination methods

### 3.1. The IEC standard

In short, the standard method according to IEC 61400-12-1 (in the revised version, see [2]) is characterized by relating



**Figure 1.** Reconstructed power curves for simulated data, (a) complete curve and (b) cutout, for different values of turbulence intensity. Results according to IEC standard are given by open symbols, stationary states according to dynamical method by full symbols (rectangle for  $\zeta = 0.10$  and circle for  $\zeta = 0.20$ ). The solid line (red) denotes the exact power curve, i.e. the input characteristic for our model.

the averages of wind speed and power output over 10 min, i.e.  $\langle u(t) \rangle_{10 \text{ min}} \mapsto \langle P(t) \rangle_{10 \text{ min}}$ , and averaging in a second step all values lying in a wind speed bin of the width of normally  $0.5 \text{ m s}^{-1}$ . Already in [14] it has been argued that this procedure does not account for the nonlinearity of the power characteristic. Expressing the wind speed as  $u(t) = V + v(t)$  where  $V$  is the mean value, i.e.  $V = \langle u(t) \rangle$ , and  $v(t)$  the corresponding short-time fluctuation around this value with  $\langle v(t) \rangle = 0$ , the power output  $P$  as a function of  $u$  can be expanded in the Taylor series

$$P(u) = P(V) + \frac{\partial P(V)}{\partial u}v + \frac{1}{2} \frac{\partial^2 P(V)}{\partial u^2}v^2 + \mathcal{O}(v^3). \quad (8)$$

Assuming that the fluctuations  $v(t)$  are symmetric around  $V$ , which is in general questionable for experimental data (see the discussion in section 3.3), and neglecting the terms  $\mathcal{O}(v^3)$ , one obtains for the averaged power output

$$\langle P(u) \rangle = P(V) + \frac{1}{2} \frac{\partial^2 P(V)}{\partial u^2} \cdot \sigma^2 \quad (9)$$

with  $\sigma^2 = \langle v^2(t) \rangle$ .

It follows  $\langle P(u) \rangle \neq P(\langle u \rangle)$  for a nonlinear function  $P(V)$  and non-vanishing  $\sigma$ . Defining the turbulence intensity  $\zeta = \sigma/V$ , one rather finds a correction term proportional to  $\zeta^2$  for  $P(V)$  known. The inequality above indicates that symmetric wind speed fluctuations, as they are assumed, are transferred to asymmetric power fluctuations due to the nonlinearity of the power curve. It follows that a linear averaging procedure does not give exact results.

To illustrate this discrepancy, we simulated wind speed and power output data, using a simple relaxation model as given by (2) with typical parameter values we obtained from the analysis of measured data. We performed the simulation for different values of turbulence intensity, and reconstructed the power characteristic according to both the IEC standard procedure and the dynamical method. Results are shown in figure 1.

The systematic deviations of the results following the IEC recommendations from the real power curve, i.e. the input characteristic for our model, are evident. Even though the result of the dynamical method is not affected by the mentioned averaging problem, there is similarly a small discrepancy. This is due to a non-stationarity of the data induced by the predetermined wind speed binning, and increases with the turbulence intensity. However, it is much smaller than the errors of the IEC method, and for our set of measured data (see section 3.3.) it is negligible.

### 3.2. Maximum principle according to Rauh

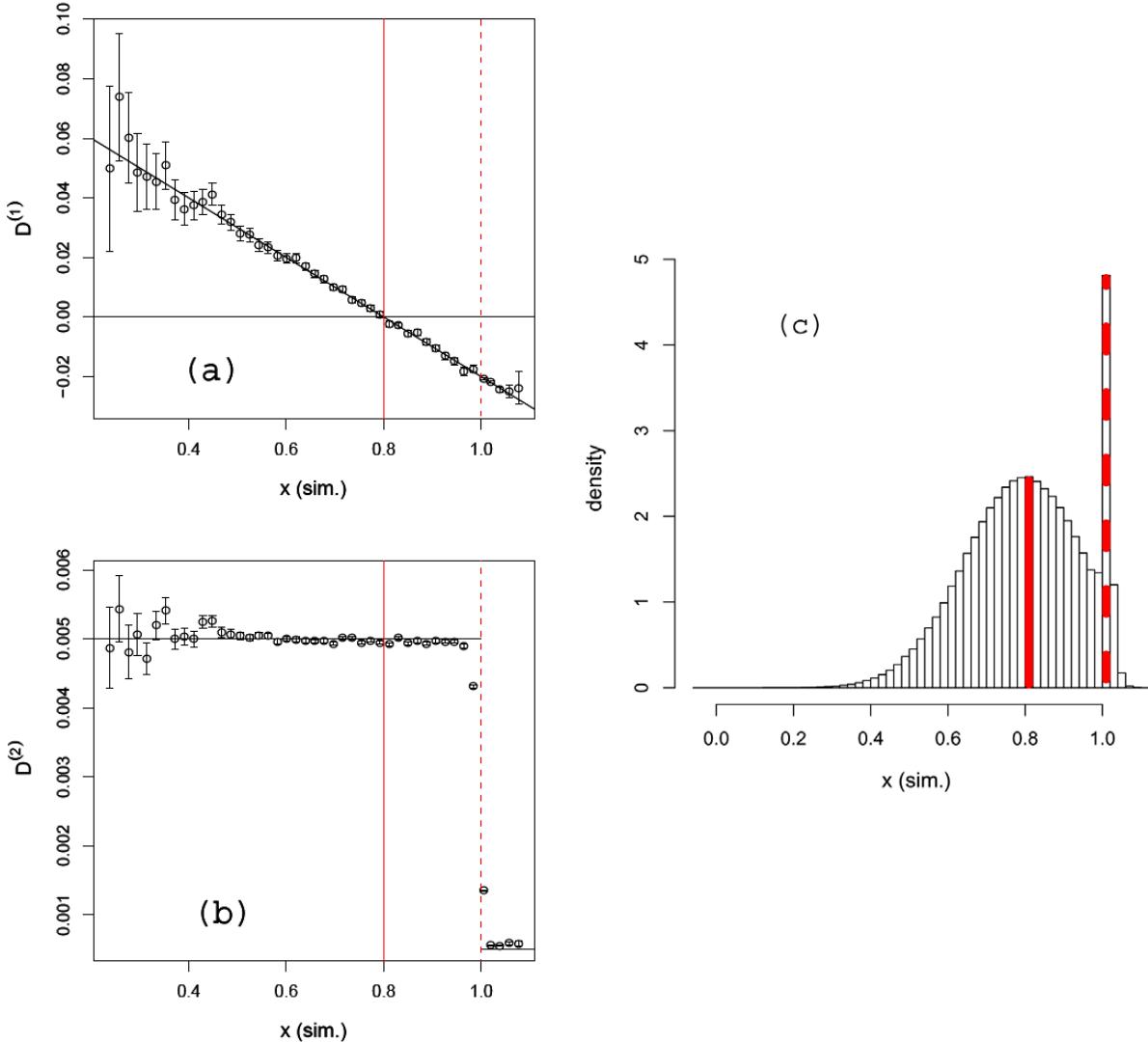
Rauh *et al* propose in [7] an even simpler method to determine the power curve for a wind turbine. The idea is to define an empirical power curve by the location where, in a given wind speed bin, the maximal density of points  $P(t_i)$  is found. I.e. the power curve is given by the points  $\{u_j, P_{k(j)}\}$ , where  $j$  is the number of the speed bin and  $k(j)$  denotes the power bin with

$$N_k := \sum_i \Theta(P(t_i) - P_k) \Theta(u(t_i) - u_j) \quad \text{and} \quad N_{k(j)} \geq N_k, \quad (10)$$

where  $\Theta(x)$  is a Heaviside function defined by

$$\Theta(x) = \begin{cases} 1 & \text{if } -\Delta/2 \leq x < \Delta/2 \\ & \text{with the particular bin width } \Delta \\ 0 & \text{else.} \end{cases} \quad (11)$$

In words, we determine for each power bin  $k$  the number of events  $N_k$ , counting the points  $P(t_i)$  lying in the respective bin. The largest number of points is denoted by  $N_{k(j)}$ , where  $k(j)$  is the sought bin with the extremal property and, correspondingly,  $P_{k(j)}$  the point of the power curve. (The values  $u_j$  and  $P_k$  can be defined either by the middle of the bin or by the mean value of the points lying in it.) Rauh *et al* argue that this extremal property is expected if the power curve is an attractor.



**Figure 2.** Simulated data and reconstruction of its dynamical coefficients to show the weakness of Rauh's maximum principle—(a) drift coefficient  $D^{(1)}(x)$ , (b) diffusion coefficient  $D^{(2)}(x)$  and (c) histogram of data. The fixed point is marked with a solid line, the maximum with a dashed line, both in red.

To show the weakness of this method, we assume again that  $P(t)$  can be described by a diffusion process and follows an equation as given by (1). As alternative to the Langevin equation, a diffusion process can also be described by a Fokker–Planck equation (see [10])

$$\begin{aligned} \frac{\partial p(P, t)}{\partial t} &= \left\{ -\frac{\partial}{\partial P} D^{(1)}(P) + \frac{\partial^2}{\partial P^2} D^{(2)}(P) \right\} p(P, t) \\ &= -\frac{\partial}{\partial P} S(P, t) \end{aligned} \quad (12)$$

$D^{(1)}(P)$  and  $D^{(2)}(P)$  are the coefficients defined in (1), the additional parameter  $u$  is omitted here. The specific kinds of description by a Fokker–Planck equation and a Langevin equation are equivalent. While the Langevin equation is a stochastic differential equation for the state variable and describes the actual trajectory of this variable, the Fokker–Planck equation is a partial differential equation for the probability density  $p(P, t)$  of the state variable evolving in

time. For  $S = \text{const}$ , we obtain the stationary solution

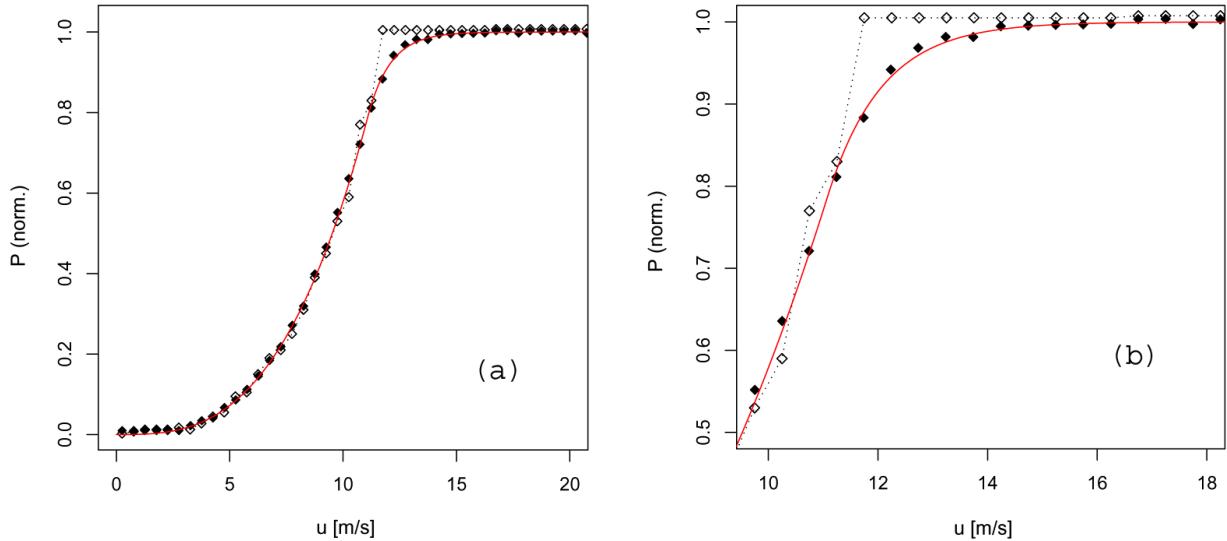
$$p_{\text{stat}}(P) = \frac{\mathcal{N}}{D^{(2)}(P)} \exp \left( \int^P \frac{D^{(1)}(\tilde{P})}{D^{(2)}(\tilde{P})} d\tilde{P} \right) \quad (13)$$

with the normalization constant  $\mathcal{N}$ .

The procedure is now to investigate under which conditions the point of maximal density  $P_{\max}$  is equal to the stationary state  $P_{\text{stat}}$ , i.e. the dynamical power curve equals the power curve according to Rauh's maximum principle. For this purpose, we differentiate  $p_{\text{stat}}(P)$  with respect to  $P$  and set it to zero. Since the exponential term is always  $> 0$  and assuming that the lower integration term vanishes, we end up with

$$\frac{\mathcal{N}}{(D^{(2)}(P_{\max}))^2} \left[ D^{(1)}(P_{\max}) - \frac{\partial}{\partial P} D^{(2)}(P_{\max}) \right] = 0. \quad (14)$$

Because  $D^{(2)}(P) > 0$  essentially,  $P_{\max}$  has to fulfil  $D^{(1)}(P_{\max}) = \partial/\partial P\{D^{(2)}(P_{\max})\}$ . The definition of the



**Figure 3.** Reconstructed power curves for simulated data, (a) complete curve and (b) cutout. Results according to Rauh's maximum principle are given by open symbols and dotted line, stationary states according to dynamical method by full symbols. The solid line (red) denotes the exact power curve, i.e. the input characteristic for our model. The turbulence intensity of the simulated wind speed data is 0.10.

stationary fixed point provides  $D^{(1)}(P_{\text{stat}}) \equiv 0$ . That means that the point of maximal density  $P_{\max}$  equals only the stationary state if its derivative with respect to  $P$  vanishes, i.e. for  $D^{(2)}(P) = \text{const}$  or if  $D^{(2)}(P)$  has an extremum for  $P_{\text{stat}}$ . We call this type of dynamical noise ideal noise. Consequently, if a diffusion function  $D^{(2)}$  does not satisfy this condition, the corresponding noise is called non-ideal.

The consequences for the reconstructed points of the power curve are, first, to be demonstrated with a one-dimensional example of simulated data, see figure 2. We integrated the Langevin equation for a process with  $D^{(1)}(x) = -0.1x + \text{const}$ , and a step function for  $D^{(2)}(x)$  as realization of non-ideal noise. This behaviour is motivated by our observations for experimental data (see section 3.3). The original functions as well as the reconstructed points, according to (3)–(7), are shown in figures 2(a) and (b). Looking at the probability density function, shown in figure 2(c), we find that the point of maximal density does not equal the actual fixed point of relaxation, but is affected essentially by the shape of  $D^{(2)}$ . The reconstruction of  $D^{(1)}$ , however, is not influenced and provides the correct result for the fixed point. Large errors of the reconstructed coefficients for small  $x$  are due a small amount of data (see histogram)—errors are proportional to  $1/\sqrt{N}$ , where  $N$  is the number of considered data points.

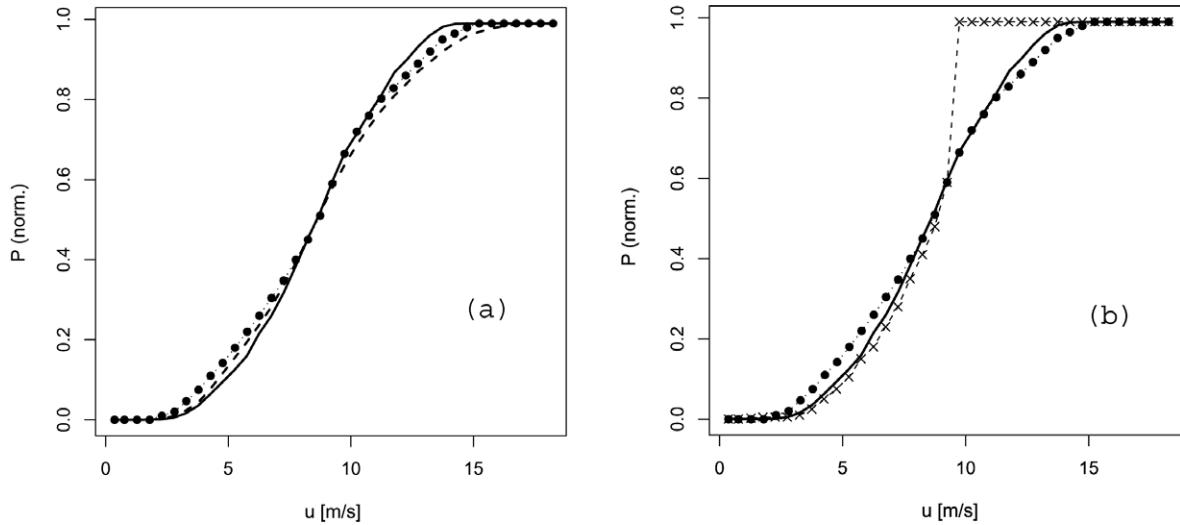
To transfer this one-dimensional example to a two-dimensional model for the power characteristic, we use again the relaxation model given by (2), and replace the constant diffusion coefficient  $\beta$  by a step function as shown in figure 2(b) with a decrease of diffusion strength to one tenth behind the step. As shown in figure 3, Rauh's maximum principle overestimates the points of the power curve in the region of transition to the rated power, and seems overall to be less accurate than the dynamical method.

### 3.3. A look at real data

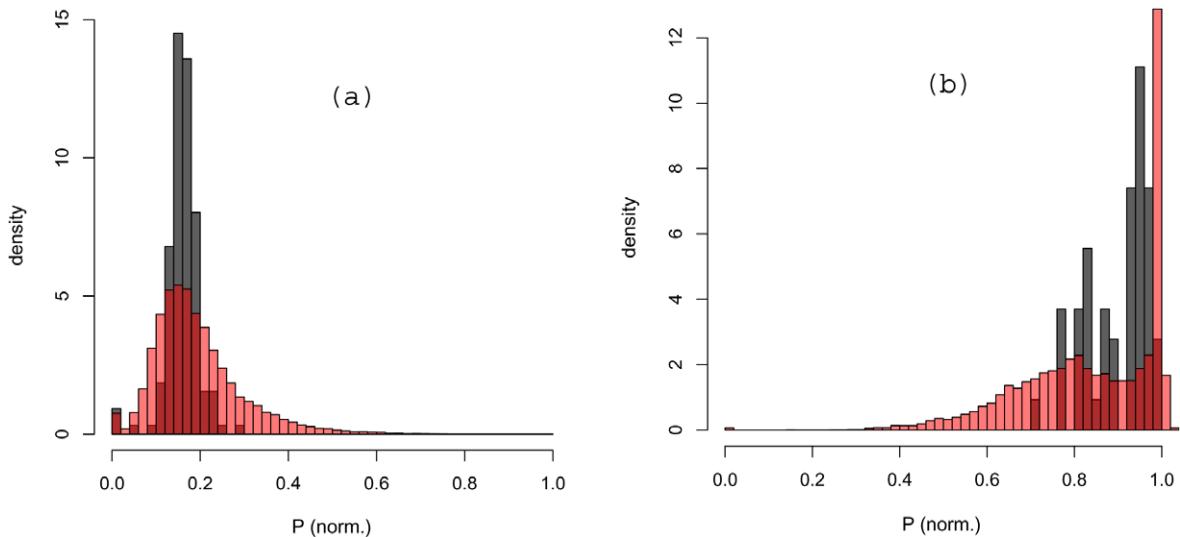
To show the actual impact of the effects explained in the last two subsections, we refer to a set of measured wind speed and power output data of a wind turbine. The data were obtained from a commercial MW-class turbine, located in a wind park in the mid-western part of Germany. The wind speed data were recorded by an ultrasonic anemometer placed on a met mast, satisfying the requirements stated in the IEC 61400-12-1 norm. The analysed data set consists of approximately 1800 000 data points, the averaged turbulence intensity of the wind speed data is 0.135.

The sampling frequency of the data is 1 Hz, defined by the recording of the power data. Our analyses showed that it is sufficient to reconstruct the drift coefficient, following (4)–(6), but too low to capture the diffusion coefficient accurately. As remarked in [11], the estimated diffusion coefficient in particular has to be corrected for finite or not sufficiently small values of  $\tau$  to get the correct functional relation  $D^{(2)}(P)$ , depending otherwise also on the drift coefficient  $D^{(1)}(P)$ .

At first, we compare the power curve estimated due to the IEC 61400-12-1 standard method with the reconstructed fixed points. Applying both methods to the measured data, we do not observe the expected results according to 3.1 (see figure 4(a) in comparison to figure 1). The deviations are rather directly opposed—for small wind speeds and a positive curvature of the power curve the fixed points lie above the IEC curve, for large wind speeds up to the rated value and a negative curvature they lie below. In figure 4(a), we have additionally compared the IEC curve with a power curve that is obtained by simply averaging the high-frequency data in each wind speed bin instead of taking the average values over 10 min. The result of this alternative approach fits much better with our expectations. At least for high wind speeds, the influence of the nonlinearity on the averaging procedure can be clearly seen in comparison to the fixed points. From this we conclude that there is another



**Figure 4.** Schematic comparison of power curves due to IEC standard (solid line), reconstructed fixed points (full symbols) and (a) bin averages of 1 Hz data (dashed line), respectively, (b) maxima according Rauh's method (crosses) for measured data.



**Figure 5.** Histograms of power output data for the bins (a)  $u = (5.75 \pm 0.25) \text{ m s}^{-1}$  and (b)  $u = (12.25 \pm 0.25) \text{ m s}^{-1}$ . Distribution of average values over 10 min in dark grey, and of the high-frequency data (red) behind.

aspect, in addition to the nonlinearity of the power curve, that influences the results due to the IEC standard substantially. In figures 5(a) and (b), we have compared the distribution of the original power data sampled at 1 Hz to the distribution of the averages over 10 min (binned according to the corresponding wind speed averages) for two different values of wind speed. The histograms for the high-frequency data are in both cases highly asymmetric. For low wind speeds, there is an additional peak at the left side of the spectrum, for high wind speeds at the right side—corresponding to the cut-in and rated values of the power curve. The averaging procedure seems to cut off the part of the other side of the distribution in each case. Hence, the mean value of the averages over 10 min is shifted to the peak value.

To inspect the application of Rauh's maximum principle, we estimated the probability distribution of the highly sampled

power output data for each single wind speed bin, as exemplarily shown in figures 5(a) and (b). For wind speeds in the range of the rated value, the histograms have basically the same shape as the one for the simulated data in figure 2 (see figure 5(b)). The additional peaks result in a distinctive kink for the whole power curve, that is not present for the other approaches (see figure 4(b)).

For the simulated data, the additional peak in the histogram has been traced back to non-ideal noise, implemented as a step function for the diffusion coefficient. Our observations let us suppose that similar dynamics underlie the measured data. An explanation for this specific diffusion dynamics are the control mechanisms of the wind turbine, especially at the point of transition to the rated power. As stated above, we could not investigate the diffusion function  $D^{(2)}$  in more detail due to the (at least for this analysis) too low sampling

frequency of the data. But we can conclude that  $D^{(2)}$  captures the transfer of the wind fluctuations to the short-time fluctuations of power output, while the drift coefficient  $D^{(1)}$  describes the response behaviour of the wind turbine. In other words, the dynamics of the system is separated into a fast and a slow varying part, and both parts provide valuable information on the complex system wind turbine.

#### 4. Conclusions and discussion

To describe the performance of a WECS in an appropriate manner, that is the main conclusion of our investigations, it is essential to grasp the actual dynamics of the process. As demonstrated, the location of both the maximum and the mean value is substantially influenced by the shape of the distribution of data. Simple sample statistics seems not to be sufficient to capture its specific characteristics; a detailed analysis of the underlying dynamics and a more flexible definition of a characteristic stationary state is rather necessary. Therefore, we have introduced a so-called dynamical approach, estimating this stationary state or fixed point by extracting the actual deterministic dynamics of the wind turbine. Stochastic influences are handled as noise and separated from this information. The dynamical approach corresponds to a generic procedure, i.e. it is flexible in application. It not only promises more accurate results than the two other presented methods. Beyond the stationary states of the process, i.e. the power curve, it provides additional relevant information, namely the complete drift characteristic, and potentiates e.g. a detailed analysis of the short-time dynamics of the WECS (see [8]). Just for this reason, the dynamical method is of another type than the IEC standard and cannot replace it without further adjustment. Instead of mean values we estimate fixed points, which has essential implications on the application of the resulting power curves.

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