GENERAL ASSEMBLY

INTRODUCTION TO PROBABILITY

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DATA SCIENCE PROCESS

1. Define problem.
2. Gather data.
3. Explore data.
4. Model with data.
5. Evaluate model.
6. Answer problem.

DEFINITIONS

- **Experiment**: A procedure that can be repeated infinitely many times and has a well-defined set of outcomes.

- **Event**: Any collection of outcomes of an experiment.

- **Sample Space**: The set of all possible outcomes of an experiment, denoted $\mathcal{S}$.

EXAMPLES

- Experiment: Flip a coin twice.
- Sample Space $\mathcal{S}$:
- Event:

- Experiment: Rolling a single die.
- Sample Space $\mathcal{S}$:
- Event:

DEFINITIONS

- **Set**: An unordered collection of distinct objects.
  - $\{Derek\ Jeter, \pi, ☺\}$

- **Element**: An object that is a member of a set.
  - Derek Jeter
  - π
  - ☺

Example taken from Justin Gash's MAT 181 course at Franklin College.

SET OPERATIONS

- **Union**: $A \cup B =$ the set of elements in A or B
- **Intersection**: $A \cap B =$ the set of elements in A and B
- Example:
  - $A =$ even numbers between 1 and 10 $= \{2,4,6,8\}$
  - $B =$ prime numbers between 1 and 10 $= \{2,3,5,7\}$

  - $A \cup B =$

  - $A \cap B =$

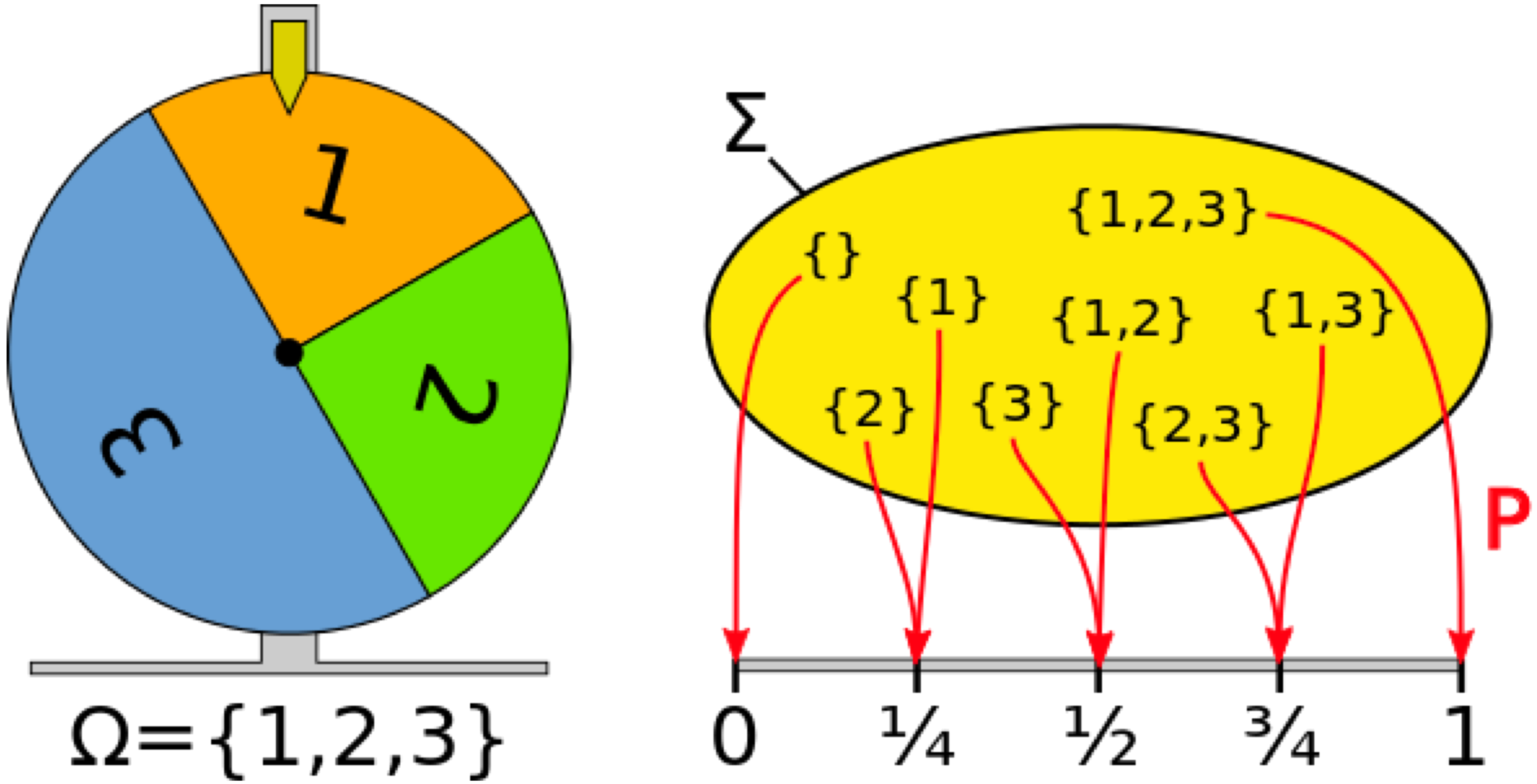
PROBABILITY – PRACTICE

- A = “a U.S. birth results in twin females”
- B = “a U.S. birth results in identical twins”
- C = “a U.S. birth results in twins”

- In words, what does $P(A \cap C)$ mean?

- In words, what does $P(A \cap B \cap C)$ mean?

PROBABILITY BASICS



https://commons.wikimedia.org/wiki/File:Probability-measure.svg

PROBABILITY RULES

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  - Venn diagrams can help to illustrate this – but remember that Venn diagrams are not proofs!
  - If A and B are disjoint, then $P(A \cap B) = 0 \Rightarrow P(A \cup B) = P(A) + P(B)$.

PROBABILITY RULES

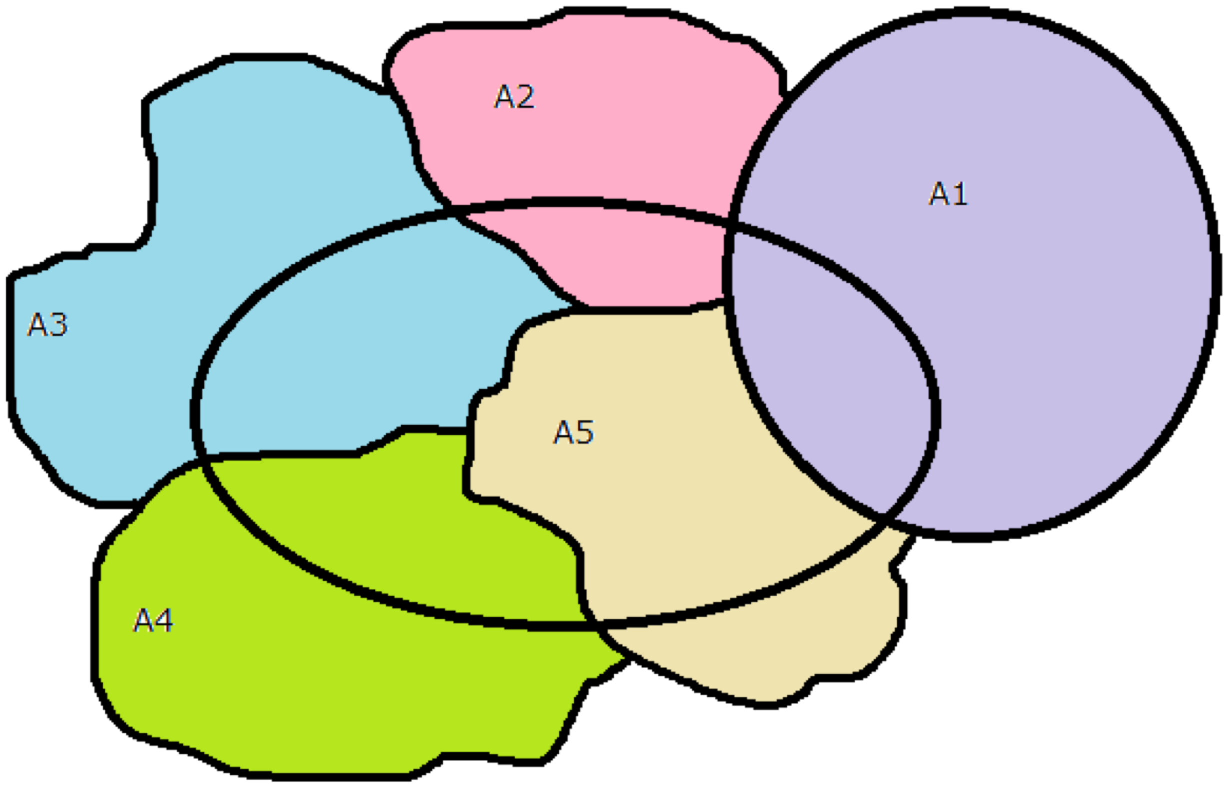
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
  - Note: $A|B$ means "A given B" or "A conditional on the fact that B happens."

PROBABILITY RULES

- $P(A \cap B) = P(A|B)P(B)$
  - We just took the last rule and multiplied both sides by $P(B)$.
  - We can rearrange these, as well! $P(B \cap A) = P(B|A)P(A)$
  - This isn't limited to two events: $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$

PROBABILITY RULES

- $P(B) = \sum_{i=1}^{n} P(B \cap A_i)$
  - "Law of Total Probability"



PROBABILITY RULES – SUMMARY

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- $P(A \cap B) = P(A|B)P(B)$
- $P(B) = \sum_{i=1}^{n} P(B \cap A_i)$

PRACTICE: INTERVIEW QUESTION

- There are 24 balls in a bucket: 12 red and 12 black.
- If you draw one ball, then draw a second ball, what is the probability of drawing two balls of the same color?

Interview question shared by DSI alum interviewing with Outbound Engine.

WHEN BY HAND IS TOUGH…

- Oftentimes, we won't evaluate probabilities by hand.
  - It's still very important to understand the ideas behind probability – as we move forward, it's critical to:
    - a) know probability's relationship with statistics and machine learning.
    - b) identify potentially bad assumptions.

- We often think of probability as how frequently an event occurs.
  - We can use simulations to give us a good approximation of the true probability of some event.

SUPPLEMENTAL SECTION

BAYES' THEOREM

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

WHAT IS $P(A)$?

- We've talked a lot about probabilities of certain events, but what does this actually mean?
- There are two broad classes of probabilistic interpretations.

TWO INTERPRETATIONS OF $P(A)$

- In the long run, how many times will A occur relative to how many times we conduct our experiment?

$$P(A) = \lim_{\#\ of\ exp's \to \infty} \frac{\#\ of\ times\ A\ occurs}{\#\ of\ experiments}$$

$$P(heads) = \lim_{\#\ of\ coin\ tosses \to \infty} \frac{\#\ of\ heads}{\#\ of\ coin\ tosses}$$

- This is called the **frequentist** interpretation of probability.

TWO INTERPRETATIONS OF $P(A)$

- What is one's degree of belief in the statement A, possibly given evidence?

$$P(A) = \text{“How likely is it that } A \text{ is true?”}$$

$$P(heads) = \text{“How likely is it that I flip a heads?”}$$

- This is called the **Bayesian** interpretation of probability.

TWO INTERPRETATIONS OF $P(A)$

- Neither interpretation of P(A) is more or less correct.

- However, these different interpretations can give rise to different ways of analyzing our data, as we’ll see later!