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Advanced Algorithms for NISQ Devices – Emphasizing Classical Shadows

Introduction

Noisy Intermediate-Scale Quantum (NISQ) devices, such as IBM's cloud-based quantum processors, are limited in qubit count, coherence time, and gate fidelity. To achieve meaningful results on current **IBM Quantum free-tier hardware** (typically 5–7 qubits with modest circuit depth), we must leverage algorithms that maximize **cost-per-accuracy** efficiency and minimize required measurements (shot count). This report identifies **advanced research-grade algorithms** that address real-world problems in high-impact domains (quantum chemistry, optimization, benchmarking, metrology) while **emphasizing the role of *classical shadows*** for improved measurement efficiency. Each proposed algorithm is evaluated on its **feasibility** on today's hardware, **potential impact** in its domain, **novelty** and patentability, compatibility with **IBM's NISQ constraints**, and **extensibility toward fault-tolerant** quantum systems. We then outline how these techniques can be integrated into a revised *QuartumSE* roadmap that prioritizes early-stage R&D, iterative algorithm refinement, hardware validations, and clear publication/patent milestones.

Candidate Algorithms for NISQ Implementation

Classical Shadows for Efficient State Property Estimation

Figure: Concept of the classical shadows technique – random quantum measurements produce a compact "shadow" of the state, which a classical algorithm can use to predict many observables without additional quantum runs. This enables a "measure-once, predict-many" capability that greatly reduces the number of measurements needed [1] [2].

One of the most promising recent advances for NISQ devices is the **classical shadows** protocol. In essence, instead of performing separate measurements for each observable of interest, one performs **randomized measurements** on the quantum state to obtain a succinct classical representation or "shadow" of that state [1]. From this stored classical shadow, a wide range of properties (expectation values, fidelities, correlation functions, etc.) can be estimated *efficiently* in post-processing, without further quantum measurements [3] [2]. This paradigm dramatically improves **shot-efficiency**, as the *same* measurement data can be reused to evaluate many observables simultaneously – enabling one to "measure first and ask questions later" [4].

Importantly, classical shadows directly tackle the **readout bottleneck** that plagues quantum simulations. In quantum chemistry and materials science, for example, a Hamiltonian may have hundreds of Pauli terms whose expectations must be measured to compute an energy or other property. Traditional approaches require partitioning terms and running a large number of circuits; indeed, the measurement step is a "core bottleneck in many schemes for quantum simulation" [5]. Classical shadow algorithms bypass the need for full tomography and instead allow efficient estimation of relevant observables (like all terms in a Hamiltonian or all elements of a reduced density matrix) with far fewer state preparations [6]. Recent research shows that for an *n*-qubit fermionic system, one can estimate all \$k\$-particle reduced density matrix (k-RDM) elements – sufficient to determine molecular energies – with polynomially scaling number of state copies using classical shadows [7] [8]. This represents a significant improvement in **cost-per-accuracy** for quantum chemistry simulations.

Another key advantage is that classical shadows can be made **noise-aware and error-mitigated**, critical for NISQ viability. The base protocol is inherently *sample-efficient* and somewhat noise-resilient (since random Clifford measurements effectively twirl certain errors) [9]. Building on this, researchers have developed **robust shadow estimation** techniques that incorporate error mitigation. For instance, an *error-mitigated classical shadow* method was proposed for fermionic systems, using a calibration routine and probabilistic error cancellation to counteract systematic noise [10] [11]. This error-mitigated approach was shown to be **robust against common noise channels** (depolarizing, amplitude damping, etc.), achieving accurate expectation values with similar sample counts as the noiseless case [12] [13]. In 2025, a team experimentally demonstrated a *"robust shallow shadows"* protocol on a 127-qubit IBM processor, using shallow random circuits plus Bayesian post-processing to learn and mitigate noise [2] [14]. The robust shadows scheme provided unbiased estimates of global properties like fidelity and entanglement entropy with up to 5× fewer samples than naive methods, even in the presence of realistic noise [14] [15]. These results validate that classical shadows can yield **order-of-magnitude improvements in shot-efficiency**, making previously infeasible characterization tasks possible on NISQ hardware.

From a **feasibility** standpoint, classical shadows are well-suited to IBM's free-tier devices: they mainly require random single-qubit Cliffords or shallow random circuits followed by computational basis measurements [2] [10] – operations readily implemented on small superconducting qubit chips. This technique can be immediately applied to **quantum chemistry VQE experiments** (to estimate energy and other observables with fewer shots), to **quantum error mitigation and benchmarking** (e.g. efficiently estimating state fidelities and cross-entropies for device validation [10]), and even to **quantum machine learning** tasks where many outcome statistics are needed [10]. The concept of classical shadows was only introduced in 2020 [1] and has since seen rapid development, meaning there is room for **novel extensions (and patents)** – for example, custom shadow protocols for specific domains, or integrating classical shadows with adaptive measurement scheduling. Finally, because classical shadows fundamentally address how to extract information from quantum states, the technique will remain relevant as we scale toward larger, **fault-tolerant** systems. In a fault-tolerant future, error-corrected circuits will produce more complex states, and classical shadows will be an indispensable tool to efficiently **learn properties of large entangled states** without exhaustive measurements [3]. In short, classical shadows offer an advanced yet implementable algorithmic tool that aligns perfectly with NISQ constraints and dramatically improves measurement efficiency – a cornerstone for the QuartumSE roadmap.

Variational Quantum Eigensolver (VQE) for Quantum Chemistry (Enhanced Readout)

The **Variational Quantum Eigensolver (VQE)** is a flagship NISQ algorithm targeting quantum chemistry and materials – high-visibility domains where quantum computers promise to solve classically intractable molecular electronic structure problems. VQE is a hybrid approach: a parameterized quantum circuit prepares trial wavefunctions, and a classical optimizer adjusts parameters to minimize the measured expectation value of the Hamiltonian (the energy). VQE's importance is evidenced by early breakthroughs on NISQ hardware: in 2017, an IBM team used a 6-qubit device to find the ground-state energies of molecules up to BeH_2 (a six-atom molecule), involving **over 100 Pauli terms** in the Hamiltonian [16]. This hardware-efficient VQE experiment successfully optimized a problem far beyond exact classical brute-force, demonstrating the method's feasibility on small devices. Subsequent work extended VQE to larger molecules and quantum magnets, showing that results from the quantum processor agree with simulated noisy models and providing a blueprint for scaling to more complex systems [17].

To fit within IBM's free-tier limitations, VQE algorithms employ **hardware-efficient ansätze** (using entangling gates native to the chip topology) and often **adaptive ansätze**. For example, the ADAPT-VQE approach builds the trial state iteratively by adding gates that most improve the energy, yielding shorter-depth circuits for a target accuracy. These techniques reduce the circuit depth and number of parameters, which improves resilience to noise on limited-coherence devices [18] [19]. Moreover, VQE has been a testbed for **error mitigation strategies** to boost accuracy without increasing hardware demands. Notably, IBM demonstrated that *zero-noise extrapolation* and other error mitigation can "extend the reach" of a noisy processor, achieving chemical accuracy in cases that would otherwise be too noisy [20]. Such error-mitigated VQE runs, reported in Nature in 2019, effectively pushed the frontier of what NISQ hardware could reliably simulate [20].

Despite these advances, a major overhead in VQE is the need to measure many Hamiltonian terms. Here **classical shadows can be leveraged** to dramatically improve VQE's shot efficiency. Instead of grouping and measuring each subset of Pauli terms separately, one can perform randomized measurements to construct a classical shadow and then extract *all* needed term expectations from the same data [6] [21]. Recent research in **fermionic classical shadows** confirms that energies (which depend on 2-RDMs and 4-RDMs) can be obtained with polynomial samples, and error-mitigated shadow techniques can handle the noise in these measurements [12] [22]. By integrating classical shadow readout into VQE, *QuartumSE* can demonstrate **orders-of-magnitude reduction in cost-per-accuracy** – for instance, obtaining an energy estimate to a given precision with far fewer circuit shots than standard methods. This directly addresses the cost metrics important to end-users (e.g. achieving chemical accuracy at lower computational cost).

In terms of **impact**, a successful VQE implementation on current hardware, even for small molecules, is highly publishable and relevant to industries like pharmaceuticals and materials design. Quantum chemistry has been a poster child for quantum advantage, and improvements here garner significant attention [23]. The **novelty** lies in the enhancements: while basic VQE is established, demonstrating VQE *with advanced measurement (classical shadows)* or *with new ansatz adaptations* on real hardware could be a first in literature, opening the door to patent claims on the specific integration of these techniques. For example, a "Classical-Shadow VQE" procedure or a method for dynamically switching measurement bases during VQE optimization might be patentable if unique. In the long run, as devices scale and error correction comes online, the **extensibility** of VQE is twofold. First, VQE itself can tackle larger systems (it naturally scales with more qubits for bigger molecules). Second, many lessons from VQE – variational optimization and ansatz design – carry over to future algorithms (like quantum phase estimation or quantum Monte Carlo) in the fault-tolerant regime. In fact, VQE may serve as a warm-start or subroutine in future error-corrected algorithms. By investing in VQE now (with cutting-edge enhancements), QuartumSE sets the stage to remain at the forefront of quantum simulation both in the NISQ era and beyond.

Quantum Approximate Optimization Algorithm (QAOA) for Combinatorial Optimization

The **Quantum Approximate Optimization Algorithm (QAOA)** is a leading candidate for demonstrating a practical quantum advantage on NISQ devices [24] [25]. QAOA targets combinatorial optimization problems (MAX-CUT, scheduling, portfolio optimization, etc.) by encoding the problem into a cost Hamiltonian H_C and alternating between applying H_C and a mixing Hamiltonian in a parameterized fashion. For domains like logistics, finance, and machine learning, even a modest improvement over classical heuristics would be highly impactful – making QAOA a high-visibility algorithm. Notably, QAOA is *shallow-depth by construction* (low circuit depth for low algorithm parameter p), aligning with NISQ hardware limitations. It has been touted as an algorithm that can run on today's noisy hardware and potentially deliver useful solutions [26].

Initial experiments have realized QAOA on small graphs using superconducting qubits and trapped ions. For instance, IBM researchers executed QAOA optimization on problems with up to 7 and 27 variables (qubits) using an IBM backend, leveraging Qiskit Runtime for efficient closed-loop optimization [27]. More recently, in 2025, a partially fault-tolerant QAOA was demonstrated on a trapped-ion system with **20 logical qubits protected by an error-detection code**, achieving better results than the unprotected circuit [28]. This represents the *largest-scale algorithmic run with some quantum error detection to date*, and it showed that adding even a modest level of error correction can improve QAOA performance [29] [30]. Such results are promising for **extensibility**: they indicate a path for QAOA to bridge the NISQ-to-fault-tolerant gap, by incrementally incorporating error correction as hardware permits. QAOA's structure is compatible with future fault-tolerant implementation (where p can be increased to boost solution quality), so techniques developed now – like parameter optimization strategies or error mitigation – will carry forward.

In terms of **potential impact**, QAOA covers a broad range of applications. It has been applied to portfolio optimization in finance, network routing, scheduling, and more [31]. In some cases, QAOA has even shown evidence of a quantum speedup or improved scaling compared to classical algorithms for specific problem instances [31]. Each incremental progress (e.g. solving a slightly larger Max-CUT than classical simulators can handle) is publishable and of interest to industry stakeholders. To maximize **cost-per-accuracy** on present hardware, research has focused on improving QAOA's efficiency. One challenge identified is the **large number of shots** required for each iteration of the classical optimizer (to estimate the cost expectation with enough precision) [32]. QAOA typically needs many circuit evaluations per parameter update, which can be costly. Strategies to mitigate this include using *shot-frugal optimizers* (adaptive shot allocation) and leveraging gradient information. Here, classical shadows might assist indirectly: if the cost function comprises many terms (e.g. weighted sum of Pauli Z interactions), a shadow or related randomized measurement could estimate the cost and maybe certain gradients in one run, though this is less straightforward than in VQE. Nonetheless, acknowledging and reducing the shot footprint is key – for example, using *analytic gradients* or low-variance estimators can cut down the total number of circuit executions needed.

Another avenue for enhancing QAOA on NISQ devices is **problem-tailored circuit design**. Researchers are exploring custom mixers (beyond the standard transverse field) to incorporate problem constraints or biases, as well as **layer-wise learning** techniques to initialize parameters for deeper circuits. These refinements keep QAOA at the research frontier and present opportunities for **patentable IP**. For instance, QuartumSE could develop a proprietary QAOA variant that uses a classical shadow to adaptively choose which clauses of a constraint to enforce at each round, or a QAOA with dynamic circuit depth adjustment based on intermediate performance. Since QAOA was first proposed in 2014, it's a well-known algorithm, but such novel extensions could be protected if they show a unique advantage.

On IBM's free-tier hardware, QAOA is **viable for small problem instances** – e.g. MAX-CUT on graphs of 5–7 nodes can be encoded on 5–7 qubits and run with $p=1$ or 2 layers within coherence limits. These demonstrations might not outdo classical solvers, but they are critical for **hardware validation** of algorithms. Each NISQ experiment teaches us about noise effects on algorithm performance; for QAOA, decoherence can wash out the advantage of deeper circuits, so finding the optimal shallow depth is part of the learning process [33]. By conducting such experiments early (and publishing results in, say, *Quantum* or *PRX Quantum*), QuartumSE can position itself as a leader in quantum optimization. Furthermore, by implementing error mitigation (like Pauli Twirling or mid-circuit resets) within QAOA or even employing small error-detecting codes as in the “Iceberg code” example [28], [34], the company can hit key **milestones**: demonstrating the *first error-corrected variational optimization*, for instance, would be a landmark publication and patent opportunity.

In summary, QAOA stands out as an algorithm that is **highly visible**, directly relevant to real-world optimization tasks, and incrementally improvable on NISQ hardware. With thoughtful enhancements focusing on shot efficiency and noise mitigation, QAOA experiments will not only yield scientific publications but also guide how we eventually scale these heuristic algorithms toward the fault-tolerant era when they can tackle industry-scale problems.

Quantum Benchmarking and Error Mitigation Protocols

A crucial category of “algorithms” for any quantum computing effort is **benchmarking and error mitigation techniques**. While these do not solve a user’s optimization or chemistry problem, they solve the very real problem of **characterizing and improving the quantum hardware**, which is essential for all other applications to succeed. In the NISQ context, benchmarking protocols have high visibility – for example, IBM’s **Quantum Volume** test (a randomized-circuit benchmark) became a de facto standard reported in press releases, and Google’s 2019 **quantum supremacy** experiment was fundamentally a benchmarking task (sampling from a random circuit distribution to compare against classical simulation). For *QuartumSE*, demonstrating advanced benchmarking on IBM free-tier devices can attract attention and also provide internal feedback to guide algorithm design choices.

Traditional benchmarking algorithms include **Randomized Benchmarking (RB)** to measure average gate fidelity and **Cross-Entropy Benchmarking (XEB)** to estimate the fidelity of random circuit outputs. These methods have been implemented on small IBM devices easily (RB on 5 qubits, etc.). However, **research-grade benchmarking** goes further: for instance, **cycle benchmarking** can pinpoint errors in specific gate layers, and **quantum volume** combines a series of randomized tests to give a single-number metric of performance. Implementing a new highest quantum volume on a free-tier device (using clever calibration or dynamical decoupling to push performance) could itself be a noteworthy result.

Moreover, there is a strong **synergy with classical shadows** in this domain. The classical shadows framework can be applied to **device characterization** by efficiently estimating many correlators or fidelities at once [10]. For example, *direct fidelity estimation* is a technique where one measures a set of random Pauli observables to estimate the fidelity of a prepared state to a target state – essentially a primitive form of classical shadows focused on fidelity [35]. Classical shadows generalize this: with one set of randomized measurements, one could extract fidelities with multiple target states or entanglement metrics that serve as benchmarks for the device. In fact, Hu *et al.* (2025) use robust shallow classical shadows to **benchmark an IBM processor’s noise** by extracting quantities like multi-qubit entropies and purities, showing their protocol’s advantage over standard single-basis measurements [14] [10]. This represents a new class of **“shadow benchmarking”** protocols that could be both publishable and patentable. For example, QuartumSE might devise a protocol where a classical shadow is used to simultaneously estimate the effective logical error rates of dozens of Pauli error channels on the device – providing a detailed noise fingerprint much faster than running separate calibration routines for each.

On the **error mitigation** side, algorithms such as **zero-noise extrapolation**, **probabilistic error cancellation**, and **digital dynamical decoupling** are all research hotspots. These techniques don’t require additional qubits (unlike full error correction) and thus are deployable on free-tier hardware. A notable example was Kandala *et al.*’s error mitigation experiment, which showed that extrapolating results from increased noise levels can recover accurate estimates of observables on a 5-qubit IBM system [20]. Integrating error mitigation into our algorithms can directly improve cost-per-accuracy – effectively reducing the number of shots or circuit repetitions needed to reach a given accuracy target. For instance, **error-mitigated classical shadows** (as discussed earlier) could become a core part of QuartumSE's toolkit, ensuring that the measurement data we gather is as reliable as possible [13] [14]. Additionally, combining benchmarking with mitigation (e.g. using interleaved RB to gauge and then cancel coherent errors) could be an innovative approach worth pursuing.

From a **feasibility** perspective, IBM's open devices allow users to run custom calibration sequences and collect raw data, which is sufficient for implementing most benchmarking algorithms. Free-tier devices have been used in the past by the community to measure quantities like coherence, crosstalk, and even small quantum error-correcting code performance. Thus, QuartumSE can confidently carry out **early hardware tests** of any new benchmarking idea on available machines. The **impact** of this line of work, while indirect for end-users, is high within the quantum industry: improved benchmarking translates to better understanding and faster improvement of hardware. It's also a path to high-profile publications – e.g. demonstrating a new record in quantum volume [36] or introducing a novel method to characterize noise could land in journals like *Nature Communications* or *PRX Quantum* (and strengthen the credibility of QuartumSE's experimental capabilities).

Finally, as quantum processors evolve, robust benchmarking and error mitigation will **scale alongside them**. In the fault-tolerant future, one will benchmark logical qubits and error-corrected gates, and still need efficient ways to do so. The techniques developed now (randomized measurements, shadow-based characterizations, etc.) can be adapted to very large systems where conventional tomography is impossible. In short, focusing on this "meta-algorithmic" domain in the roadmap ensures that QuartumSE not only solves problems on today's devices but also masters the tools to keep those devices (and future ones) working at peak performance. Such expertise is itself a valuable product, potentially yielding patented software for automated calibration or error mitigation workflows.

Variational Quantum Metrology (Entangled Sensor Networks)

Quantum metrology is a domain where quantum computers or devices promise **quantitative improvements in measurement precision**, with applications ranging from atomic clocks and gravitational wave detectors to medical imaging and navigation systems [37]. While IBM's superconducting qubit processors are not deployed as sensors per se, they can be used to *simulate and design quantum sensing protocols*, and small entangled states on these devices can demonstrate the principles of quantum-enhanced measurement. We propose exploring **variational quantum metrology algorithms**, which use NISQ devices to *prepare and optimize entangled probe states and measurement settings* for sensing a parameter of interest.

The core idea is that by leveraging quantum entanglement and adaptive optimization, one can exceed classical limits of precision (e.g. beat the standard shot-noise limit) [38]. For example, a simple metrological task is phase estimation: using N qubits, the best classical strategy yields uncertainty $\propto 1/\sqrt{N}$, whereas an entangled GHZ state can in principle achieve uncertainty $\propto 1/N$ (Heisenberg limit). On a small quantum device, one can prepare a GHZ state of 3–5 qubits, insert a controlled phase shift, and measure – this setup emulates an interferometer. A **variational algorithm** can be wrapped around it to tune the state preparation circuit for maximum sensitivity given the presence of noise. In 2024, MacLellan *et al.* introduced an **end-to-end variational quantum sensing framework**, combining parameterized quantum circuits for state preparation and measurement with classical neural networks for estimation [39]. The approach is general and hardware-informed: it can adapt to the specific noise and connectivity of a given device, optimizing the entire sensing protocol (state **and** readout) to be as robust and precise as possible [38] [40]. They demonstrate this concept on realistic models for trapped-ion and photonic sensor platforms, showing that variational algorithms can indeed find configurations that balance noise effects and maximize metrological gain [39].

For *QuartumSE*, implementing a prototype variational metrology experiment on an IBM device would emphasize early-phase **research innovation**. For instance, consider using a 5-qubit chip to detect a small Z-rotation angle on one qubit (mimicking a magnetic field sensing). We could prepare an entangled state (like a GHZ or a more general circuit ansatz), let the rotation act, then measure in an entangled basis that concentrates information about the angle. The parameters of the state prep and measurement circuit can be variationally optimized to maximize the Fisher information or minimize estimation error. Such an experiment would validate the concept of **quantum-enhanced sensing on NISQ hardware** – a compelling result to publish, as it touches on quantum metrology, a field with "transformative impacts across science and engineering" [38]. It also intersects with quantum machine learning if one uses a classical network to process the measurement data (as in end-to-end schemes).

The **feasibility** of this on IBM's free-tier devices is reasonable: creating 3- or 4-qubit GHZ states is a matter of a few CNOT gates (IBM has even provided this as a textbook example). The main challenge is dealing with noise that may rapidly degrade the theoretical advantage. However, this *is precisely the point* of variational metrology – the algorithm can learn a strategy that is optimal under the noise constraints. For example, maybe a slightly error-resilient entangled state (not a perfect GHZ) yields better results on a noisy device; the variational approach would find that. Additionally, error mitigation techniques (like readout error mitigation or purification of measurement data) could be applied to further enhance the sensing fidelity.

From a **novelty** perspective, variational quantum metrology is an emerging idea. Early papers (2021–2024) have begun to explore it [41] [42], but there is ample room for new contributions – potentially patentable – especially in the context of specific applications (e.g. a patent on a quantum algorithm for magnetic field sensing with a hybrid quantum-classical loop). By staking out this area in the roadmap, QuartumSE can aim to produce **high-impact publications** (quantum sensing results often publish in journals like *Nature Physics* or *PRL*) and at the same time develop IP around bespoke sensing algorithms. Collaborations with academic or national lab partners (who may have physical sensing setups) could also amplify the real-world relevance of this work, by transferring learned circuits from the IBM device to, say, a quantum sensor platform.

Finally, regarding **extensibility**, any advances made in NISQ metrology algorithms will become even more powerful on better hardware. In a fault-tolerant regime, one could imagine large entangled states (protected from decoherence by error correction) being used as exquisite sensors – for instance, a 1000-qubit error-corrected GHZ state as an interferometer. The principles and algorithms we develop now (variational optimization of probe states, machine-learning-based error correction of estimates, etc.) will directly inform how such future sensors are designed. Thus, focusing on quantum metrology in the early R&D phases not only diversifies QuartumSE's portfolio into another high-impact domain, but also future-proofs the roadmap by aligning with an area that scales favorably with improved qubit quality. It's a strategic investment in know-how that could pay off when quantum hardware transitions from just computing to broader quantum technology applications.

Comparative Analysis of Proposed Algorithms

The table below summarizes the key characteristics of each proposed algorithm, comparing their feasibility on NISQ hardware, potential impact, novelty, hardware compatibility, and long-term extensibility:

Algorithm	Feasibility (NISQ)	Potential Impact	Novelty	Hardware Compatibility	Extensibility (Fault-Tolerance)
Classical Shadows for State Estimation	High – Demonstrated on real devices (e.g. 127-qubit IBM) with shallow random circuits [14]. Ready to apply on 5–7 qubit systems for measuring many observables at once.	High – Boosts all applications by dramatically reducing measurement overhead. Enables efficient estimation of energies, fidelities, entropies etc., turning infeasible measurements feasible [3] [2].	Very novel – Concept introduced 2020, with ongoing research (robust and fermionic shadows in 2024–25). New variants (custom ensembles, noise-tailored shadows) could be patented.	Excellent – Uses random single-qubit or Clifford ops and simple measurements [2]; well within gate capabilities of IBM devices. Low-depth requirements suit NISQ noise limitations.	Strong – Remains useful as system size grows. In fault-tolerant era, allows quick characterization of large states without full tomography. Can integrate with error-corrected circuits to inform feedback and control.
Variational Quantum Eigensolver (VQE)	Medium-High – Proven on small molecules (H_2, LiH, BeH_2) using 4–6 qubits [16]. Feasible on IBM 5-qubit for simple molecules; requires error mitigation for accuracy. Measurement enhancements (shadows) further improve feasibility.	High – Solves quantum chemistry problems (molecular energies, reaction dynamics) of great interest to chemistry and pharma. Even small-scale demonstrations validate the approach for future larger systems.	Moderate – VQE (2014) is established, but **hardware-tailored ansätze** and **shadow-based measurement** are cutting-edge. Adaptive VQE and error-mitigated VQE are recent innovations [18] [12], offering publication and IP opportunities.	Good – Circuit depth can be kept low with hardware-efficient ansatz [16]. 5–7 qubits can encode small molecules. Runs within coherence time if optimized (IBM qubits ~100 μs). Classical optimizer overhead manageable via cloud runtime.	Conditional – In long term, full quantum phase estimation might replace VQE for exact solutions, but VQE methods can still prepare trial states or serve as subroutines. Techniques from VQE (ansatz design, adaptive strategies) will inform algorithms on error-corrected devices.

Algorithm	Feasibility (NISQ)	Potential Impact	Novelty	Hardware Compatibility	Extensibility (Fault-Tolerance)
Quantum Approximate Optimization (QAOA)	Medium – Implemented on up to ~20 qubits in experiments with shallow p [28] ; easily run on 5–7 qubits for toy problems. Limited by gate errors as depth grows, but workable at $p=1,2$. Requires many shots for optimization (can be mitigated) [32] .	High – Addresses combinatorial optimization across finance, logistics, scheduling. A leading candidate for showing a quantum advantage in the near term [31] . Even modest successes (faster or better solutions on small instances) would be impactful in the optimization community.	Moderate – QAOA (2014) is known, but **enhancements** (custom mixers, error mitigation, partial error correction in circuit [29]) are novel. First demonstrations of error-detected QAOA or problem-specific QAOA variants are recent, offering fresh research contributions.	Moderate – QAOA circuits are shallow but involve entangling gates matching problem graph connectivity. On IBM hardware, limited connectivity may require SWAP networks, adding depth [43] . Free-tier devices can handle small graphs natively or with minor qubit reordering.	Strong – QAOA will benefit greatly from improved qubit counts and coherence. As fault tolerance comes in, QAOA can scale to larger problems and higher depth (potentially becoming as powerful as adiabatic algorithms). Moreover, techniques like layer-wise optimization and error suppression used now will integrate with error-corrected versions to tackle industry-scale problems.

Algorithm	Feasibility (NISQ)	Potential Impact	Novelty	Hardware Compatibility	Extensibility (Fault-Tolerance)
Benchmarking & Error Mitigation Protocols	High – Standard RB/XEB easily run on small devices; advanced protocols (cycle benchmarking, shadow fidelity estimation) already tested on IBM hardware [14] [10]. Free-tier access allows full control of experiments needed for custom benchmarks.	High (indirect) – Improves device reliability and guides hardware improvements. Critical for claiming any quantum advantage (must verify results). High visibility in quantum computing milestones (e.g. quantum volume records, “supremacy” tests).	Moderate – Core ideas (RB, XEB) are established, but **comprehensive, efficient benchmarking** is an active research area. New methods (e.g. using classical shadows for channel characterization [10]) are novel. Integration of error mitigation in benchmarking (or vice versa) is innovative and potentially patentable.	Excellent – Compatible with hardware: these protocols are by definition tailored to hardware operations. They often involve running random or structured circuits at various lengths – doable within calibration routines on IBM devices. No special hardware features needed beyond what’s available.	Strong – Benchmarking evolves with hardware: even in fault-tolerant era, we’ll benchmark logical qubits and error rates. The efficient characterization techniques developed now (randomized and sampling-based) will be even more vital as systems grow (where exhaustive testing is impossible). Error mitigation strategies will transition into aiding error-correcting codes and improving fault-tolerant overhead.

Algorithm	Feasibility (NISQ)	Potential Impact	Novelty	Hardware Compatibility	Extensibility (Fault-Tolerance)
Variational Quantum Metrology	Medium – Small-scale demonstrations (3–5 qubit entangled states) are feasible now. IBM devices can create GHZ states and simple interferometry circuits. Noise may limit absolute gain, but variational approach can compensate to some extent. Requires multiple runs for training.	High – Could revolutionize how sensors are designed. Applicable to high-impact areas like navigation (atomic gyroscopes), astronomy (interferometry), and defense (radar) by improving measurement sensitivity [37] . Even a proof-of-concept quantum enhancement on real hardware would be a notable scientific result.	High – Variational sensing frameworks are very recent (mid-2020s) [39] . Few experiments exist. A QuartumSE implementation would likely be among the first, yielding novel data. This space is ripe for patents (e.g. specific variational algorithms for multi-parameter estimation or adaptive sensing).	Moderate – Uses standard gates to prepare entangled probes and measurements. 5-qubit device sufficient for basic GHZ or other entangled states. Readout noise is an issue for phase estimation but can be mitigated. No special hardware modifications needed, though more qubits = better signal.	Strong – Quantum sensing advantages compound with more qubits. Techniques developed in NISQ (state prep, error-aware optimization) will directly translate to enhanced precision on larger, error-corrected sensors. In fault-tolerant era, one could utilize hundreds of qubits in entangled sensors – our variational methods would scale to optimize those large systems. Additionally, the combination of quantum computing and sensing may open hybrid applications (e.g. quantum computers analyzing data from quantum sensors).

(Sources: Key points synthesized from references 【9】 【23】 【2】 【34】 【36】 【21】 【24】 【14】 as cited above.)

Integrating into the QuartumSE Roadmap

To effectively harness these algorithms, QuartumSE should realign its development roadmap to emphasize **research and prototyping in the early phases**, followed by iterative refinement and clear milestones for hardware validation, publications, and patent filings. Below is a phased integration plan:

1. **Exploratory Research & Algorithm Design (Phase 1)** – *"R&D Front-Load"*. In the initial stage, devote significant effort to **analyzing and simulating** the proposed algorithms. The team should conduct paper studies and classical simulations (using Qiskit Aer or similar) for classical shadows, VQE, QAOA, etc., on representative small problems. The goal is to identify promising configurations (e.g. which classical shadow ensemble to use, which ansatz for VQE, what depth for QAOA) before running on real hardware. During this phase, maintain a strong focus on **algorithmic innovation**: tweak known algorithms (e.g. implement the robust shadow protocol with Bayesian inference, or try a new QAOA mixer) and evaluate their potential. Regular internal seminars and brainstorming sessions will help iterate on ideas quickly. By the end of Phase 1, QuartumSE should have a **portfolio of algorithm prototypes** (validated in simulation) and a clear sense of their performance trade-offs and novelty. This will form the basis for initial technical write-ups, which can later be turned into publications or patent drafts. *Milestone*: Produce an internal whitepaper or technical report on each algorithm's simulated performance and novelty, and identify at least one patentable concept emerging from this research (e.g. a specific improvement to classical shadow processing or an adaptive VQE strategy).

2. **Hardware Demonstration & Iterative Refinement (Phase 2)** – *"Prototype on IBM Hardware"*. In this phase, move to **implementation on actual IBM free-tier devices**. Prioritize running simplified versions of each algorithm to get real-world data: for example, use IBM's 7-qubit machine to execute a classical shadows experiment (measuring the energy of H_2 or LiH), or run QAOA on a 5-qubit Max-CUT problem, or perform a basic GHZ-phase estimation trial. The focus here is on **hardware validation** – uncover how noise, calibration errors, and resource limits affect the algorithms. Expect to iterate frequently: data from the device might suggest modifying the algorithm (e.g. adding error mitigation steps if results are too noisy, or adjusting the variational optimizer). It's important to work closely with IBM's tools (like Qiskit's runtime and Ignis calibration routines) to optimize performance. During this stage, the roadmap should allow time for **debugging and optimization cycles** – effectively a loop of "run -> measure -> adjust". QuartumSE's developers and researchers should document all results and refine the algorithms in response to observed errors (for instance, if certain two-qubit gates have high error, adjust circuit ordering or apply purification to shadow estimates). *Milestone*: Achieve a concrete proof-of-concept result on hardware for each algorithm – e.g. "demonstrated VQE convergence to within 0.05 Ha of true energy for H_2", "prepared an entangled sensor state that beat the classical shot-noise limit in estimating a phase by 20%", or "measured 10 different observables' expectations from one set of classical shadow experiments with >90% accuracy of direct measurement". These results, even if small scale, are crucial early **validation points** and can form the core of external communications.

3. **Publication and Patent Pipeline (Phase 3)** – *"Disseminate and Protect IP"*. With successful hardware experiments in hand, QuartumSE should rapidly transition to **publishing and patenting** its innovations. The roadmap in this phase emphasizes writing and submission timelines. For each algorithm, target a high-impact journal or conference. For example, a classical shadows result could be written up for *npj Quantum Information* or *Physical Review X Quantum*, while an optimization result might go to *Nature Communications* or *Quantum*. Simultaneously, work with IP counsel to file **patent applications** on the novel aspects of the algorithms. This could include the specific classical shadow noise-mitigation technique, a unique

adaptive VQE protocol, or an integrated workflow for quantum optimization with error suppression. It's important that the patent filings occur before or parallel to publication to ensure no loss of rights. QuartumSE's roadmap should include time for patent drafting and revision after examiner feedback. On the publication side, plan for at least **one flagship paper per algorithm**. Coordinate these releases with marketing for maximum visibility – for instance, a press release highlighting that *QuartumSE has achieved a record in shot-efficiency for quantum chemistry using classical shadows*, citing the publication [44]. By the end of Phase 3, QuartumSE aims to have built a reputation via peer-reviewed results and secured IP around its core techniques, which together validate the company's R&D-centric approach.

4. **Scaling Up and Strategic Partnerships (Phase 4)** – *"Towards Quantum Advantage"*. After the initial demonstrations, the roadmap shifts towards scaling these algorithms for larger problems and integrating them with emerging hardware. As IBM (and others) release devices with more qubits or special features (like mid-circuit measurement or error mitigation APIs), QuartumSE should be ready to **upgrade its implementations**. For example, if a 20-qubit device becomes available on the free tier or through a partnership, aim to run a next-level problem (perhaps a medium-sized molecule on VQE or a 10-node graph on QAOA). This phase may involve **cross-team efforts**: combining the algorithms (e.g. using classical shadows within QAOA to measure many constraint terms, or applying variational algorithms on error-mitigated logical qubits if small quantum error correction becomes feasible). It's also wise to pursue **partnerships with academic or industry groups** for co-authored studies – for instance, collaborating with a national lab on quantum sensor experiments, or with a pharma company on a specific molecular simulation. Such partnerships can provide additional validation environments and potentially access to better hardware (some partners have premium access to larger IBM systems). The roadmap should include milestone goals like *"quantum advantage demonstration attempt"*: using our algorithms to try solving a problem notably faster or more accurately than the best classical method. Even if full advantage isn't reached, pushing in that direction guides our R&D. *Milestone*: Achieve a notable scale-up such as running a patented QuartumSE algorithm on a 20+ qubit device with successful results, and prepare a "quantum advantage" case study whitepaper on the findings. This will set the stage for attracting further investment or customers interested in the practical performance of QuartumSE's solutions.

Throughout all phases, QuartumSE's revised roadmap should maintain an **R&D-focused ethos**: allocate time for learning from each experiment, encourage publication (ensuring our scientists stay at the cutting edge), and use those publications as a marketing tool to establish thought leadership. Regularly update the roadmap to incorporate new scientific insights – for example, if a new paper in 2025 reports a better algorithm in our domains, evaluate it and consider merging it with our approach (continuous innovation). By emphasizing algorithm iteration, hardware validation at every step, and concrete publication/patent outcomes, QuartumSE will build a robust foundation for long-term success. This approach not only leads to short-term milestones (working prototypes, papers, IP) but also primes the company for the eventual era of fault-tolerant quantum computing, having accumulated deep expertise and credibility in the algorithms that matter.

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