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1. Density function for normal distribution with mean = θ_1 and variance θ_2 is :-

$$\text{Pdf} = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\left(\frac{(x_i - \theta_1)^2}{2\theta_2}\right)}$$

The likelihood function L for a sample of n from normal distribution with mean θ_1 and variance θ_2 is :-

$$L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\left(\frac{(x_i - \theta_1)^2}{2\theta_2}\right)}$$

Taking natural log on both sides :-

$$\ln(L) = n \ln\left(\frac{1}{\sqrt{2\pi\theta_2}}\right) + \ln\left(e^{-\left(\frac{(x_i - \theta_1)^2}{2\theta_2}\right)}\right)$$

$$\boxed{\ln(L) = -n \ln\left(\sqrt{2\pi\theta_2}\right) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2} \quad \text{— (1)}$$

For Maximum Likihood :-

(a) Diffrentiating (1) w.r.t. θ_1 :-

$$\frac{\partial \ln(L)}{\partial \theta_1} = -\frac{1}{2\theta_2} \sum_{i=1}^{n} 2(x_i - \theta_1)(-1) = 0$$

$$= \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\boxed{\theta_1 = \frac{\sum_{i=1}^{n} x_i}{n}}$$

(b) Diffrentiating ① w.r.t. θ_2 :-

$$\frac{\partial \ln(L)}{\partial \theta_2} = \left(-\frac{n}{2} \times \frac{1}{\cancel{2\pi}\theta_2} \times \cancel{2\pi}\right) - \left(\frac{-1}{2\theta_2^2}\sum_{i=1}^{n}(x_i-\theta_1)^2\right) = 0$$

$$= -\frac{n}{\cancel{2}\theta_2} + \frac{1}{\cancel{2}\theta_2^{\cancel{2}}}\sum_{i=1}^{n}(x_i-\theta_1)^2 = 0$$

$$\Rightarrow \boxed{\theta_2 = \frac{\sum_{i=1}^{n}(x_i-\theta_1)^2}{n}}$$

∴ Maximum Likehood ~~of~~

- mean $(\theta_1) = \frac{\sum_{i=1}^{n} x_i}{n}$
- variance $(\theta_2) = \frac{\sum_{i=1}^{n}(x_i-\theta_1)^2}{n}$

Ans//

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2. Density function for Binomial distribution $B(m,\theta)$ is :-

$$\boxed{\text{Pdf} = {}^{m}C_{x_i}\, \theta^{x_i}(1-\theta)^{m-x_i}}$$

Likelihood function L for a random sample of size n is given by :-

$$\boxed{L = \prod_{i=1}^{n} {}^{m}C_{x_i}\, \theta^{x_i}(1-\theta)^{m-x_i}}$$

Taking natural log on both sides :

$$\boxed{\ln L = \sum_{i=1}^{n} \left(\ln \left({}^{m}C_{x_i} \right) + x_i \ln\theta + (m - x_i) \ln(1-\theta) \right)} \quad \text{—— (1)}$$

To find Maximum Likelihood :-

Differentiating (1) w.r.t. θ and setting derivative to 0 :-

$$\frac{\partial \ln L}{\partial \theta} = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} + \frac{(m - x_i)(-1)}{(1-\theta)} \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right) = 0$$

$$\Rightarrow \frac{\Sigma x_i}{\theta} = \frac{nm - \Sigma x_i}{1-\theta}$$

$$\Sigma x_i - \cancel{\theta \Sigma x_i} = nm\theta - \cancel{\theta \Sigma x_i}$$

$$\boxed{\theta = \frac{\sum_{i=1}^{n} x_i}{nm}}$$

Ans//