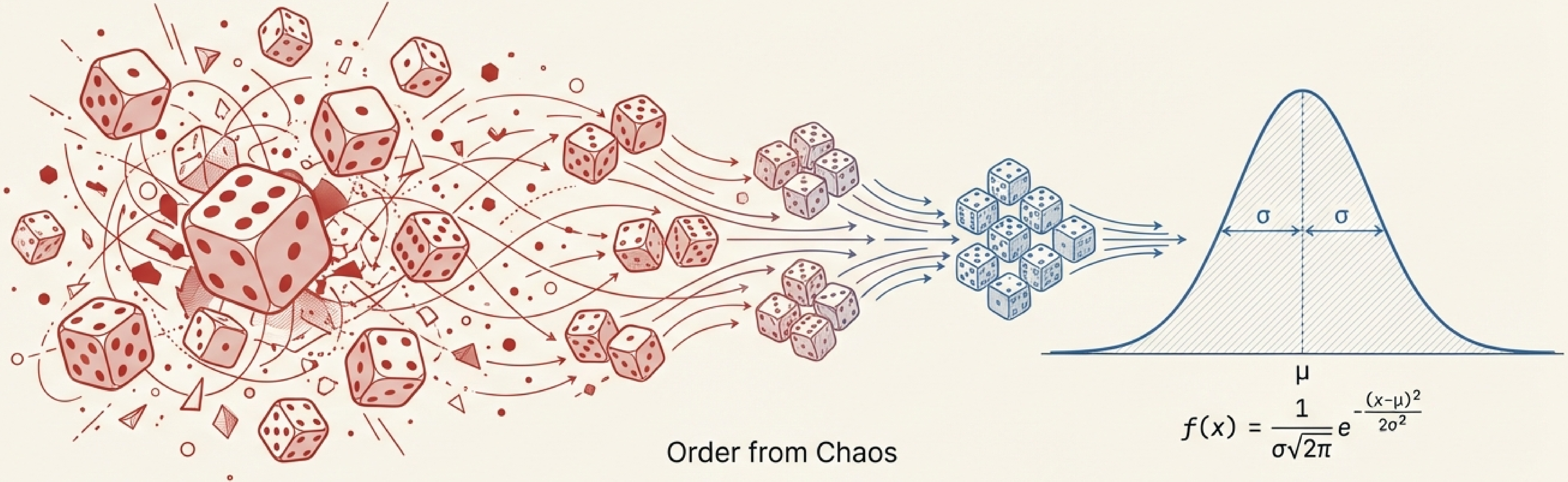
The Science of Chance:
The Science of Chance: From Intuition to Algorithm

A comprehensive journey through the mechanics, history, and philosophy of probability theory



Order from Chaos

Inter

Prepared for [Reader Name] | Based on the works of Grinstead, Snell, and historical archives.

Quantifying Uncertainty Through Ratios

Vocabulary

Experiment: A trial or operation producing an outcome (e.g., rolling a die).

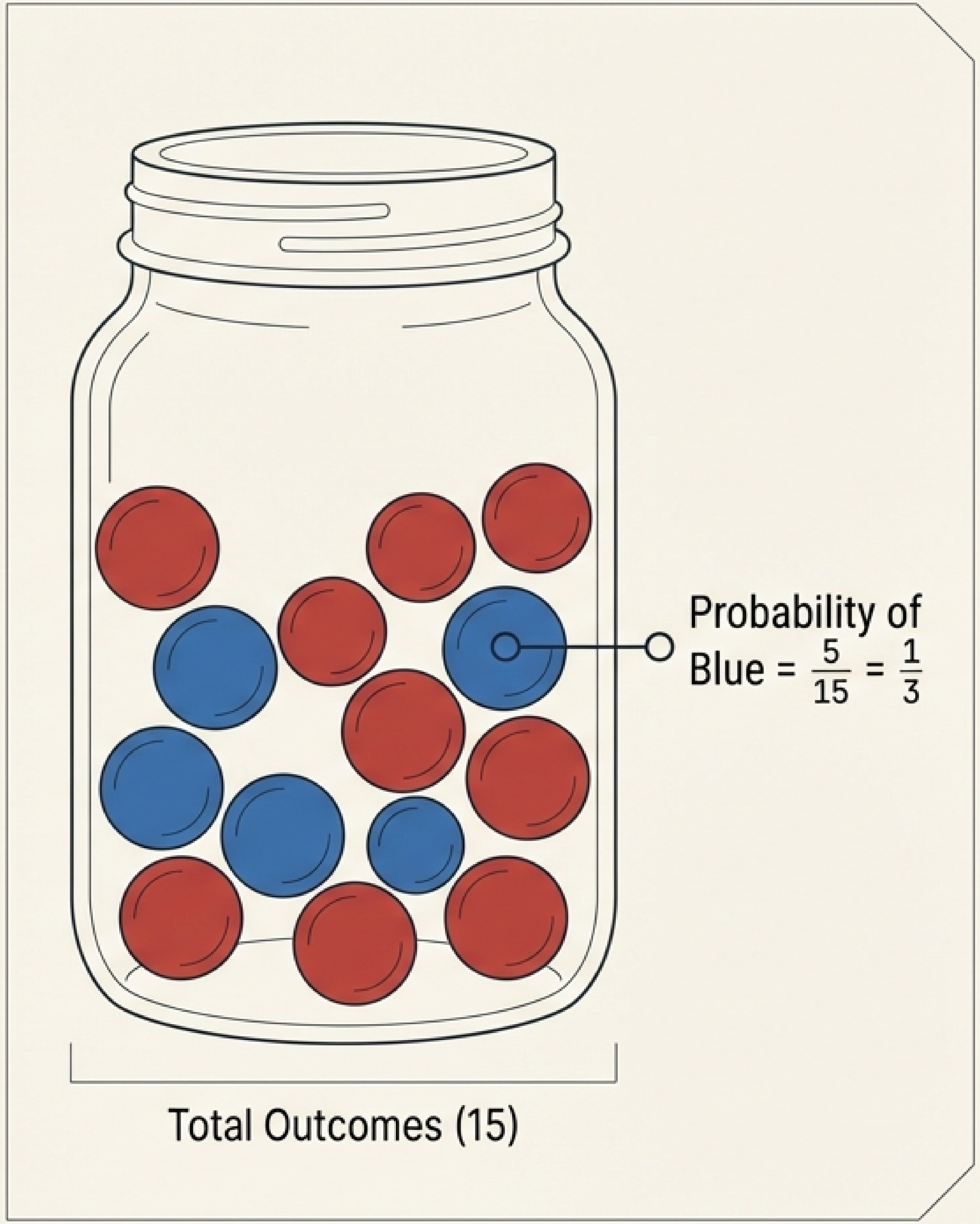
Sample Space (Ω): The set of all possible outcomes. For a die, Ω = {1, 2, 3, 4, 5, 6}.

Event: A subset of the sample space (e.g., rolling an even number {2, 4, 6}).

Probability is the ratio of favourable outcomes to the total number of possible outcomes in an experiment.

$$P(E) = \frac{n(E)}{n(S)}$$

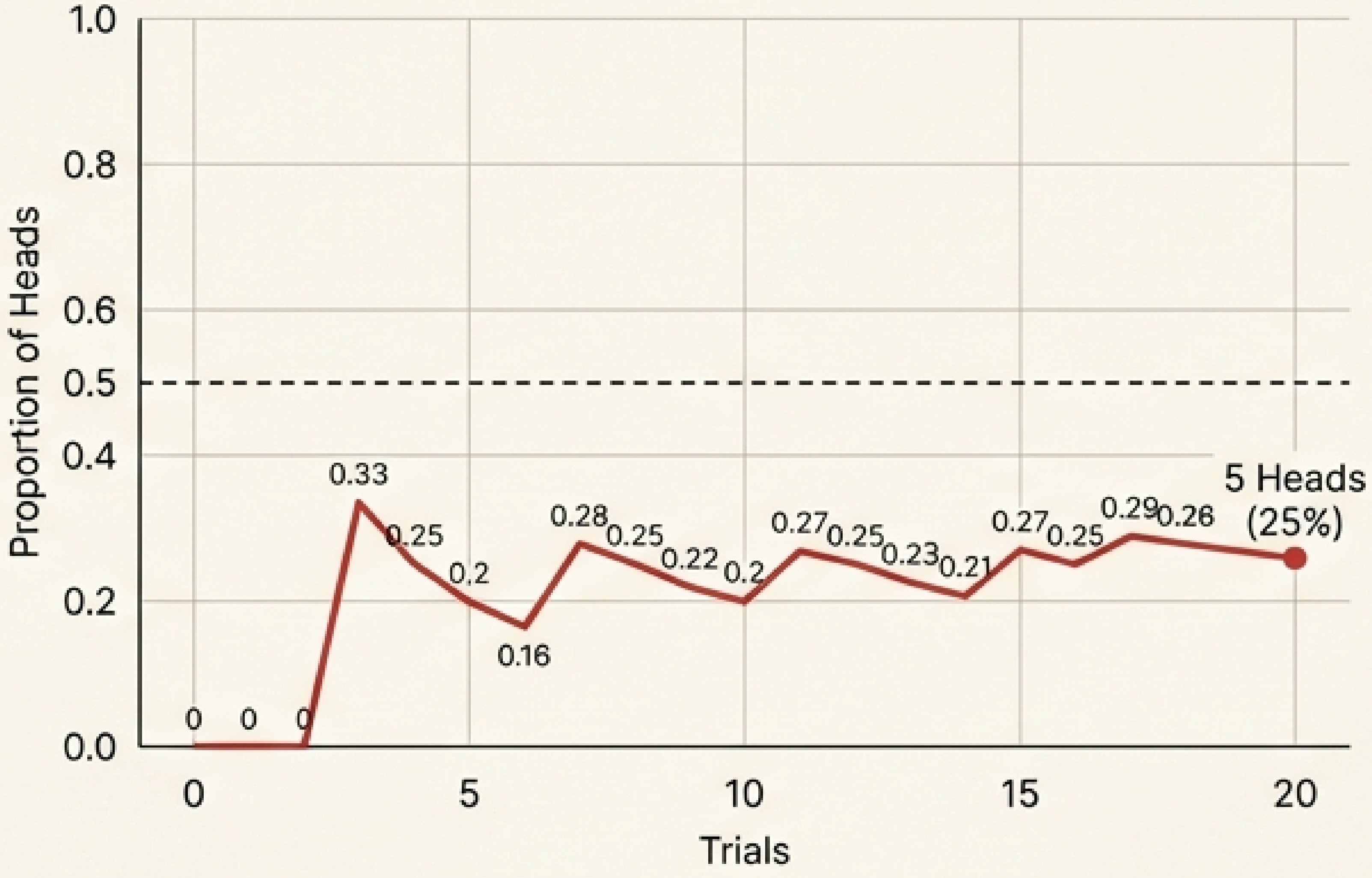
Where $n(E)$ is the number of favourable outcomes and $n(S)$ is the total outcomes in the sample space.



Frequency Stabilises Over Time

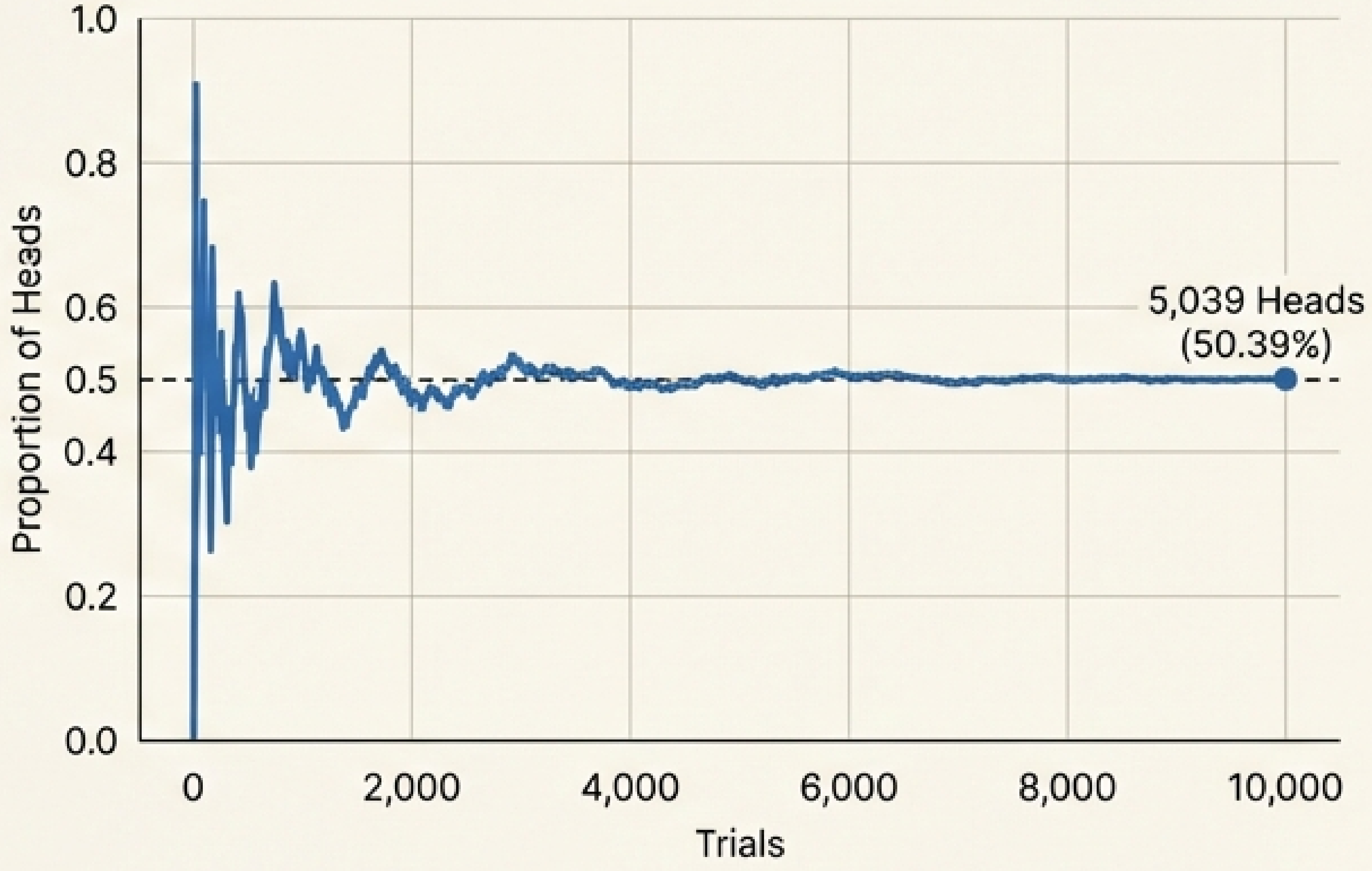
The Law of Large Numbers: As trials increase ($n \to \infty$), observed frequency converges on theoretical probability. in Inter and `JetBrains Mono`.

Short Run Chaos (n=20)



THTTTHTTTTHTTTTTTHHTT

Long Run Stability (n=10,000)

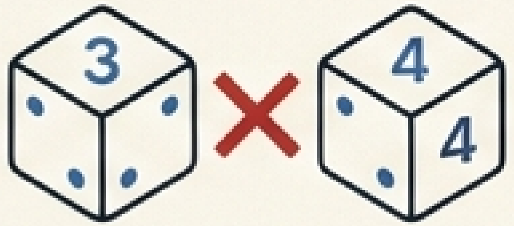


The Logic of Compound Events

Are the events combined?

Mutually Exclusive
(The "**OR**" Rule)

Events cannot happen simultaneously.



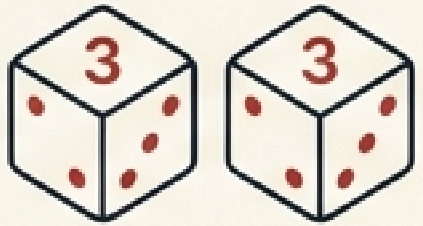
$$P(A \cup B) = P(A) + P(B)$$

Rolling a 3 OR a 4 on a single die:

$$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

Independent Events
(The "**AND**" Rule)

One event does not affect the other.



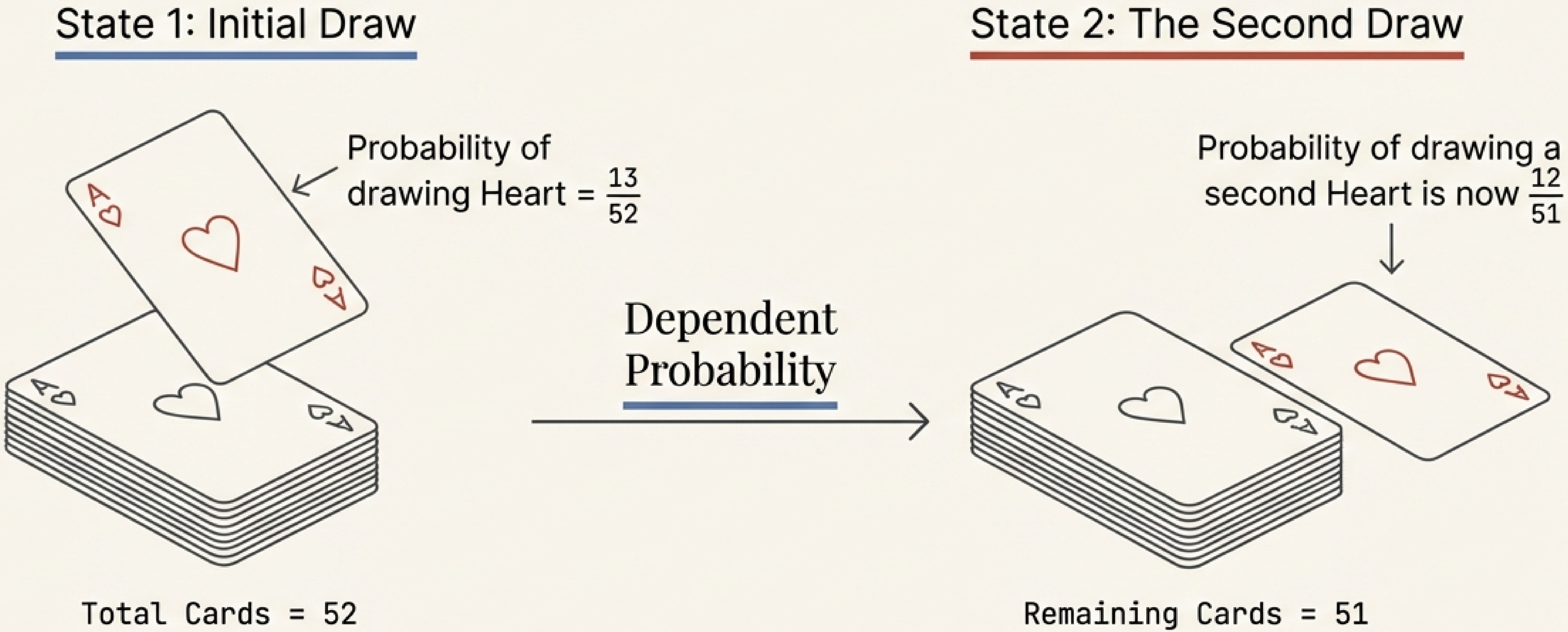
$$P(A \cap B) = P(A) \times P(B)$$

Rolling a 3 AND a 3 on two dice:

$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Conditional Probability and the Shrinking Sample Space

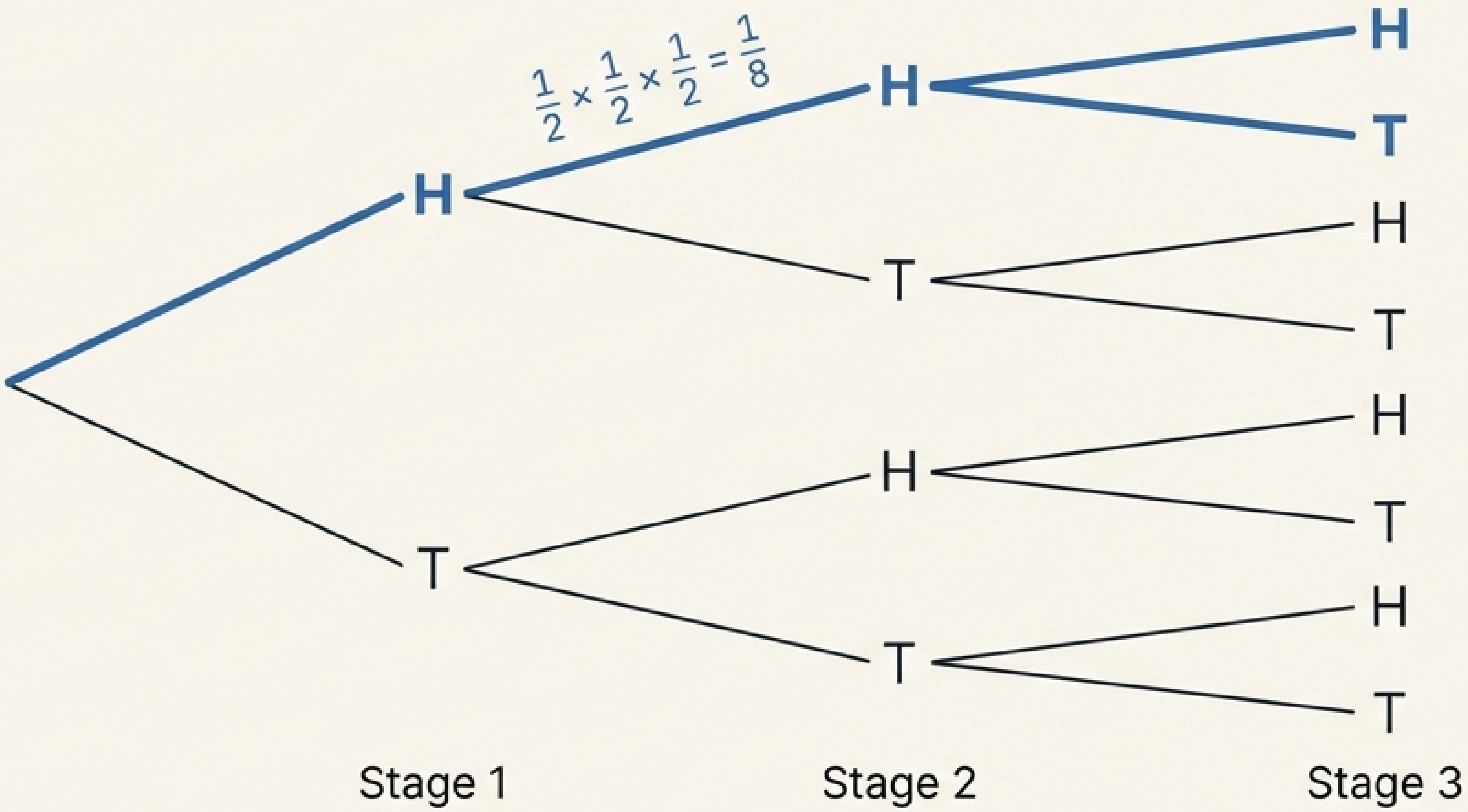
When new information fundamentally alters the math ($P(B|A)$).



$$P(A \cap B) = P(A) \times P(B|A)$$

Mapping Outcomes with Tree Diagrams

Visualising the sample space of 3 Coin Tosses

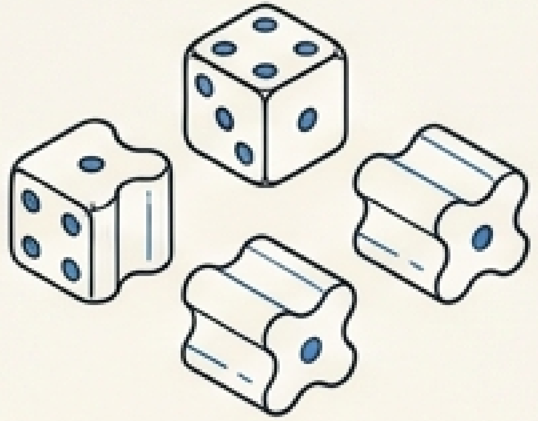


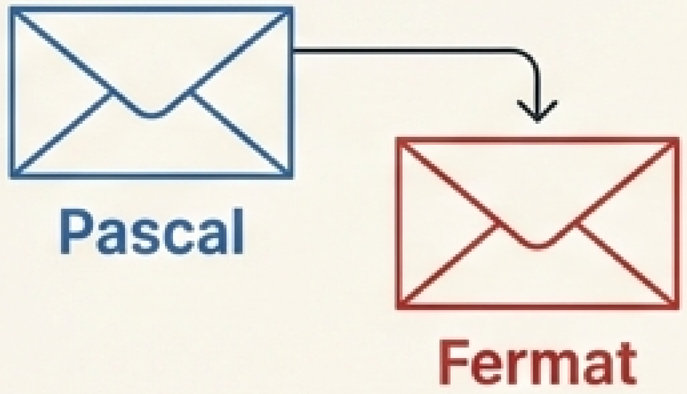
Sample Space:
{HHH, HHT,
HHT,
HTT,
THH,
THT,
TTH,
TTT}

Total outcomes (2^3) = 8.
Probability of at least one Head = $\frac{7}{8}$.

The Late Arrival of Probability

A 5,000-year gap between gambling and mathematics.





3500 B.C. - Ancient Egypt

Games like Hounds and Jackals use bone dice. Randomness is seen as fate, not math.

1494 - Paccioli

Discusses the "Problem of Points" in *Summa de Arithmetica*. Solution is incorrect.

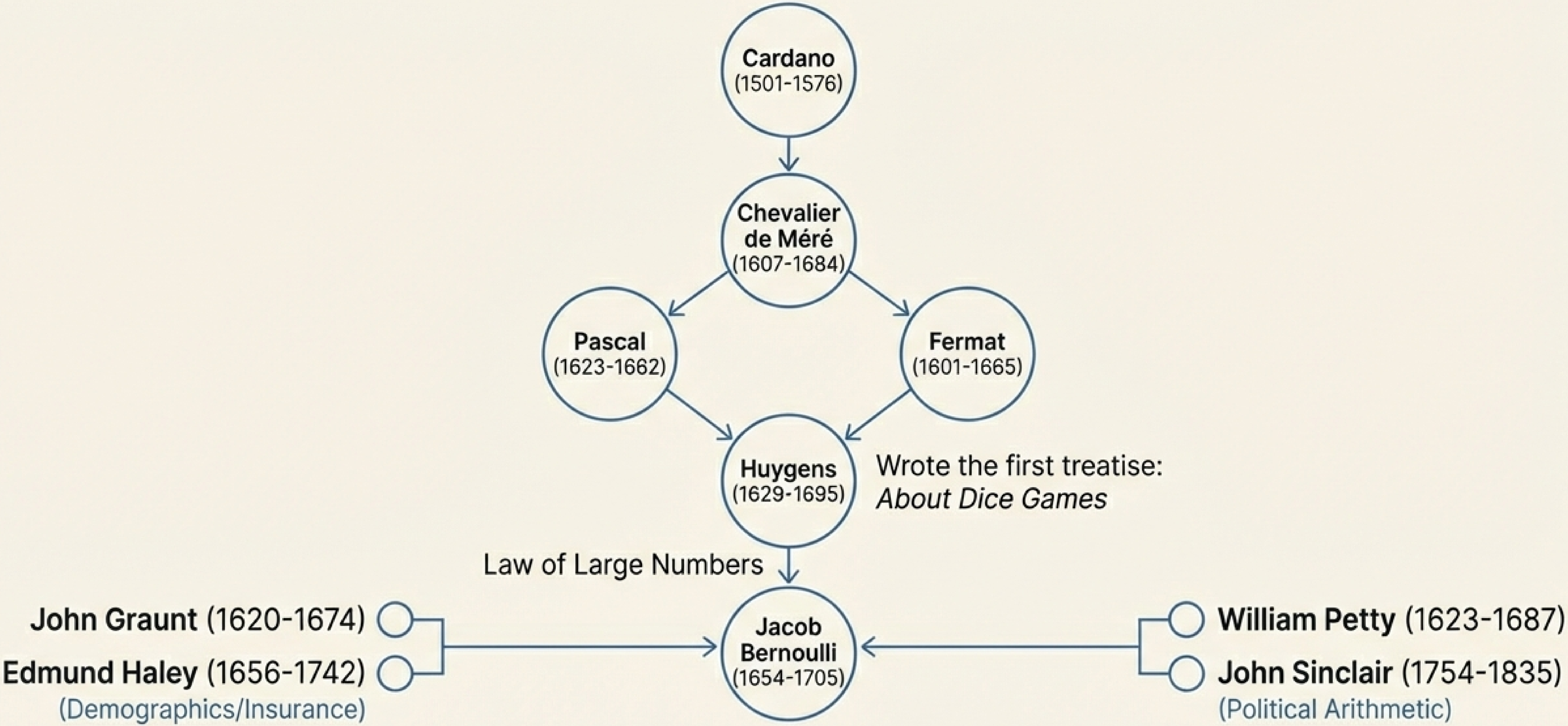
1526 - Cardano

Writes *Liber de Ludo Aleae*. Defines probability as a ratio, but the work is obscure.

1654 - The Turning Point

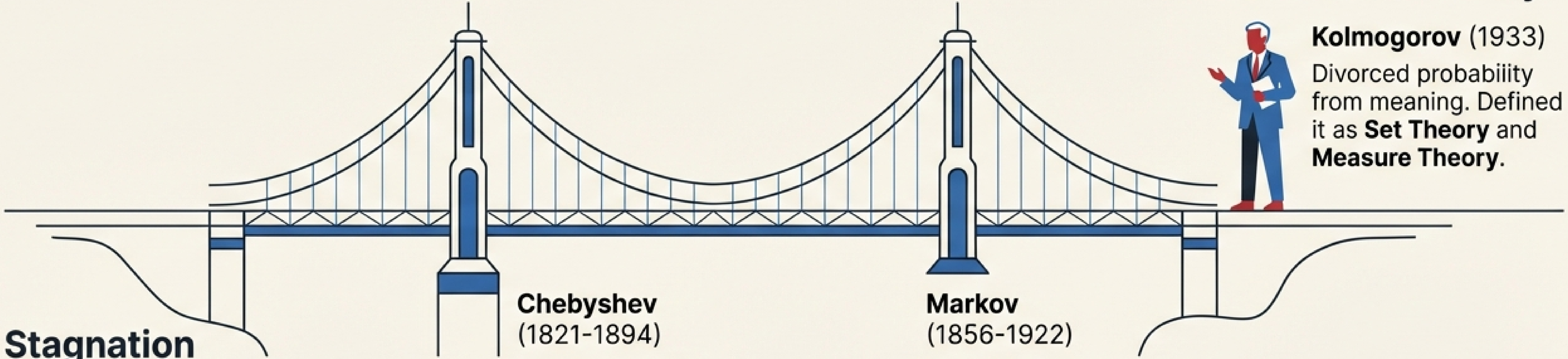
The Pascal-Fermat Correspondence solves the Problem of Points. Probability theory is born.

From Gambling Dens to Demographics



The Russian School and Axiomatic Rigor

Bridging 19th-century stagnation and 20th-century modern mathematics.



Modern Probability

Kolmogorov (1933)
Divorced probabiity from meaning. Defined it as **Set Theory** and **Measure Theory**.

Stagnation

Loose definitions

Chebyshev
(1821-1894)
Inequalities and limits.

Markov
(1856-1922)
Dependent variables (Chains).

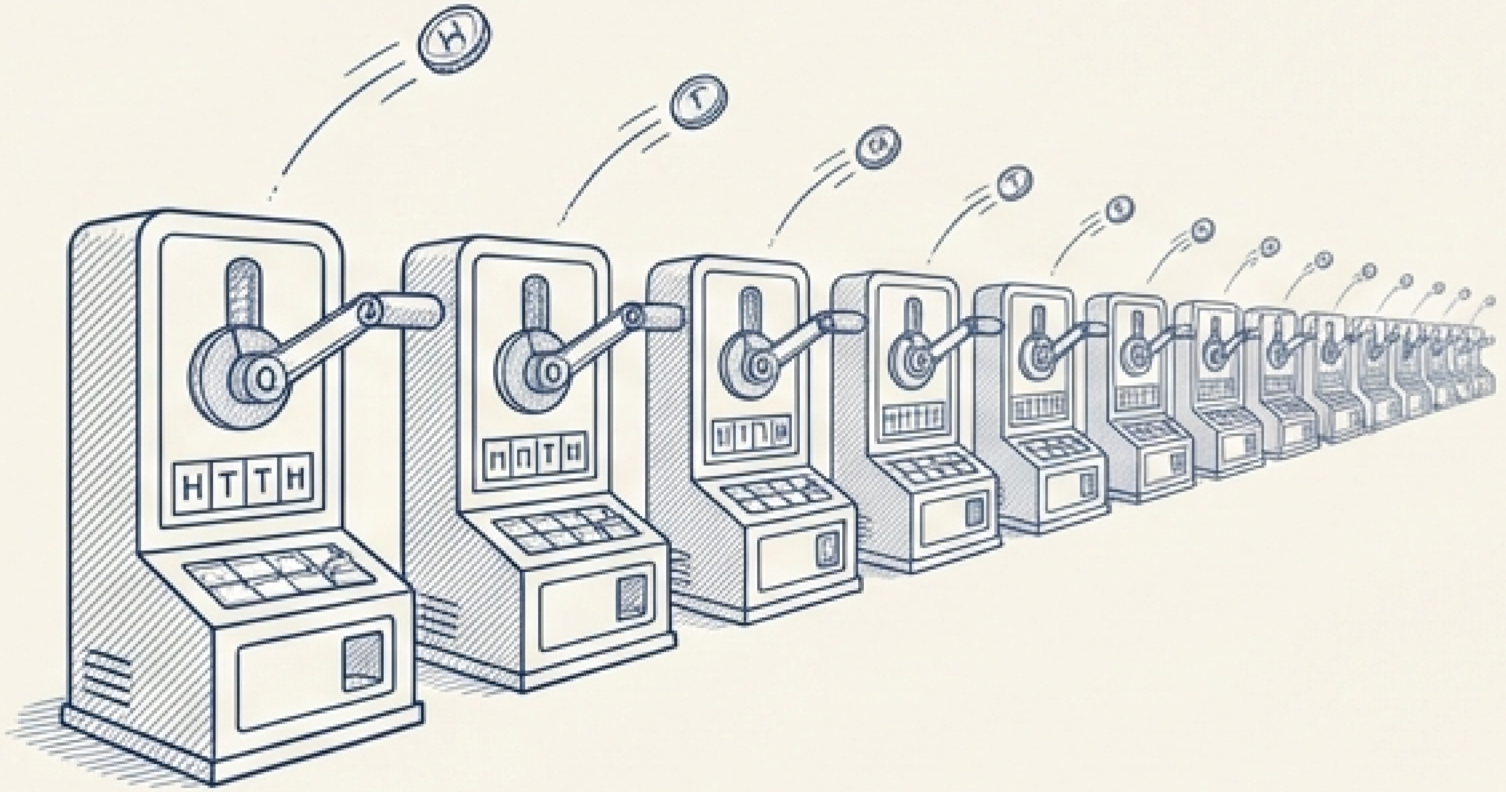
“The theory of probability… is a system of sets which which satisfy certain conditions.” — Kolmogorov

The Philosophical Divide: Objective vs. Subjective

The Frequentist View (Objective)

Definition: Probability is the relative frequency of an event over a long run of repeated trials.

Key trait: Parameters are **fixed constants**. Data is random.



The Bayesian View (Subjective)

Definition: Probability is a degree of belief in a proposition.

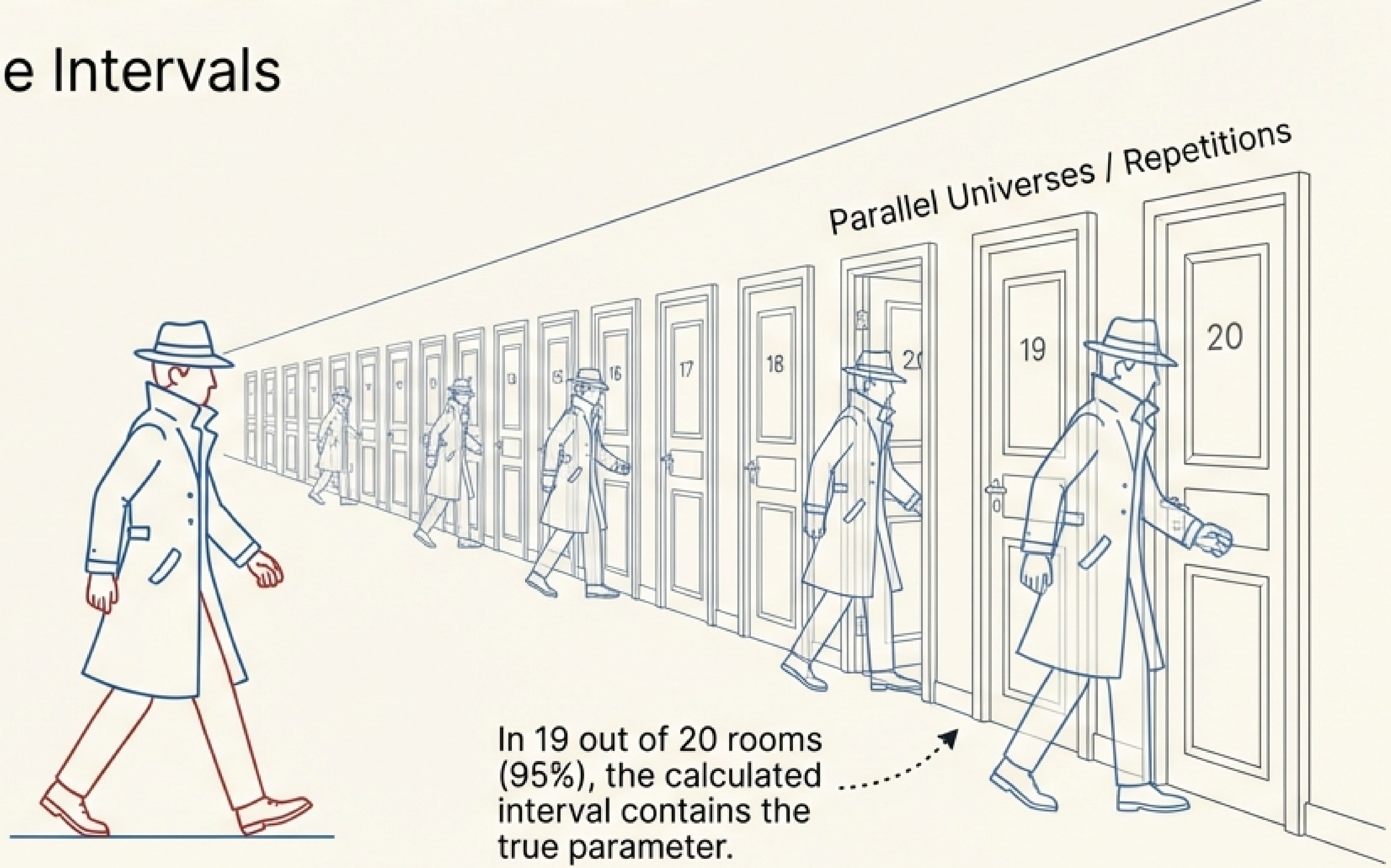
Key trait: Parameters are **random variables**. Data is fixed.

The Frequentist View: The Parallel Universe Walker

Understanding Confidence Intervals

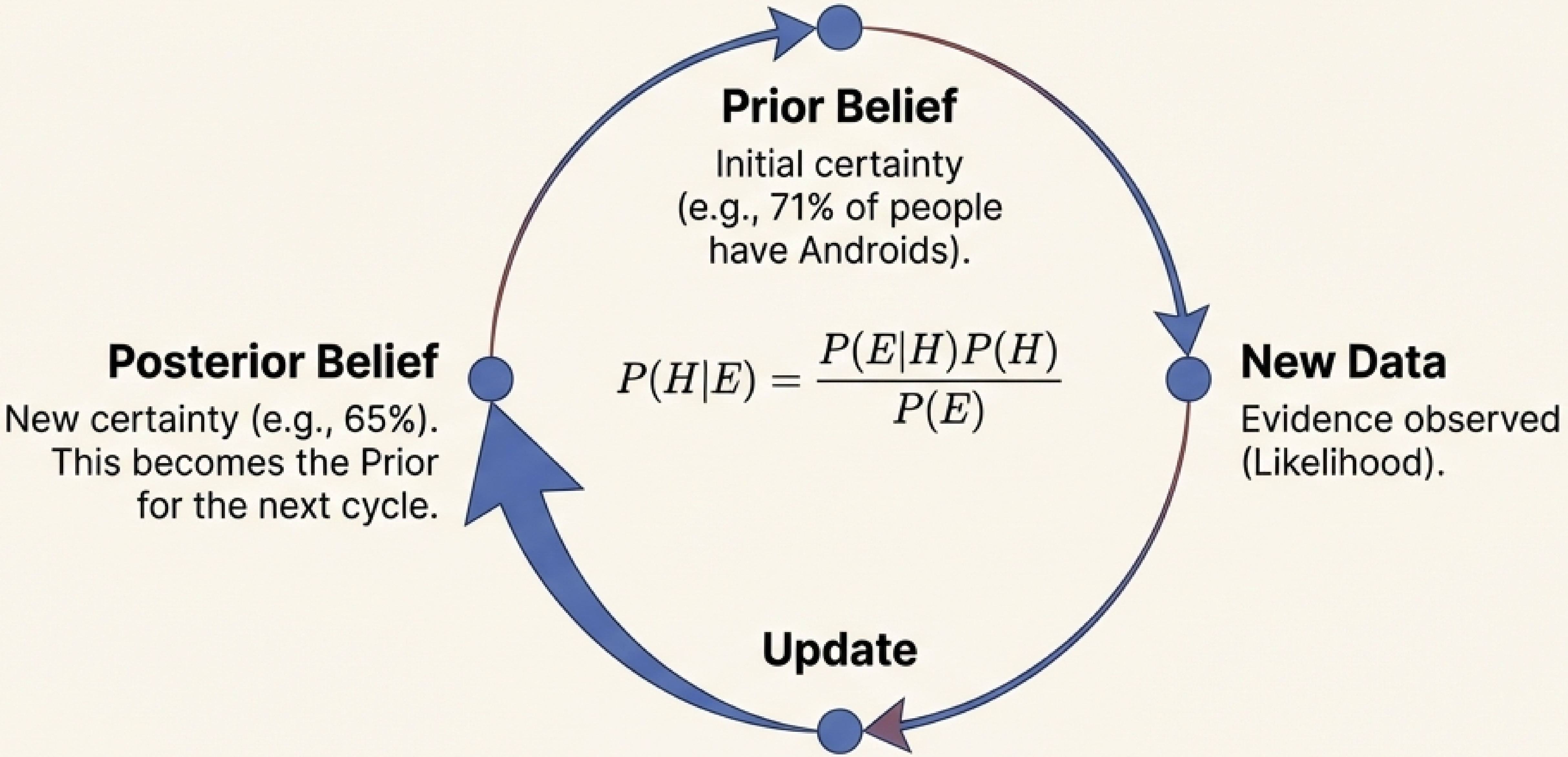
Philosophy: ‘If we repeated this experiment infinite times, what would happen?’

Constraint: We cannot calculate the probability of a specific hypothesis being true. It is either true or false. We can only measure the accuracy of the method.



The Bayesian View: The Architecture of Belief

Updating certainty with evidence.



Philosophy: Probability is a measure of knowledge. We can assign probability to single, non-repeatable events.

Case Study: The 1001st Person

Scenario: 1000 people surveyed. 543 have Androids. Is the 1001st person an Android user?

Frequentist Approach

- The person either *is* or *is not* an Android user. True probability is 0 or 1.
- We use the frequency (0.543) to guess, but we cannot assign a probability to the person, only to the accuracy of our guessing method over time.

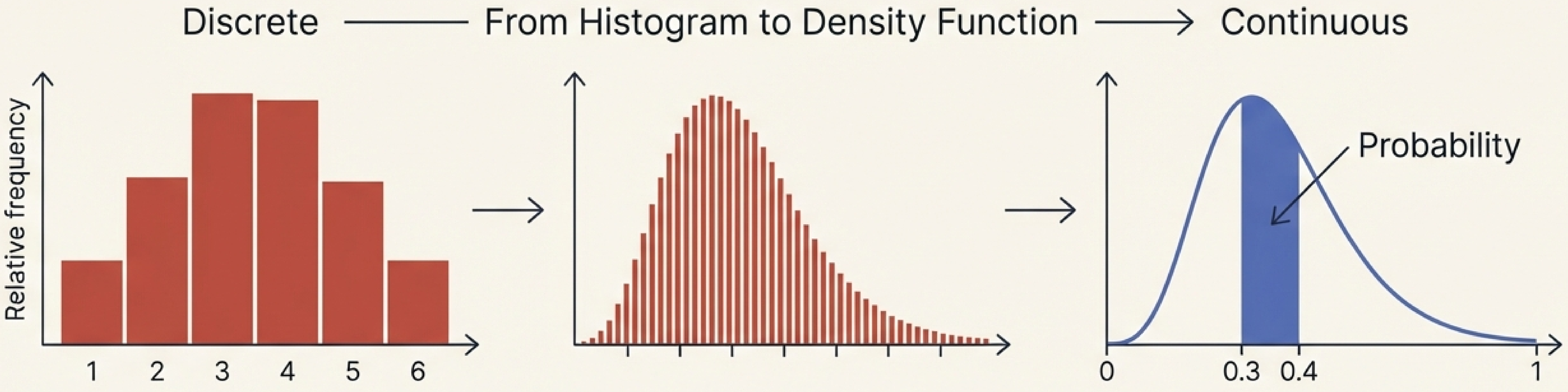
Bayesian Approach

- We incorporate **Prior Information** (e.g., **General** population is 71% Android).
- We combine **Prior + Data** (543/1000).
- **Conclusion:** We update our belief to a **Posterior Probability of 0.65.**
- **Verdict:** There is a 65% probability this specific person is an Android user.

NotebookLM

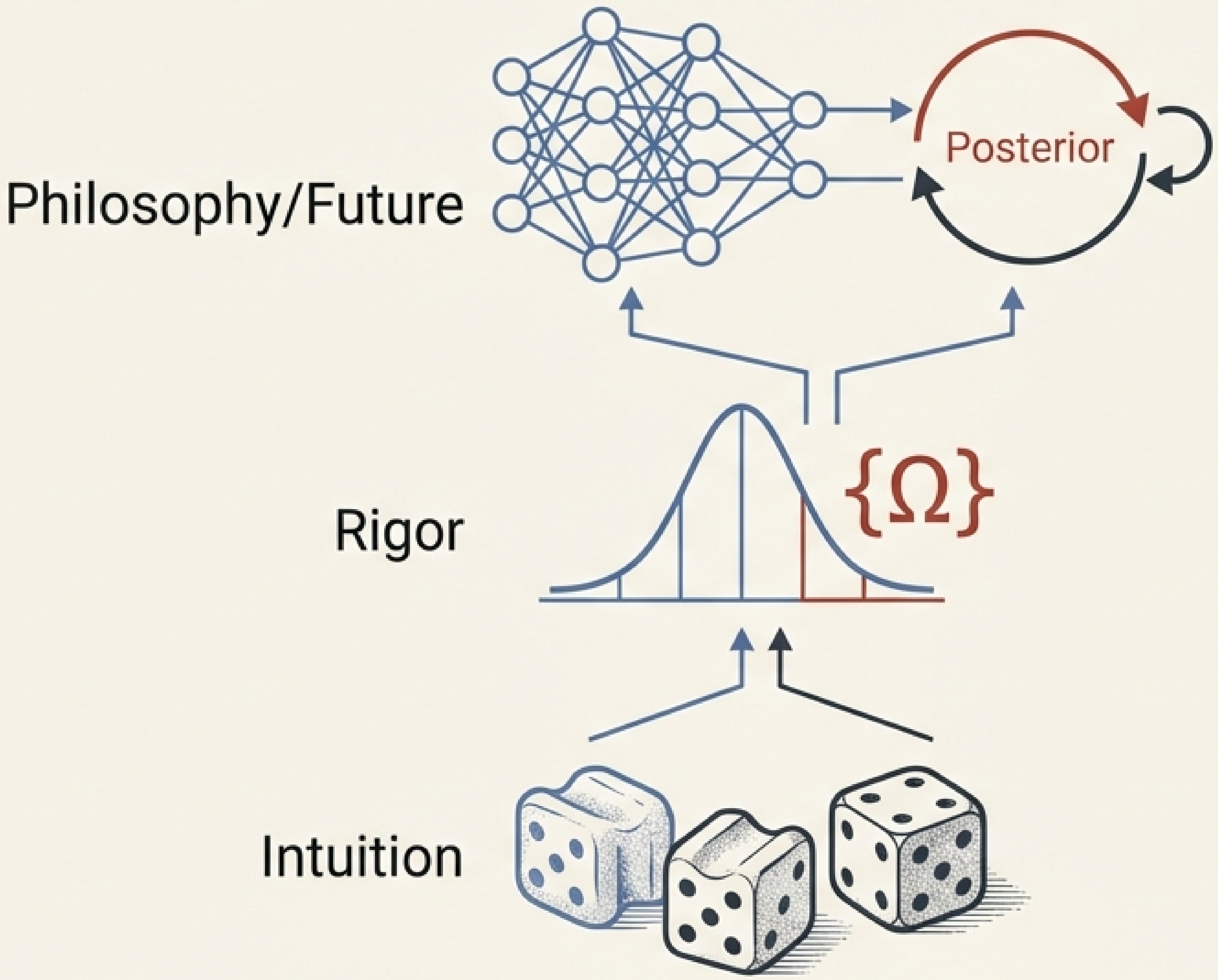
“Beyond the **Discrete:** Continuous Probability" in Playfair Display

From dice rolls (integers) to spinners (real numbers).



The Spinner Problem: The probability of landing on *exactly* 0.555... is 0. We measure the probability of landing within an **interval** (e.g., between 0.3 and 0.4).

The Architecture of Prediction



We have moved from the intuition of the gambler to the rigor of the mathematician and the philosophy of the data scientist.

Probability is the logic of science and the engine of modern Artificial Intelligence.