

## **Part I**

# **A First Course in General Relativity<sup>1</sup>**



# Chapter 1

## Special Relativity

### On “Principle of relativity (Galileo)”

#### Galilean invariance

Newton’s laws of motion hold in all frames related to one another by a Galilean transformation. In other words, all frames related to one another by such a transformation are inertial (meaning, Newton’s equation of motion is valid in these frames).<sup>2</sup> The proof has been given by the book on page 2.

### 1.5 - Construction of the coordinates used by another observer

#### Why would the tangent of the angle is the speed in Fig. 1.2?

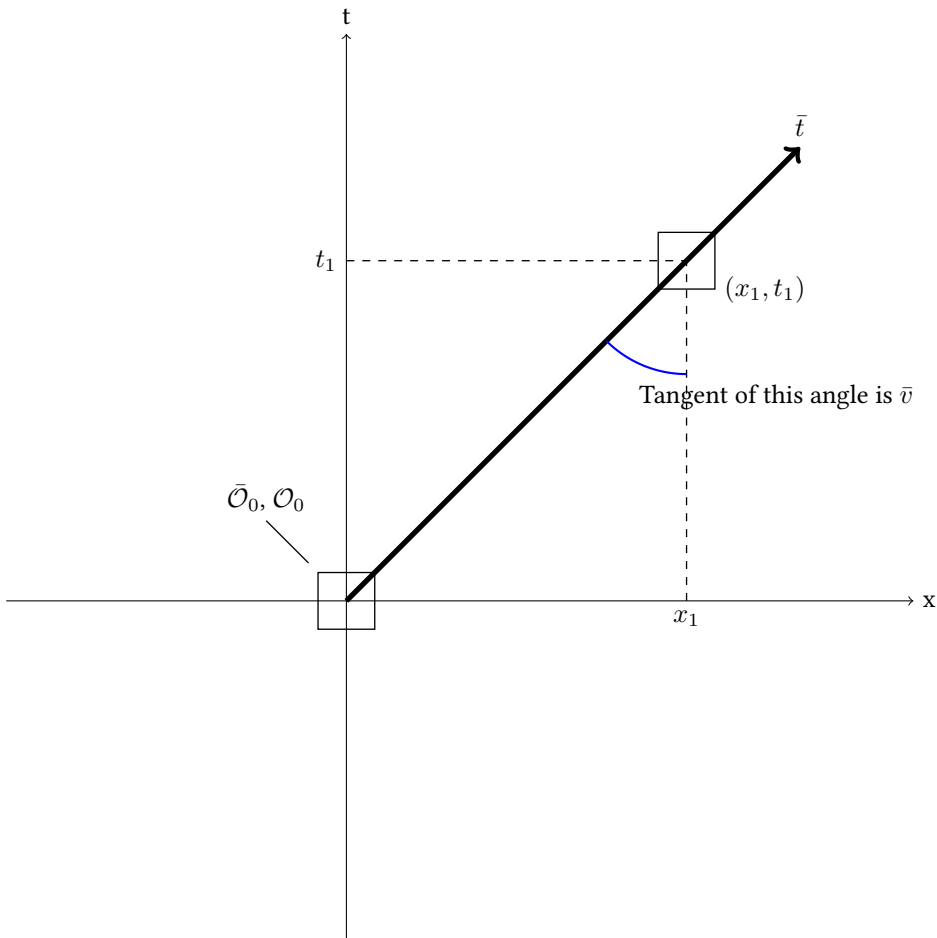
Suppose  $\mathcal{O}$  and  $\bar{\mathcal{O}}$  both start out at the same position where  $\bar{\mathcal{O}}$  moves along the  $x$  at some speed. After  $t_1$ , observer  $\mathcal{O}$  sees  $\bar{\mathcal{O}}$  at position  $x_1$ :

$$\bar{\mathcal{O}}_1 = (x_1, t_1)$$

Observer  $\bar{\mathcal{O}}$ , however, still sees themselves at  $x = 0$ :

$$\bar{\mathcal{O}}_1 = (0, t_1)$$

By definition where “ $\bar{t}$  is the locus of events at constant  $\bar{x} = 0$ ”,  $\bar{t}$  is the straight line that passes the origin and the  $(x_1, t_1)$ :



## 1.6 - Why $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (\Delta t)^2 = 0$ for two events in the same light beam?

Let's say, in a simplified 1D case, event  $\mathcal{E} = (x_0, t_0)$  and  $\mathcal{P} = (x_1, t_1)$ .

$$(\Delta x)^2 - (\Delta t)^2 = (x_1 - x_0)^2 - (t_1 - t_0)^2$$

Since the speed of light is 1,

$$(x_1 - x_0)^2 - (t_1 - t_0)^2 = (x_1 - x_0)^2 - (t_1 \times 1 - t_0 \times 1)^2 = (x_1 - x_0)^2 - (x_1 - x_0)^2 = 0$$

## Why do we have a 2nd term in equation 1.3 on p.10?

$$\Delta \bar{s}^2 = \sum_{\alpha=0}^3 \sum_{\beta=0}^3 M_{\alpha\beta} (\Delta x^\alpha) (\Delta x^\beta) \quad (1.1)$$

$$= \sum_{\alpha=0}^0 \sum_{\beta=0}^3 M_{\alpha\beta} (\Delta x^\alpha) (\Delta x^\beta) + \sum_{\alpha=0}^3 \sum_{\beta=0}^0 M_{\alpha\beta} (\Delta x^\alpha) (\Delta x^\beta) + \sum_{\alpha=1}^3 \sum_{\beta=1}^3 M_{\alpha\beta} (\Delta x^\alpha) (\Delta x^\beta) \quad (1.2)$$

$$= \sum_{\beta=0}^3 M_{0\beta} \Delta t (\Delta x^\beta) + \sum_{\alpha=0}^3 M_{\alpha 0} (\Delta x^\alpha) \Delta t + \sum_{\alpha=1}^3 \sum_{\beta=1}^3 M_{\alpha\beta} (\Delta x^\alpha) (\Delta x^\beta) \quad (1.3)$$

$$= 2M_{00} (\Delta t)^2 + \sum_{\beta=1}^3 M_{0\beta} \Delta t (\Delta x^\beta) + \sum_{\alpha=1}^3 M_{\alpha 0} (\Delta x^\alpha) \Delta t + \sum_{\alpha=1}^3 \sum_{\beta=1}^3 M_{\alpha\beta} (\Delta x^\alpha) (\Delta x^\beta) \quad (1.4)$$

$$= 2M_{00} (\Delta t)^2 + 2 \left[ \sum_{i=1}^3 M_{0i} \Delta t (\Delta x^i) \right] + \sum_{\alpha=1}^3 \sum_{\beta=1}^3 M_{\alpha\beta} (\Delta x^\alpha) (\Delta x^\beta) \quad (1.5)$$

$$(1.6)$$



# Bibliography

<sup>1</sup> Bernard Schutz. *[A First Course on General Relativity](#)*. 2009.

<sup>2</sup> Wikipedia. [Galilean invariance](#).