Part I

A First Course on General Relativity

By studying General Relativity, I spotted my weakness of being too submerged in details and not being able to solve problems systematically and as a whole

A person being able to master GR has the best advantage in analyzing business and tech problems right, which is why I study GR and Physics in general (Sorry, Einstein)

Chapter 1

Special Relativity

On "Principle of relativity (Galileo)"

Galilean invariance

Newton's laws of motion hold in all frames related to one another by a Galilean transformation. In other words, all frames related to one another by such a transformation are inertial (meaning, Newton's equation of motion is valid in these frames).¹ The proof has been given by the book on page 2.

1.5 - Construction of the coordinates used by another observer

Why would the tangent of the angle is the speed in Fig. 1.2?

Suppose \mathcal{O} and $\bar{\mathcal{O}}$ both start out at the same position where $\bar{\mathcal{O}}$ moves along the x at some speed. After t_1 , observer \mathcal{O} sees $\bar{\mathcal{O}}$ at position x_1 :

$$\bar{\mathcal{O}}_1 = (x_1, t_1)$$

Observer $\bar{\mathcal{O}}$, however, still sees themself at x=0:

$$\bar{\mathcal{O}}_1 = (0, t_1)$$

By definition where " \bar{t} is the locus of events at constant $\bar{x}=0$ ", \bar{t} is the straight line that passes the origin and the (x_1,t_1) :

¹Galilean invariance



1.6 Invariance of the interval

Why does the equation contains only $M_{\alpha\beta}+M_{\beta\alpha}$ terms when $\alpha\neq\beta$, which guarantees $M_{\alpha\beta}=M_{\beta\alpha}$?

$$\Delta \bar{s}^{2} = \sum_{\alpha=0}^{3} \sum_{\beta=0}^{3} M_{\alpha\beta} \left(\Delta x^{\alpha} \right) \left(\Delta x^{\beta} \right)$$

Before spending too much time on expanding the equation, we can pick up a pair of indices of $(\alpha, \beta) = (\alpha^*, \beta^*)$ where $\alpha^* \neq \beta^*$. Then we would definitely have the following 2 terms in the expansion:

$$\boldsymbol{M}_{\alpha^*\beta^*} \left(\Delta x^{\alpha^*}\right) \left(\Delta x^{\beta^*}\right)$$

$$\boldsymbol{M}_{eta^*lpha^*}\left(\Delta x^{eta^*}\right)\left(\Delta x^{lpha^*}\right)$$

Since

$$\left(\Delta x^{\alpha^*}\right)\left(\Delta x^{\beta^*}\right) = \left(\Delta x^{\beta^*}\right)\left(\Delta x^{\alpha^*}\right)$$

We can then group these 2 terms and factor out the product, leaving

$$\left(\Delta x^{\alpha^*}\right)\left(\Delta x^{\beta^*}\right)\left(\boldsymbol{M}_{\alpha^*\beta^*}+\boldsymbol{M}_{\beta^*\alpha^*}\right)$$

The terms of expanded $\Delta \bar{s}^2$ can be expressed in a matrix of

$$\begin{bmatrix} \boldsymbol{M}_{00} \Delta x^0 \Delta x^0 & \boldsymbol{M}_{01} \Delta x^0 \Delta x^1 & \boldsymbol{M}_{02} \Delta x^0 \Delta x^2 & \boldsymbol{M}_{03} \Delta x^0 \Delta x^3 \\ \boldsymbol{M}_{10} \Delta x^1 \Delta x^0 & \boldsymbol{M}_{11} \Delta x^1 \Delta x^1 & \boldsymbol{M}_{12} \Delta x^1 \Delta x^2 & \boldsymbol{M}_{13} \Delta x^1 \Delta x^3 \\ \boldsymbol{M}_{20} \Delta x^2 \Delta x^0 & \boldsymbol{M}_{21} \Delta x^2 \Delta x^1 & \boldsymbol{M}_{22} \Delta x^2 \Delta x^2 & \boldsymbol{M}_{23} \Delta x^2 \Delta x^3 \\ \boldsymbol{M}_{30} \Delta x^3 \Delta x^0 & \boldsymbol{M}_{31} \Delta x^3 \Delta x^1 & \boldsymbol{M}_{32} \Delta x^3 \Delta x^2 & \boldsymbol{M}_{33} \Delta x^3 \Delta x^3 \end{bmatrix}$$

Because the off-diagonal terms always appear in paris above, we could effectively replace them with their mean value:

$$oldsymbol{M}_{lpha^*eta^*} = oldsymbol{M}_{eta^*lpha^*} = rac{(oldsymbol{M}_{lpha^*eta^*} + oldsymbol{M}_{eta^*lpha^*})}{2}$$

And since $oldsymbol{M}_{lphaeta}=oldsymbol{M}_{etalpha}$ if lpha=eta , we conclude that

$$oldsymbol{M}_{lphaeta} = oldsymbol{M}_{etalpha}$$
 for all $lpha$ and eta

Why do we have a 2nd term in equation 1.3 on p.10?

$$\Delta \bar{s}^2 = \sum_{\alpha=0}^{3} \sum_{\beta=0}^{3} \boldsymbol{M}_{\alpha\beta} \left(\Delta x^{\alpha} \right) \left(\Delta x^{\beta} \right) \tag{1.1}$$

$$=\sum_{\alpha=0}^{0}\sum_{\beta=0}^{3}\boldsymbol{M}_{\alpha\beta}\left(\Delta x^{\alpha}\right)\left(\Delta x^{\beta}\right)+\sum_{\alpha=0}^{3}\sum_{\beta=0}^{0}\boldsymbol{M}_{\alpha\beta}\left(\Delta x^{\alpha}\right)\left(\Delta x^{\beta}\right)+\sum_{\alpha=1}^{3}\sum_{\beta=1}^{3}\boldsymbol{M}_{\alpha\beta}\left(\Delta x^{\alpha}\right)\left(\Delta x^{\beta}\right)$$
(1.2)

$$=\sum_{\beta=0}^{3}\boldsymbol{M}_{0\beta}\Delta t\left(\Delta x^{\beta}\right)+\sum_{\alpha=0}^{3}\boldsymbol{M}_{\alpha 0}\left(\Delta x^{\alpha}\right)\Delta t+\sum_{\alpha=1}^{3}\sum_{\beta=1}^{3}\boldsymbol{M}_{\alpha \beta}\left(\Delta x^{\alpha}\right)\left(\Delta x^{\beta}\right)\tag{1.3}$$

$$= \boldsymbol{M}_{00} \left(\Delta t\right)^{2} + \sum_{\beta=1}^{3} \boldsymbol{M}_{0\beta} \Delta t \left(\Delta x^{\beta}\right) + \sum_{\alpha=1}^{3} \boldsymbol{M}_{\alpha 0} \left(\Delta x^{\alpha}\right) \Delta t + \sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} \boldsymbol{M}_{\alpha \beta} \left(\Delta x^{\alpha}\right) \left(\Delta x^{\beta}\right)$$
(1.4)

$$= \boldsymbol{M}_{00} \left(\Delta t\right)^{2} + 2 \left[\sum_{i=1}^{3} \boldsymbol{M}_{0i} \Delta t \left(\Delta x^{i}\right)\right] + \sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} \boldsymbol{M}_{\alpha\beta} \left(\Delta x^{\alpha}\right) \left(\Delta x^{\beta}\right)$$

$$(1.5)$$

1.6 - Why $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (\Delta t)^2 = 0$ for two events in the same light beam?

Let's say, in a simplified 1D case, event $\mathcal{E}=(x_0,t_0)$ and $\mathcal{P}=(x_1,t_1)$.

$$(\Delta x)^2 - (\Delta t)^2 = (x_1 - x_0)^2 - (t_1 - t_0)^2$$

Since the speed of light is 1,

$$(x_1 - x_0)^2 - (t_1 - t_0)^2 = (x_1 - x_0)^2 - (t_1 \times 1 - t_0 \times 1)^2 = (x_1 - x_0)^2 - (x_1 - x_0)^2 = 0$$