Part I

A First Course on General Relativity

Chapter 1

Special Relativity

On "Principle of relativity (Galileo)"

Galilean invariance

Newton's laws of motion hold in all frames related to one another by a Galilean transformation. In other words, all frames related to one another by such a transformation are inertial (meaning, Newton's equation of motion is valid in these frames).[2] The proof has been given by the book on page 2.

1.5 - Construction of the coordinates used by another observer

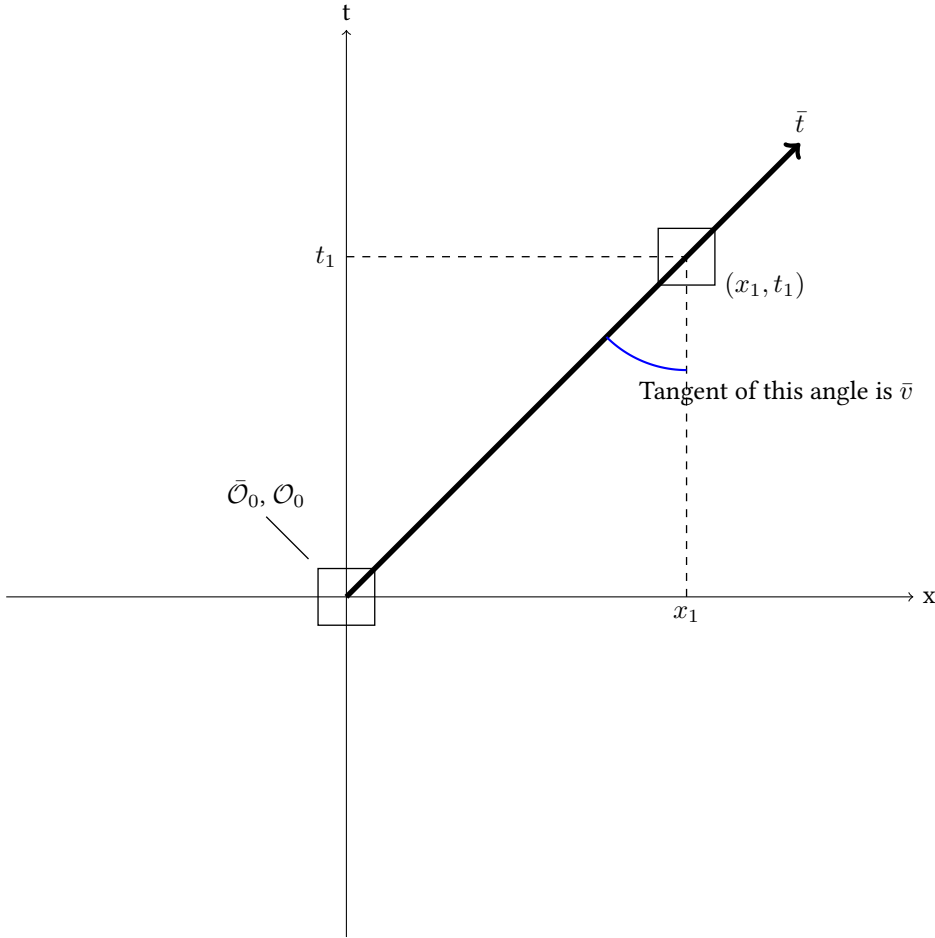### Why would the tangent of the angle is the speed in Fig. 1.2?

Suppose $\mathcal{O}$ and $\bar{\mathcal{O}}$ both start out at the same position where $\bar{\mathcal{O}}$ moves along the x at some speed. After t_1, observer $\mathcal{O}$ sees $\bar{\mathcal{O}}$ at position x_1:

$$\bar{\mathcal{O}}_1 = (x_1, t_1)$$

Observer $\bar{\mathcal{O}}$, however, still sees themself at $x = 0$:

$$\bar{\mathcal{O}}_1 = (0, t_1)$$

By definition where "$\bar{t}$ is the locus of events at constant $\bar{x} = 0$", $\bar{t}$ is the straight line that passes the origin and the (x_1, t_1):

1.6 Invariance of the interval

$$\Delta \bar{s}^2 = \sum_{\alpha=0}^{3} \sum_{\beta=0}^{3} M_{\alpha\beta} \left(\Delta x^{\alpha}\right) \left(\Delta x^{\beta}\right)$$

1.6 - Why $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (\Delta t)^2 = 0$ **for two events in the same light beam?**

Let's say, in a simplified 1D case, event $\mathcal{E} = (x_0, t_0)$ and $\mathcal{P} = (x_1, t_1)$.

$$(\Delta x)^2 - (\Delta t)^2 = (x_1 - x_0)^2 - (t_1 - t_0)^2$$

Since the speed of light is 1,

$$(x_1 - x_0)^2 - (t_1 - t_0)^2 = (x_1 - x_0)^2 - (t_1 \times 1 - t_0 \times 1)^2 = (x_1 - x_0)^2 - (x_1 - x_0)^2 = 0$$

Why do we have a 2nd term in equation 1.3 on p.10?

$$\Delta \bar{s}^2 = \sum_{\alpha=0}^{3} \sum_{\beta=0}^{3} \boldsymbol{M}_{\alpha\beta} \left(\Delta x^\alpha\right)\left(\Delta x^\beta\right) \tag{1.1}$$

$$= \sum_{\alpha=0}^{0} \sum_{\beta=0}^{3} \boldsymbol{M}_{\alpha\beta} \left(\Delta x^\alpha\right)\left(\Delta x^\beta\right) + \sum_{\alpha=0}^{3} \sum_{\beta=0}^{0} \boldsymbol{M}_{\alpha\beta} \left(\Delta x^\alpha\right)\left(\Delta x^\beta\right) + \sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} \boldsymbol{M}_{\alpha\beta} \left(\Delta x^\alpha\right)\left(\Delta x^\beta\right) \tag{1.2}$$

$$= \sum_{\beta=0}^{3} \boldsymbol{M}_{0\beta} \Delta t \left(\Delta x^\beta\right) + \sum_{\alpha=0}^{3} \boldsymbol{M}_{\alpha 0} \left(\Delta x^\alpha\right) \Delta t + \sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} \boldsymbol{M}_{\alpha\beta} \left(\Delta x^\alpha\right)\left(\Delta x^\beta\right) \tag{1.3}$$

$$= \boldsymbol{M}_{00} \left(\Delta t\right)^2 + \sum_{\beta=1}^{3} \boldsymbol{M}_{0\beta} \Delta t \left(\Delta x^\beta\right) + \sum_{\alpha=1}^{3} \boldsymbol{M}_{\alpha 0} \left(\Delta x^\alpha\right) \Delta t + \sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} \boldsymbol{M}_{\alpha\beta} \left(\Delta x^\alpha\right)\left(\Delta x^\beta\right) \tag{1.4}$$

$$= \boldsymbol{M}_{00} \left(\Delta t\right)^2 + 2 \left[\sum_{i=1}^{3} \boldsymbol{M}_{0i} \Delta t \left(\Delta x^i\right) \right] + \sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} \boldsymbol{M}_{\alpha\beta} \left(\Delta x^\alpha\right)\left(\Delta x^\beta\right) \tag{1.5}$$

Bibliography

[1] Bernard Schutz. 2009.

[2] Wikipedia. Galilean invariance.