Part I

A First Course on General Relativity

By studying General Relativity, I spotted my weakness of being too submerged in details and not being able to solve problems systematically and as a whole

A person being able to master GR has the best advantage in analyzing business and tech problems right, which is why I study GR and Physics in general (Sorry, Einstein)

Chapter 1

Special Relativity

On "Principle of relativity (Galileo)"

Galilean invariance

Newton's laws of motion hold in all frames related to one another by a Galilean transformation. In other words, all frames related to one another by such a transformation are inertial (meaning, Newton's equation of motion is valid in these frames).[1] The proof has been given by the book on page 2.

1.5 - Construction of the coordinates used by another observer

Why would the tangent of the angle is the speed in Fig. 1.2?

Suppose $\mathcal{O}$ and $\bar{\mathcal{O}}$ both start out at the same position where $\bar{\mathcal{O}}$ moves along the x at some speed. After t_1, observer $\mathcal{O}$ sees $\bar{\mathcal{O}}$ at position x_1:

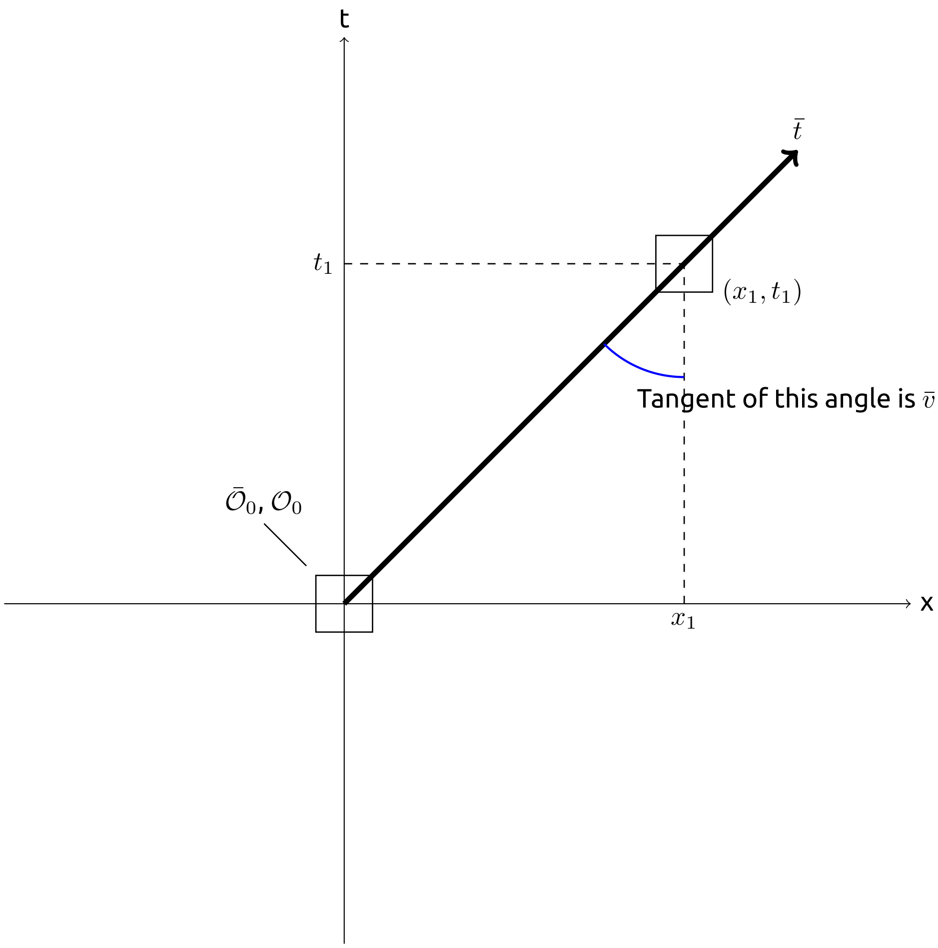
$$\bar{\mathcal{O}}_1 = (x_1, t_1)$$

Observer $\bar{\mathcal{O}}$, however, still sees themself at $x = 0$:

$$\bar{\mathcal{O}}_1 = (0, t_1)$$

By definition where "$\bar{t}$ is the locus of events at constant $\bar{x} = 0$", $\bar{t}$ is the straight line that passes the origin and the (x_1, t_1):

[1] Galilean invariance



1.6 Invariance of the interval

Why does the equation contains only $M_{\alpha\beta} + M_{\beta\alpha}$ terms when $\alpha \neq \beta$, which guarantees $M_{\alpha\beta} = M_{\beta\alpha}$?

$$\Delta\bar{s}^2 = \sum_{\alpha=0}^{3}\sum_{\beta=0}^{3} M_{\alpha\beta}\left(\Delta x^{\alpha}\right)\left(\Delta x^{\beta}\right)$$

Before spending too much time on expanding the equation, we can pick up a pair of indices of $(\alpha, \beta) = (\alpha^*, \beta^*)$ where $\alpha^* \neq \beta^*$. Then we would definitely have the following 2 terms in the expansion:

$$M_{\alpha^*\beta^*}\left(\Delta x^{\alpha^*}\right)\left(\Delta x^{\beta^*}\right)$$

$$M_{\beta^*\alpha^*}\left(\Delta x^{\beta^*}\right)\left(\Delta x^{\alpha^*}\right)$$

Since

$$\left(\Delta x^{\alpha^*}\right)\left(\Delta x^{\beta^*}\right) = \left(\Delta x^{\beta^*}\right)\left(\Delta x^{\alpha^*}\right)$$

We can then group these 2 terms and factor out the product, leaving

$$\left(\Delta x^{\alpha^*}\right)\left(\Delta x^{\beta^*}\right)\left(\boldsymbol{M}_{\alpha^*\beta^*}+\boldsymbol{M}_{\beta^*\alpha^*}\right)$$

The terms of expanded $\Delta\bar{s}^2$ can be expressed in a matrix of

$$\begin{bmatrix}
\boldsymbol{M}_{00}\Delta x^0\Delta x^0 & \boldsymbol{M}_{01}\Delta x^0\Delta x^1 & \boldsymbol{M}_{02}\Delta x^0\Delta x^2 & \boldsymbol{M}_{03}\Delta x^0\Delta x^3 \\
\boldsymbol{M}_{10}\Delta x^1\Delta x^0 & \boldsymbol{M}_{11}\Delta x^1\Delta x^1 & \boldsymbol{M}_{12}\Delta x^1\Delta x^2 & \boldsymbol{M}_{13}\Delta x^1\Delta x^3 \\
\boldsymbol{M}_{20}\Delta x^2\Delta x^0 & \boldsymbol{M}_{21}\Delta x^2\Delta x^1 & \boldsymbol{M}_{22}\Delta x^2\Delta x^2 & \boldsymbol{M}_{23}\Delta x^2\Delta x^3 \\
\boldsymbol{M}_{30}\Delta x^3\Delta x^0 & \boldsymbol{M}_{31}\Delta x^3\Delta x^1 & \boldsymbol{M}_{32}\Delta x^3\Delta x^2 & \boldsymbol{M}_{33}\Delta x^3\Delta x^3
\end{bmatrix}$$

Because the off-diagonal terms always appear in paris above, we could effectively replace them with their mean value:

$$\boldsymbol{M}_{\alpha^*\beta^*}=\boldsymbol{M}_{\beta^*\alpha^*}=\frac{\left(\boldsymbol{M}_{\alpha^*\beta^*}+\boldsymbol{M}_{\beta^*\alpha^*}\right)}{2}$$

where $\alpha^* \neq \beta^*$
And since $\boldsymbol{M}_{\alpha\beta}=\boldsymbol{M}_{\beta\alpha}$ if $\alpha=\beta$, we conclude that

$$\boxed{\boldsymbol{M}_{\alpha\beta}=\boldsymbol{M}_{\beta\alpha} \text{ for all } \alpha \text{ and } \beta}$$

Why do we have a 2nd term in equation 1.3 on p.10?

$$\Delta\bar{s}^2=\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}\boldsymbol{M}_{\alpha\beta}\left(\Delta x^\alpha\right)\left(\Delta x^\beta\right) \tag{1.1}$$

$$=\sum_{\alpha=0}^{0}\sum_{\beta=0}^{3}\boldsymbol{M}_{\alpha\beta}\left(\Delta x^\alpha\right)\left(\Delta x^\beta\right)+\sum_{\alpha=0}^{3}\sum_{\beta=0}^{0}\boldsymbol{M}_{\alpha\beta}\left(\Delta x^\alpha\right)\left(\Delta x^\beta\right)+\sum_{\alpha=1}^{3}\sum_{\beta=1}^{3}\boldsymbol{M}_{\alpha\beta}\left(\Delta x^\alpha\right)\left(\Delta x^\beta\right) \tag{1.2}$$

$$=\sum_{\beta=0}^{3}\boldsymbol{M}_{0\beta}\Delta t\left(\Delta x^\beta\right)+\sum_{\alpha=0}^{3}\boldsymbol{M}_{\alpha 0}\left(\Delta x^\alpha\right)\Delta t+\sum_{\alpha=1}^{3}\sum_{\beta=1}^{3}\boldsymbol{M}_{\alpha\beta}\left(\Delta x^\alpha\right)\left(\Delta x^\beta\right) \tag{1.3}$$

$$=\boldsymbol{M}_{00}\left(\Delta t\right)^2+\sum_{\beta=1}^{3}\boldsymbol{M}_{0\beta}\Delta t\left(\Delta x^\beta\right)+\sum_{\alpha=1}^{3}\boldsymbol{M}_{\alpha 0}\left(\Delta x^\alpha\right)\Delta t+\sum_{\alpha=1}^{3}\sum_{\beta=1}^{3}\boldsymbol{M}_{\alpha\beta}\left(\Delta x^\alpha\right)\left(\Delta x^\beta\right) \tag{1.4}$$

$$=\boldsymbol{M}_{00}\left(\Delta t\right)^2+2\left[\sum_{i=1}^{3}\boldsymbol{M}_{0i}\Delta t\left(\Delta x^i\right)\right]+\sum_{\alpha=1}^{3}\sum_{\beta=1}^{3}\boldsymbol{M}_{\alpha\beta}\left(\Delta x^\alpha\right)\left(\Delta x^\beta\right) \tag{1.5}$$

1.6 - Why $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (\Delta t)^2 = 0$ for two events in the same light beam?

Let's say, in a simplified 1D case, event $\mathcal{E} = (x_0, t_0)$ and $\mathcal{P} = (x_1, t_1)$.

$$(\Delta x)^2 - (\Delta t)^2 = (x_1 - x_0)^2 - (t_1 - t_0)^2$$

Since the speed of light is 1,

$$(x_1 - x_0)^2 - (t_1 - t_0)^2 = (x_1 - x_0)^2 - (t_1 \times 1 - t_0 \times 1)^2 = (x_1 - x_0)^2 - (x_1 - x_0)^2 = 0$$