# Part I A First Course in General Relativity 1

### **Chapter 1**

## **Special Relativity**

#### On "Principle of relativity (Galileo)"

#### Galilean invariance

Newton's laws of motion hold in all frames related to one another by a Galilean transformation. In other words, all frames related to one another by such a transformation are inertial (meaning, Newton's equation of motion is valid in these frames).<sup>2</sup> The proof has been given by the book on page 2.

#### 1.5 - Construction of the coordinates used by another observer

#### Why would the tangent of the angle is the speed in Fig. 1.2?

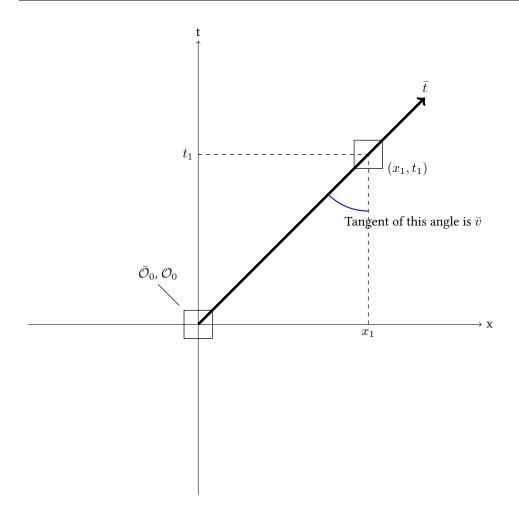
Suppose  $\mathcal{O}$  and  $\bar{\mathcal{O}}$  both start out at the same position where  $\bar{\mathcal{O}}$  moves along the x at some speed. After  $t_1$ , observer  $\mathcal{O}$  sees  $\bar{\mathcal{O}}$  at position  $x_1$ :

$$\bar{\mathcal{O}}_1 = (x_1, t_1)$$

Observer  $\bar{\mathcal{O}}$ , however, still sees themself at x=0:

$$\bar{\mathcal{O}}_1 = (0, t_1)$$

By definition where " $\bar{t}$  is the locus of events at constant  $\bar{x}=0$ ",  $\bar{t}$  is the straight line that passes the origin and the  $(x_1,t_1)$ :



# 1.6 - Why $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (\Delta t)^2 = 0$ for two events in the same light beam?

Let's say, in a simplified 1D case, event  $\mathcal{E}=(x_0,t_0)$  and  $\mathcal{P}=(x_1,t_1)$ .

$$(\Delta x)^2 - (\Delta t)^2 = (x_1 - x_0)^2 - (t_1 - t_0)^2$$

Since the speed of light is 1,

$$(x_1 - x_0)^2 - (t_1 - t_0)^2 = (x_1 - x_0)^2 - (t_1 \times 1 - t_0 \times 1)^2 = (x_1 - x_0)^2 - (x_1 - x_0)^2 = 0$$

#### Why do we have a 2nd term in equation 1.3 on p.10?

$$\Delta \bar{s}^2 = \sum_{\alpha=0}^3 \sum_{\beta=0}^3 \boldsymbol{M}_{\alpha\beta} \left( \Delta x^{\alpha} \right) \left( \Delta x^{\beta} \right) \tag{1.1}$$

$$= \sum_{\alpha=0}^{0} \sum_{\beta=0}^{3} \boldsymbol{M}_{\alpha\beta} \left( \Delta x^{\alpha} \right) \left( \Delta x^{\beta} \right) + \sum_{\alpha=0}^{3} \sum_{\beta=0}^{0} \boldsymbol{M}_{\alpha\beta} \left( \Delta x^{\alpha} \right) \left( \Delta x^{\beta} \right) + \sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} \boldsymbol{M}_{\alpha\beta} \left( \Delta x^{\alpha} \right) \left( \Delta x^{\beta} \right)$$
(1.2)

$$= \sum_{\beta=0}^{3} \boldsymbol{M}_{0\beta} \Delta t \left( \Delta x^{\beta} \right) + \sum_{\alpha=0}^{3} \boldsymbol{M}_{\alpha 0} \left( \Delta x^{\alpha} \right) \Delta t + \sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} \boldsymbol{M}_{\alpha \beta} \left( \Delta x^{\alpha} \right) \left( \Delta x^{\beta} \right)$$
(1.3)

$$= \boldsymbol{M}_{00} \left(\Delta t\right)^{2} \left[ \sum_{\beta=1}^{3} \boldsymbol{M}_{0\beta} \Delta t \left(\Delta x^{\beta}\right) + \sum_{\alpha=1}^{3} \boldsymbol{M}_{\alpha 0} \left(\Delta x^{\alpha}\right) \Delta t \right] + \sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} \boldsymbol{M}_{\alpha \beta} \left(\Delta x^{\alpha}\right) \left(\Delta x^{\beta}\right)$$
(1.4)

# **Bibliography**

 $<sup>^{\</sup>rm 1}$  Bernard Schutz. A First Course on General Relativity. 2009.

 $<sup>^2\,\</sup>mathrm{Wikipedia.}$  Galilean invariance.