

Part I

A First Course on General Relativity

By studying General Relativity, I spotted my weakness of being too submerged in details and not being able to solve problems systematically and as a whole

A person being able to master GR has the best advantage in analyzing business and tech problems right, which is why I study GR and Physics in general (Sorry, Einstein)

Chapter 1

Special Relativity

On “Principle of relativity (Galileo)”

Galilean invariance

[Newton's laws of motion](#) hold in all frames related to one another by a [Galilean transformation](#). In other words, all frames related to one another by such a transformation are inertial (meaning, Newton's equation of motion is valid in these frames).¹ The proof has been given by the book on page 2.

1.5 - Construction of the coordinates used by another observer

Why would the tangent of the angle is the speed in Fig. 1.2?

Suppose \mathcal{O} and $\bar{\mathcal{O}}$ both start out at the same position where $\bar{\mathcal{O}}$ moves along the x at some speed. After t_1 , observer \mathcal{O} sees $\bar{\mathcal{O}}$ at position x_1 :

$$\bar{\mathcal{O}}_1 = (x_1, t_1)$$

Observer $\bar{\mathcal{O}}$, however, still sees themselves at $x = 0$:

$$\bar{\mathcal{O}}_1 = (0, t_1)$$

By definition where “ \bar{t} is the locus of events at constant $\bar{x} = 0$ ”, \bar{t} is the straight line that passes the origin and the (x_1, t_1) :

¹Galilean invariance



1.6 Invariance of the interval

Why does the equation contains only $M_{\alpha\beta} + M_{\beta\alpha}$ terms when $\alpha \neq \beta$, which guarantees $M_{\alpha\beta} = M_{\beta\alpha}$?

$$\Delta \bar{s}^2 = \sum_{\alpha=0}^3 \sum_{\beta=0}^3 M_{\alpha\beta} (\Delta x^{\alpha}) (\Delta x^{\beta})$$

Before spending too much time on expanding the equation, we can pick up a pair of indices of $(\alpha, \beta) = (\alpha^*, \beta^*)$ where $\alpha^* \neq \beta^*$. Then we would definitely have the following 2 terms in the expansion:

$$M_{\alpha^*\beta^*} (\Delta x^{\alpha^*}) (\Delta x^{\beta^*})$$

$$M_{\beta^*\alpha^*} (\Delta x^{\beta^*}) (\Delta x^{\alpha^*})$$

Since

$$(\Delta x^{\alpha^*}) (\Delta x^{\beta^*}) = (\Delta x^{\beta^*}) (\Delta x^{\alpha^*})$$

We can then group these 2 terms and factor out the product, leaving

$$(\Delta x^{\alpha*}) (\Delta x^{\beta*}) (M_{\alpha*\beta*} + M_{\beta*\alpha*})$$

The terms of expanded $\Delta \bar{s}^2$ can be expressed in a matrix of

$$\begin{bmatrix} M_{00}\Delta x^0\Delta x^0 & M_{01}\Delta x^0\Delta x^1 & M_{02}\Delta x^0\Delta x^2 & M_{03}\Delta x^0\Delta x^3 \\ M_{10}\Delta x^1\Delta x^0 & M_{11}\Delta x^1\Delta x^1 & M_{12}\Delta x^1\Delta x^2 & M_{13}\Delta x^1\Delta x^3 \\ M_{20}\Delta x^2\Delta x^0 & M_{21}\Delta x^2\Delta x^1 & M_{22}\Delta x^2\Delta x^2 & M_{23}\Delta x^2\Delta x^3 \\ M_{30}\Delta x^3\Delta x^0 & M_{31}\Delta x^3\Delta x^1 & M_{32}\Delta x^3\Delta x^2 & M_{33}\Delta x^3\Delta x^3 \end{bmatrix}$$

Because the off-diagonal terms always appear in pairs above, we could effectively replace them with their mean value:

$$M_{\alpha*\beta*} = M_{\beta*\alpha*} = \frac{(M_{\alpha*\beta*} + M_{\beta*\alpha*})}{2}$$

where $\alpha* \neq \beta*$. And since $M_{\alpha\beta} = M_{\beta\alpha}$ if $\alpha = \beta$, we conclude that

$$M_{\alpha\beta} = M_{\beta\alpha} \text{ for all } \alpha \text{ and } \beta$$

Why do we have a 2nd term in equation 1.3 on p.10?

$$\Delta \bar{s}^2 = \sum_{\alpha=0}^3 \sum_{\beta=0}^3 M_{\alpha\beta} (\Delta x^{\alpha}) (\Delta x^{\beta}) \quad (1.1)$$

$$= \sum_{\alpha=0}^0 \sum_{\beta=0}^3 M_{\alpha\beta} (\Delta x^{\alpha}) (\Delta x^{\beta}) + \sum_{\alpha=0}^3 \sum_{\beta=0}^0 M_{\alpha\beta} (\Delta x^{\alpha}) (\Delta x^{\beta}) + \sum_{\alpha=1}^3 \sum_{\beta=1}^3 M_{\alpha\beta} (\Delta x^{\alpha}) (\Delta x^{\beta}) \quad (1.2)$$

$$= \sum_{\beta=0}^3 M_{0\beta} \Delta t (\Delta x^{\beta}) + \sum_{\alpha=0}^3 M_{\alpha 0} (\Delta x^{\alpha}) \Delta t + \sum_{\alpha=1}^3 \sum_{\beta=1}^3 M_{\alpha\beta} (\Delta x^{\alpha}) (\Delta x^{\beta}) \quad (1.3)$$

$$= M_{00} (\Delta t)^2 + \sum_{\beta=1}^3 M_{0\beta} \Delta t (\Delta x^{\beta}) + \sum_{\alpha=1}^3 M_{\alpha 0} (\Delta x^{\alpha}) \Delta t + \sum_{\alpha=1}^3 \sum_{\beta=1}^3 M_{\alpha\beta} (\Delta x^{\alpha}) (\Delta x^{\beta}) \quad (1.4)$$

$$= M_{00} (\Delta t)^2 + 2 \left[\sum_{i=1}^3 M_{0i} \Delta t (\Delta x^i) \right] + \sum_{\alpha=1}^3 \sum_{\beta=1}^3 M_{\alpha\beta} (\Delta x^{\alpha}) (\Delta x^{\beta}) \quad (1.5)$$

1.6 - Why $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (\Delta t)^2 = 0$ for two events in the same light beam?

Let's say, in a simplified 1D case, event $\mathcal{E} = (x_0, t_0)$ and $\mathcal{P} = (x_1, t_1)$.

$$(\Delta x)^2 - (\Delta t)^2 = (x_1 - x_0)^2 - (t_1 - t_0)^2$$

Since the speed of light is 1,

$$(x_1 - x_0)^2 - (t_1 - t_0)^2 = (x_1 - x_0)^2 - (t_1 \times 1 - t_0 \times 1)^2 = (x_1 - x_0)^2 - (x_1 - x_0)^2 = 0$$