Question 1:

A rocket is traveling vertically and expels fuel at a velocity (u) of 2000 m/s at a consumption rate (q) of 2100 kg/s. The initial mass of the rocket (m_0) is 140,000 kg. The rocket starts from the rest at $t = 0$.

(a) Establish that the expression for the velocity of the rocket (in m/s) as a function of time is as follows:

$$v = 2000 \log_e\left(\frac{14 \times 10^4}{14 \times 10^4 - 2100t}\right) - 9.8t$$

(Hint: you can do force balance on the rocket body; upward force is due to the fuel expulsion that can be expressed as the product of velocity at which the fuel is being expelled and rate at which fuel is expelled (consumed); mass of rocket is changing with time) **[3 marks]**

Using force balance,

$$\boxed{uq - mg = m\frac{dv}{dt}}$$

$$\boxed{m = m_0 - qt}$$

Thus, $uq - (m_0 - qt)g = (m_0 - qt)\frac{dv}{dt}$

$$\left(\frac{uq}{m_0 - qt} - g\right)dt = dv$$

$m_0 - qt = P$

$-q\,dt = dP$;

$$\int_{m_0}^{m_0 - qt} \frac{-u\,dP}{P} - gt = \int_0^v dv$$

$$\Rightarrow -u\log_e(P)\Big|_{m_0}^{m_0 - qt} - gt = v$$

$$\boxed{v = u\log_e\left(\frac{m_0}{m_0 - qt}\right) - gt}$$

Saurabh