Lab class 5 Foundations of Machine Learning

Moritz Beyer 3705919

Exercise 1

(a)

P(A)=$\frac{|A|}{|\Omega|} = \frac{4}{8} = \frac{1}{2}$
P(B)=$\frac{1}{2}$
P(C)=$\frac{3}{8}$
P(D)=$\frac{1}{2}$

(b)

$P(A \cap B) = \frac{1}{4}$
$P(A \cap C) = \frac{1}{8}$
$P(B \cap C) = \frac{1}{4}$
$P(B \cap D) = \frac{1}{4}$

(c)

yes
no
no
yes

(d)

$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{8}}{\frac{3}{8}} = \frac{1}{3}$
$P(B|C) = \frac{2}{3}$
$P(A \cap B|C) = \frac{1}{3}$

(e)

$P(B|D) = \frac{1}{2}$
$P(C|D) = \frac{1}{2}$
$P(B \cap C|D) = \frac{1}{4}$

(f)

no
yes

Exercise 2

(a)

$P(D_1) = \frac{1}{8}$
$P(D_2) = \frac{1}{4}$
$P(D_3) = \frac{3}{8}$
$P(D_4) = \frac{1}{8}$
$P(D_5) = \frac{1}{8}$

(b)

$P(D_i|S_4) = \frac{P(D_i)P(S_4|D_i)}{\sum_{j=1}^{5} P(D_j)P(S_4|D_j)}$
$P(D_1|S_4) = \frac{\frac{1}{8}\cdot 0}{\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{8}\cdot 1} = 0$
$P(D_2|S_4) = \frac{\frac{1}{4}\cdot\frac{1}{2}}{\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{8}\cdot 1} = \frac{1}{2}$
$P(D_3|S_4) = 0$
$P(D_4|S_4) = \frac{1}{2}$
$P(D_5|S_4) = 0$

Exercise 3

1. $P(A) + P(\bar{A}) \overset{A\cap\bar{A}=\emptyset}{=} P(A \cup \bar{A}) = P(\Omega) = 1$
2. $P(\emptyset) \overset{1.}{=} 1 - P(\bar{\emptyset}) = 1 - P(\Omega) \overset{II}{=} 1 - 1 = 0$
3. $A \subseteq B \Rightarrow B = A \cup C$ mit $A \cap C = \emptyset$
$\overset{III}{\Rightarrow} P(B) = P(A) + \underbrace{P(C)}_{\geq 0} \Rightarrow P(B) \geq P(A)$
4. Idee: $A \cap B = C$, $A \cup B = (A \setminus C) \cup (B \setminus C) \cup C$ und $P(A \setminus B) = P(A) - P(A \cap B)$
zu lang
5. Induktion
IV: $P(A_1 \cup A_2 \cup ... \cup A_k) = P(A_1) + P(A_2) + ... + P(A_k)$
IA: $P(A_1 \cup A_2) \overset{III}{=} P(A_1) + P(A_2)$
IS:$k \to k+1 : P(A_1 \cup A_2 \cup ... \cup A_{k+1}) = P((A_1 \cup A_2 \cup ... \cup A_k) \cup A_{k+1}) \overset{III}{=} P(A_1 \cup A_2 \cup ... \cup A_k) + P(A_{k+1}) \overset{IV}{=}$
$P(A_1) + P(A_2) + ... + P(A_{k+1})$