

常微分方程自测题 2

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摘要: 本文主要介绍了矩阵分解的三种办法, 分别讨论了分解的存在性及唯一性的问题. 在三角分解中, 通过 Gauss 消元法引出了分解, 并给出了消元过程能进行到底的条件, 最后得到了 LDU 基本定理. 在 QR 分解中, 分别使用了 Gram-Schmidt 正交化方法、Givens 变换和 Householder 变换来得到分解式. 在最大秩分解中, 通过使用初等行变换将矩阵化为阶梯形矩阵获得了最大秩分解.

关键字: 矩阵; LU 分解; QR 分解; 最大秩分解

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第一部分 部分

1 一级标题

1.1 二级标题

1.1.1 三级标题

Hello L^AT_EX

你好 L^AT_EX

方程 1 $a + b = c$,

方程 3 $g \div h = i$.

Newton-Leibniz 公式

$$\int_a^b f(x) = F(b) - F(a). \quad (1.1)$$

质能方程

$$E = mc^2.$$

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线性组合

$$\boldsymbol{\gamma} = \lambda_1 \boldsymbol{\alpha} + \lambda_2 \boldsymbol{\beta}.$$

正弦定理

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$\forall \exists \{ \}$

$\varepsilon \partial \cdots \aleph \rtimes$

\lim

矩阵的秩 a rank \boldsymbol{M} rank

最大值 \max_i

$$\max_i$$

$$\max_i \int_a^b$$

$$\text{积分} \int_a^b$$

$$\int_a^b$$

$$\int_a^b$$

$$\int_a^b$$

$$\int_a^b$$

$$\sum_{\substack{0\leqslant i\\0<j<n}}$$

练习 1

$$(\lim_B f(x) = A) := (\forall V(a) \subset Y \exists B \in \mathcal{B}(f(B) \subset f(A))).$$

$$(\lim_{n\rightarrow\infty} x_n = A) := \forall \varepsilon > 0, \exists N \in \mathbb{N}, \forall n \geqslant N (|x_n - A| < \varepsilon).$$

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勾股定理

$$c^2 = a^2 + b^2. \tag{1.2}$$

勾股定理

$$c^2=a^2+b^2.$$

$$\left. \begin{array}{ll} \text{常数)} & y=c \\ \text{抛物线)} & y=cx+d \\ \text{直线)} & y=bx^2+cx+d \end{array} \right\} \text{多项式}$$

$$x_k=\left\{\begin{array}{ll}\eta,&k=0,\\a+(k-1)h,&k=1,2,\cdots,n,\end{array}\right.$$
$$y_k=\left\{\begin{array}{ll}\eta,&k=0,\\y_{k-1}+\frac{h}{8}(t_{1,k}+3t_{2,k}+3t_{3,k}+t_{4,k}),&k=1,2,\cdots,n,\end{array}\right.$$

$$y=\mathrm{e}^x\tag{1.3}$$

$$x=\ln y\tag{1.4}$$

$$\left\{\begin{array}{l}y=\mathrm{e}^x\\x=\ln y\end{array}\right.\text{与}\left\{\begin{array}{l}y=x+1\\x=y-1\end{array}\right.\tag{1.5}$$

$$c_{11}=a_{11}\times b_{11}\qquad\qquad c_{12}=a_{12}\times b_{12}\tag{1.6}$$

$$c_{21}=a_{21}\times b_{21}\qquad\qquad c_{22}=a_{22}\times b_{22}\tag{1.7}$$

$$\begin{array}{ll}c_{11}=a_{11}\times b_{11}&c_{12}=a_{12}\times b_{12}\\c_{21}=a_{21}\times b_{21}&c_{22}=a_{22}\times b_{22}\end{array}\tag{1.8}$$

$$c_{11}=a_{11}\times b_{11}\qquad\qquad\qquad c_{12}=a_{12}\times b_{12}\tag{1.9}$$

$$c_{21}=a_{21}\times b_{21}\qquad\qquad\qquad c_{22}=a_{22}\times b_{22}\tag{1.10}$$

向量相加

$$\boldsymbol{a} = \boldsymbol{b} + \boldsymbol{c}$$

(1.11a)

$$a_1 = b_1 + c_1$$

(1.11b)

$$a_2 = b_2 + c_2$$

(1.11c)

$$a_2 = b_2 + c_2$$

(1.11d)

完全平方展开计算

$$\begin{aligned} f(x) &= 2(x+1)^2 - 1 \\ &= 2(x^2 + 2x + 1) - 1 \\ &= 2x^2 + 4x + 1 \end{aligned}$$

(1.12)

$$|x| = \begin{cases} x & x \geqslant 0 \\ -x & x \leqslant 0 \end{cases}$$

(1.13)

练习 2 真空中电磁场的麦克斯韦方程组:

$$\begin{aligned} \operatorname{div} \boldsymbol{E} &= \frac{\rho}{\varepsilon_0}, & \operatorname{div} \boldsymbol{B} &= 0, \\ \operatorname{rot} \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t}, & \operatorname{rot} \boldsymbol{B} &= \frac{\boldsymbol{j}}{\varepsilon_0 c^2} + \frac{1}{c^2} \frac{\partial \boldsymbol{E}}{\partial t}. \end{aligned}$$

(1.14)

$$U = Q + W$$

[T.1]

[T.1]

$$U = Q + W$$

$$U = Q + W$$

$$U = Q + W$$

(*)

$$U = Q + W$$

*

$$U = Q + W$$

*

绝对值函数 1.13

绝对值函数 (1.13)

分块矩阵

$$\left(\begin{array}{cc|ccc} \boxed{\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix}} & & & & \\ & \boldsymbol{O} & & & \\ \hline & & \boxed{\begin{matrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{matrix}} & & \\ \boldsymbol{O} & & & & \end{array} \right)$$

(1.15)

$$\begin{array}{ccc} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{array} \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$T_{ij} = \begin{pmatrix} 1 & & & & \\ & \ddots & & & \\ & & 1 & & \\ & & c & & s \\ & & & 1 & \\ & & & \ddots & \\ & -s & & 1 & \\ & & c & & \\ & & & 1 & \\ & & & & \ddots \\ & & & & & 1 \end{pmatrix}, \quad (i < j)$$

$$ab \quad c$$

d

$$\frac{a}{b} \frac{a}{b} \frac{a}{b} \frac{a}{b} \frac{a}{b}$$

$$\frac{a}{b} \frac{a}{b} \frac{a}{b} \frac{a}{b} \frac{a}{b}$$

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}.$$

二项式系数 $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

$$\left(\sum_{i=1}^n u_i v_i\right)^2 \leq \left(\sum_{i=1}^n u_i^2\right) \left(\sum_{i=1}^n v_i^2\right).$$

柯西不等式

$$\left(\sum_{i=1}^n u_i v_i \right)^2 \leq \left(\sum_{i=1}^n u_i^2 \right) \left(\sum_{i=1}^n v_i^2 \right).$$

练习 3 (Cauchy-Binet 公式) 设 $A = (a_{ij})$ 是 $m \times n$ 矩阵, $B = (b_{ij})$ 是 $n \times m$ 矩阵. $A \begin{pmatrix} i_1 & \cdots & i_s \\ j_1 & \cdots & j_s \end{pmatrix}$ 表示 A 的一个 s 阶子式, 它由 A 的第 i_1, \dots, i_s 行与第 j_1, \dots, j_s 列交点上的元素按原次序排列组成的行列式. 同理可定义 B 的 s 阶子式.

(1) 若 $m > n$, 则必有 $|AB| = 0$;

(2) 若 $m \leq n$, 则必有

$$|AB| = \sum_{1 \leq j_1 < j_2 < \cdots < j_m \leq n} A \begin{pmatrix} 1 & 2 & \cdots & m \\ j_1 & j_2 & \cdots & j_m \end{pmatrix} B \begin{pmatrix} j_1 & j_2 & \cdots & j_m \\ 1 & 2 & \cdots & m \end{pmatrix}.$$

练习 4

$$\begin{aligned} \varphi^* \omega(t)(\tau_1, \tau_2) &:= \omega(\varphi(\tau_1, \tau_2)) = dx^{i_1} dx^{i_2}(\xi_1, \xi_2) \\ &= \begin{vmatrix} \xi_1^{i_1} & \xi_1^{i_2} \\ \xi_2^{i_1} & \xi_2^{i_2} \end{vmatrix} = \begin{vmatrix} \frac{\partial x^{i_1}}{\partial t^{j_1}} \tau_1^{j_1} & \frac{\partial x^{i_2}}{\partial t^{j_2}} \tau_1^{j_2} \\ \frac{\partial x^{i_1}}{\partial t^{j_1}} \tau_2^{j_1} & \frac{\partial x^{i_2}}{\partial t^{j_2}} \tau_2^{j_2} \end{vmatrix} \\ &= \sum_{j_1, j_2=1}^m \frac{\partial x^{i_1}}{\partial t^{j_1}} \frac{\partial x^{i_2}}{\partial t^{j_2}} \begin{vmatrix} \tau_1^{j_1} & \tau_1^{j_2} \\ \tau_2^{j_1} & \tau_2^{j_2} \end{vmatrix} \\ &= \sum_{j_1, j_2=1}^m \frac{\partial x^{i_1}}{\partial t^{j_1}} \frac{\partial x^{i_2}}{\partial t^{j_2}} dt^{j_1} \wedge dt^{j_2}(\tau_1, \tau_2) \\ &= \sum_{1 \leq j_1 < j_2 \leq m} \left(\frac{\partial x^{i_1}}{\partial t^{j_1}} \frac{\partial x^{i_2}}{\partial t^{j_2}} - \frac{\partial x^{i_1}}{\partial t^{j_2}} \frac{\partial x^{i_2}}{\partial t^{j_1}} \right) dt^{j_1} \wedge dt^{j_2}(\tau_1, \tau_2) \\ &= \sum_{1 \leq j_1 < j_2 \leq m} \begin{vmatrix} \frac{\partial x^{i_1}}{\partial t^{j_1}} & \frac{\partial x^{i_2}}{\partial t^{j_2}} \\ \frac{\partial x^{i_1}}{\partial t^{j_2}} & \frac{\partial x^{i_2}}{\partial t^{j_1}} \end{vmatrix} (t) dt^{j_1} \wedge dt^{j_2}(\tau_1, \tau_2). \end{aligned}$$

$$\begin{array}{ccc} a & \xrightarrow{j} & b \\ \downarrow & & \downarrow \lim P \\ c & \xlongequal{\quad} & d \end{array} \quad (1.16)$$

$$\begin{array}{ccc} A & \longrightarrow & B \\ \downarrow & & \uparrow \\ B & \longrightarrow & C \end{array} \quad (1.17)$$

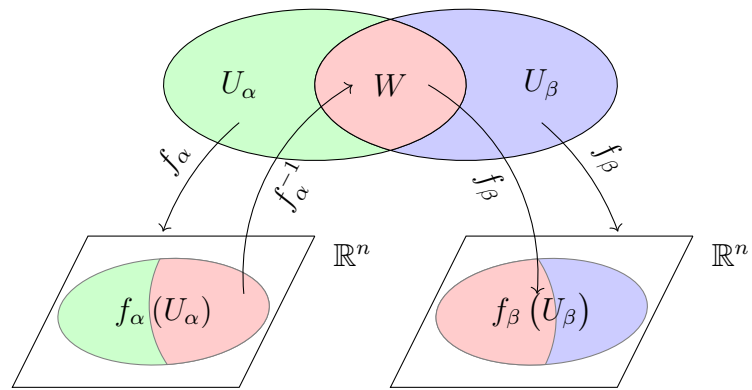
$$\begin{array}{ccccccc} & & & & a & & \\ & & & & & & \\ bbbbbb & & & & & & a \\ & c & & b & & & \\ & & & & c & & \end{array}$$


图 1.1: Condition

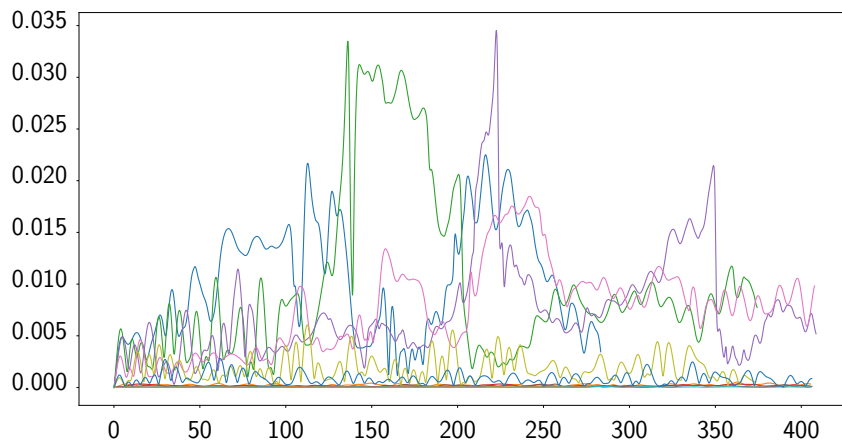


Diagram illustrating a transition from 'aaa' to 'aaa' with an arrow pointing up and to the right.



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```
1  #include <stdio.h>
2
3  int main()
4  {
5      printf("Hello_world!");
6  }
```

“”
%



算法 1 粒子群算法

输入: 直杆影子端点坐标 $(x_i^*, y_i^*), i = 1, 2, \dots, n$.

输出: 纬度 ϕ , 经度 ψ .

1: 在纬度 $\phi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, 地方时 $t \in [720, 1380]$ 的范围内随机创建粒子.

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Matrix decomposition
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Abstract .
Key words Matrix; LU decomposition;

附录

1 Some Appendix2

The contents...