常微分方程自测题 2

阙嘉豪[1]

(1. 北京师范大学 数学科学学院, 北京 100875)

摘要: 本文主要介绍了矩阵分解的三种办法, 分别讨论了分解的存在性及唯一性的问题. 在三角分解中, 通过 Gauss 消元法引出了分解, 并给出了消元过程能进行到底的条件, 最后得到了 LDU 基本定理. 在 QR 分解中, 分别使用了 Gram–Schmidt 正交化方法、Givens 变换和 Householder 变换来得到分解式. 在最大秩分解中, 通过使用初等行变换将矩阵化为阶梯形矩阵获得了最大秩分解.

关键字: 矩阵; LU 分解; QR 分解; 最大秩分解

中图分类号: O 151.21 **文献标识码**: A

目录

第一部分 部分

1 一级标题

1.1 二级标题

1.1.1 三级标题

Hello LaTeX

你好 LaTeX

方程 1 $a + b = c$,

方程 3 $g \div h = i$.

Newton-Leibniz 公式

$$\int_a^b f(x) = F(b) - F(a). \tag{1.1}$$

质能方程

$$E = mc^2.$$

作者简介: 阙嘉豪 (1999—), 男, 广东深圳人, 北京师范大学数学科学学院本科生.

线性组合

$$\boldsymbol{\gamma} = \lambda_1 \boldsymbol{\alpha} + \lambda_2 \boldsymbol{\beta}.$$

正弦定理

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$\forall \exists$ { }

$\varepsilon \partial \cdots \aleph \rtimes$

$\lim$

矩阵的秩 $a \operatorname{rank} \boldsymbol{M} rank$

$$\text{最大值 } \max_i$$

$$\max_i$$

$$\max_i$$

$$\text{积分 } \int_a^b$$

$$\int_a^b$$

$$\int_a^b$$

$$\int\limits_a^b$$

$$\int\limits_a^b$$

$$\sum_{\substack{0 \leqslant i \\ 0<j<n}}$$

练习 1

$$(\lim_{\mathcal{B}} f(x) = A) := (\forall V(a) \subset Y \exists B \in \mathcal{B}(f(B) \subset f(A))).$$

$$(\lim_{n\to\infty} x_n = A) := \forall \varepsilon > 0, \exists N \in \mathbb{N}, \forall n \geqslant N(|x_n - A| < \varepsilon).$$

$(\lim_{n\to\infty} x_n = A) := \forall \varepsilon > 0, \exists N \in \mathbb{N}, \forall n \geqslant N(|x_n - A| < \varepsilon).$

$(\lim\limits_{n\to\infty} x_n = A) := \forall \varepsilon > 0, \exists N \in \mathbb{N}, \forall n \geqslant N(|x_n - A| < \varepsilon).$

勾股定理

$$c^2 = a^2 + b^2. \tag{1.2}$$

勾股定理

$$c^2 = a^2 + b^2.$$

$$\left.\begin{array}{rl}\text{常数)} & y{=}c\\ \text{抛物线)} & y{=}cx+d\\ \text{直线)} & y{=}bx^2+cx+d\end{array}\right\}\text{多项式}$$

$$x_k = \begin{cases} \eta, & k=0,\\ a+(k-1)h, & k=1,2,\cdots,n,\end{cases}$$
$$y_k = \begin{cases} \eta, & k=0,\\ y_{k-1}+\dfrac{h}{8}(t_{1,k}+3t_{2,k}+3t_{3,k}+t_{4,k}), & k=1,2,\cdots,n,\end{cases}$$

$$y = \mathrm{e}^x \tag{1.3}$$
$$x = \ln y \tag{1.4}$$

$$\begin{cases} y = \mathrm{e}^x \\ x = \ln y \end{cases} \text{与} \begin{cases} y = x+1 \\ x = y-1 \end{cases} \tag{1.5}$$

$$c_{11} = a_{11}\times b_{11} \qquad c_{12} = a_{12}\times b_{12} \tag{1.6}$$
$$c_{21} = a_{21}\times b_{21} \qquad c_{22} = a_{22}\times b_{22} \tag{1.7}$$

$$\begin{aligned} c_{11} &= a_{11}\times b_{11} & c_{12} &= a_{12}\times b_{12}\\ c_{21} &= a_{21}\times b_{21} & c_{22} &= a_{22}\times b_{22}\end{aligned} \tag{1.8}$$

$$c_{11} = a_{11}\times b_{11} \qquad c_{12} = a_{12}\times b_{12} \tag{1.9}$$
$$c_{21} = a_{21}\times b_{21} \qquad c_{22} = a_{22}\times b_{22} \tag{1.10}$$

向量相加

$$\boldsymbol{a}=\boldsymbol{b}+\boldsymbol{c} \tag{1.11a}$$

$$a_1=b_1+c_1 \tag{1.11b}$$

$$a_2=b_2+c_2 \tag{1.11c}$$

$$a_2=b_2+c_2 \tag{1.11d}$$

完全平方展开计算

$$\begin{aligned} f(x)&=2(x+1)^2-1\\ &=2(x^2+2x+1)-1\\ &=2x^2+4x+1 \end{aligned} \tag{1.12}$$

$$|x|=\begin{cases} x & x\geqslant 0\\ -x & x\leqslant 0 \end{cases} \tag{1.13}$$

练习 2 真空中电磁场的麦克斯韦方程组:

$$\begin{aligned} &\mathrm{div}\boldsymbol{E}=\frac{\rho}{\varepsilon_0}, && \mathrm{div}\boldsymbol{B}=0,\\ &\mathrm{rot}\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t}, && \mathrm{rot}\boldsymbol{B}=\frac{\boldsymbol{j}}{\varepsilon_0 c^2}+\frac{1}{c^2}\frac{\partial\boldsymbol{E}}{\partial t}. \end{aligned} \tag{1.14}$$

$$U=Q+W \tag*{[T.1]}$$

$$[T.1] \qquad U=Q+W$$

$$U=Q+W$$

$$U=Q+W \tag{$*$}$$

$$U=Q+W \tag*{$*$}$$

$$U=Q+W \tag*{$*$}$$

绝对值函数 1.13
绝对值函数 (1.13)
分块矩阵

$$\begin{pmatrix} \boxed{\begin{matrix} a_{11} & a_{12}\\ a_{21} & a_{22} \end{matrix}} & \boldsymbol{O}\\ \boldsymbol{O} & \boxed{\begin{matrix} b_{11} & b_{12} & b_{13}\\ b_{21} & b_{22} & b_{23}\\ b_{31} & b_{32} & b_{33} \end{matrix}} \end{pmatrix} \tag{1.15}$$

三阶循环矩阵

$$\begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{matrix} \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} \quad \begin{Vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{Vmatrix} \quad \begin{smallmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{smallmatrix}$$

$$\boldsymbol{T}_{ij} = \begin{array}{c} \begin{pmatrix} 1 & & & & & & & & & & \\ & \ddots & & & & & & & & & \\ & & 1 & & & & & & & & \\ & & & c & & & & s & & & \\ & & & & 1 & & & & & & \\ & & & & & \ddots & & & & & \\ & & & & & & 1 & & & & \\ & & & -s & & & & c & & & \\ & & & & & & & & 1 & & \\ & & & & & & & & & \ddots & \\ & & & & & & & & & & 1 \end{pmatrix} \end{array} \begin{array}{c} \\ \\ \\ i \\ \\ \\ \\ j \\ \\ \\ \\ \end{array} , \quad (i < j)$$

$$ab \quad c$$

$$d$$

分数

$$\frac{a}{b}\frac{a}{b}\frac{a}{b}\tfrac{a}{b}\tfrac{a}{b}$$

$$\tfrac{a}{b}\frac{a}{b}\frac{a}{b}\tfrac{a}{b}\tfrac{a}{b}$$

二项式系数

$$\binom{n+1}{k} = \tbinom{n}{k} + \binom{n}{k-1}.$$

$$\text{二项式系数 } \tbinom{n+1}{k} = \tbinom{n}{k} + \binom{n}{k-1}.$$

柯西不等式

$$(\sum_{i=1}^{n} u_i v_i)^2 \leqslant (\sum_{i=1}^{n} u_i^2)(\sum_{i=1}^{n} v_i^2).$$

柯西不等式

$$\left(\sum_{i=1}^{n} u_i v_i\right)^2 \leqslant \left(\sum_{i=1}^{n} u_i^2\right)\left(\sum_{i=1}^{n} v_i^2\right).$$

练习 3 (Cauchy-Binet 公式) 设 $\boldsymbol{A}=\left(a_{ij}\right)$ 是 $m \times n$ 矩阵, $\boldsymbol{B}=\left(b_{ij}\right)$ 是 $n \times m$ 矩阵. $\boldsymbol{A}\begin{pmatrix} i_1 & \cdots & i_s \\ j_1 & \cdots & j_s \end{pmatrix}$ 表示 $\boldsymbol{A}$ 的一个 s 阶子式, 它由 $\boldsymbol{A}$ 的第 $i_1, \cdots, i_s$ 行与第 $j_1, \cdots, j_s$ 列交点上的元素按原次序排列组成的行列式. 同理可定义 $\boldsymbol{B}$ 的 s 阶子式.

(1) 若 $m>n$, 则必有 $|\boldsymbol{AB}|=0$;

(2) 若 $m \leqslant n$, 则必有

$$|\boldsymbol{AB}|=\sum_{1 \leqslant j_1<j_2<\cdots<j_m \leqslant n} \boldsymbol{A}\begin{pmatrix} 1 & 2 & \cdots & m \\ j_1 & j_2 & \cdots & j_m \end{pmatrix} \boldsymbol{B}\begin{pmatrix} j_1 & j_2 & \cdots & j_m \\ 1 & 2 & \cdots & m \end{pmatrix}.$$

练习 4

$$\begin{aligned}
\varphi^* \omega(t)(\boldsymbol{\tau}_1, \boldsymbol{\tau}_2) := \omega(\varphi(\boldsymbol{\tau}_1, \boldsymbol{\tau}_2)) &= \mathrm{d}x^{i_1} \mathrm{d}x^{i_2}(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2) \\
&= \begin{vmatrix} \xi_1^{i_1} & \xi_1^{i_2} \\ \xi_2^{i_1} & \xi_2^{i_2} \end{vmatrix} = \begin{vmatrix} \dfrac{\partial x^{i_1}}{\partial t^{j_1}} \tau_1^{j_1} & \dfrac{\partial x^{i_2}}{\partial t^{j_2}} \tau_1^{j_2} \\ \dfrac{\partial x^{i_1}}{\partial t^{j_1}} \tau_2^{j_1} & \dfrac{\partial x^{i_2}}{\partial t^{j_2}} \tau_2^{j_2} \end{vmatrix} \\
&= \sum_{j_1, j_2=1}^{m} \frac{\partial x^{i_1}}{\partial t^{j_1}} \frac{\partial x^{i_2}}{\partial t^{j_2}} \begin{vmatrix} \tau_1^{j_1} & \tau_1^{j_2} \\ \tau_2^{j_1} & \tau_2^{j_2} \end{vmatrix} \\
&= \sum_{j_1, j_2=1}^{m} \frac{\partial x^{i_1}}{\partial t^{j_1}} \frac{\partial x^{i_2}}{\partial t^{j_2}} \mathrm{d}t^{j_1} \wedge \mathrm{d}t^{j_2}(\boldsymbol{\tau}_1, \boldsymbol{\tau}_2) \\
&= \sum_{1 \leqslant j_1<j_2 \leqslant m} \left(\frac{\partial x^{i_1}}{\partial t^{j_1}} \frac{\partial x^{i_2}}{\partial t^{j_2}}-\frac{\partial x^{i_1}}{\partial t^{j_2}} \frac{\partial x^{i_2}}{\partial t^{j_1}}\right) \mathrm{d}t^{j_1} \wedge \mathrm{d}t^{j_2}(\boldsymbol{\tau}_1, \boldsymbol{\tau}_2) \\
&= \sum_{1 \leqslant j_1<j_2 \leqslant m} \begin{vmatrix} \dfrac{\partial x^{i_1}}{\partial t^{j_1}} & \dfrac{\partial x^{i_2}}{\partial t^{j_1}} \\ \dfrac{\partial x^{i_1}}{\partial t^{j_2}} & \dfrac{\partial x^{i_2}}{\partial t^{j_2}} \end{vmatrix}(t) \mathrm{d}t^{j_1} \wedge \mathrm{d}t^{j_2}(\boldsymbol{\tau}_1, \boldsymbol{\tau}_2).
\end{aligned}$$

$$\begin{tikzcd}
a \arrow[r, "j"] \arrow[d] & b \arrow[d, "\lim P"] \\
c \arrow[r, equal] & d
\end{tikzcd} \tag{1.16}$$

$$\begin{tikzcd}
A \arrow[r] \arrow[d] & B \\
B \arrow[r] & C \arrow[u]
\end{tikzcd} \tag{1.17}$$

8

引用参考文献超链接 [1]

text

$$\begin{array}{rl} a & \\ bbbbbbbbbb & a \\ c & b \\ & c \end{array}$$

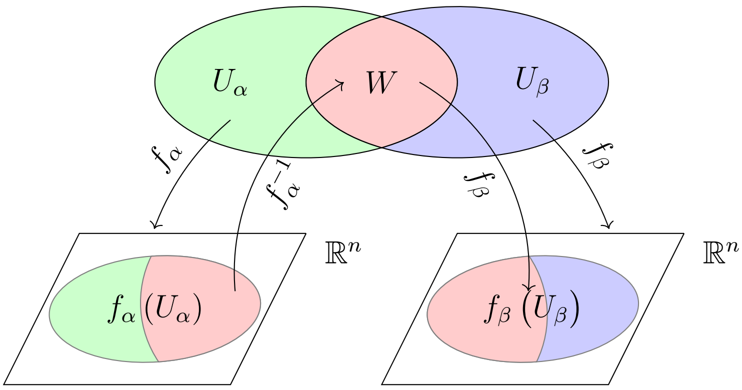
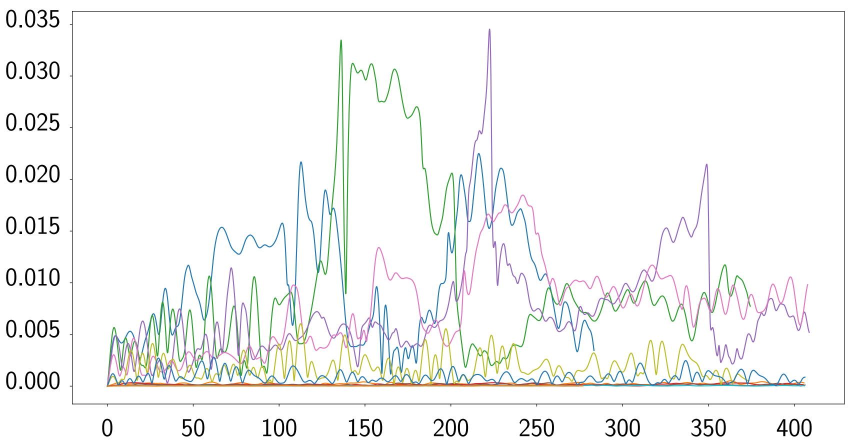


图 1.1: Condition



aaa aaa

欢迎扫码加群及关注公众号

```c
#include <stdio.h>

int main()
{
    printf("Hello␣world!");
}
```

“”

%

算法 1 粒子群算法

输入: 直杆影子端点坐标 (x_i^*, y_i^*) , $i = 1, 2, \cdots, n$.
输出: 纬度 ϕ, 经度 ψ.

1: 在纬度 $\phi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, 地方时 $t \in [720, 1380]$ 的范围内随机创建粒子.

参考文献

[1] 袁荣. 常微分方程 [M]. 北京: 高等教育出版社, 2012:59,62.

Matrix decomposition

QUE Jia-hao

(School of Mathematical Sciences, Beijing Normal University, Beijing 100875, China)

Abstract .

Key words Matrix; LU decomposition;

附录

1 Some Appendix2

The contents...