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Quantum tensor networks
Problem set 1

0. Using a provided code generate random quantum states and ground states of a classical Ising mode for n spins $\frac{1}{2}$. The states are represented by a tensor of probability amplitudes $\psi_{i_0,i_1,\ldots,i_{n-1}}$. The Ising model is defined by a Hamiltonian

$$H = \sum_{\langle i,j \rangle} Z_i Z_j, \tag{1}$$

where Z_i is Pauli matrix acting at a site i, and $\langle i,j \rangle$ denotes a sum over the nearest neighbors ($j = i+1$). The Pauli matrix is defined as

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{2}$$

1. Write a function transforming $\psi_{i_0,i_1,\ldots,i_{n-1}}$ to a left-canonical matrix product state (MPS). Apply it to a generated state.

2. Write a function returning bond dimensions of a MPS. Check the bond dimensions of the generated MPS.

3. Write a function checking if an MPS is left-canonical. Use it to test correctness of the generated MPS.

4. Write functions computing a norm of left-canonical MPS and an expectation value of a single site operator acting at the last site. Use them to check correctness of the generated MPS. Compare the obtained expectation value with the one returned by a provided code employing $\psi_{i_0,i_1,\ldots,i_{n-1}}$.

5. Modify the function transforming $\psi_{i_0,i_1,\ldots,i_{n-1}}$ to a left-canonical MPS adding an option to truncate its bond-dimension discarding singular values smaller than a numerical precision (i.e. $s_i/s_0 < \epsilon$, $\epsilon \sim 10^{-15}$). Apply it a random quantum state and an Ising model ground state. Compare their bond dimensions.

6. Write functions transforming $\psi_{i_0,i_1,\ldots,i_{n-1}}$ to a mixed-canonical MPS with a central site j. Write functions checking if an MPS has the mixed-canonical form, and computing the norm of such an MPS. Use them to check correctness of the obtained representation.

7. Write functions computing expectation values of a single-site operator acting on a site j and a two-site nearest-neighbor operator acting at sites $j, j+1$ using the mixed-canonical form. Check their correctness using a provided code which uses $\psi_{i_0,i_1,\ldots,i_{n-1}}$.

8. Plot mean and standard deviation of $\langle Z_{n/2} \rangle$, and $\langle = Z_{n/2} Z_{n/2+1} \rangle$ versus $n = 2, 4, 6, 8, 10, \ldots$ for random quantum states and the Ising model ground states. Use a sample of 100 states for each n and the mixed-canonical MPS representation.

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