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Quantum tensor networks
Problem set 2

1. Let's consider a one-dimensional quantum transverse Ising model (QTIM) with open boundary conditions defined by a Hamiltonian

$$H = -\sum_{i=0}^{n-2} Z_i Z_{i+1} - h \sum_{i=0}^{n-1} X_i - h_Z \sum_{i=0}^{n-1} Z_i. \tag{1}$$

   Write the Hamiltonian as an MPO with $\chi_H = 3$. Notice that this Hamiltonian is a modification of the XXZ model discussed in the lecture notes.

2. Consider an MPS. Compute the expectation value of the QTIM Hamiltonian contracting a corresponding tensor network, as explained in detail on page 19 of the notes. More precisely, prepare a function that returns all L_j, R_j tensors introduced there and compute the $\langle H \rangle$ using those tensors. Check the internal consistency of the contraction using multiple pairs of tensors. Compare the obtained energy with a reference code computing it for a state represented by a rank n tensor of probability amplitudes (use code from Set 1 to decompose the state to MPS).

3. Prepare a function computing $h_j A_j$ that defines the single-site DMRG eigenproblem (see pages 18-19 of the notes for h_j and $h_j A_j$ definition). The function should have the cost scaling as χ^3.

4. Assume that the MPS has the mixed-canonical form with the central site j. Write a function updating A_j to minimize the energy of the MPS. Use the function computing $h_j A_j$ and *scipy.sparse.linalg.LinearOperator* and *scipy.sparse.linalg.eigsh* to obtain χ^3 cost scaling. Check that the function performs energy minimization.

5. Write a function performing sequential energy minimization with respect to all MPS tensors going from left to right with a cost scaling linearly in n. Check that it minimizes the energy. Prepare a similar function optimizing all the tensors going from right to left.

6. Consider $h_Z = 0$. Perform the energy minimization for $h = 0, 0.1, 1., 10$ for increasing n and χ values. Analyze the convergence of the results with χ and n. For small enough n, compare the obtained energies to the ground state energy from a reference code using the rank-n tensor representation.

7. Add small h_Z of order $10^{-4} - 10^{-2}$. Plot behavior of the ferromagnetic order parameter

$$O = \frac{1}{n} \sum_{i=0}^{n-1} \langle Z_i \rangle \tag{2}$$

   as a function of h for the ground state.

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