

6 Notion de limite

6.1 Convergence de suites

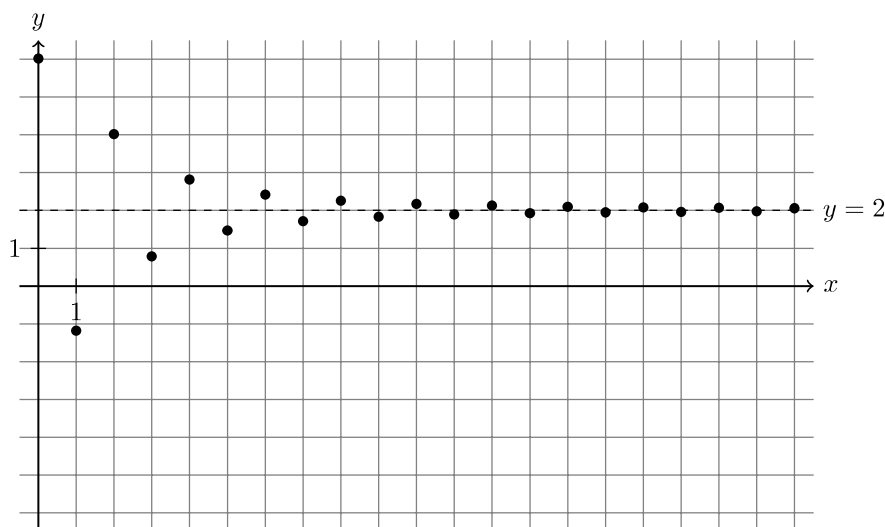
Définition 8 (Limite finie d'une suite). Soit $(u_n)_{n \in \mathbb{N}}$ une suite numérique, et l un nombre réel. On dit que la suite $(u_n)_{n \in \mathbb{N}}$ **admet l comme limite** quand les nombres u_n sont aussi proches de l que l'on veut à mesure que les indices n sont grands. On le note

$$\lim_{n \rightarrow +\infty} u_n = l$$

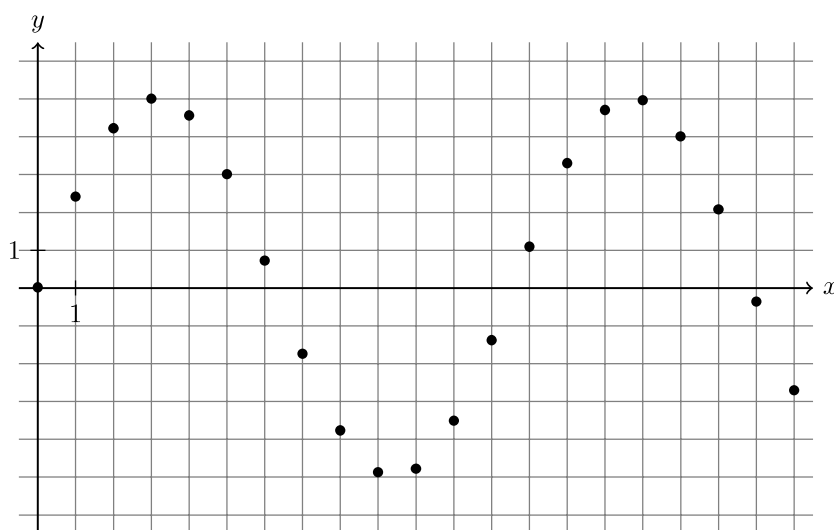
Remarque.

- Quand une suite admet une limite finie, on dit que la suite **converge**.
- Quand une suite ne converge pas, on dit qu'elle **diverge**.

Exemple. On représente une suite $(u_n)_{n \in \mathbb{N}}$ par les points de coordonnées (n, u_n) .



La suite (u_n) semble converger vers le réel 2 : plus n est grand (pour des abscisses de plus en plus grandes), et plus u_n est proche de 2 (les ordonnées des points sont de plus en plus proche de 2).



Ici, la suite représentée ne semble pas admettre de limite finie l . En effet, les ordonnées des points de coordonnées (n, u_n) ne semblent pas se rapprocher d'une valeur en particulier, à la mesure que n augmente.

Définition 9. Soit $(u_n)_{n \in \mathbb{N}}$ une suite numérique. On dit que **la suite (u_n) admet $+\infty$ comme limite** quand les valeurs de u_n sont de plus en plus grandes à la mesure où n augmente. On le note

$$\lim_{n \rightarrow +\infty} u_n = +\infty$$

$$\lim_{n \rightarrow +\infty} u_n = -\infty$$

- si elle n'admet pas de limite finie;
- ou si elle admet $+\infty$ ou $-\infty$ comme limite.

The figure consists of two vertically stacked scatter plots. Both plots have a horizontal x-axis and a vertical y-axis, with a grid of lines every 1 unit. The x-axis is labeled 'x' and the y-axis is labeled 'y'. The origin (0,0) is marked with a small circle. The number '1' is marked on both axes at the first grid line from the origin.

The top plot is for $\alpha = 0.05$. It shows a series of points starting at (0,0) and increasing as x increases. The points are approximately at (0,0), (1,0.01), (2,0.02), (3,0.03), (4,0.04), (5,0.05), (6,0.06), (7,0.07), (8,0.08), (9,0.09), (10,0.1), (11,0.11), (12,0.12), (13,0.13), (14,0.14), (15,0.15), (16,0.16), (17,0.17), (18,0.18), (19,0.19), (20,0.2), (21,0.21), (22,0.22), (23,0.23), (24,0.24), (25,0.25), (26,0.26), (27,0.27), (28,0.28), (29,0.29), (30,0.3), (31,0.31), (32,0.32), (33,0.33), (34,0.34), (35,0.35), (36,0.36), (37,0.37), (38,0.38), (39,0.39), (40,0.4), (41,0.41), (42,0.42), (43,0.43), (44,0.44), (45,0.45), (46,0.46), (47,0.47), (48,0.48), (49,0.49), (50,0.5), (51,0.51), (52,0.52), (53,0.53), (54,0.54), (55,0.55), (56,0.56), (57,0.57), (58,0.58), (59,0.59), (60,0.6), (61,0.61), (62,0.62), (63,0.63), (64,0.64), (65,0.65), (66,0.66), (67,0.67), (68,0.68), (69,0.69), (70,0.7), (71,0.71), (72,0.72), (73,0.73), (74,0.74), (75,0.75), (76,0.76), (77,0.77), (78,0.78), (79,0.79), (80,0.8), (81,0.81), (82,0.82), (83,0.83), (84,0.84), (85,0.85), (86,0.86), (87,0.87), (88,0.88), (89,0.89), (90,0.9), (91,0.91), (92,0.92), (93,0.93), (94,0.94), (95,0.95), (96,0.96), (97,0.97), (98,0.98), (99,0.99), (100,1.0).

The bottom plot is for $\alpha = 0.5$. It shows a series of points starting at (0,1) and decreasing as x increases. The points are approximately at (0,1), (1,0.99), (2,0.98), (3,0.97), (4,0.96), (5,0.95), (6,0.94), (7,0.93), (8,0.92), (9,0.91), (10,0.9), (11,0.89), (12,0.88), (13,0.87), (14,0.86), (15,0.85), (16,0.84), (17,0.83), (18,0.82), (19,0.81), (20,0.8), (21,0.79), (22,0.78), (23,0.77), (24,0.76), (25,0.75), (26,0.74), (27,0.73), (28,0.72), (29,0.71), (30,0.7), (31,0.69), (32,0.68), (33,0.67), (34,0.66), (35,0.65), (36,0.64), (37,0.63), (38,0.62), (39,0.61), (40,0.6), (41,0.59), (42,0.58), (43,0.57), (44,0.56), (45,0.55), (46,0.54), (47,0.53), (48,0.52), (49,0.51), (50,0.5), (51,0.49), (52,0.48), (53,0.47), (54,0.46), (55,0.45), (56,0.44), (57,0.43), (58,0.42), (59,0.41), (60,0.4), (61,0.39), (62,0.38), (63,0.37), (64,0.36), (65,0.35), (66,0.34), (67,0.33), (68,0.32), (69,0.31), (70,0.3), (71,0.29), (72,0.28), (73,0.27), (74,0.26), (75,0.25), (76,0.24), (77,0.23), (78,0.22), (79,0.21), (80,0.2), (81,0.19), (82,0.18), (83,0.17), (84,0.16), (85,0.15), (86,0.14), (87,0.13), (88,0.12), (89,0.11), (90,0.1), (91,0.09), (92,0.08), (93,0.07), (94,0.06), (95,0.05), (96,0.04), (97,0.03), (98,0.02), (99,0.01), (100,0).