Formalisation of Pattern 2

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Pattern 2 can be formally defined using the definition of abstract graphs.

Definition 1. *(Pattern 2) Let* $G=(S_N,S_E)$ *be an abstract graph,* T *the task to move,* $n_{new}=(\{T,\emptyset\})$ *a new abstract node containing only* T*,* n_f *the first abstract node of* G *and* n_l *the last abstract node of* G*. The set of abstract graphs generated by applying Pattern 2 to* G *and* T*, written* $gen_{P2}(G,T)$*, is defined as* $gen_{P2}(G,T)=$ **(i)** $\cup$ **(ii)** *where*

(i) $= \bigcup_{p=(n_i,\dots,n_n)\in\mathcal{P}^*_O(S_N)} (S_N\setminus p\cup\{(\{T\},\{(p,\{n_i\rightarrow n_{i+1},\dots,n_{n-1}\rightarrow n_n\})\})\},S'_E)$ *with*

$$S'_E=\begin{cases}\emptyset \textit{ if } n_n=n_l \wedge n_i=n_f\\ S_E\setminus\{n_i\rightarrow n_{i+1},\dots,n_n\rightarrow n_{n+1}\}\cup\{n_{new}\rightarrow n_{n+1}\} \textit{ if } n_n\neq n_l \wedge n_i=n_f\\ S_E\setminus\{n_{i-1}\rightarrow n_i,\dots,n_{n-1}\rightarrow n_n\}\cup\{n_{i-1}\rightarrow n_{new}\} \textit{ if } n_n=n_l \wedge n_i\neq n_f\\ S_E\setminus\{n_{i-1}\rightarrow n_i,\dots,n_n\rightarrow n_{n+1}\}\cup\{n_{i-1}\rightarrow n_{new},n_{new}\rightarrow n_{n+1}\} \textit{ otherwise}\end{cases}$$

represents the addition of T *in parallel of any ordered set of nodes of* G*, with* $S_{E'}$ *differentiating cases when the first node of the combination is the first node of the abstract graph and/or the last node of the combination is the last node of the abstract graph, and*

(ii) $= \bigcup_{n\in S_N}\bigcup_{g\in S_{G_n}}\bigcup_{g'\in gen_{P2}(g,T)} (S_N\setminus\{n\}\cup\{(S_{T_n},\ S_{G_n}\setminus\{g\}\cup\{g'\})\},\ S_E)$ *is the result of the recursive call of this function on each abstract sub-graph of each node of* G*.*