Formalisation of Pattern 3

Quentin Nivon and Gwen Salaün

Univ. Grenoble Alpes, CNRS, Grenoble INP, Inria, LIG, F-38000 Grenoble France

Pattern 3 can be formally defined using the definition of abstract graphs.

Definition 1. *(Pattern 3) Let* $G = (S_N, S_E)$ *be an abstract graph,* T *the task to move and* $n_{new} = (\{T, \emptyset\})$ *a new abstract node containing only* T*. The set of abstract graphs generated by applying Pattern 3 to* G *and* T*, written* $gen_{P3}(G,T)$*, is defined as* $gen_{P3}(G,T) =$ **(i)** $\cup$ **(ii)** $\cup$ **(iii)** *where*

(i) $= \bigcup_{n \in S_N} \bigcup_{\substack{p_T \in \mathcal{P}(S_{T_n}) \\ p_G \in \mathcal{P}(S_{G_n}) \\ |p_T|+|p_G|>1}} (S_N \setminus \{n\} \cup \{(S_{T_n} \setminus p_T, S_{G_n} \setminus p_G \cup \{(\{n_{new}, n'\}, \{n_{new} \rightarrow n'\})\})\}, S_E)$

– *with* $n' = (p_T, p_G)$ – *represents the addition of task* T *before any combination of tasks and sub-graphs of any nodes of* G*,*

(ii) $= \bigcup_{n \in S_N} \bigcup_{\substack{p_T \in \mathcal{P}(S_{T_n}) \\ p_G \in \mathcal{P}(S_{G_n}) \\ |p_T|+|p_G|>1}} (S_N \setminus \{n\} \cup \{(S_{T_n} \setminus p_T, S_{G_n} \setminus p_G \cup \{(\{n', n_{new}\}, \{n' \rightarrow n_{new}\})\})\}, S_E)$

– *with* $n' = (p_T, p_G)$ – *represents the addition of task* T *after any combination of tasks and sub-graphs of any nodes of* G*, and*

(iii) $= \bigcup_{n \in S_N} \bigcup_{g \in S_{G_n}} \bigcup_{g' \in gen_{P3}(g,T)} (S_N \setminus \{n\} \cup \{(S_{T_n}, S_{G_n} \setminus \{g\} \cup \{g'\})\}, S_E)$ *is the result of the recursive call of this function on each abstract sub-graph of each node of* G*.*