Recap

Corrigés de l'exo 10 et 13

A connaître :

$$\sum_{k=0}^{n} k = \frac{n(n+1)}{2} \quad \Big| \quad \sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} \underbrace{a^k b^{n-k}}_{a^{n-k} b^k}$$

$$(a-b)^n = (a-b)\, a^k b^{n-k-1} = (a-b)\, a^{n-k-1} b^k$$

Exemple :

$n=2$

$$a^2 - b^2 = (a-b) \sum_{k=0}^{1} a^k b^{1-k} = (a-b)(b+a) = \underbrace{(a-b)(a+b)}_{\text{connu}}$$

$n=4$

$$a^4 - b^4 = (a-b) \sum_{k=0}^{3} a^k b^{3-k} = (a-b)(b^3 + ab^2 + a^2b + a^3)$$
$$= (a-b)(a^3 + a^2 b + ab^2 + b^3)$$

Démonstration :

$$a^n - b^n = (a-b) \sum_{k=0}^{n-1} a^k b^{n-k-1}$$

$$(a-b) \sum_{k=0}^{n-1} a^k b^{n-k-1} = a \sum_{k=0}^{n-1} a^k b^{n-k-1} - b \sum_{k=0}^{n-1} a^k b^{n-k-\cancel{1}}$$

$$= \sum_{k=0}^{n-1} a^{k+1} b^{n-k-1} - \sum_{k=0}^{n-1} a^k b^{n-k}$$

$j = k+1$

$k = 0 \Rightarrow j = 1$

$k = n-1 \Rightarrow j = n$

$$= \sum_{j=1}^{n} a^{j} b^{n-j} - \sum_{k=0}^{n-1} a^{k} b^{n-k}$$

$$= \left(\cancel{\sum_{j=1}^{n-1} a^{j} b^{n-j}} + a^{n} \right) - \left(b^{n} + \cancel{\sum_{k=1}^{n-1} a^{k} b^{n-k}} \right)$$

$$= \left(a^{n} - b^{n} \right)$$