Corrigé du TD N°1 - Exercices 1 à 5

Exercice 1 :

1)

- $(3+2i)+3(5-6i) = 3+2i+15-18i = 18-16i$
- $(3-2i)+3(5+i) = 3-2i+15+3i = 18+i$
- $(6+5i)-4 = 6+5i-4 = 2+5i$
- $(2-5i)(1+3i) = 2+6i-5i\underbrace{-15i^2}_{+15} = 17+i$
- $(1+2i)^5 = \ ?$

1ère méthode : On "éclate" l'exposant 5.

$$(1+2i)^5 = \left((1+2i)^2\right)^2(1+2i)$$

Or $(1+2i)^2 = 1^2+2\times 1\times 2i+(2i)^2 = 1+4i-4 = -3+4i$

Donc : $\left((1+2i)^2\right)^2 = (-3+4i)^2 = (-3)^2+2(-3)\times 4i+(4i)^2 = 9-24i-16$
$= -7-24i$

Donc : $(1+2i)^5 = (-7-24i)(1+2i) = -7-14i-24i+48$
$= 41-38i$

2ème méthode : On utilise la formule du binôme.

Le Triangle de Pascal est :

n	
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	$\boxed{1\ 5\ 10\ 10\ 5\ 1}$

Coeff utilisés pour développer $(a+b)^5$

Donc $(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$

Donc $(1+2i)^5 = 1^5+5\times 1^4\times(2i)+10\times 1^3\times\underbrace{(2i)^2}_{-4}+10\times 1^2\times\underbrace{(2i)^3}_{-8i}+5\times 1\times\overbrace{(2i)^4}^{16}+\underbrace{(2i)^5}_{32i}$

$= 1+10i-40-80i+80+32i$

$= 41-38i$

- $\dfrac{1}{2-3i} = \dfrac{2+3i}{2^2+3^2} = \dfrac{2+3i}{13} = \dfrac{2}{13}+\dfrac{3}{13}i$
- $\dfrac{1}{i} = \dfrac{-i}{0^2+1^2} = -i$
- $\dfrac{2+i}{3+4i} = \dfrac{(2+i)(3-4i)}{3^2+4^2} = \dfrac{6-8i+3i+4}{25} = \dfrac{10}{25}-\dfrac{5}{25}i = \dfrac{2}{5}-\dfrac{1}{5}i$

- $2+3i + \dfrac{5-i}{1+4i} = \dfrac{(2+3i)(1+4i)+5-i}{1+4i}$

$= \dfrac{2+8i+3i-12+5-i}{1+4i}$

$= \dfrac{-5+10i}{1+4i} = \dfrac{(-5+10i)(1-4i)}{1^2+4^2}$

$= \dfrac{-5+20i+10i+40}{17}$

$= \dfrac{35}{17} + \dfrac{30}{17}i$

- $\dfrac{1-2i}{2+3i} + \dfrac{2-i}{3+i} = \dfrac{(1-2i)(3+i)+(2-i)(2+3i)}{(2+3i)(3+i)}$

$= \dfrac{(3+i-6i+2)+(4+6i-2i+3)}{6+2i+9i-3}$

$= \dfrac{12-i}{3+11i}$

$= \dfrac{(12-i)(3-11i)}{3^2+11^2}$

$= \dfrac{36-132i-3i+11}{130}$

$= \dfrac{25}{130} - \dfrac{135i}{130}$

$= \dfrac{5}{26} - \dfrac{27}{26}i$

2.) a) $i^2 = -1$; $i^3 = i^2 . i = -i$; $i^4 = (i^2)^2 = (-1)^2 = 1$; $i^5 = i^4 i = i$, ...

Donc, $\forall k \in \mathbb{N}$:

$$\begin{cases} i^{4k} = (i^4)^k = 1^k = 1 \\ i^{4k+1} = i^{4k}\, i = i \\ i^{4k+2} = i^{4k}\, i^2 = -1 \\ i^{4k+3} = i^{4k}\, i^3 = -i \end{cases}$$

b) • $i^{1515} = i^{4\times 378+3} = -i$

- $i^{1789} = i^{4\times 447+1} = i$

- $i^{2014} = i^{4\times 503+2} = -1$

- $(1-i)^{2008} = ((1-i)^2)^{1004} = (1-2i-1)^{1004} = (-2i)^{1004}$

$= (-2)^{1004}\, i^{1004}$

$= 2^{1004} \underbrace{i^{4\times 250}}_{1} = 2^{1004}$

$$\bullet\ (1+i)^{2013} = (1+i)^{2012} \times (1+i)$$
$$= \left((1+i)^2\right)^{1006} \times (1+i)$$
$$= (2i)^{1006} \times (1+i)$$
$$= 2^{1006} \times i^{1006} \times (1+i)$$
$$= 2^{1006} \times \underbrace{i^{4\times 250+2}}_{-1} \times (1+i)$$
$$= -2^{1006} - 2^{1006}\, i$$

Exercice 2 :

- $|2-4i| = \sqrt{2^2+(-4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$
- $|5i+2| = \sqrt{2^2+5^2} = \sqrt{4+25} = \sqrt{29}$
- $|3i+12| = \sqrt{3^2+12^2} = \sqrt{9+144} = \sqrt{153} = 3\sqrt{17}$
- $|-3i| = \sqrt{0^2+(-3)^2} = \sqrt{9} = 3$
- $\left|\cos\left(\frac{\pi}{24}\right) + i\sin\left(\frac{\pi}{24}\right)\right| = \sqrt{\cos^2\left(\frac{\pi}{24}\right) + \sin^2\left(\frac{\pi}{24}\right)} = \sqrt{1} = 1$
- $|\sqrt{2}+i| = \sqrt{(\sqrt{2})^2+1^2} = \sqrt{2+1} = \sqrt{3}$
- $\left|\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2}+\frac{1}{2}} = \sqrt{1} = 1$
- $|(1+i)(3+4i)(4+3i)| = |1+i| \times |3+4i| \times |4+3i|$
  $= \sqrt{2} \times \sqrt{25} \times \sqrt{25}$
  $= 25\sqrt{2}$
- $\left|\frac{\sqrt{3}}{5} + \frac{i}{5}\right| = \frac{1}{5}|\sqrt{3}+i| = \frac{1}{5}\sqrt{(\sqrt{3})^2+1^2} = \frac{1}{5}\sqrt{4} = \frac{2}{5}$
- $\left|\frac{1}{4-3i}\right| = \frac{1}{|4-3i|} = \frac{1}{\sqrt{4^2+(-3)^2}} = \frac{1}{\sqrt{16+9}} = \frac{1}{5}$
- $\left|\frac{1+i\sqrt{3}}{3-2i}\right| = \frac{|1+i\sqrt{3}|}{|3-2i|} = \frac{\sqrt{1^2+(\sqrt{3})^2}}{\sqrt{3^2+(-2)^2}} = \frac{2}{\sqrt{13}}$
- $|(2-i)^4| = |2-i|^4 = \left(\sqrt{2^2+(-1)^2}\right)^4 = (\sqrt{5})^4 = 5^2 = 25$
- $\left|5e^{\frac{5i\pi}{7}}\right| = |5|\,\underbrace{\left|e^{5i\pi/7}\right|}_{1} = 5$
- $\left|-7e^{2i\pi/5}\right| = |-7|\,\underbrace{\left|e^{2i\pi/5}\right|}_{1} = 7$

Exercice 3 :

1°) • $x+iy+3(2-3i) = 6-10i \iff x+iy = \underbrace{6-10i-(6-9i)}_{0-i}$

$\iff \boxed{x=0 \text{ et } y=-1}$

• $(x+iy)(2+i) = (1-i)^2 \iff x+iy = \dfrac{(1-i)^2}{2+i}$

$\iff x+iy = \dfrac{-2i}{2+i}$

$\iff x+iy = -\frac{2}{5}-\frac{4}{5}i$

$\left(\text{car } \dfrac{-2i}{2+i} = \dfrac{(-2i)(2-i)}{2^2+1^2} = \dfrac{-4i-2}{5} = -\frac{2}{5}-\frac{4}{5}i\right)$

$\iff \boxed{x=-\frac{2}{5} \text{ et } y=-\frac{4}{5}}$

• $x+2ixy+y = 10+6i \iff \begin{cases} x+y=10 \\ xy=3 \end{cases} \iff \begin{cases} y=10-x \\ x(10-x)=3 \end{cases}$

$\iff \begin{cases} y=10-x \\ x^2-10x+3=0 \end{cases}$ $(\Delta = 100-12=88 \ ; \sqrt{\Delta}=\sqrt{88}=2\sqrt{22})$

$\iff \begin{cases} y=10-x \\ x=\dfrac{10\pm 2\sqrt{22}}{2} = 5\pm\sqrt{22} \end{cases}$

$\iff \begin{cases} x=5+\sqrt{22} \text{ et } y=5-\sqrt{22} \\ \text{ou} \\ x=5-\sqrt{22} \text{ et } y=5+\sqrt{22} \end{cases}$

• $\dfrac{x}{1+i}+\dfrac{y}{1+2i} = 1 \iff \dfrac{x}{2}(1-i)+\dfrac{y}{5}(1-2i)=1$

$\left(\text{car } \dfrac{1}{1+i} = \frac{1}{2}(1-i) \text{ et } \dfrac{1}{1+2i} = \frac{1}{5}(1-2i)\right)$

$\iff \left(\frac{x}{2}+\frac{y}{5}\right)+i\left(-\frac{x}{2}-\frac{2y}{5}\right)=1$

$\iff \left[\frac{x}{2}+\frac{y}{5}=1 \text{ et } -\frac{x}{2}-\frac{2y}{5}=0\right]$

$\iff \begin{cases} 5x+2y=10 \\ 5x+4y=0 \end{cases} \iff \begin{cases} 5x+2y=10 \\ 2y=-10 \end{cases}$ $(L_2 \leftarrow L_2-L_1)$

$\iff \begin{cases} x=\frac{1}{5}(10-2y)=\frac{1}{5}(10+10)=4 \\ y=-5 \end{cases}$

$\iff \boxed{x=4 \text{ et } y=-5}$

2°) • $2iz-3 = z+i \iff (2i-1)z = 3+i \iff z = \dfrac{3+i}{2i-1}$

$\iff z = \dfrac{(3+i)(-1-2i)}{1^2+(-2)^2}$

$\iff z = \dfrac{-3-6i-i+2}{5} \iff z = -\frac{1}{5}-\frac{7}{5}i$

- $(3\bar{z}-i)(z+2+3i)=0 \iff [3\bar{z}-i=0$ ou $z+2+3i=0]$

$\iff [\bar{z}=\frac{i}{3}$ ou $z=-2-3i]$

$\iff [z=-\frac{i}{3}$ ou $z=-2-3i]$

- $3z(\bar{z}+i)=-iz \iff \begin{cases} z=0 \\ \text{ou} \\ 3(\bar{z}+i)=-i \end{cases} \iff \begin{cases} z=0 \\ \text{ou} \\ \bar{z}=\underbrace{-\frac{i}{3}-i}_{-4/3\, i} \end{cases} \iff \begin{cases} z=0 \\ \text{ou} \\ z=4/3\, i \end{cases}$

- $\frac{z-1}{iz+3}=4i \iff z-1=4i(iz+3)$

$\iff z(1-4i^2)=1+12i$

$\iff z=\frac{1+12i}{5} \iff z=\frac{1}{5}+\frac{12}{5}i$

- $z^2+5z+6=0$: $\Delta=25-24=1$ donc :

$z^2+5z+6=0 \iff z=\frac{-5\pm1}{2} \iff [z=-3$ ou $z=-2]$

- $\bar{z}^2+3\bar{z}+5=0 \iff \begin{cases} Z=\bar{z} \\ Z^2+3Z+5=0 \end{cases}$ $(\Delta=9-20=-11)$

$\iff \begin{cases} Z=\bar{z} \\ Z=\frac{-3\pm i\sqrt{11}}{2} \end{cases}$

$\iff [\bar{z}=\frac{-3+i\sqrt{11}}{2}$ ou $\bar{z}=\frac{-3-i\sqrt{11}}{2}]$

$\iff [z=\frac{-3-i\sqrt{11}}{2}$ ou $z=\frac{-3+i\sqrt{11}}{2}]$

- $z^2+3z-4=0$: $\Delta=9+16=25$ donc ;

$z^2+3z-4=0 \iff z=\frac{-3\pm\sqrt{25}}{2} \iff z=\frac{-3\pm5}{2} \iff [z=-4$ ou $z=1]$

- $z^2+12z+36=0$: $\Delta=12^2-4\times36=0$; donc

$z^2+12z+36=0 \iff z=-\frac{12}{2} \iff z=-6$

3°) - $2z+i\bar{z}=3 \iff \begin{cases} z=x+iy \quad (x,y\in\mathbb{R}) \\ 2(x+iy)+i(x-iy)=3 \end{cases}$

$\iff \begin{cases} z=x+iy \quad (x,y\in\mathbb{R}) \\ (2x+y)+(2y+x)i=3 \end{cases}$

$\iff \begin{cases} z=x+iy \quad (x,y\in\mathbb{R}) \\ 2x+y=3 \text{ et } 2y+x=0 \end{cases}$

$\iff \begin{cases} z=x+iy \quad (x,y\in\mathbb{R}) \\ y=3-2x \text{ et } \underbrace{2(3-2x)+x=0}_{6-3x=0} \end{cases}$

$\iff \begin{cases} z=x+iy \quad (x,y\in\mathbb{R}) \\ y=3-2x \text{ et } x=6/3=2 \end{cases}$

$$\iff \begin{cases} z = x+iy \quad (x,y\in\mathbb{R}) \\ x = 2 \text{ et } y = 3-2\times 2 = -1 \end{cases} \iff z = 2-i$$

- $z^2 + z\bar{z} = 0 \iff z(z+\bar{z}) = 0 \iff \begin{cases} z=0 \\ \text{ou} \\ z+\bar{z}=0 \end{cases} \iff \begin{cases} z=0 \\ \text{ou} \\ \mathrm{Re}(z)=0 \end{cases} \iff z=0$

Exercice 4 :

1) Si $|z| = 1$ alors $\sqrt{z\bar{z}} = 1$ donc $z\bar{z} = 1$; donc $\bar{z} = \frac{1}{z}$

2) a) Posons $Z = \frac{z+z'}{1+zz'}$. Alors :

car $|z| = 1$ et $|z'| = 1$

$$\bar{Z} = \overline{\frac{z+z'}{1+zz'}} = \frac{\bar{z}+\bar{z}'}{1+\bar{z}\bar{z}'} = \frac{\frac{1}{z}+\frac{1}{z'}}{1+\frac{1}{z}\times\frac{1}{z'}} = \frac{\frac{z'+z}{zz'}}{\frac{zz'+1}{zz'}} = \frac{z'+z}{1+zz'} = Z$$

Donc $Z \in \mathbb{R}$.

b) $$|Z|^2 = Z\bar{Z} = \frac{z+z'}{1+zz'} \times \frac{\bar{z}+\bar{z}'}{1+\bar{z}\bar{z}'} = \frac{z\bar{z}+z'\bar{z}'+z\bar{z}'+\bar{z}z'}{1+\bar{z}\bar{z}'+zz'+z\bar{z}z'\bar{z}'}$$

$$= \frac{2+2\,\mathrm{Re}(z\bar{z}')}{2+2\,\mathrm{Re}(zz')} = \frac{1+\mathrm{Re}(z\bar{z}')}{1+\mathrm{Re}(zz')}$$

3) $$\frac{z+z'}{1+zz'} = 1 \iff z+z' = 1+zz'$$
$$\iff z+z'-1-zz'=0$$
$$\iff (z-1)(1-z')=0$$
$$\iff [z-1=0 \text{ ou } 1-z'=0]$$
$$\iff [z=1 \text{ ou } z'=1]$$

Exercice 5 :

1) • $2 = 2e^{i0}$ car $|2| = 2$ et $\theta = 0\ [2\pi]$ $\left(\text{car } \begin{cases} \cos\theta = \frac{2}{2} = 1 \\ \sin\theta = \frac{0}{2} = 0 \end{cases}\right)$

• $-3 = 3e^{i\pi}$ car $|-3| = 3$ et $\theta = \pi\ [2\pi]$ $\left(\text{car } \begin{cases} \cos\theta = -\frac{3}{3} = -1 \\ \sin\theta = \frac{0}{3} = 0 \end{cases}\right)$

• $3i = 3e^{i\pi/2}$ car $|3i| = 3$ et $\theta = \pi/2\ [2\pi]$ $\left(\text{car } \begin{cases} \cos\theta = \frac{0}{3} = 0 \\ \sin\theta = \frac{3}{3} = 1 \end{cases}\right)$

• $-5i = 5e^{-i\pi/2}$ car $|-5i| = 5$ et $\theta = -\pi/2\ [2\pi]$ $\left(\text{car } \begin{cases} \cos\theta = \frac{0}{5} = 0 \\ \sin\theta = -\frac{5}{5} = -1 \end{cases}\right)$

• $1+i = \sqrt{2}\,e^{i\pi/4}$ car $|1+i| = \sqrt{1^2+1^2} = \sqrt{2}$ et $\theta = \frac{\pi}{4}$ $\left(\text{car } \begin{cases} \cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{cases}\right)$

- $\sqrt{3} - i = 2e^{-i\frac{\pi}{6}}$ car $|\sqrt{3}-i| = \sqrt{(\sqrt{3})^2+(-1)^2} = 2$ et $\theta = -\pi/6$
$$\left(\text{car } \begin{cases} \cos\theta = \frac{\sqrt{3}}{2} \\ \sin\theta = -\frac{1}{2} \end{cases}\right)$$

- $\frac{1}{3} + \frac{1}{3}i = \frac{\sqrt{2}}{3} e^{i\frac{\pi}{4}}$ car $1+i = \sqrt{2}\, e^{i\pi/4}$ (et donc $\frac{1}{3}+\frac{1}{3}i = \frac{1}{3}(1+i) = \frac{1}{3}\sqrt{2}\, e^{i\pi/4}$)

- $\frac{-5}{1-i\sqrt{3}} = \frac{5}{2} e^{\frac{4i\pi}{3}}$. En effet :
$$\begin{cases} -5 = 5e^{i\pi} \\ 1-i\sqrt{3} = 2e^{-i\pi/3} \end{cases} \quad \left(|1-i\sqrt{3}| = \sqrt{1^2+(-\sqrt{3})^2} = 2 \text{ et } \begin{cases} \cos\theta = \frac{1}{2} \\ \sin\theta = -\frac{\sqrt{3}}{2} \end{cases}\right)$$
Donc $\frac{-5}{1-i\sqrt{3}} = \frac{5e^{i\pi}}{2e^{-i\pi/3}} = \frac{5}{2} e^{i(\pi+\pi/3)} = \frac{5}{2} e^{4i\pi/3}$

- $\frac{1+i}{\sqrt{3}+i} = \frac{1}{\sqrt{2}} e^{i\frac{\pi}{12}}$. En effet :
$$\begin{cases} 1+i = \sqrt{2}\, e^{i\pi/4} & \left(|1+i| = \sqrt{2} \text{ et } \begin{cases} \cos\theta = 1/\sqrt{2} = \frac{\sqrt{2}}{2} \\ \sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{cases}\right) \\ \sqrt{3}+i = 2e^{i\pi/6} & \left(|\sqrt{3}+i| = \sqrt{4} = 2 \text{ et } \begin{cases} \cos\theta = \frac{\sqrt{3}}{2} \\ \sin\theta = \frac{1}{2} \end{cases}\right) \end{cases}$$
Donc $\frac{1+i}{\sqrt{3}+i} = \frac{\sqrt{2}\, e^{i\pi/4}}{2e^{i\pi/6}} = \frac{\sqrt{2}}{2} e^{i\left(\frac{\pi}{4}-\frac{\pi}{6}\right)} = \frac{\sqrt{2}}{2} e^{i\pi/12}$

- $(1-i)^9 (1+i\sqrt{3})^{-5} = \frac{1}{\sqrt{2}} e^{-\frac{47i\pi}{12}}$. En effet :

$1-i = \sqrt{2}\, e^{-i\pi/4}$ et $1+i\sqrt{3} = 2e^{i\pi/3}$; donc
$$\begin{aligned}(1-i)^9(1+i\sqrt{3})^{-5} &= \left(\sqrt{2}\, e^{-i\pi/4}\right)^9 \left(2e^{i\pi/3}\right)^{-5} \\ &= \underbrace{\sqrt{2}^9}_{2^{9/2}} e^{-\frac{9i\pi}{4}} \times 2^{-5} e^{-5i\pi/3} \\ &= 2^{9/2-5} e^{\left(-\frac{9\pi}{4}-\frac{5\pi}{3}\right)i} \\ &= 2^{-1/2} e^{-\frac{47i\pi}{12}} = \frac{1}{\sqrt{2}} e^{-\frac{47i\pi}{12}}\end{aligned}$$

- $\left(-\frac{1-i\sqrt{3}}{4}\right)^{2013} = \frac{1}{2^{2013}} e^{i0}$. En effet :
$$-\frac{1-i\sqrt{3}}{4} = \frac{1}{4}\underbrace{(-1+i\sqrt{3})}_{2e^{2i\pi/3}} = \frac{1}{2} e^{2i\pi/3}. \text{ Donc :}$$
$$\begin{aligned}\left(-\frac{1-i\sqrt{3}}{4}\right)^{2013} &= \left(\frac{1}{2} e^{2i\pi/3}\right)^{2013} = \frac{1}{2^{2013}} e^{\frac{4026}{3} i\pi} = \frac{1}{2^{2013}} e^{1342 i\pi} \\ &= \frac{1}{2^{2013}} \underbrace{e^{671\times 2i\pi}}_{1} = \frac{1}{2^{2013}}\end{aligned}$$

- $-3\,e^{\frac{7i\pi}{11}} = 3\,e^{\frac{18i\pi}{11}}$ car $-3 = 3e^{i\pi}$ et donc :

$$-3\,e^{\frac{7i\pi}{11}} = 3\,e^{i\pi}\,e^{\frac{7i\pi}{11}} = 3\,e^{i\left(\pi+\frac{7\pi}{11}\right)} = 3\,e^{\frac{18i\pi}{11}}$$

- $i\,e^{\frac{3i\pi}{5}} = e^{\frac{11i\pi}{10}}$ car $i = e^{i\pi/2}$ et donc :

$$i\,e^{3i\pi/5} = e^{i\pi/2}\,e^{3i\pi/5} = e^{i\left(\pi/2+\frac{3\pi}{5}\right)} = e^{\frac{11i\pi}{10}}$$

- $\tan\left(\frac{3\pi}{7}\right)+i = \dfrac{1}{\cos\left(\frac{3\pi}{7}\right)}\,e^{\frac{i\pi}{14}}$. En effet :

$$\tan\left(\frac{3\pi}{7}\right)+i = \frac{\sin\left(\frac{3\pi}{7}\right)}{\cos\left(\frac{3\pi}{7}\right)}+i = \frac{1}{\cos\left(\frac{3\pi}{7}\right)}\left(\sin\left(\frac{3\pi}{7}\right)+i\cos\left(\frac{3\pi}{7}\right)\right)$$

Or
$$\begin{cases}\sin\left(\frac{3\pi}{7}\right) = \cos\left(\frac{\pi}{2}-\frac{3\pi}{7}\right) = \cos\left(\frac{\pi}{14}\right)\\ \cos\left(\frac{3\pi}{7}\right) = \sin\left(\frac{\pi}{2}-\frac{3\pi}{7}\right) = \sin\left(\frac{\pi}{14}\right)\end{cases}$$

Donc : $\tan\left(\frac{3\pi}{7}\right)+i = \dfrac{1}{\cos\left(\frac{3\pi}{7}\right)}\left(\cos\left(\frac{\pi}{14}\right)+i\sin\left(\frac{\pi}{14}\right)\right)$

$$= \frac{1}{\cos\left(\frac{3\pi}{7}\right)}\,e^{i\frac{\pi}{14}}$$

(qui est la forme exponentielle car de la forme $re^{i\theta}$ où $r = \dfrac{1}{\cos\left(\frac{3\pi}{7}\right)} > 0$ et $\theta\in\mathbb{R}$)

2.) a) $z_1 = \dfrac{\sqrt{6}-i\sqrt{2}}{2} = \sqrt{2}\left(\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i\right) = \sqrt{2}\,e^{-i\pi/6}$

$z_2 = \dfrac{1+i}{2} = \dfrac{1}{\sqrt{2}}\left(\dfrac{1}{\sqrt{2}}+\dfrac{i}{\sqrt{2}}\right) = \dfrac{1}{\sqrt{2}}\,e^{i\pi/4}$

b) D'une part :

$$z_1z_2 = \frac{\sqrt{6}-i\sqrt{2}}{2}\times\frac{1+i}{2} = \frac{(\sqrt{6}-i\sqrt{2})(1+i)}{4}$$
$$= \frac{\sqrt{6}+\sqrt{6}i-i\sqrt{2}+\sqrt{2}}{4}$$
$$= \frac{\sqrt{6}+\sqrt{2}}{4}+i\,\frac{\sqrt{6}-\sqrt{2}}{4}$$

D'autre part :

$$z_1z_2 = \sqrt{2}\,e^{-\frac{i\pi}{6}}\,\frac{1}{\sqrt{2}}\,e^{i\pi/4} = e^{i\left(-\frac{\pi}{6}+\frac{\pi}{4}\right)} = e^{\frac{i\pi}{12}}$$
$$= \cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)$$

Donc
$$\begin{cases}\cos\left(\frac{\pi}{12}\right) = \mathrm{Re}(z_1z_2) = \dfrac{\sqrt{6}+\sqrt{2}}{4}\\ \sin\left(\frac{\pi}{12}\right) = \mathrm{Im}(z_1z_2) = \dfrac{\sqrt{6}-\sqrt{2}}{4}\end{cases}$$