Paper / Subject Code: 49311 / Engineering Mathematics-III

SE/ AIDS/Sem-III/ CBCGS/ R-19/EM-III

(Time: 3 hours)

Max. Marks: 80

[5]

N.B. (1) Question No. 1 is compulsory.

- (2) Answer any three questions from Q.2 to Q.6.
- (3) Use of Statistical Tables permitted.
- (4) Figures to the right indicate full marks.

Q1 (a) Find Laplace transform of
$$\frac{c \delta s \sqrt{t}}{\sqrt{t}}$$
 given that $L\{\sin \sqrt{t}\} = \frac{\sqrt{\pi}}{2s^3/2} e^{-(1/4s)}$ [5]

(b)	Calculate Spearman's rank correlation coefficient for the following data:											
	X	32	.55	49	60	43	37	43	49	10	20	•
κ,	Y	40	30	70	20	30 .	50	-72	60	45	25	
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(c) Find inverse Laplace transform of
$$\frac{2s-1}{s^2+8s+29}$$
 [5]

(d) If
$$f(z) = qx^2y + 2x^2 + ry^3 - 2y^2 - i(px^3 - 4xy - 3xy^2)$$
 is analytic, find the values of p, q, and r [5]

Find Laplace transform of
$$e^{3t}$$
 f(t) where $f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \\ 0, & otherwise \end{cases}$ [6]

- (b) Two unbiased dice are thrown. If X represents sum of the numbers on the two dice.
 Write probability distribution of the random variable X and find mean, standard deviation, and P(|X-7|≥3)
- Obtain Fourier series for $f(x) = x \sin x$ in the interval $0 \le x \le 2\pi$.
- Q3 (a) Using Milne-Thompson's method construct an analytic function f(z)=u+iv in terms of z where $u+v=e^x(\cos y+\sin y)+\frac{x-y}{x^2+y^2}$ [6]

Using convolution theorem find the inverse Laplace transform of
$$\frac{(s+3)^2}{(s^2+6s+5)^2}$$
 [6]

(c) Fit a parabola
$$y=a+bx+cx^2$$
 to the following data and estimate y when $x=10$

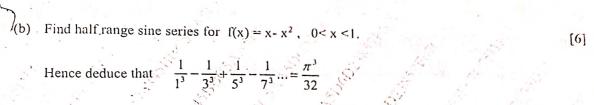
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у	2	6	7	.8	10	11 5	11	10	9

Q4 (a) Find Laplace transform of
$$e^{-(1/2)t}$$
 t f(3t) if $L\{f(t)\} = \frac{1}{5\sqrt{5+1}}$ [6]

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[8]



- (c) Given regression lines 6y=5x+90, 15x=8y+130, $\sigma_x^2=16$. Find i) \bar{x} and \bar{y} , ii) r, iii) σ_y^2 and iv) angle between the regression lines
- (a) Can the function $u = r + \frac{a^2}{r} \cos \theta$ be considered as real or imaginary part of an analytic function? If yes, find the corresponding analytic function.
- (b) An unbiased coin is tossed three times. If X denotes the absolute difference between the number of heads and the number of tails, find moment generating function of X and hence obtain the first moment about origin and the second moment about mean.
 - (c) Evaluate $\int_0^\infty e^{-2t} \cosh t \int_0^t u^2 \sinh u \cosh u du dt$ [8]
- Q6 (a) Find inverse Laplace transform of $\frac{1}{(s-2)^4(s+3)}$ using method of partial fractions. [6]
 - (b) If a continuous random variable X has the following probability density function $f(x) = \begin{cases} k e^{-\frac{x}{4}}, & \text{for } x > 0 \\ 0, & \text{elsewhere} \end{cases}$ find k, mean and variance.
- (c) Find half range cosine series for f(x) = x, $0 \le x \le 2$. Hence deduce that i) $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \dots = \frac{\pi^4}{96}$

ii)
$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots = \frac{\pi^4}{90}$$
