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★ Absolute & Conditional Convergence

a) $\sum |a_n|$ or $\sum a_n$ can say $\sum a_n$ is absolutely

b)

c) Q: Can we find $\sum a_n$ where we say that $\sum a_n$ is conditionally

★ Test Absolutely Not Total.

Q: If $\sum |a_n|$ is, then $\sum a_n$ is. Let a_n be series. Absolutely conditionally

Proof: $-|a_n| \le a_n \le |a_n|$

$\Rightarrow 0 \le a_n + |a_n| \le 2|a_n|$ —(1)

But:

$\sum 2|a_n| = 2\sum |a_n|$ is conv then

by Basic comparison, $\sum (1)$ $\sum a_n + |a_n|$ also conv

$\sum a_n + \sum |a_n|$ conv

$\Rightarrow \sum a_n + \sum |a_n| - \sum |a_n|$

$\Rightarrow \sum a_n$ is conv

★ The opposite is not true. Proof

(ex): Given $\sum$ conv yet abs conv

(a) $\sum \frac{(-1)^n}{n}$ (and this not conv-Test)

but $\sum |a_n| = \sum \frac{1}{n}$ (not conv)

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$ conv

$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{n}$

$\sum_{n=1}^{\infty} \frac{1}{n^{1+0}}$

This is not convergent. So $\sum a_n$ is conv

(Absolutely conditional)

by test $\sum \frac{1}{n^{1+\epsilon}}$ (not conv)

(c) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ conv

$|a_n| = \sum_{n=1}^{\infty} \frac{|\sin n|}{n^{1+0.1}}$

$0 \le \frac{|\sin n|}{n^2} \le \frac{1}{n^{1+0.1}}$

$\sum \frac{1}{n^{1+0.1}}$ conv

By direct test

$\Rightarrow \frac{|\sin n|}{n^2}$ conv

$\Rightarrow \sum |a_n| \Rightarrow \sum \sin$ conv (Absolutely conv)

(d) $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n^{1+0.1}}$ conv

$|a_n| = \sum_{n=1}^{\infty} \frac{|\sin n|}{n^{1+0.1}}$

$\sum \frac{1}{n^{1+0.1}}$ (if $q > 1$)

Now $\frac{a_n}{b_n} = \frac{\sin n}{n^{1+0.1}} \cdot \frac{n^{1+0.1}}{1}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sin n}{n^{1+0.1}} = 1 > 0$

By $\sum a_n$ & $\sum b_n$

$\Rightarrow$ Since Series is absolutely conv

now to check for $\sum a_n$

(i) $\sum \frac{(-1)^n \sin n}{n^{1+0.1}}$ is $\mathcal{O}$ (Alternating Series)

(ii) $\lim_{n\to\infty} \frac{\sin n}{n^{1+0.1}} = 0$

(iii) and $\sum$ ↓

$f_n(x) = \frac{1}{x^{1+0.1}} \frac{\sin(x) - x\cos(x)}{(x^{1+0.1})^2}$

$\therefore \frac{(x^{1+0.1})^2}{(x^{1+0.1})^2} < 0$ $\left(\frac{f'(x)}{g'(x)}\right)$

$\Rightarrow$ a_n is conv

$\Rightarrow$ a_n is conditionally convergent

(e) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{n^3+1}$ conv

$\Rightarrow |a_n| = \sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$

Using ratio test:

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{(n+1)^3+1} \cdot \frac{n^3+1}{n^2+1}$

$\lim_{n\to\infty} \frac{(n+1)^2}{(n+1)^3+1} \cdot \frac{(n^3+1)}{(n^2+1)} = \boxed{1} < 1$

$\Rightarrow \sum |a_n| \Rightarrow$ conv

$\Rightarrow \sum (-1)^n \frac{n^2+1}{n^3+1}$ also conv & $\sum |a_n|$ conv

$\Rightarrow$ series are absolutely

(f) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{n^3+1}$ conv

$= \sum_{n=1}^{\infty} \frac{n^2}{n^3} + \frac{1}{n^3} \Big]$ conv series

$\sum_{n=1}^{\infty} \frac{1}{n^3}$ (conv)

$\sum_{n=1}^{\infty} \frac{1}{n^3}$ (conv series) convergent.

$\sum_{n=1}^{\infty} \frac{1}{n^3}$

$\sum_{n=1}^{\infty} \frac{n^2}{n^3} = \sum_{n=1}^{\infty} \frac{n^2}{n^3} \frac{(1)}{(1)}$

$\sum_{n=1}^{\infty} \left(\frac{1}{n}\right) = \sum_{n=1}^{\infty} \frac{1}{n^{1+0}}$ → (1) series is conv

$\Rightarrow$ Convergent

$\Rightarrow$ (Absolutely conv)

$\Rightarrow$ $\sum a_n$ is conv

$\Rightarrow$ Since parts $\sum$ converges

so overall series is conv

(Conditionally conv)

(ex 8) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{0.1}}$

$= (-1)^1 + (-1)^2 \frac{1}{2^{0.1}} + \dots$

Because $\sum \frac{1}{n^{0.1}} = \frac{1}{1^{0.1}} + \frac{1}{2^{0.1}} + \frac{1}{3^{0.1}} + \dots$ is not p-series $p < 1$

& the series can be written as

$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^{0.1}}$

$a_n = \frac{1}{n^{0.1}}$

$a_n > a_{n+1}$ True

$\lim a_n \to 0$ ✓

So $\sum$ ✓

$\Rightarrow$ a_n is conv (By alt test Test)

★ If power series radius of convergence $R > 0$ then $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$ (1)

(a) If power series converges at $x = 5$ then $\sum$ this power