BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: BE
BRANCH: CS

SEMESTER: III
SESSION : MO/2017

SUBJECT: CS4101 DISCRETE MATHEMATICAL STRUCTURE

TIME: 1.5 HOURS

FULL MARKS: 25

INSTRUCTIONS:

1. The total marks of the questions are 30.
2. Candidates may attempt for all 30 marks.
3. In those cases where the marks obtained exceed 25 marks, the excess will be ignored.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. The missing data, if any, may be assumed suitably.

---

Q1 (a) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logcally equivalent by developing a series of logical equivalences. [2]

(b) Show that $\forall x(P(x) \wedge Q(x))$ and $\forall x\, P(x) \wedge \forall x Q(x))$ are logically equivalent. [3]

Q2 (a) When the mixed quantifiers (i) $\forall x \exists y P(x, y)$ (ii) $\forall x \forall y\, P(x, y)$ are false? [2]

(b) Show that the hypothesis "It is not sunny this afternoon and it is colder than yesterday", "We will go swimming only if it is sunny", "If we do not go swimming, then we will take a canoe trip", and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset". [3]

Q3 (a) Find the matrix representing the relation $R^2$, where the matrix representing R is [2]

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(b) If the relation R on a set A is transitive then $R^n \subseteq R$, where n is a natural number. [3]

Q4 Let $R = \{(a,d),(b,a),(b,c),(c,a),(c,d),(d,c)\}$ be the relation defined on a set $A = \{a,b,c,d\}$. Find the transitive closure of R using Warshall's Algorithm. [5]

Q5 (a) Define equivalence classes. What are the equivalence classes of 0 and 1 for congruence modulo 4? [2]

(b) Let R be an equivalence relation on a set A. Prove that the statements (i) a R b (ii) $[a] = [b]$ (iii) $[a] \cap [b] \neq \phi$ are equivalent. [3]

Q6 (a) How many reflexive and symmetric relations are there on a set with n elements? [2]

(b) Prove by contradiction that the number of primes are infinite. [3]