

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(MID SEMESTER EXAMINATION MO/SP20\*\*)

CLASS: BTECH / IMSc.  
BRANCH: ALL / FT

SEMESTER : I  
SESSION : MO/22

SUBJECT: MA103 MATHEMATICS - I

TIME: 02 HOURS

FULL MARKS: 25

INSTRUCTIONS:

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

- Q.1(a) Find whether the sequence  $\{a_n\}$  is monotonically increasing, bounded and convergent or not.  $a_n = \frac{3n}{n+1}$  for all  $n \in \mathbb{N}$  [2] CO BL CO1 1
- Q.1(b) Test the convergence of the series  $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \dots$  [3] CO1 5
- Q.2(a) Find whether the following series is convergent or not  $\sum_{n=1}^{\infty} \frac{(n - \ln n)^n}{2^n \cdot n^n}$  [2] CO1 1
- Q.2(b) Test the convergence of the series  $1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \dots (x > 0)$  [3] CO2 5
- Q.3(a)  $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & b \end{bmatrix}$ . Find for which value of b, rank of A is 3. [2] CO2 2
- Q.3(b) Find for which real value of b the following equations have a non zero solution  $x + 2y + 3z = bx$ ,  $3x + y + 2z = by$  and  $2x + 3y + z = bz$  [3] CO2 2
- Q.4(a) Can the vector  $(7, 7, 9, 11)$  be expressed as a linear combination of vectors  $(2, 0, 3, 1)$ ,  $(4, 1, 3, 2)$  and  $(1, 3, -1, 3)$ ? If, so, find the linear combination. [2] CO2 3
- Q.4(b) Applying Cayley Hamiltonian theorem find  $A^{-1}$ .  $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$  [3] CO2 3
- Q.5(a) Evaluate whether following limit exists or not.  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^2 + y^8}$  [2] CO3 1
- Q.5(b) If  $u = x^z$ ; check whether  $\frac{\partial^2 u}{\partial x^2 \partial y} = \frac{\partial^2 u}{\partial x \partial y \partial x}$  or not. [3] CO2 3

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