

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) If the number $\frac{\pi}{14} = 0.785398163$ is approximated by $\frac{11}{14}$, then [5]

- (i) Find the number of digits up to which, this approximation is accurate
- (ii) Find the absolute and percentage errors

Q.1(b) Perform four iterations of the Newton-Raphson method to obtain the approximate value of $(17)^{1/3}$ starting with the initial approximation $x_0 = 2$, correct upto 5 decimal places. [5]

Q.2(a) Solve the system of equations by gauss-Jordan method: [5]

$$\begin{aligned} x + y + z + u &= 2 \\ 2x - y + 2z - u &= -5 \\ 3x + 2y + 3z + 4u &= 7 \\ x - 2y - 3z + 2u &= 5 \end{aligned}$$

Q.2(b) Using power method find the dominant eigen value and the corresponding eigen vector for the matrix [5]

$$A = \begin{pmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{pmatrix}$$

Correct results upto 3 decimal places at each iteration and proceed till 4th iteration.

Q.3(a) Find the value of $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ at $x = 1$, from the table given below: [5]

x	-1	1	2	3
F(x)	7	5	19	51

Q.3(b) Using the following data, find the value of x corresponding to $y = f(x) = 100$ [5]

x	3	5	7	9	11
y	6	24	58	108	174

Q.4(a) Find the value of the integral $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $1/3^{rd}$ rule with step size $h = \frac{1}{4}$ correct to 5 [5]
decimal places. Further, estimate the value of π from the integral.

Q.4(b) If Δ is the forward difference operator, then find $\Delta^4 y_0$. Hence find the general expression for $\Delta^n y_0$ in [5]
terms of y_0, y_1, \dots, y_n that does not involve the difference operator

Q.5(a) For the initial value problem $\frac{dy}{dx} = y, y(0) = \alpha$. If $y(1) = 1$, find the value of k_3 obtained by [5]
Runge-Kutta fourth order method, correct to 4 decimal places.

Q.5(b) Determine the value of y correct to 4 decimal places using Euler's method when $x=0.04$ given that [5]
 $y' = x^2 + y, y(0) = 1$, when $h=0.2$ and compare with the exact answer.