BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: BTECH/IMSC
BRANCH: ALL/FT

SEMESTER: I/BL
SESSION : MO/2019

SUBJECT : MA103 MATHEMATICS - I

TIME: 2.00 HOURS

FULL MARKS: 25

INSTRUCTIONS:
1. The total marks of the questions are 25.
2. Candidates may attempt for all 25 marks.
3. Before attempting the question paper, be sure that you have got the correct question paper.
4. The missing data, if any, may be assumed suitably.

Q1 (a) Determine whether the sequence $\{a_n = 4 + (3/4)^n\}$ is monotonic, bounded and convergent. [2]

Q1 (b) Test the behaviour of the infinite series: $\sum_{n=1}^{\infty}\left((n^3+1)^{1/3} - n\right)$ [3]

Q2 (a) Apply integral test to test the convergence of the series $\sum_{n=2}^{\infty}\left[\frac{1}{n^2}\sin\left(\frac{\pi}{n}\right)\right]$ [2]

Q2 (b) Check the series for absolutely or conditionally convergence: $\sum_{n=1}^{\infty}\left(\frac{n\cos n\pi}{n^2+1}\right)$ [3]

Q3 (a) Reduce the following matrix into its Echelon form and hence find the rank [2]

$$\begin{bmatrix} 1 & 2 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

Q3 (b) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ [3]

Q4 (a) Find the value of λ for which the system of equations: 3x-y+4z=3, x+2y-3z=-2 and 6x+5y+ λz=-3 will have infinite number of solutions and solve them with that value of λ. [2]

Q4 (b) Verify Cayley Hamilton theorem for the matrix [3]

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}.$$ Also find A^{-1} and A^4.

Q5 (a) Verify that $u_{xy} = u_{yx}$ where $u(x,y) = (x^2+y^2)^{3/2}$ [2]

Q5 (b) If u= $f(r)$, where $r^2 = x^2 + y^2$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$ [3]

:::::: 14/10/2019 M ::::::