BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: BE
BRANCH: CSE

SEMESTER : III
SESSION : MO/16

SUBJECT: CS4101 DISCRETE MATHEMATICAL STRUCTURES

TIME: 1.5 HOURS

FULL MARKS: 25

INSTRUCTIONS:
1. The total value of the questions are 30 marks.
2. Candidates may attempt for all 30 marks.
3. In those cases where the marks obtain exceed 25 marks. The excess will be ignored.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. The missing data, if any, may be assumed suitably.

Q1. (a) Construct a truth table for the compound proposition: [2]
$(p \vee q) \rightarrow (p \wedge q)$

(b) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences. [3]

Q2. (a) Let M (x,y) be "x has sent y an e-mail message". and T(x,y) be "x has telephoned y", where the domain consists of all students in your class. Use quantifiers to express the statement: [3]
(i) There is a student in your class who has sent everyone else in your class an e-mail message.
(ii) Everyone in your class has either telephoned Avi or sent him an e-mail message.

(b) Use quantifiers and predicates with more than one variable to express the statement "Every user has access to exactly one mailbox". [2]

Q3. (a) Show that the hypothesis "it is not sunny this afternoon and it is colder than yesterday", "we will go swimming only if it is sunny", "if we do not go swimming, then we will take a canoe trip", and "if we take a canoe trip, then we will home by sunset" lead to conclusion "we will be home by sunset". [2]

(b) Let A, B, C be three sets, R a relation from A to B, and S a relation from B to C. Then prove that $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$. [3]

Q4. Define connectivity relation. Let $A = \{1, 2, 3, 4\}$ and let $R = \{(1,2), (2,3), (3,4), (2,1)\}$. Find the transitive closure of R by using the definition of connectivity relation. [5]

Q5. (a) Define equivalence class. Let R be an equivalence relation on a set A. Then prove that the statements given below for elements a and b of A are equivalent: [3]
$(i)\, a R b$ $(ii)\, [a]_R = [b]_R$ $(iii) [a]_R \wedge [b]_R \neq \phi$

(b) Let R_1 and R_2 be equivalence relations. Prove that $R_1 \wedge R_2$ is an equivalence relation but $R_1 \vee R_2$ is not necessarily an equivalence relation. [2]

Q6. (a) Consider the poset {1,2,3,4,5,6,7,8,9,10,11,12} with integer division as the partial order. Draw Hasse diagram for this poset. [3]

(b) Proof by contraposition that if $x^3 - x > 0$ then $x > -1, \forall x \in R$. [2]

*****26.09.16*****E