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I. Мьютон (КР N°1)

Программа в Excel, в Wolfram

1. Составить интерполяционный многочлен Ньютона

x	f
0.69	-4.78
2.52	5.56
4.2	-2.75
6.41	-6.5

Найти $F'(x=1.2)$.

x_0	f_0	D_0	x_1	f_1	D_1	x_2	f_2	D_2	x_3	f_3	D_3
0.69	-4.78		2.52	5.56	5.65	4.2	-2.75	-3.0188	6.41	-6.5	0.6738

$$f_{01} = \frac{f_1 - f_0}{x_1 - x_0} = \frac{5.56 - (-4.78)}{2.52 - 0.69} = 5.65$$

$$f_{12} = \frac{f_2 - f_1}{x_2 - x_1} = \frac{-2.75 - 5.56}{4.2 - 2.52} = -4.946$$

$$f_{23} = \frac{f_3 - f_2}{x_3 - x_2} = \frac{-6.5 - (-2.75)}{6.41 - 4.2} = -1.6968$$

$$f_{02} = \frac{f_2 - f_0}{x_2 - x_0} = \frac{-2.75 - (-4.78)}{4.2 - 0.69} = -3.0188$$

$$f_{13} = \frac{f_3 - f_1}{x_3 - x_1} = \frac{-6.5 - 5.56}{6.41 - 2.52} = 0.8353$$

$$f_{03} = \frac{f_3 - f_0}{x_3 - x_0} = \frac{-6.5 - (-4.78)}{6.41 - 0.69} = 0.6738$$

$$F(x) = D_0 + D_1(x-x_0) + D_2(x-x_0)(x-x_1) + D_3(x-x_0)(x-x_1)(x-x_2) \quad F(x) = 1.4951$$

2. Найти кубические интерполяции по Эрмиту на двух отрезках

x	f	f'
0.39	-1.15	-0.22
2.89	-6.93	0.43
4.29	-4.31	-0.49

Найти $F''(x=2.89-)$ и $F''(x=2.89+)$

x_0	f_0	D_0	x_1	f_1	D_1	x_2	f_2	D_2	x_3	f_3	D_3
0.39	-1.15		2.89	-6.93	-2.312	4.29	-4.31	1.0968			

$$D = \frac{f_1 - f_0}{x_1 - x_0} = \frac{-6.93 - (-1.15)}{2.89 - 0.39} = -2.312$$

$$= (D - f'_0) / (x_1 - x_0) = -2.312 - (-0.22) / 2.89 - 0.39 = -0.8368$$

$$\dots = (f'_1 - D) / x_1 - x_0 = 0.43 - (-2.312) / 2.89 - 0.39 = 1.0968$$

$$\dots = \cancel{f''^2} \dots = \dots / x_1 - x_0 = 1.0968 - (-0.8368) / 2.89 - 0.39 = 0.77344$$

$$F'' = 2D_2 + D_3(6x - 2x_1 - 4x_0)$$

$$F''_+ = 2 \cdot (-0.8368) + 0.77344(6 \cdot 2.89 - 2 \cdot 2.89 - 4 \cdot 0.39) = 5.0608$$

2.2.	x_0	x_1	x_2	x_3	x_4	x_5	x_6
	2.89	2.89	4.29	4.29	4.29	4.29	4.29
	-6.93	0.43	-4.31	-0.49	-0.49	-0.49	-0.49
	$\underline{-6.93}$	$\underline{0.43}$	$\underline{1.8714}$	$\underline{1.0295}$	$\underline{-1.6867}$	$\underline{-1.940143}$	$\underline{-1.940143}$

$$D = \frac{-4.31 - (-6.93)}{x_1 - x_0} = 1.8714$$

$$F'' = 2D_2 + D_3(6x - 2x_1 - 4x_0)$$

$$F''_+ = 2 \cdot 1.0295 - 1.940143(6 \cdot 2.89 - 2 \cdot 4.29 - 4 \cdot 2.89) = 7.4914$$

Приложение

$$f(x) = D_0 + D_1(x - x_0) + D_2(x - x_0)^2 + D_3(x - x_0)^2(x - x_1)$$

$$f''(x) = 2D_2 + D_3(6x - 2x_1 - 4x_0)$$

Тема: Ф4Д (КР№2)

Дана ф-ия: $3 + 8x - 8x^2 + x^4$

$$x_0 = 1$$

$$h = 2.6$$

1. Найти f' двухточечной Ф4Д

$$f' = 8 - 16x + 4x^3 \quad x_1 = 2.6 + 1 = 3.6$$

$$f'' = -16 + 12x^2 \quad x_2 = 3.6 + 2.6 = 6.2$$

$$f' = \frac{f_1 - f_0}{x_1 - x_0} = \frac{96.0816 - 4}{3.6 - 1} = 35.416 = D_1$$

2. Найти f' трехточечной Ф4Д

$$D_2 = \frac{f_2 - 2f_1 + f_0}{2h^2} = \frac{1222.7136 - 2 \cdot 96.0816 + 4}{2 \cdot 2.6^2} \Rightarrow f_2 = 1222.7136$$

$$\Rightarrow D_2 = 76.52$$

$$f' = D_1 + D_2(2x - x_0 - x_1) = 35.416 + 76.52(2x - 1 - 3.6) = -316.576 + 153.04x =$$
$$= x - 2.0686$$

$$f'(x=x_0) = D_1 - hD_2 = 35.416 - 2.6 \cdot 76.52 = -163.536$$

$$f'(x=x_1) = D_1 + hD_2 = 35.416 + 2.6 \cdot 76.52 = 234.368$$

3. Найти f'' трехточечной Ф4Д

$$f'' = 2D_2 = 2 \cdot 76.52 = 153.04$$

$$f''(x_0) = -16 + 12 \cdot 1 = -4$$

$$f''(x_1) = -16 + 12 \cdot 3.6^2 = 139.52$$

4. Найти h_{opt} для п.1. при $\delta = 0.001$

$$h_{opt} = \sqrt{4\delta / M_2}$$

$$M_2 = 139.52$$

$$h_{opt} = \sqrt{4 \cdot 0.001 / 139.52} = 0.005354$$

III Интеграл (КР №3)

содержит программа КР-3. и др. от

Найти интеграл $\int_a^b f(x) dx$ формулами Ньютона-Котеса и Гаусса
($n=1, 2, 3$).

$$f(x) = e^{3x} \cdot x$$

$$a = 0.3 \quad b = 0.9$$

$$H = b - a = 0.9 - 0.3 = 0.6$$

$$x_0 = a = 0.3, \quad x_1 = \frac{a+b}{2} = \frac{0.3+0.9}{2} = 0.6, \quad x = \frac{a+b}{2} + \frac{b-a}{2}t$$

$$f(x_0) = e^{3 \cdot 0.3} \cdot 0.3 = 0.73788$$

$$f(x_1) = e^{3 \cdot 0.6} \cdot 0.6 = 3.62979 \quad (\text{для } n=3)$$

$$f(x_2) = e^{3 \cdot 0.9} \cdot 0.9 = 13.39176$$

Формула Ньютона-Котеса.

$$1) \ n=1 \quad S_{\text{эл}}(f) = H \cdot f_0 = 0.6 \cdot 0.73788 = \underline{0.44273}$$

$$2) \ n=2 \quad S_{\text{эл}}(f) = H \left(\frac{1}{2} f_0 + \frac{1}{2} f_1 \right) = 0.6 \left(\frac{1}{2} \cdot 0.73788 + \frac{1}{2} \cdot 3.62979 \right) = \underline{4.23892}$$

$$3) \ n=3 \quad S_{\text{эл}}(f) = H \left(\frac{1}{6} f_0 + \frac{4}{6} f_1 + \frac{1}{6} f_2 \right) = \underline{2.86488}$$

Формула Гаусса

$$1) \ n=1 \quad S_{\text{эл}}(f) = H f(t=0) = 0.6 \cdot 3.62979 = \underline{2.17787}$$

$$2) \ n=2 \quad S_{\text{эл}}(f) = \frac{H}{2} [f(-\frac{1}{\sqrt{3}}) + f(\frac{1}{\sqrt{3}})]$$

$$x = \frac{a+b}{2} + \frac{b-a}{2}t$$

$$x_1 = \frac{a+b}{2} + \frac{b-a}{2}t = 0.6 + 0.3 \left(-\frac{1}{\sqrt{3}} \right) = 0.426795$$

$$f(-\frac{1}{\sqrt{3}}) = f(0.426795) = e^{3 \cdot 0.426795} \cdot 0.426795 = 1.535501$$

$$x_2 = 0.6 + 0.3 \left(\frac{1}{\sqrt{3}} \right) = 0.773205$$

$$f(\frac{1}{\sqrt{3}}) = f(0.773205) = e^{3 \cdot 0.773205} \cdot 0.773205 = 7.864854$$

$$S_{\text{эл}}(f) = \frac{0.6}{2} [1.535501 + 7.864854] = \underline{2.820107}$$

3) $n=3$

$$S_{\text{эл}}(f) = \frac{H}{18} [5f(-\frac{\sqrt{3}}{5}) + 8f(0) + 5f(\frac{\sqrt{3}}{5})]$$

$$x_1 = 0.6 + 0.3 \left(-\frac{\sqrt{3}}{5} \right) = 0.367621$$

$$f(-\frac{\sqrt{3}}{5}) = e^{3 \cdot 0.367621} \cdot 0.367621 = 1.107561$$

$$x_2 = 0.6 + 0.3 \left(\frac{\sqrt{3}}{5} \right) = 0.832379$$

$$f(\frac{\sqrt{3}}{5}) = e^{3 \cdot 0.832379} \cdot 0.832379 = 10.111462$$

$$S_{\text{эл}}(f) = \underline{2.837781}$$

IV Диф. ур. (КР 4) + программа Wolfram

Решить диф. ур. $x'' = F$ методом Эйлера, Хойна, Рунге и Рунге-Кутты

$$F = -5t + x + \sin[x'] \quad t_0 = 2, h = 0.01, x(t_0) = 1, x'(t_0) = y(t_0) = -2.$$

1. Метод Эйлера

$$K_0 = f(t_n, y_n)$$

$$y_{n+1} = y_n + h K_0$$

$$x_1 = x_0 + h y_0 = 1 + 0.01 \cdot (-2) = 0.98$$

$$K_0 = -5 \cdot 2 + 1 + \sin(-2) = -9.909297$$

$$y_1 = y_0 + h K_0 = -2 + 0.01 \cdot (-9.909297) = -2.099093$$

2. Метод Хойна

$$K_0 = f(t_n, y_n)$$

$$K_1 = f(t_n + h, y_n + h K_0)$$

$$y_{n+1} = y_n + \frac{h}{2} (K_0 + K_1)$$

$$K_0 = \dots = -9.909297$$

$$K_1 = -5(2 + 0.01) + (1 + 0.01 \cdot (-9.909297)) + \sin(-2 + 0.01 \cdot (-9.909297)) = -10.01276$$

$$x_1 = x_0 + \frac{h}{2} (y_0 + y_1) = 1 + \frac{0.01}{2} (-2 + (-2.099093)) = 0.9799$$

$$y_1 = y_0 + \frac{h}{2} (K_0 + K_1) = -2 + \frac{0.01}{2} (-9.909297 - 10.01276) = -2.099361$$

3. Метод Рунге

$$K_0 = f(t_n, y_n)$$

$$K_1 = f(t_n + \frac{h}{2}, y_n + h \frac{K_0}{2})$$

$$y_{n+1} = y_n + h K_1$$

$$K_0 = -9.909297$$

$$K_1 = -5(2 + \frac{0.01}{2}) + (1 + 0.01 \cdot \frac{-9.909297}{2}) + \sin(-2 + 0.01 \cdot \frac{-9.909297}{2}) = -9.962118$$

$$x_1 = x_0 + h (y_0 + h \frac{y_0}{2}) = 1 + 0.01 (-2 + 0.01 \cdot \frac{-2}{2}) = 0.9799$$

$$y_1 = y_0 + h K_1 = -2 + 0.01 (-9.962118) = -2.099621$$

4. Метод Рунге-Кутты

$$K_0 = f(t_n, y_n)$$

$$K_1 = f(t_n + \frac{h}{2}, y_n + h \frac{K_0}{2})$$

$$K_2 = f(t_n + \frac{h}{2}, y_n + h \frac{K_1}{2})$$

$$K_3 = f(t_n + h, y_n + h K_2)$$

$$y_{n+1} = y_n + \frac{h}{6} (K_0 + 2K_1 + 2K_2 + K_3)$$

для x

$$K_0 = y_0 = -2$$

$$K_1 = y_0 + h \frac{y_0}{2} = -2.01$$

$$K_2 = y_0 + h \frac{K_1}{2} = -2.010005$$

$$K_3 = y_0 + h K_2 = -2.020101$$

$$x_1 = x_0 + \frac{h}{6} (2 + 2(-2.01) + 2(-2.010005) + (-2.020101)) = 0.9799$$

$$y_1 = -2 + \frac{0.01}{6} (-9.909297 + 2(-9.962118) + 2(-9.96226) + (-10.013022)) = -2.099618$$

для y

$$K_0 = -9.909297$$

$$K_1 = -9.962118$$

$$K_2 = -5(2 + \frac{0.01}{2}) + (1 + 0.01 \cdot \frac{-9.962118}{2}) + \sin(-2 + 0.01 \cdot \frac{-9.962118}{2}) = -9.96226$$

$$K_3 = -5(2 + 0.01) + (1 + 0.01(-9.96226)) + \sin(-2 + 0.01(-9.96226)) = -10.013022$$