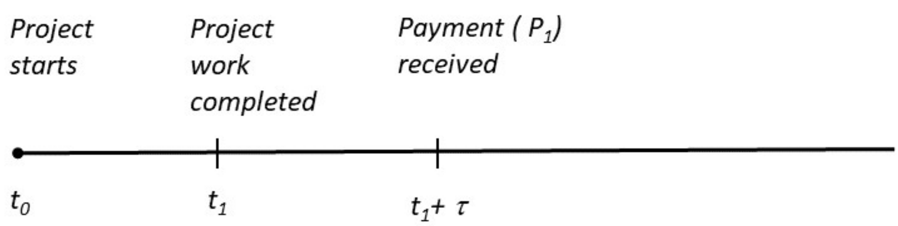
Explaining the early payment trade-off: two models

Simple model: one-stage

Assume that a project starts at time t_0. The work is completed at time t_1. The contractor is paid after τ time units, at time $t_1 + \tau$. The payment that the contractor receives is P_1.

The contractor can reduce the amount of time it takes to complete the work. In other words, t_1 is a decision variable. The cost associated with taking t_1 time units is $c(t_1) = \frac{k}{t_1}$. (We could add a fixed cost, but we will leave it out for simplicity.)



Assume that the supplier's cost of capital is r, and it is continuously compounded.

The value of the project to the supplier is $V = P_1 e^{-r(t_1+\tau)} - \frac{k}{t_1}$. We can solve for the optimal t_1 from the contractor's perspective. The optimal t_1 is a function of (P_1, r, τ, k). We are interested in how τ affects t_1.

Note that the cross-partial of V, with respect to (t_1, τ) is $e^{(-r(t_1+\tau))} P_1 r^2$. This is positive. In other words, V is super-modular (has increasing differences) in (t_1, τ). This means that t_1 and τ are strategic substitutes and this implies that if τ decreases then t_1 will decrease and, also, the optimal t_1 will decrease.

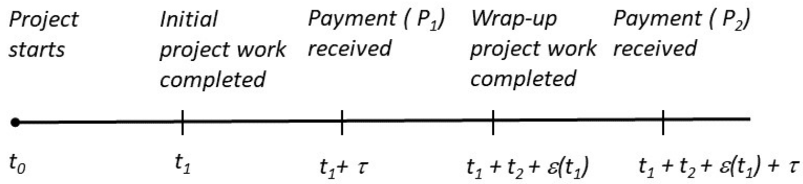
The simple model illustrates a clear relationship between payment delay and optimal project (work) length. The relationship comes directly from the fact that, for the contractor, project length and payment delay are strategic substitutes. Reducing payment delay makes it optimal to finish the project faster, because the marginal benefit of doing so increases as payment delay decreases.

The simple model shows a one-directional relationship between payment delay and project length. However, we want to capture a richer interaction. In particular, we want to capture the idea that payment delays -- by inducing the contractor to speed up work on the project -- may have a negative effect on project length. This negative effect is a result of poor quality work, which may result in rework. We try to capture this argument in the next model.

Less simple model: two stage

Now consider a two-stage model, but the work is now completed in two stages: an "initial" stage and "wrap-up" stage. The project starts at time t_0 and the "initial" stage work is completed at time t_1. The contractor is paid P_1 for this work after τ time units, at time $t_1 + \tau$. However, in this model, the work is not yet completed. The contractor has some wrap-up work to do, and this takes a minimum of t_2 additional time units. But the work may take longer than t_2 units, and this extra time, $\epsilon(t_1)$, is a decreasing function of t_1. In other words, the faster the contractor finishes the initial work, the more likely it is that it will have more rework. After the completion of the project the buyer pays the contractor for the wrap-up work, and this payment, P_2, is received at time $t_1 + t_2 + \epsilon(t_1) + \tau$.

Assume, for simplicity, that both t_2 and $\epsilon(t_1)$ cannot be affected after time t_1, so the only decision variable that the contractor controls in this model is still t_1. Also, for the purposes of analysis, assume $\epsilon(t_1) = \frac{z}{t_1}$. The figure below illustrates this model.



The value of the project to the supplier is $V = P_1 e^{-r(t_1+\tau)} - \frac{k}{t_1} + P_2 e^{-r(t_1+t_2+\frac{z}{t_1}+\tau)}$. We can again solve for the optimal t_1 from the contractor's perspective. The optimal t_1 is a function of $(P_1, r, \tau, k, t_2, z)$. We are interested in how τ affects t_1.

The cross-partial of V, with respect to (t_1, τ) is $e^{(-r(t_1+\tau))} P_1 r^2 + e^{(-r(t_1+t_2+\tau+\frac{z}{t_1}))} P_2 r^2 (1 - \frac{z}{t_1^2})$. This is no longer positive. It depends on the value of z. Specifically, for low values of z, the cross-partial is positive and t_1 and τ are strategic substitutes (as in the simple model above). But for high enough values of z, the sign of the cross-partial switches and becomes negative. In this case, t_1 and τ are strategic complements. This implies that as τ decreases then t_1 will increase.

To understand the intuition behind this, consider what happens when z increases. As z increases, the amount of rework (for any given initial project length) increases. If τ is high, the contractor optimally chooses a t_1 to maximize the total value of the project. If $z = 0$, i.e. there is no rework, then when the buyer reduces τ it is optimal for the contractor to reduce t_1. This follows from the strategic substitutes argument of the simple model.

But if z is high, then there is a different argument. For any t_1, the contractor has to consider the amount of rework and the impact on the delayed payment of P_2. Balancing off all the considerations, the contractor chooses an optimal t_1. If the buyer now reduces τ, the buyer will get paid earlier (holding all else constant). The contractor may now (for high enough z) find it optimal to increase t_1, getting paid P_1 a little later, but getting paid P_2 earlier because of lower rework.

Note that the other parameters like P_1 and P_2 also affect the optimal decisions, and the impact of τ. For example, for any z, a higher P_2 (relative to P_1) would increase the range of values of z where the cross-partial is negative.

Finally, note that in this model, there is no moral hazard. The contractor's decision to increase or decrease t_1 is the outcome of a trade-off between more or less re-work -- which is affected by various parameters including how fast the contractor is paid.