Project stage-dependent delay effect

Consider a project that has two sequential tasks, where the second task cannot start until the first task is completed. Upon completion, each task generates a deliverable with associated payments P_1 and P_2.

The contractor determines the completion times of the tasks by controlling its effort level. Let t_i denote the completion time of task i ($i = 1, 2$). A shorter completion time implies a higher effort level and thus a higher cost for the contractor. Let $c_i(t)$ denote the cost of completing task i with time t. The two tasks may have different costs but both c_1 and c_2 are increasing convex in t. The different costs c_1 and c_2 imply that it more be easier to change the completion time of one task than the other.

Let τ denote the payment delay. Then completion times (t_1, t_2) imply that the first task is completed at t_1, the second task at $t_1 + t_2$. The contractor receives P_1 at $t_1 + \tau$ and the contractor receives P_2 at $t_1 + t_2 + \tau$. (This is the first-best scenario where the contractor is not financially constrained, i.e., it does not rely on the payment to continue the project.)

Let r denote the discount rate. The contractor determines (t_1, t_2) given delay τ by maximizing the present value of its profit:

$$v(t_1, t_2) = P_1 e^{-r(t_1+\tau)} + P_2 e^{-r(t_1+t_2+\tau)} - c(t_1) - c(t_2) e^{-rt_1} \tag{1}$$

The optimal (t_1^*, t_2^*) satisfies the first-order conditions:

$$\frac{\partial v(t_1^*, t_2^*)}{\partial t_1} = 0, \qquad \frac{\partial v(t_1^*, t_2^*)}{\partial t_2}. \tag{2}$$

The second-order conditions are: $\frac{\partial^2 v(t_1^*, t_2^*)}{\partial t_1^2} < 0$, $\frac{\partial^2 v(t_1^*, t_2^*)}{\partial t_2^2} < 0$.

The optimal completion times (t_1^*, t_2^*) are functions of the payment delay τ. Using equ. (2) and totally differentiating wrt to (t_1, τ) and (t_2, τ) yields:

$$\frac{\partial t_1^*}{\partial \tau} = -\frac{\partial^2 v(t_1^*, t_2^*)}{\partial t_1 \partial \tau} \Big/ \frac{\partial^2 v(t_1^*, t_2^*)}{\partial t_1^2}, \tag{3}$$

$$\frac{\partial t_2^*}{\partial \tau} = -\frac{\partial^2 v(t_1^*, t_2^*)}{\partial t_2 \partial \tau} \Big/ \frac{\partial^2 v(t_1^*, t_2^*)}{\partial t_2^2}. \tag{4}$$

Note that

$$\frac{\partial^2 v(t_1, t_2)}{\partial t_1 \partial \tau} = r^2 e^{-r(t_1+\tau)} P_1 + r^2 e^{-r(t_1+t_2+\tau)} P_2, \qquad \frac{\partial^2 v(t_1, t_2)}{\partial t_2 \partial \tau} = r^2 e^{-r(t_1+t_2+\tau)} P_2. \tag{5}$$

Thus, under the second-order condition, $\frac{\partial t_1^*}{\partial \tau} > 0$, $\frac{\partial t_2^*}{\partial \tau} > 0$. But their magnitudes may differ. Clearly, the numerator in equ. (3) is greater than that in equ. (4). But the relative magnitude of the denominators in (3) and (4) is not clear.

$$-\frac{\partial^2 v(t_1, t_2)}{\partial t_1^2} = c''(t_1) + r^2 c(t_2) e^{-rt_1} - r^2 P_1 e^{-r(t_1+\tau)} - r^2 P_2 e^{-r(t_1+t_2+\tau)}, \tag{6}$$

$$-\frac{\partial^2 v(t_1, t_2)}{\partial t_2^2} = c''(t_2) e^{-rt_1} - r^2 P_2 e^{-r(t_1+t_2+\tau)}. \tag{7}$$

Thus, reducing payment delay τ generally has an *uneven* effect on the project progress depending on the stage of the project.