Treatment intensity

We want the treatment intensity to represent the extent to which a particular project/firm is affected by QuickPay *when QuickPay is implemented*.

Our data:

- QuickPay was implemented on April 27, 2011.
- We have five quarters before the implementation and five quarters after the implementation, from Q1 of 2010 to Q2 of 2012. Label these ten quarters at $t = 1, 2, \ldots, 10$. Then $Post_t = Ind(t \geq 6)$, where $Ind(\cdot)$ is the indicator function.

Consider project i owned by firm j at time t. Recall that we define ρ_{ijt} as

$$\rho_{ijt} = \frac{\sum_{k \in \mathcal{I}_{jt}} FAO_{kt}}{Sales_{jt}} \times Treat_i, \tag{1}$$

where

- $FAO_{kt} =$ total federal action obligation on project k in period t.
- $\mathcal{I}_{jt}$ = set of firm j's projects in which firm j is categorized as "small business" and thus benefit from QuickPay.
- $Sales_{jt} =$ total sales of firm j in period t.
- ρ_{ijt} is the weight of all small-business projects in firm j's business portfolio if project i is a small-business project. ρ_{ijt} is zero if project i is a large-business project.

Estimate treatment intensity of QuickPay

There are a few alternatives:

1. **Use the ρ_{ijt} computed in the *last quarter before* QuickPay implementation.** This would be the value ρ_{ij5} at $t = 5$, Q1 of 2011.
2. **Use the average weight in the *last year before* QuickPay implementation.** This means that in equ. (1), we compute the total FAO_{kt} on project k over the four quarters at $t = 2, 3, 4, 5$, then divide it the total sales over the four quarters. This might be better than using the first metric because it looks at the entire fiscal year, so we may have less of a problem in government allocating too much cash in a particular quarter and taking some cash back in another quarter. In other words, it smooths out the fluctuation in government obligation.
3. **Use the average weight in *four consecutive quarters before* QuickPay implementation, excluding the last quarter before QuickPay.** This is similar to the second approach but we exclude the last quarter before QuickPay and use $t = 1, 2, 3, 4$. The exclusion of the last quarter guards against a firm's actions, e.g., take on more small-business projects, in anticipation of the implementation of QuickPay. So the exogeneity of the treatment intensity is more likely to be true.

I don't think using ρ_{ijt} *after* QuickPay implementation ($t \geq 6$) is appropriate. The reason is that the very implementation of QuickPay may incentivize a firm to get more small-business projects. Furthermore, if QuickPay does help the firm, then the firm's sales should also be affected. Therefore, the value of ρ_{ijt} *after* QuickPay implementation embodies the treatment effect itself.

In contrast, ρ_{ijt} values *before* QuickPay implementation can be viewed as exogeneous. So by using ρ_{ijt} *before* QuickPay, our treatment intensity metric captures the different extent to which QuickPay would affect a firm.

Model

Let $\hat{\rho}_{ij}$ denote the treatment intensity computed from ρ_{ijt} using one of the three approaches mentioned above. Note that $\hat{\rho}_{ij}$ may be undefined for certain projects. This happens if we use the third approach and a small-business project i' of firm j starts in the last quarter ($t = 5$) before QuickPay implementation. In this case, if firm j has small-business projects before $t = 5$, then we have the value of $\hat{\rho}_{ij}$ for firm j and can use that for the new project i'. If firm j does not have any small-business project before $t = 5$, then we would not have such a value. I am not sure what to do in that scenario. (Of course, if we think that the exclusion of last quarter is not necessary, then we wouldn't have this issue. This is because all our projects start before QuickPay and end after QuickPay.)

The value $\hat{\rho}_{ij}$ is invariant in time and $\hat{\rho}_{ij} = 0$ for all project i in the control group.

Note that $\hat{\rho}_{ij}$ varies with firm j. All small-business projects in firm j have the same value of $\hat{\rho}_{ij}$. Henceforth, write $\hat{\rho}_{ij}$ as $\hat{\rho}_{\cdot j}$ for clarity.

Model 1: Continuous treatment intensity

Let N denote the number of firms in the treatment group. Let α_j denote the empirical distribution of $\hat{\rho}_{\cdot j}$. Specifically, sort all N values of $\hat{\rho}_{\cdot j}$ in increasing order. The firm with the lowest value of $\hat{\rho}_{\cdot j}$ value has $\alpha_j = 1/N$, the firm with the second lowest $\hat{\rho}_{\cdot j}$ has $\alpha_j = 2/N$, ..., the firm with the highest $\hat{\rho}_{\cdot j}$ has $\alpha_j = 1$.

Define θ_i denote the treatment intensity of project i owned by firm j:

$$\theta_i = Treat_i \times \alpha_j \qquad \forall i \in \cup_{t=1,\ldots,5} \mathcal{I}_{jt}. \tag{2}$$

Run the following regression model:

$$Delay_{it} = \eta_t + \beta_1 \theta_i + \beta_2 Post_t \times \theta_i + \beta_3 Post_t \times X_i + \beta_4 X_i + \gamma_i + \epsilon_{it} \tag{3}$$

The parameter of interest of β_2, which captures the treatment effect on the treated group.

Model 2: Discrete treatment intensity

Using α_j computed above, we can divide $\{\alpha_j : j = 1, \ldots, N\}$ into K brackets ($K = 2, 3$). Let $\Theta_i \in \{1, \ldots, K\}$ denote the categorical variable that indicates the treatment intensity of project i.

The model is:

$$Delay_{it} = \eta_t + \beta_1 \Theta_i + \beta_2 Post_t \times \Theta_i + \beta_3 Post_t \times X_i + \beta_4 X_i + \gamma_i + \epsilon_{it} \tag{4}$$

The parameter of interest is β_2, which is a K-dim vector. The components in β_2 estimate the effect of different treatment intensities.