The problem: understanding the impact of QuickPay on project performance

The US government instituted QuickPay, an initiative designed to pay its contractors faster, in 2011. Traditionally, the government paid its invoices in 30 days. Under QuickPay, this was reduced to 15 days for some contractors.

We are interested in studying the impact of QuickPay on project performance. Did QuickPay result in improved project performance? We use project delays as a measure of project performance.

Payment delays impact a contractor in multiple ways. Its most basic effect is to reduce the value of a project to a contractor because of the discounting of cash flows. Furthermore, payment delays may force contractors to raise outside capital to finance its operations. Since different contractors may be more or less able to do so, payment delays may have a different affect on different firms.

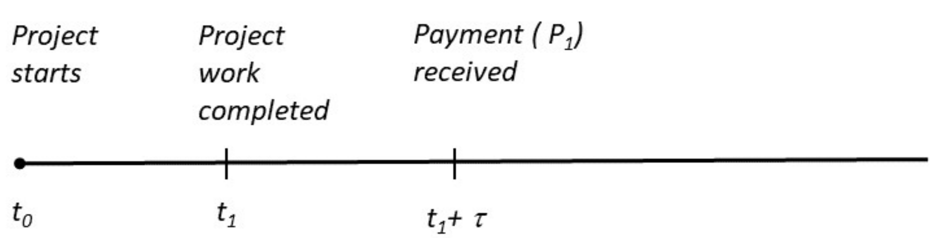
We want to empirically estimate the impact that QuickPay had on project delays. A specific question: under what conditions are payment delay and project length substitutes and when are they complements?

Theories

To understand the impact of QuickPay, we outline a few simple models to capture the impact of payment acceleration.

Model 1

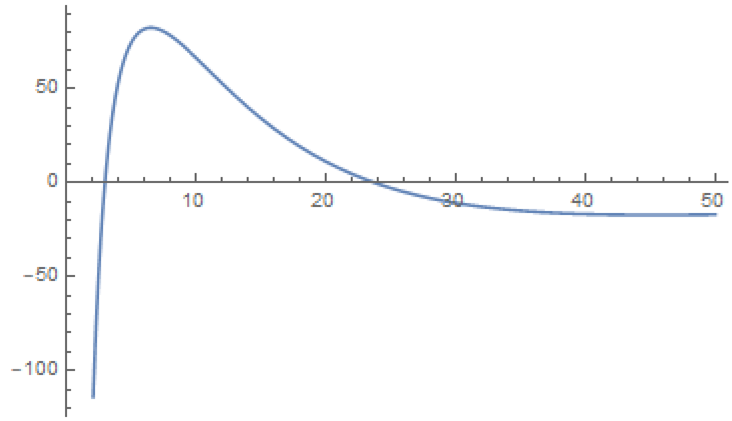
Assume that a project starts at time $t_0 = 0$. The work is completed at time $t_1 > 0$. The contractor is paid after $\tau \geq 0$ time units, at time $t_1 + \tau$. The payment that the contractor receives is P_1. See Figure below.



The contractor can determine the amount of time it takes to complete the work. In other words, t_1 is a decision variable. The cost associated with taking t_1 time units is $c(t_1) = \frac{k}{t_1}$. (We could add a fixed cost, but we will leave it out for simplicity.)

Assume that the supplier's cost of capital is r, and it is continuously compounded. The value of the project to the supplier is $V = P_1 e^{-r(t_1+\tau)} - \frac{k}{t_1}$. We can solve for the optimal t_1 from the contractor's perspective. The optimal t_1, t_1^*, is a function of (τ, P_1, r, k). We are interested in how τ affects t_1^*.

In summary, the contractor solves the following problem in Model 1: $\max_{t_1} V(t_1, \tau; P_1, r, k)$.



The figure above shows a graph of the contractors value function (with t_1 on the x-axis).

Note that the cross-partial of V, with respect to (t_1, τ) is $e^{(-r(t_1+\tau))} P_1 r^2$. This is positive. In other words, V is super-modular (has increasing differences) in (t_1, τ). This means that t_1 and τ are strategic complements and this implies that if τ decreases then t_1 will decrease and, also, the t_1^* will decrease.

The simple model illustrates a clear relationship between payment delay and optimal project (work) length. The relationship comes directly from the fact that, for the contractor, project length and payment delay are strategic complements. Reducing payment delay makes it optimal to finish the project faster, because the marginal benefit of doing so increases as payment delay decreases.

Hypothesis from Model 1

Hypothesis 1: QuickPay will result in shorter project lengths.

Question: Does this result hold up as a baseline? Across all firms, what is the average impact of QuickPay?

However, the simple interaction between payment delay and project length ignores many factors with can affect the relationship between these two variables.

We want to capture a richer interaction between Quickpay and project performance. We present other models below.

Model 2

We expect that the effect of payment delays on financially constrained firms will be different from the effect on financially unconstrained firms.

To capture this effect, we modify Model 1 by incorporating the following feature. Once a contractor starts working on a project, it will incur certain financial liabilities on this project (e.g. payroll) that are due by a certain time, t_L. Ideally, the firm would receive payment from the project in order to meet this liability. However, this may not be possible since the time to complete the project and receive payment may exceed t_L.

Assume, further, that the contractor can delay paying the liability to a time αt_L, where $\alpha \geq 1$. The value of α can be interpreted as being related to the financial health of the contractor. For example, a financially unconstrained contractor can raise external funds to meet this liability and can therefore delay the time by which it needs to be paid to αt_L where $\alpha > 1$. A financially constrained contractor, on the other hand, would (by definition) not be able to raise funds easily to meet this financial commitment and has to rely on receiving payment from the government contract in order to meet its financial liability. For a completely financially constrained contractor, $\alpha = 1$.

To Model 1 (above), we incorporate the fact that if the payment is received after time αt_L, i.e. if $t_1 + \tau > \alpha t_L$, the contractor experiences an extra financing cost, δ, for the duration $t_1 + \tau - \alpha t_L$. In other words, the excess financing cost is incurred for the duration $(t_1 + \tau - \alpha t_L)^+$; which can be zero.

In summary, the contractor's objective function in Model 2 is
$\max_{t_1} V(t_1, \tau) = P_1 e^{-(r(t_1+\tau)+\delta(t_1+\tau-\alpha t_L)^+)} - \frac{k}{t_1}$.

Comment:

Note that as $\delta \to \infty$, then we can write the contractor's problem as a constrained optimization problem: $P_1 e^{-r(t_1+\tau)} - \frac{k}{t_1}$ with the constraint $t_1 + \tau \leq \alpha t_L$. In other words, the objective function is the same as Model 1, but with the constraint that the project payment has to be received at (or before) the time when the liability is due. A "high enough" δ transforms the contractor's problem to a constrained version of Model 1. However, we consider the more general version of Model 2 given above with the assumption that δ is not "too high".

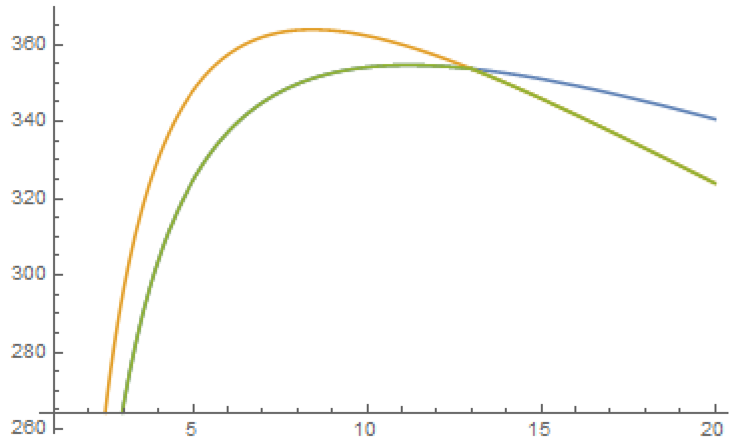
Now suppose that αt_L is "very high". For example, assume a fixed α and $t_L \to \infty$. In this case, $(t_1 + \tau - \alpha t_L)^+ = 0$ and the contractor's problem reduces to Model 1, i.e. $P_1 e^{-r(t_1+\tau)} - \frac{k}{t_1}$. In other words, if αt_L is "high enough", the financial constraint/penalty is never going to kick in and we are back to Model 1 (no impact of financial constraint).

On the other hand, suppose αt_L is "very low", i.e. $(t_1 + \tau - \alpha t_L)^+ = (t_1 + \tau - \alpha t_L)$. For example, assume a fixed α and $t_L = \epsilon$ where $\epsilon > 0$ and arbitrarily close to zero. In this case, the financial constraint/penalty is always binding. The contractor's problem now is to maximize $P_1 e^{-(r(t_1+\tau)+\delta(t_1+\tau-\alpha t_L))} - \frac{k}{t_1}$. This can be re-written as $P_1 e^{-(r+\delta)(t_1+\tau)} e^{\delta \alpha t_L} - \frac{k}{t_1}$. Note that, in this case, the objective function similar to Model 1 but with a higher discount rate ($r + \delta > r$) and the revenue component is multiplied by a constant term ($e^{-\delta \alpha t_L}$). The interpretation is that the contractor always pays the higher cost of capital ($r + \delta$) but receives a discount of δ for the first αt_L periods.

More generally, if $t_1 + \tau - \alpha t_L \leq 0$, then the contractor's objective function is $P_1 e^{-r(t_1+\tau)} - \frac{k}{t_1}$; and if $t_1 + \tau - \alpha t_L > 0$, then the contractor's objective function is $P_1 e^{-(r+\delta)(t_1+\tau)} e^{\delta \alpha t_L} - \frac{k}{t_1}$. Furthermore, note that $P_1 e^{-(r+\delta)(t_1+\tau)} e^{\delta \alpha t_L} - \frac{k}{t_1} < P_1 e^{-r(t_1+\tau)} - \frac{k}{t_1}$ when $t_1 + \tau - \alpha t_L > 0$ and the inequality is reversed for $t_1 + \tau - \alpha t_L \leq 0$.

Based on this, we can rewrite the contractor's objective function in Model 2 as
$\max_{t_1} V(t_1, \tau) = \min[P_1 e^{-r(t_1+\tau)} - \frac{k}{t_1}, P_1 e^{-(r+\delta)(t_1+\tau)} e^{\delta \alpha t_L} - \frac{k}{t_1}]$.

The figure below illustrates the function. The x-axis is t_1 and the y-axis is V.



Note that the objective function we are interested in is the green line, and it is the minimum of the blue line (which is the Model 1 objective function) and the orange line. Note that that the kink in the green line represents the point where the higher cost of financing kicks in.

Lemmas:

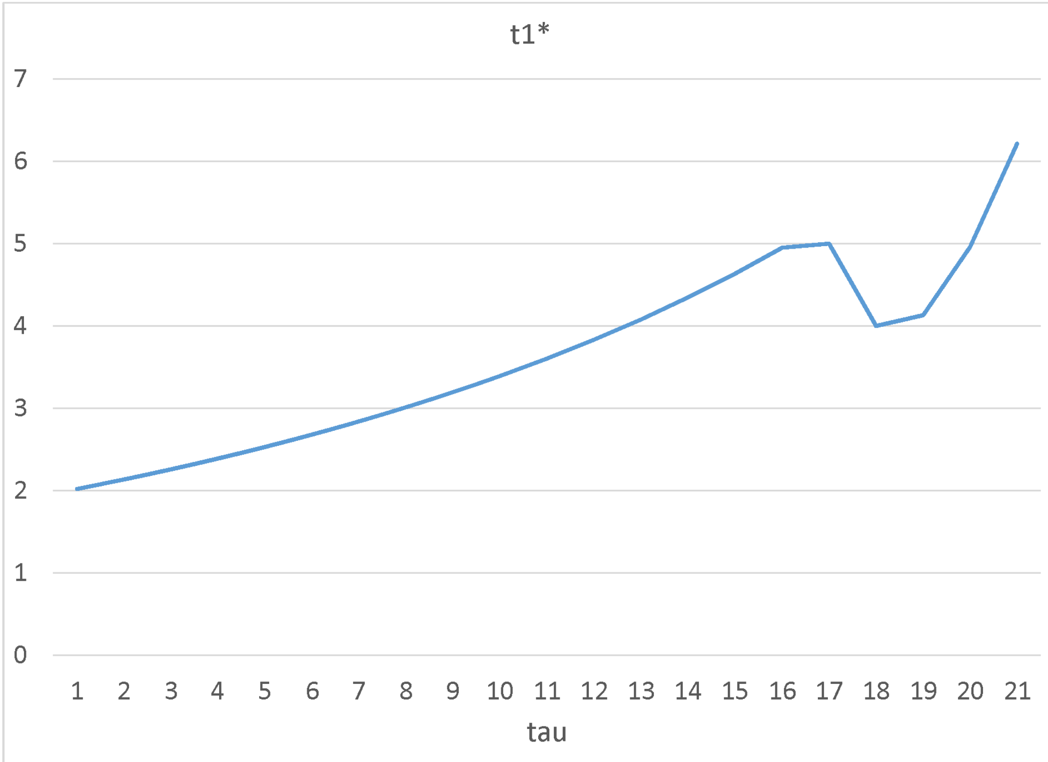
- Both the blue line and the orange line are quasi-concave (*can show this by showing that the slope for both functions are monotone*; therefore the green line is also quasi-concave.
- Note that optimal t_1 for the blue line is always greater than the optimal t_1 for the orange line. (*Can prove this by comparing the slopes.*)

Proposition:

- When the firm is fully unconstrained (e.g. αt_L is very high), then the optimal t_1 lies on the blue line only and is at the optimal point of the blue line. In this case, t_1 and τ are strategic complements.
- When the firm is fully constrained (e.g. αt_L is very low), then the optimal t_1 lies on the orange line only and is at the optimal point of the orange line. In this case, t_1 and τ are strategic complements.
- There is a range of αt_L ("in the middle"), where the optimal t_1 lies at the intersection of the blue line and the orange line. In this case, t_1 and τ are strategic substitutes.

Illustration:

The graph below provides an illustration of the above model. In this graph, the following parameters are chosen: $r = 0.1$, $P_1 = 30$ and $k = 10$. Also, $t_L = 21$ and $\alpha = 1$. Then, τ is varied between 0 and 20, and the optimal value of t_1 is calculated. Note that when τ is low, the liability constraint is not binding and the optimal value of t_1 is the unconstrained value. This value rises as τ rises. However, when τ gets to 16, the liability constraint starts binding. Any further increases in τ need to be compensated with a decrease in t_1. However, once τ increases beyond a second threshold, the contractor is fully constrained and the optimal t_1 again increases as τ increases.



Sketch of proof:

- The green and the orange lines intersect at the point where $t_1 + \tau = \alpha t_L$
- Below this point, the blue line is the min; above this line the orange line is the min
- Suppose αt_L is high. A high αt_L means that the two lines will intersect at a point to the right of the optimal points. And the blue line will be the min to the left of this intersection point. So the optimal point is the optimal point of the blue line.
- Suppose αt_L is low. A low αt_L means that the two lines will intersect at a point to the left of the optimal points. And the orange line will be the min to the right of the intersection point. So the optimal point is the optimal point of the orange line.
- Suppose αt_L is "in the middle"; such that they intersect in between blue optimum and the orange optimum. In this region, the optimal point is always the point of intersection; the orange line will be falling and the blue line will be rising. So the optimal point is $t_1 + \tau = \alpha t_L$. Therefore, since αt_L is fixed, increasing τ will reduce t_1.

Hypothesis from Model 2

For firms that are fully unconstrained or fully constrained, reducing τ it will reduce t_1. For firms that are "moderately constrained", reducing τ *may* increase t_1.

Question:

- How do we empirically test this?
- How do we know which firms are fully unconstrained, which are fully constrained, and which are in the middle?

- Would we see firms that are fully constrained? Would these firms be unable to compete and therefore not win contracts?
- So we should either see fully unconstrained firms or firms that are at the cusp.

If so, we get the following result:

- For financially unconstrained firms, Quickpay will decrease project length.

- For financially constrained firms (at the cusp), Quickpay will increase project length.

This is what we seem to be seeing in the data.

But this may be a naïve conclusion because the above is a "marginal-effect" argument. Since the treatment is a fixed (15-day) reduction, a more nuanced view is important. To illustrate a more nuanced view, consider the following example. Suppose τ is 17 (in the above example). In this case, the firm is financially constrained (i.e. the liability constraint is binding). Reducing τ to 16 will increase project length (consistent with the above prediction). But if we go from $\tau = 17$ to $\tau = 10$, note that the project length will decrease. This is because the firm was financially constrained at $\tau = 17$ but is not once τ drops below 16. At $\tau = 10$, the net effect is a decrease in project length (because we have moved from the constrained region to the unconstrained region and the net effect of decreasing τ from 17 to 10 is a decrease in project length.

This more nuanced view is important because what we observe in the data is a decrease in τ of a specific number of days (from 30 to 15). What is our prediction about impact on project length? It depends on several specifics. However, on average, firms that are more likely to be financially constrained (in the above model, that is firms with α close to 1) will see a smaller decrease in project length as compared to firms that are less likely to be constrained.

Model 1a and 2a

This model is a brief detour, assuming uncertainty. Suppose there is uncertainty in payment time. I will illustrate this here with Model 1, but this could be extended to Model 2. Think of the model below as Model 1a.

Suppose payment time has a random component. Let the payment time be $(1+\epsilon)\tau$, where $\epsilon \geq 0$ is a random variable with mean μ. Note that in Model 1, $\epsilon = 0$. Note that the above formulation implies that the magnitude of the uncertain term is increasing in τ.

$V = P_1 e^{-r(t_1+(1+\epsilon)\tau)} - \frac{k}{t_1} = P_1 e^{-r(t_1+\tau)} e^{-r\epsilon\tau} - \frac{k}{t_1}$

The expected value of the project is $EV = P_1 e^{-r(t_1+\tau)} e^{-r\mu\tau} - \frac{k}{t_1}$.

Note that as τ goes down (e.g. under QuickPay), the value of the contract goes up. This would further enhance the strategic complementarity effect of Model 1.

Model 2 can similarly be extended to incorporate variability.

Why discuss variability? We had earlier had a discussion that the impact of QuickPay may be due to the reduction in payment time variability rather than payment time.

Also, see below.

Model 3: portfolio of contracts

Consider the fact that firms are usually working on a portfolio of contracts, where the portfolio consists of government and private contracts. What impact does a portfolio of contracts have on how QuickPay affects government project completion time?

There are two proposed models for the portfolio of contracts. Roughly speaking, I will categorize them as follows:

- The "critical path" argument (which is the key point in Vlad's argument).
- The "uncertain private project" argument (which is the key point in Jie's argument).

The "critical path" argument:

- Consider a firm that holds a government project (indexed by "g") and a private project (indexed by "p").
- To illustrate the critical path argument, assume that the firm will receive value only on the completion of *both* projects. This is an unrealistic assumption, but can be motivated by assuming that the firm borrows money to finance operations to run both projects and will receive value only when both projects are completed.
- Let $P = P_g + P_p$ be the total value on completion of both projects.
- The value to the firm is $V = Pe^{-rMax[t_g+\tau_g, t_p+\tau_p]} - \frac{k_p}{t_p} - \frac{k_g}{t_g}$
- Note that in this simple model it is optimal to finish both projects at the same time, i.e. $t_g + \tau_g = t_p + \tau_p$
- Note that if $\tau_g > \tau_p$, then the firm will be focused on lowering t_g; if $\tau_g = \tau_p$, then the firm will be focused on lowering both t_g and t_p; if $\tau_g < \tau_p$, then the firm will be focused on lowering t_p
- Assume that, before QuickPay both payment times are equal, i.e. $\tau_g = \tau_p$; after QuickPay, $\tau_g < \tau_p$
- Note that QuickPay will primarily cause the firm to focus on lowering time of the private project
- More details to be worked out
- *Hypothesis:* QuickPay will have less of an effect on the time of government projects for firms that hold a portfolio of contracts

The "uncertain private project" argument:

- The key assumption is that the private project is uncertain but the government project is not
- Assume projects are worked on sequentially
- Payoffs are as follows:
  - Work on government project first: $V_{gp} = P_g e^{-r(t_g+\tau_g)} - \frac{k_g}{t_g} + P_p e^{-r(t_g+t_p+\tau_p)} e^{-r\mu\tau_p} - \frac{k_p}{t_p}$
  - Work on private project first: $V_{pg} = P_p e^{-r(t_p+\tau_p)} e^{-r\mu\tau_p} - \frac{k_p}{t_p} + P_g e^{-r(t_p+t_g+\tau_g)} - \frac{k_g}{t_g}$
- In general, it should make sense to work on the more certain (government) project first

Notes on models of strategic (or dynamic) effects

There are three dynamic effects we have discussed; one is an "ex post" effect and two are "ex ante" effects

1. The "rework" effect: speeding up projects can lead to "rework" which will end add time to the project; this is an ex post effect and we had sketched a model earlier. We can revive that model, but note that it makes a distinction between phase 1 ("planned" completion time for project) and phase 2 ("actual" completion time for project).

   Hypothesis: QuickPay will incentivize firms to speed up projects but this may end up backfiring on the project and lead to longer completion times (due to rework)

2. The aggressive bidding or competitive effect: explained clearly by Vlad. Note again that a model will need a phase 1 and phase 2.

   Hypothesis: Also provided by Vlad; basic idea: QuickPay will lead to more "delays" because of aggressive bidding. (Note that more "delays" may not mean longer project completion times.)

3. More aggressive uptake of project and the congestion effect: again explained clearly by Vlad. Note again that a model will need a phase 1 and phase 2.

   Hypothesis: QuickPay will lead to longer project completion times because of congestion.