Combining two treatment periods together

Need to consider the effect of QuickPay on large businesses

I proposed that we focus on the 2011-implementation of QuickPay that only treats small businesses. Now I disagree with my earlier proposal.

Looking only at the 2011-implementation of QuickPay provides an incomplete picture. So far, we have obtained strong evidence that financing constraints affect project delay due to QuickPay. Intuitively, the financing constraints intensify in the order of *large business* < *large business with contract financing* < *small business* < *small business with contract financing*. We miss the left half of the spectrum if we only consider the 2011-QuickPay on small businesses.

The left half of the spectrum is important because it provides empirical evidence for Theory 1 on the strategic complementary between payment days and project duration. We show in our data that large businesses who do not receive contract financing indeed finish ahead of time under QuickPay (see here). Leaving this empirical evidence out would harm us given the importance of Theory 1.

In addition, the competition result from large businesses provides empirical evidence for our competition theory. (Maybe we will be able to understand the opposite result from small businesses by incorporating financial constraints to the theory?)

Issue

If we are to consider the effect of QuickPay on large businesses, I feel the current approach is not satisfactory. The reason is that we analyze the treatment effect on small and large businesses in separate DiD models. This approach feels fragmented and is not easy to draw comparisons or present.

In what follows I propose a way to combine the two models together and use all data to estimate the treatment effects on small and large businesses.

Basic idea

The basic idea is to generalize our current "spatial" model along the time dimension.

Take the model with contract financing as an example:

$$Y_{it} = \beta_1 + \beta_2 Post_t \times Treat_i \times CF_i + \beta_3 Post_t \times Treat_i \times NCF_i + \beta_4 CF_i + \beta_5 Post_t \times CF_i + \beta_6 X_i + \beta_7 Post_t \times X_i + \eta_t + \epsilon_{it} \quad (1)$$

This model allows us to combine two models into one by allowing the treatment effect to vary with a characteristic: whether or not a project receives contract financing. The key thing in equ. (1) is that **within each value of the characteristic (CF=1 or CF=0), we have a treated group and a control group**. This way, we can delineate the different treatment effects and utilize all data to get more statistical model.

The contract financing charateristic can be viewed as a "spatial" feature. We can generalize it by letting the characteristic to be time-dependent.

An example

Consider the following example with two groups: A and B.

- At period 0-1, no group is treated.
- At period 1-2, group A is treated.
- At period 2-3, no group is treated.
- At period 3-4, group B is treated.

The treatment at period 1 is the same as that in period 3.

To study the treatment effect, we can run two separate DiD models (assuming that parallel trend assumption holds).

1. On periods 0-2: Group A is treated and group B is control. Estimate the treatment effect on group A.
2. On periods 2-4: Group B is treated and group A is control. Estimate the treatment effect on group B.

We cannot directly combine all the data together and run a DiD because all units are treated eventually so there is no control group.

BUT we can adopt the idea in equ. (1) and estimate the treatment effects on A and B in one regression as follows. Similar to the contract financing characteristic, **define the characteristic as whether or not observation i is treated in period 1**. Let $T1_i$ be an indicator that observation i is treated in period 1. Let $NT1_i$ be an indicator that observation i is *not* treated in period 1 and thus is treated in period 3. Then $T1_i$ and $NT1_i$ resemble CF_i and NCF_i in equ. (1). Importantly, **for each value of the characteristic ($T1_i = 0$ and $T1_i = 1$), we have a treated group and a control group.**

A model similar to equ. (1) is

$$Y_{it} = \eta_t + (\beta_1 Post_t^1 \times T1_i + \beta_2 Post_t^1) + (\beta_3 Post_t^2 \times NT1_i + \beta_4 Post_t^2) + \beta_5 T1_i + \beta_6 NT1_i + \epsilon_{it}, \tag{2}$$

where

- $Post_t^1$ is an indicator of whether period t is after the first treatment, i.e., $t \geq 1$.
- $Post_t^2$ is an indicator of whether period t is after the second treatment, i.e., $t \geq 3$.
- η_t is time fixed effect

Parameter interpretation

Period	Group	$E[Y_{it}]$	Notation
0	A	$\eta_0 + \beta_5$	$\bar{Y}_{A0}$
0	B	$\eta_0 + \beta_6$	$\bar{Y}_{B0}$
1	A	$\eta_1 + \beta_1 + \beta_2 + \beta_5$	$\bar{Y}_{A1}$
1	B	$\eta_1 + \beta_2 + \beta_6$	$\bar{Y}_{B1}$
2	A	$\eta_2 + \beta_1 + \beta_2 + \beta_5$	$\bar{Y}_{A2}$
2	B	$\eta_2 + \beta_2 + \beta_6$	$\bar{Y}_{B2}$
3	A	$\eta_3 + \beta_1 + \beta_2 + \beta_4 + \beta_5$	$\bar{Y}_{A3}$
3	B	$\eta_3 + \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_6$	$\bar{Y}_{B3}$
4	A	$\eta_4 + \beta_1 + \beta_2 + \beta_4 + \beta_5$	$\bar{Y}_{A4}$
4	B	$\eta_3 + \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_6$	$\bar{Y}_{B4}$

Treatment effect on group A = $(\bar{Y}_{A1} - \bar{Y}_{A0}) - (\bar{Y}_{B1} - \bar{Y}_{B0})$

=

$[(\eta_1 + \beta_1 + \beta_2 + \beta_5) - (\eta_0 + \beta_5)] - [(\eta_1 + \beta_2 + \beta_6) - (\eta_0 + \beta_6)]$

= β_1

Treatment effect on group B = $(\bar{Y}_{B3} - \bar{Y}_{B2}) - (\bar{Y}_{A3} - \bar{Y}_{A2})$

=

$[(\eta_3 + \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_6) - (\eta_2 + \beta_2 + \beta_6)] - [(\eta_3 + \beta_1 + \beta_2 + \beta_4 + \beta_5) - (\eta_2 + \beta_1 + \beta_2 + \beta_5)]$

= β_3

Thus, we can run one single regression to find the treatment effects on the two groups.

Applying to our data

Assumption

The QuickPay policy is implemented in the same way in Apr. 2011 and Aug. 2014 in the sense that payment days is reduced from 30 days to 15 days for treated firms.

Model

Concatenate the two DiD models for 2011- and 2014-implementation of QuickPay in a similar way as equ. (2). Then we will be able to estimate the treatment effects in one model and directly compare them.