## Treatment as a continuous variable

In some empirical tests, e.g., the portfolio conjecture and the effect of QuickPay on initial value of contract, we need to take a finer look at QuickPay rather than treating it as a binary variable. The reason is that the payment acceleration under QuickPay may have a bigger impact on a business if government projects with accelerated payments make up a bigger fraction of the business's sales.

Therefore, we can use the weight of treated government projects in a business portfolio as a proxy for the *intensity of treatment*, which is a continuous variable. The control group are the firms that are not treated at all (zero treatment intensity).

A benefit of using such a continuous treatment variable is that we can delineate the treatment effect from other policies that have a fixed effect, such as the JOBS act. So if we find that the treatment effect changes with the treatment intensity, then it is more convincing that the change is caused by the treatment.

## **Treatment intensity**

Let  $\rho_{it}$  denote the proxy of treatment intensity on firm i in period t.

$$\rho_{it} = \frac{\sum_{j \in \mathcal{I}_{it}} FAO_{jt}}{Sales_{it}},\tag{1}$$

where

- $FAO_{it} =$ total federal action obligation on project j in period t
- $\mathcal{I}_{it}$  = set of firm i's projects in which firm i is categorized as "small business" and thus benefit from QuickPay
- $Sales_{it} = \text{total sales of firm } i \text{ in period } t$

Thus,  $\rho_{it}$  estimates the weight of QuickPay-affected revenue in firm i's business portfolio.  $\rho_{it}$  is a firm-level variable. Intuitively, the higher the value of  $\rho_{it}$ , the higher impact QuickPay has on firm i

## Model to test the portfolio conjecture

#### **Basic model**

$$Y_{it} = \beta_0 + \beta_1 Post_t + \beta_2 Post_t \cdot \rho_{it} + \delta_i + \eta_t + e_{it}$$
 (2)

#### Model with interaction

$$Y_{jt} = \beta_0 + \beta_1 Post_t \cdot CF_{jt} + \beta_2 Post_t \cdot CF_{jt} \cdot \rho_{it} + \beta_3 Post_t \cdot NCF_{jt} \cdot \rho_{it} + \delta_j + \eta_t + e_{jt} \quad (3)$$

where

- $Y_{jt} = \text{Delay of project } j \text{ in period } t$
- ullet  $CF_{jt}=$  indicator of whether project j receives contract financing in period t
- ullet  $NCF_{jt}=1-CF_{jt}$ : indicator of whether project j receives no contract financing in period t
- ullet  $Post_t = ext{indicator of whether period } t ext{ is post QuickPay implementation}$
- $ho_{it} =$  treatment intensity defined in equ. (1) of firm i that owns project j in period t

#### Parameters of interest

- $\beta_2$ : the average rate at which the delay increases as the impact of QuickPay increases on a firm in a project that receives financing. If  $\beta_2 > 0$ , then a higher weight of treated projects in a firm's business leads to longer delays.
- $\beta_3$ : symmetric rate of  $\beta_2$  for projects that do not receive financing.

### **Assumptions**

- Delay of projects that receive contract financing in the treatment group should have parallel trend as delay of projects that receive contract financing in the control group pre-treatment.
- Delay of projects that do not receive contract financing in the treatment group should have parallel trend as delay of projects that do not receive contract financing in the control group pre-treatment.

#### **Notes**

- Equ. (2) and (3) make the linear rate assumption. We can add a term  $ho_{it}^2$  to allow nonlinear effect.
- Adding  $CF_{jt}$  in the model allows different treatment effects based on financial constraints. This is also what we have observed.

# Model to test the effect of QuickPay on the project value of a firm

#### **Basic model**

$$Y_{it} = \beta_0 + \beta_1 Post_t + \beta_2 Post_t \cdot \rho_{it} + \delta_i + \eta_t + e_{it}$$

$$\tag{4}$$

#### Model with interaction

$$Y_{it} = \beta_0 + \beta_1 Post_t \cdot CF_{it} + \beta_2 Post_t \cdot CF_{it} \cdot \rho_{it} + \beta_3 Post_t \cdot NCF_{it} \cdot \rho_{it} + \delta_i + \eta_t + e_{it} \quad (5)$$

where

- ullet  $Y_{it}=$  Avg./median/max. of the initial values of all projects owned by firm i in period t
- $CF_{it} = \text{indicator of whether firm } i \text{ receives financial assistance in period } t$
- $NCF_{it} = 1 CF_{it}$ : indicator of whether firm i receives no financial assistance in period t
- ullet  $Post_t = ext{indicator of whether period } t ext{ is post QuickPay implementation}$
- $\rho_{it}$  = treatment intensity defined in equ. (1).

#### **Parameters of interest**

- $\beta_2$ : the average rate at which the government contract value increases as the impact of QuickPay increases on a firm that receives financial assistance. If  $\beta_2 > 0$ , then a higher weight of treated projects in a firm's business leads to higher-valued contracts.
- $\beta_3$ : symmetric rate of  $\beta_2$  for firms that do not receive financial assistance.

#### **Assumptions**

- The avg./median/max. of the initial project values of firms that receive financial assistance in the treatment group has parallel trend as that of firms that receive financial assistance in the control group
- The avg./median/max. of the initial project values of firms that do not receive financial assistance in the treatment group has parallel trend as that of firms that do not receive financial assistance in the control group