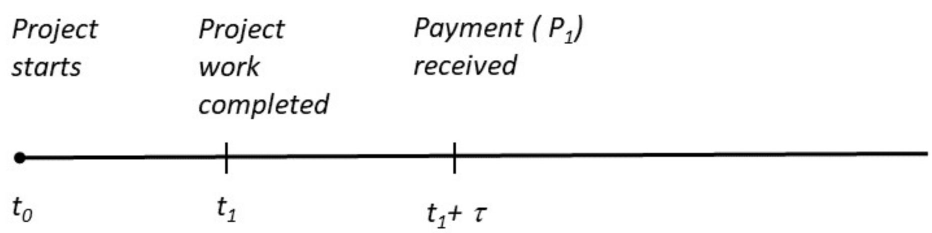
Explaining the early payment trade-off: two models

Theory 1: Simple one-stage model

Assume that a project starts at time t_0. The work is completed at time t_1. The contractor is paid after τ time units, at time $t_1 + \tau$. The payment that the contractor receives is P_1.

The contractor can reduce the amount of time it takes to complete the work. In other words, t_1 is a decision variable. The cost associated with taking t_1 time units is $c(t_1) = \frac{k}{t_1}$. (We could add a fixed cost, but we will leave it out for simplicity.)



Assume that the supplier's cost of capital is r, and it is continuously compounded.

The value of the project to the supplier is $V = P_1 e^{-r(t_1+\tau)} - \frac{k}{t_1}$. We can solve for the optimal t_1 from the contractor's perspective. The optimal t_1, t_1^*, is a function of (P_1, r, τ, k). We are interested in how τ affects t_1^*.

Note that the cross-partial of V, with respect to (t_1, τ) is $e^{(-r(t_1+\tau))} P_1 r^2$. This is positive. In other words, V is super-modular (has increasing differences) in (t_1, τ). This means that t_1 and τ are strategic complements and this implies that if τ decreases then t_1 will decrease and, also, the t_1^* will decrease.

The simple model illustrates a clear relationship between payment delay and optimal project (work) length. The relationship comes directly from the fact that, for the contractor, project length and payment delay are strategic complements. Reducing payment delay makes it optimal to finish the project faster, because the marginal benefit of doing so increases as payment delay decreases.

The simple model shows a one-directional relationship between payment delay and project length. However, we want to capture a richer interaction. In particular, we want to capture the idea that payment delays -- by inducing the contractor to speed up work on the project -- may have a negative effect on project length. This negative effect is a result of poor quality work, which may result in rework. We try to capture this argument in the next model.

Theory 2: Less simple two stage model with rework

Now consider a two-stage model, but the work is now completed in two stages: an "initial" stage and "wrap-up" stage. The project starts at time t_0 and the "initial" stage work is completed at time t_1. The contractor is paid P_1 for this work after τ time units, at time $t_1 + \tau$. However, in this model, the work is not yet completed. The contractor has some wrap-up work to do, and this takes a minimum of t_2 additional time units. But the work may take longer than t_2 units, and this extra time, $\epsilon(t_1)$, is a decreasing function of t_1. In other words, the faster the contractor finishes the initial work, the more likely it is that it will have more rework. After the completion of the project the buyer pays the contractor for the wrap-up work, and this payment, P_2, is received at time $t_1 + t_2 + \epsilon(t_1) + \tau$.

Assume, for simplicity, that both t_2 and $\epsilon(t_1)$ cannot be affected after time t_1, so the only decision variable that the contractor controls in this model is still t_1. Also, for the purposes of analysis, assume $\epsilon(t_1) = \frac{z}{t_1}$. The figure below illustrates this model.

The value of the project to the supplier is $V = P_1 e^{-r(t_1+\tau)} - \frac{k}{t_1} + P_2 e^{-r(t_1+t_2+\frac{z}{t_1}+\tau)}$. We can again solve for the optimal t_1 from the contractor's perspective. The optimal t_1, t_1^*, is a function of $(P_1, r, \tau, k, t_2, z)$. We are interested in how τ affects t_1^*.

The cross-partial of V, with respect to (t_1, τ) is $e^{(-r(t_1+\tau))} P_1 r^2 + e^{(-r(t_1+t_2+\tau+\frac{z}{t_1}))} P_2 r^2 (1 - \frac{z}{t_1^2})$. This is no longer positive. It depends on the value of z. Specifically, for low values of z, the cross-partial is positive and t_1 and τ are strategic complements (as in the simple model above). But for high enough values of z, the sign of the cross-partial switches and becomes negative. In this case, t_1 and τ are strategic substitutes. This implies that, for high enough value of z, as τ decreases then t_1^* will increase.

To understand the intuition behind this, consider what happens when z increases. As z increases, the amount of rework (for any given initial project length) increases. If τ is high, the contractor optimally chooses a t_1 to maximize the total value of the project. If $z = 0$, i.e. there is no rework, then when the buyer reduces τ it is optimal for the contractor to reduce t_1. This follows from the strategic complements argument of the simple model.

But if z is high, then there is a different argument. For any t_1, the contractor has to consider the amount of rework and the impact on the delayed payment of P_2. Balancing off all the considerations, the contractor chooses an optimal t_1. If the buyer now reduces τ, the buyer will get paid earlier (holding all else constant). The contractor may now (for high enough z) find it optimal to increase t_1, getting paid P_1 a little later, but getting paid P_2 earlier because of lower rework.

Note that the other parameters like P_1 and P_2 also affect the optimal decisions, and the impact of τ. For example, for any z, a higher P_2 (relative to P_1) would increase the range of values of z where the cross-partial is negative.

Finally, note that in this model, there is no moral hazard. The contractor's decision to increase or decrease t_1 is the outcome of a trade-off between more or less re-work -- which is affected by various parameters including how fast the contractor is paid.

Theory 3: One stage model with a liability constraint

The two-stage model above was necessary to show that early payment could increase or decrease project length. The key was impact of project length on rework. If finishing the contractor finished the initial stage of the work too quickly, then this could have a negative affect on total project length by adding rework to the second "wrap-up" stage of the project.

There is a certain intuitive logic to this model. But it may be hard to empirically test this theory. How do we know which projects are more susceptible to rework if the initial stage of the project is expedited? It may be possible that some types of work require more care and diligence to avoid rework but, in general, this theory may be hard to test empirically.

So is there another factor that we can consider? Another model that has some intuitive logic is one where we can create a relationship between early payment and the financial strength of contractors. Arguably, the impact of early payment on financially constrained contractors would be different when compared to the impact on financially secure contractors.

To model this relationship, consider the simple (one-stage) model presented above and add the following financial constraint. A contractor is working on a government project incurs certain financial liabilities on this project (e.g. payroll) that are due by a certain time, t_L. Ideally, the firm would receive payment from the project in order to meet this liability. However, this may not be possible since the time to complete the project and receive payment may exceed t_L.

Assume, further, that a firm can delay paying the liability to a time αt_L, where $\alpha \geq 1$. The value of α can be interpreted as being related to the financial health of the contractor. For example, a financially unconstrained contractor can raise external funds to meet this liability and can therefore delay the time by which it needs to be paid to αt_L where $\alpha > 1$. A financially constrained contractor, on the other hand, would (by definition) not be able to raise funds easily to meet this financial commitment and has to rely on receiving payment from the government contract in order to meet its financial liability. For a completely financially constrained contractor, $\alpha = 1$.

To the model in Theory 1 (above), we now add the financial constraint $t_1 + \tau \leq \alpha t_L$. In other words, the constrained optimization problem in Theory 3 is $\max_{t_1} V(t_1, \tau) = P_1 e^{-r(t_1+\tau)} - \frac{k}{t_1}$, subject to the constraint $t_1 + \tau \leq \alpha t_L$.

Note that if α is sufficiently high, this constraint will not be binding and the optimal t_1 would be the optimal project completion time as defined in Theory 1. For clarity, we refer to this value as t_1^U where the U denotes that this is the unconstrained optimal project duration. However, if α is low enough, the constraint will be binding, i.e. $t_1 + \tau = \alpha t_L$, and it may not be feasible to achieve the optimal $t_1 = t_1^U$. In this case, the constrained optimal project duration will be $t_1^C = \alpha t_L - \tau$.

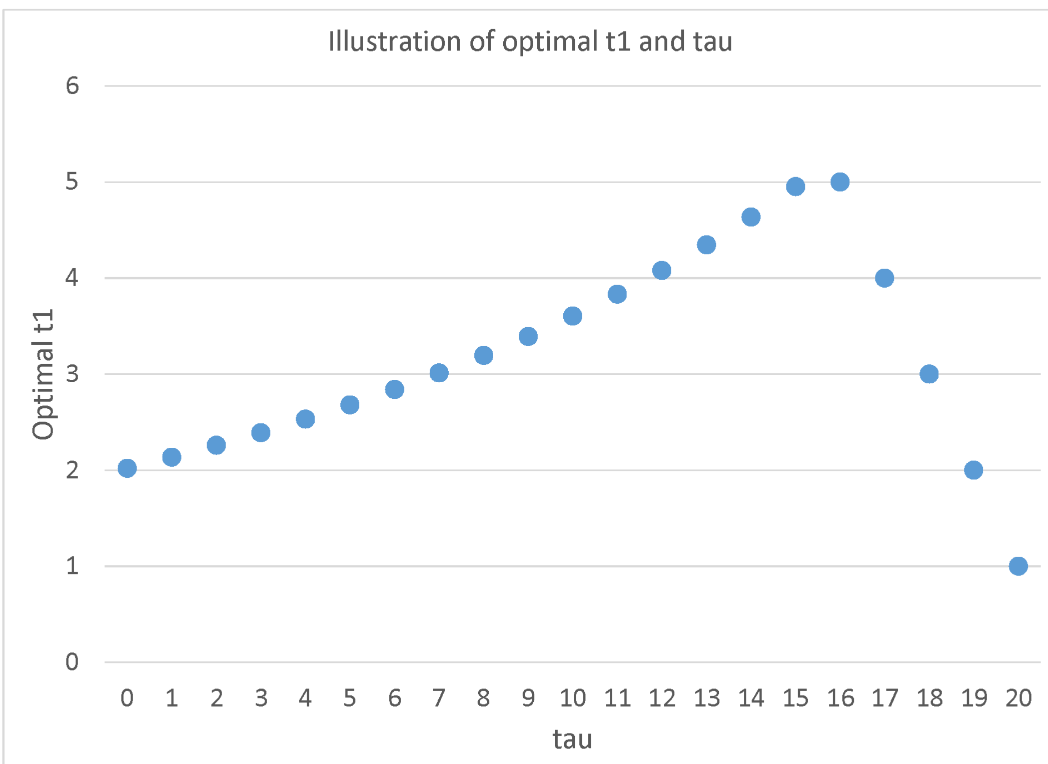
Some additional restrictions

1. Note that in Theory 1, we had implicitly assumed that $t_1 > 0$. Of course, the project has to have a strictly non-negative duration. In this model, to enforce this condition, we need the assumption that $\tau < \alpha t_L$. Otherwise the constraint will never be satisfied and the problem would be infeasible. To ensure that we meet this constraint for all α, we simply assume that $\tau < t_L$.
2. Furthermore, we assume that, $t_L > t_1^U$ when $\tau = 0$. In other words, the (earliest) time that the liability payment is due is greater than the unconstrained optimal project duration when the buyer pays immediately ($\tau = 0$). If this were not true, then even if $\tau = 0$ (the payment is made immediately) the financially constrained contractor will always face a binding financial constraint. We want a model where changing τ will determine whether this constraint binds. For

a high τ (long payment delay), the constraint should bind. But for a very short payment delay (τ close to 0), this constraint should not bind.

Under these assumptions, for the financially unconstrained contractor (say $\alpha >> 1$), the optimal project duration is always t_1^U. As in Theory 1, for these contractors, payment time and project duration are strategic complements, i.e. $t_1^U(\tau)$ is increasing in τ. If the buyer increases payment time, the buyer will complete the project later. And, conversely, if the buyer reduces payment time, the contractor will finish the project sooner. For financially constrained contractors, for whom the liability constraint is binding, recall that the constrained optimum $t_1^C = \alpha t_L - \tau$. In other words, as τ decreases, the project duration t_1^C increases.

The graph below provides one illustration of the above model. In this graph, the following parameters are chosen: $r = 0.1$, $P_1 = 30$ and $k = 10$. Also, $t_L = 21$ and $\alpha = 1$. Then, τ is varied between 0 and 20, and the optimal value of t_1 is calculated. Note that when τ is low, the liability constraint is not binding and the optimal value of t_1 is the unconstrained value. This value rises as τ rises. However, when τ gets to 16, the liability constraint starts binding. Any further increases in τ need to be compensated with a decrease in t_1.



The hypothesis that comes out of this is:

- For financially unconstrained firms, Quickpay will decrease project length.
- For financially constrained firms, Quickpay will increase project length.

This is what we seem to be seeing in the data.