7. Exploring and Reporting

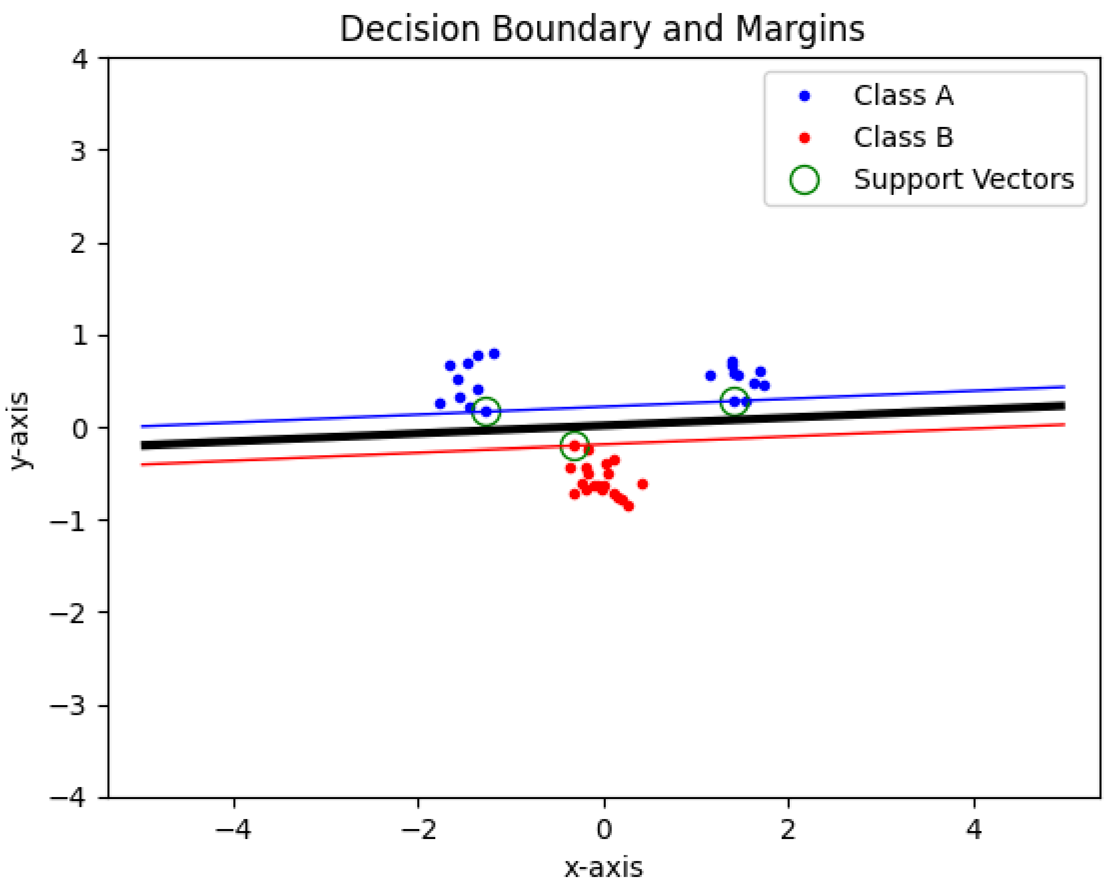
7.1

1. Move the clusters around and change their sizes to make it easier or harder for the classifier to find a decent boundary. Pay attention to when the optimizer (minimize function) is not able to find a solution at all

In [9]:
```
from IPython.display import Image, display
```

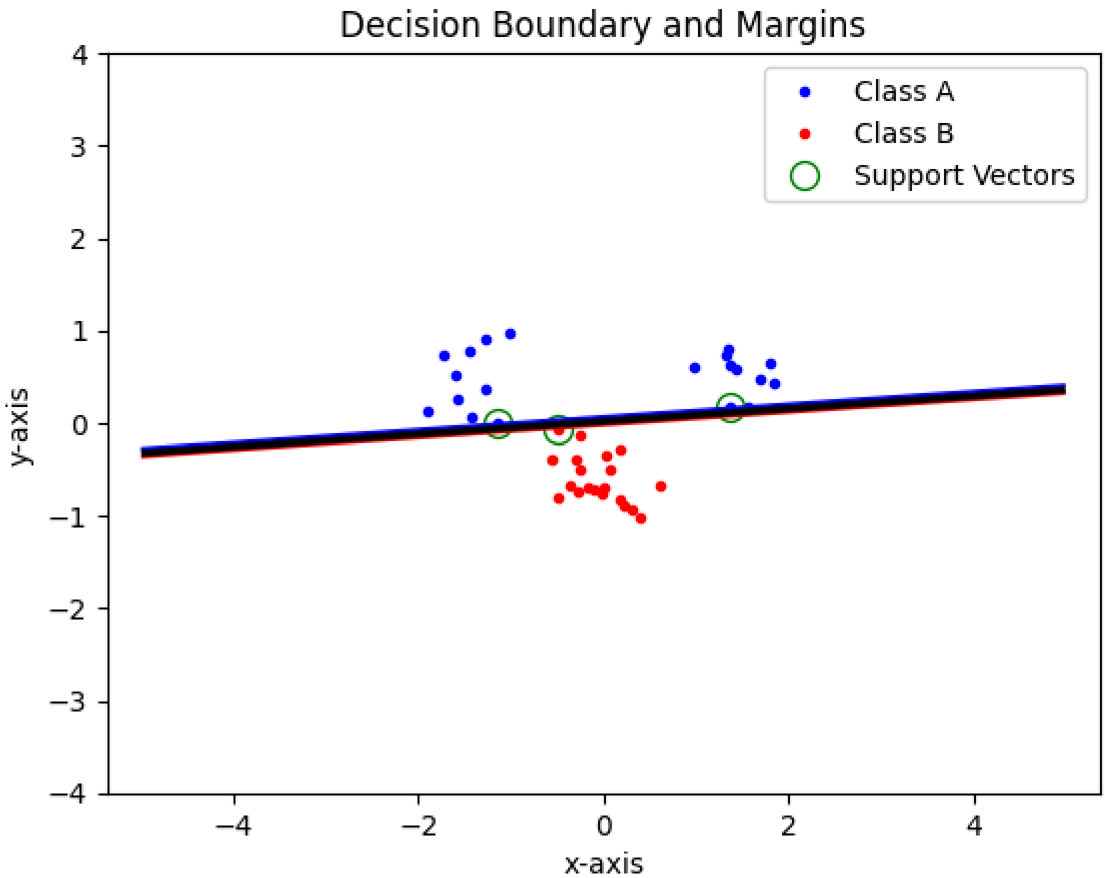
Here is the output when std = 0.2, C = None, using linear kernel function

In [10]:
```
display(Image('src/7_1_linear_origin.png'))
```



Here is the output when std = 0.3, C = None, using linear kernel function, which is the edge case where the linear kernel could find the hard boundary without slack.

In [11]:
```
display(Image('src/7_1_linear_0.3.png'))
```

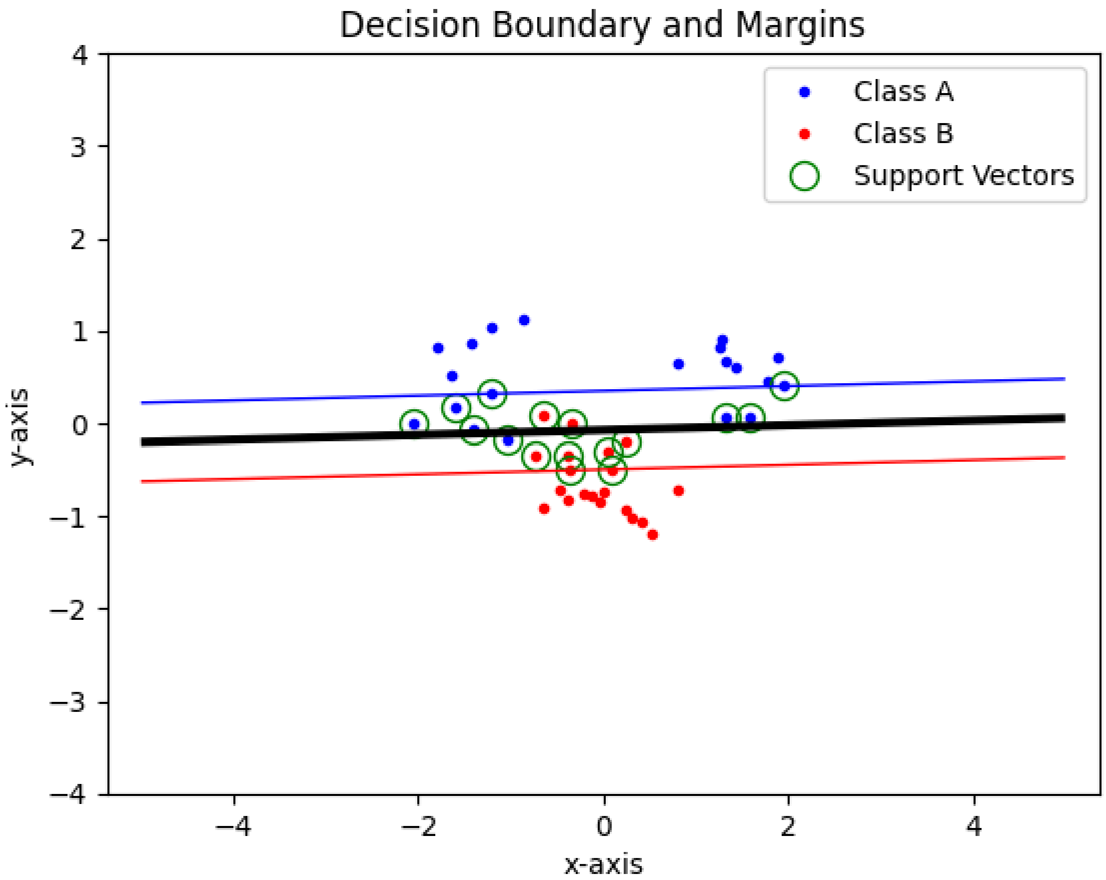


Increasing std = 0.4, we can see that the optimizer can not find the solution. The error message is attached as follows:

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[124], line 8
      6 # The string 'success' instead holds a boolean representing if the optimizer has found a solution
      7 if (not success):
----> 8     raise ValueError(f'Cannot find optimizing solution when std is {std} and C is {C}')
      9 print(f"Optimization successful: {success} when std is {std} and C is {C}")

ValueError: Cannot find optimizing solution when std is 0.4 and C is None
```

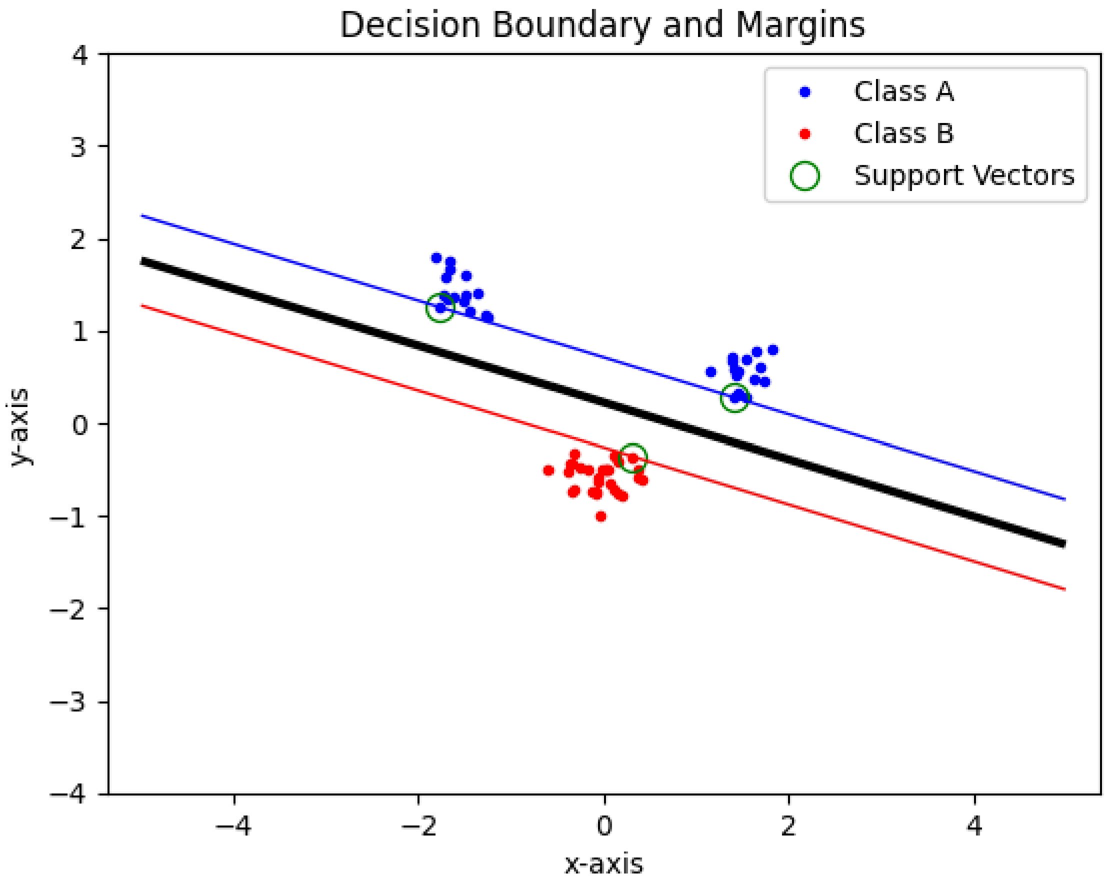
Let 'C = 1', we get the following result:

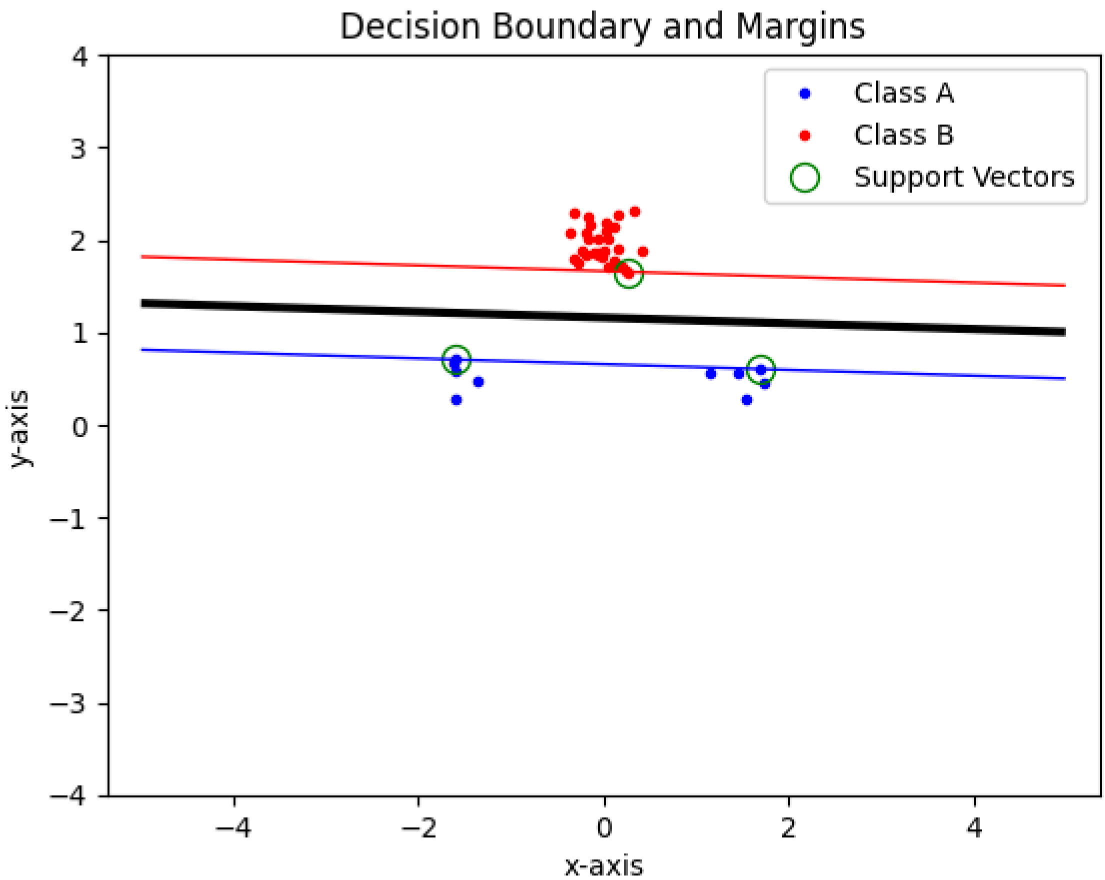
In [12]:
```
display(Image('src/7_1_linear_0.4_C_0.1.png'))
```



We changed the parameters when generating the data, here are examples when we tried to change the center of data sets.

In [13]:
```
display(Image('src/7_1_linear_change_size_move_centerA.png'))
display(Image('src/7_1_linear_change_size_move_centerB.png'))
```





7.2

- Implement the two non-linear kernels. You should be able to clas sify very hard data sets with these.

```python
def poly_kernel(x, y, p=2):
    return (np.dot(x, y) + 1.0)**p

def rbf_kernel(x, y, sigma=3):
    # The parameter is used to control the smoothness of the boundary
    diff = x - y
    return np.exp(-np.dot(diff, diff) / (2*sigma**2))
```

- Using the original datasets, we will display the results together in 7.3.

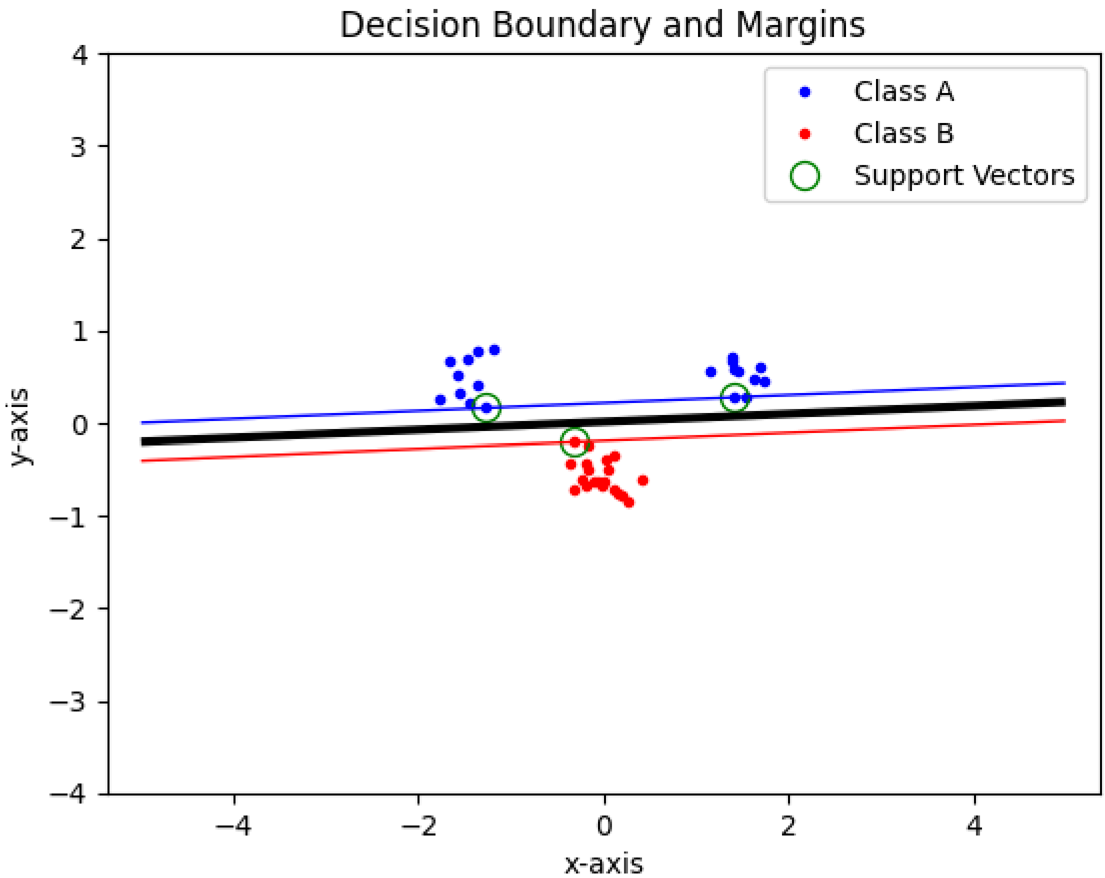
7.3

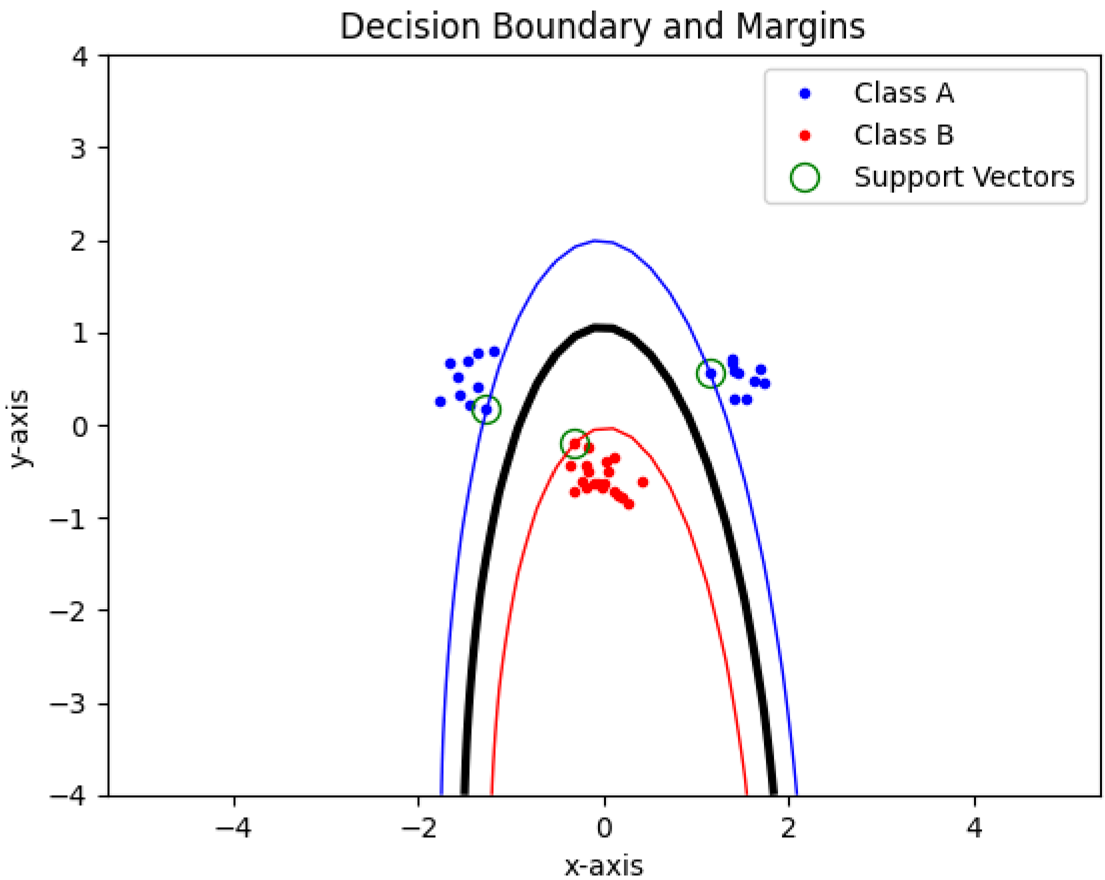
- Low Complexity (High Bias, Low Variance):
    - This occurs with simple kernels (like linear) or when non-linear kernel parameters are set to make the boundary smoother (e.g., a large σ in the RBF kernel, or a low degree p in the polynomial kernel).
    - The model is too simple (high bias), potentially underfitting the training data, but it generalizes well to new data (low variance).
- High Complexity (Low Bias, High Variance):
    - This occurs when the parameters allow the boundary to become very complex and curvy (e.g., a small σ in the RBF kernel, or a high degree p in the polynomial kernel).
    - The model is flexible enough to perfectly fit the training data (low bias), but it becomes overly specialized to the noise in the training set, leading to poor generalization on new data (high variance).
- We will set Slack parameter C as none, and the results are as follows:

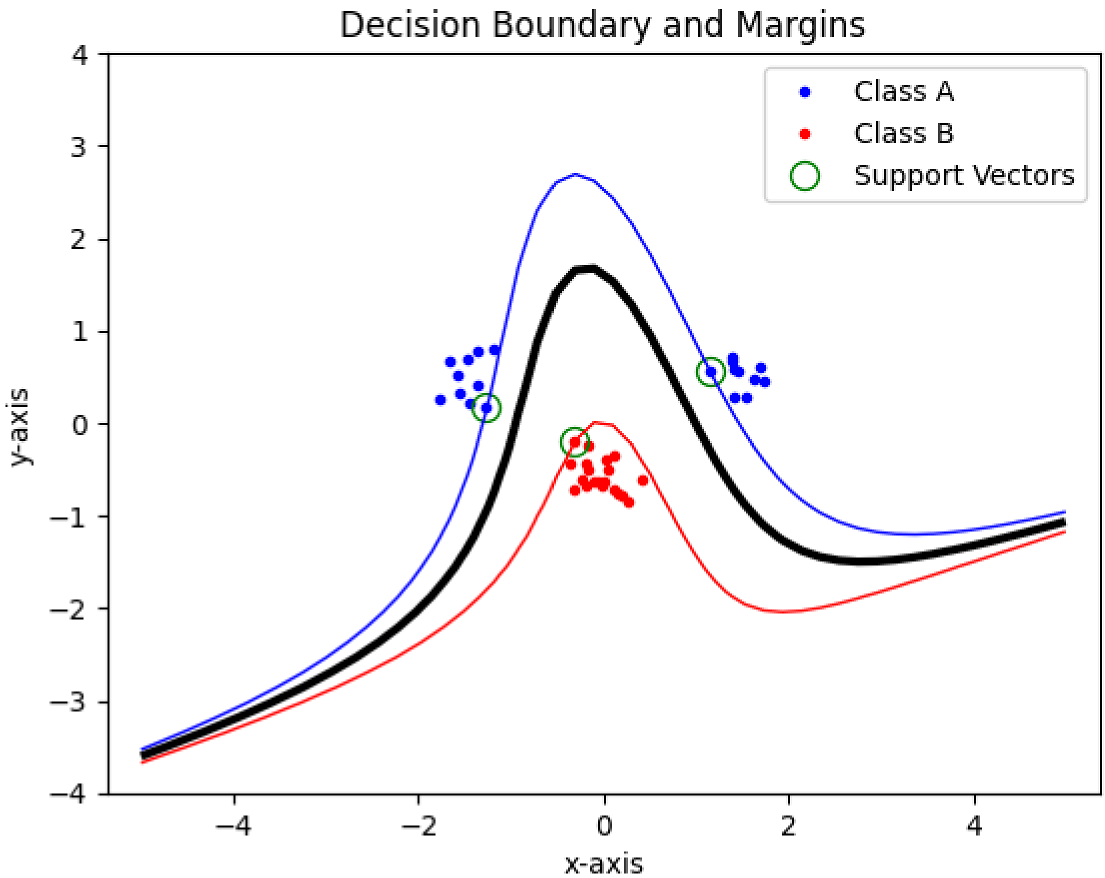
7.3.1 Poly kernel

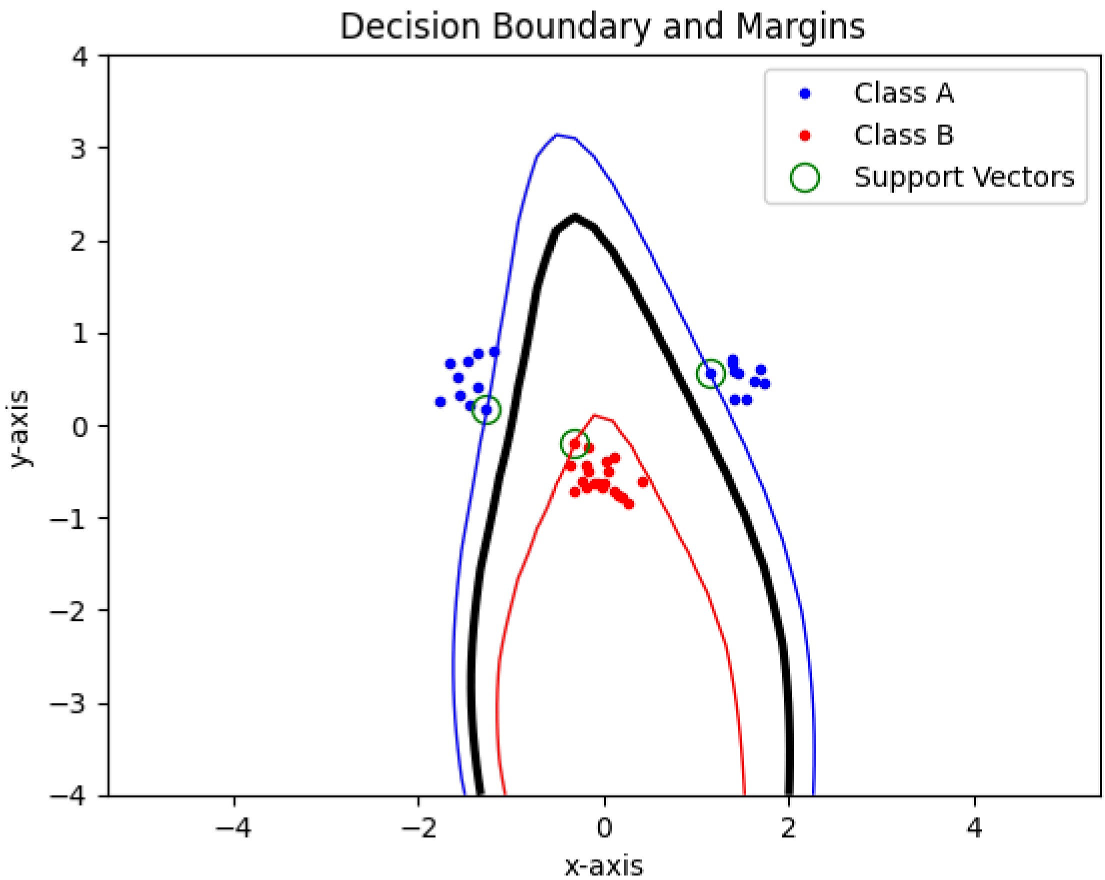
- High p: complex decision boundary -> overfitting -> high variance & low bias
- Low p: smooth decision boundary -> underfitting -> low variance & high bias

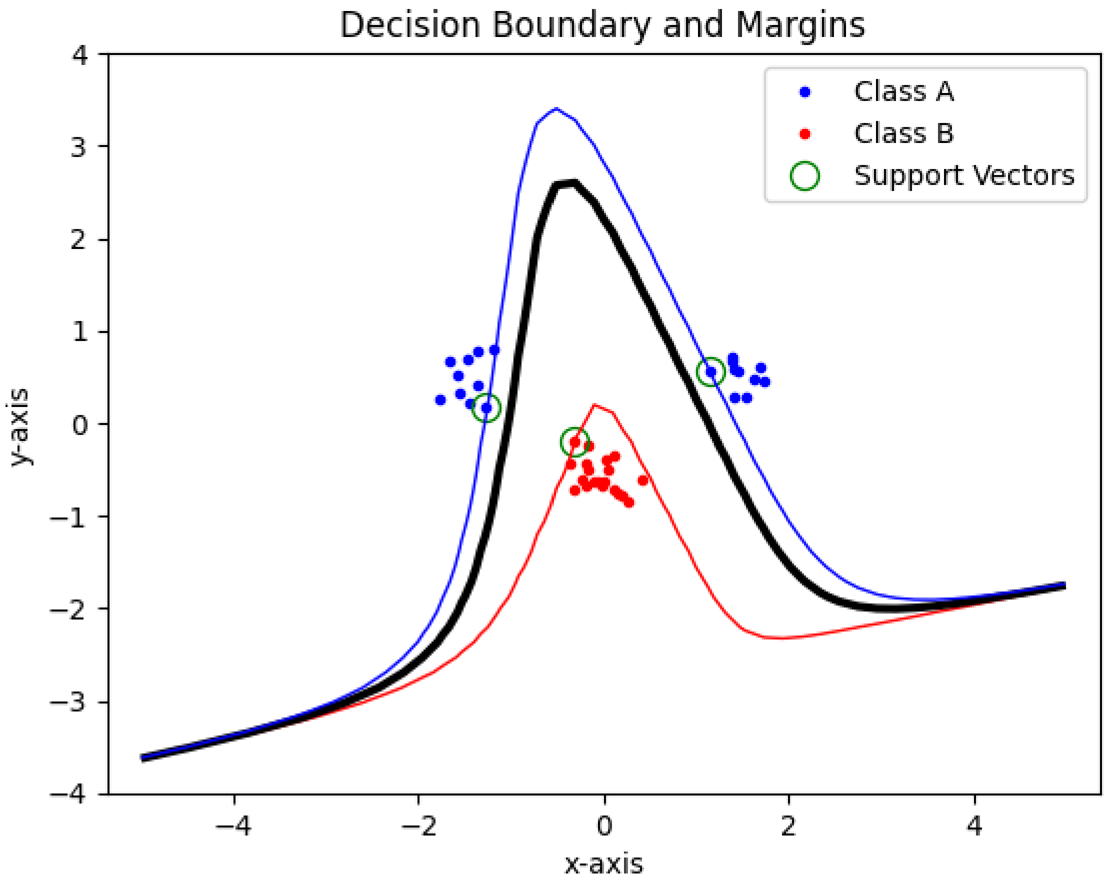
In [14]:
```python
display(Image('src/7_3_poly_1.png'))
display(Image('src/7_3_poly_2.png'))
display(Image('src/7_3_poly_3.png'))
display(Image('src/7_3_poly_4.png'))
display(Image('src/7_3_poly_5.png'))
```


Decision Boundary and Margins
Class A
Class B
Support Vectors
y-axis
x-axis
4
3
2
1
0
−1
−2
−3
−4
−4
−2
0
2
4


Decision Boundary and Margins
Class A
Class B
Support Vectors
y-axis
x-axis
4
3
2
1
0
−1
−2
−3
−4
−4
−2
0
2
4


Decision Boundary and Margins
Class A
Class B
Support Vectors
y-axis
x-axis
4
3
2
1
0
−1
−2
−3
−4
−4
−2
0
2
4

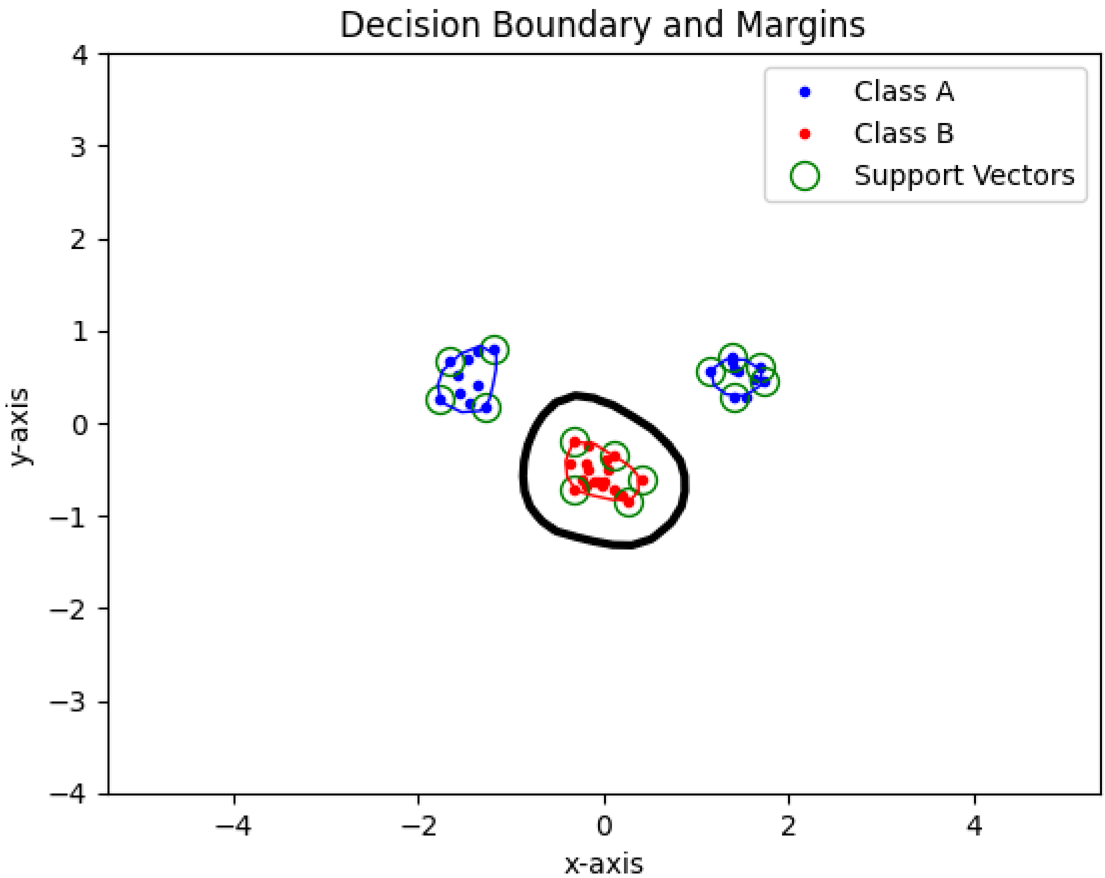

Decision Boundary and Margins
Class A
Class B
Support Vectors
y-axis
x-axis
4
3
2
1
0
−1
−2
−3
−4
−4
−2
0
2
4

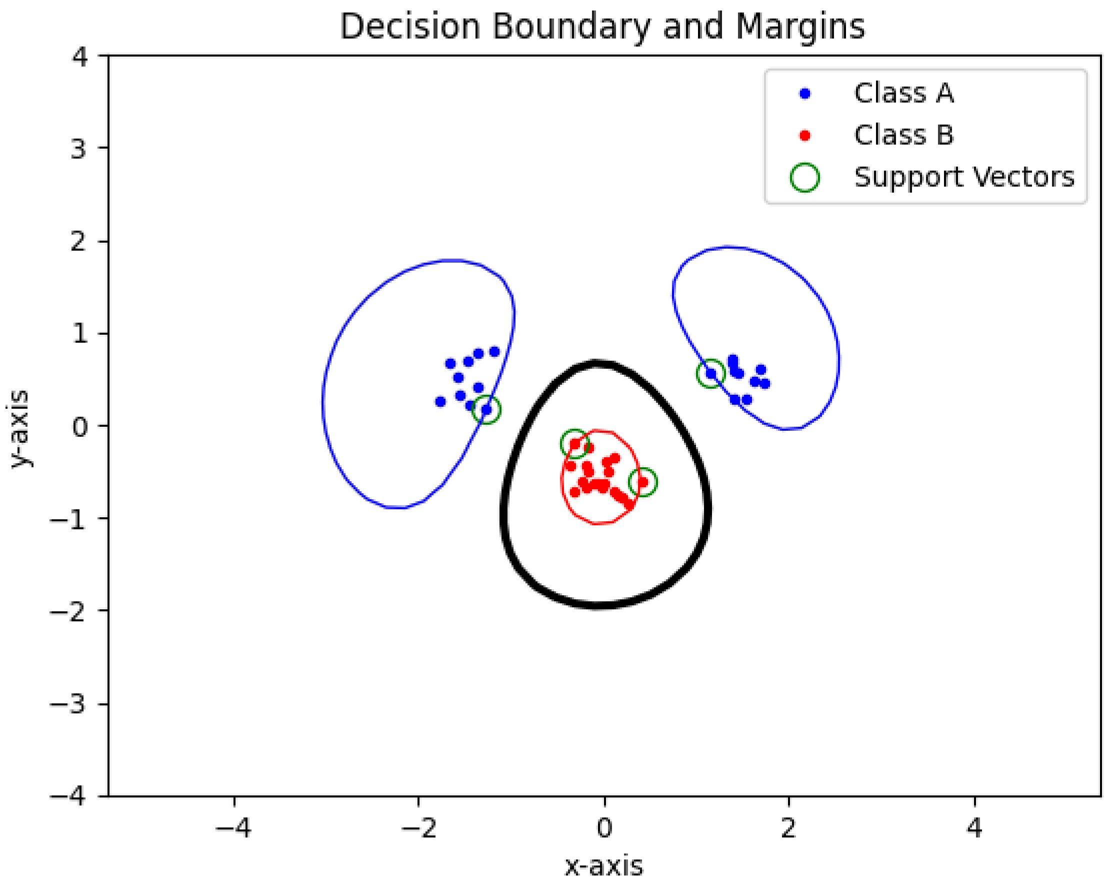


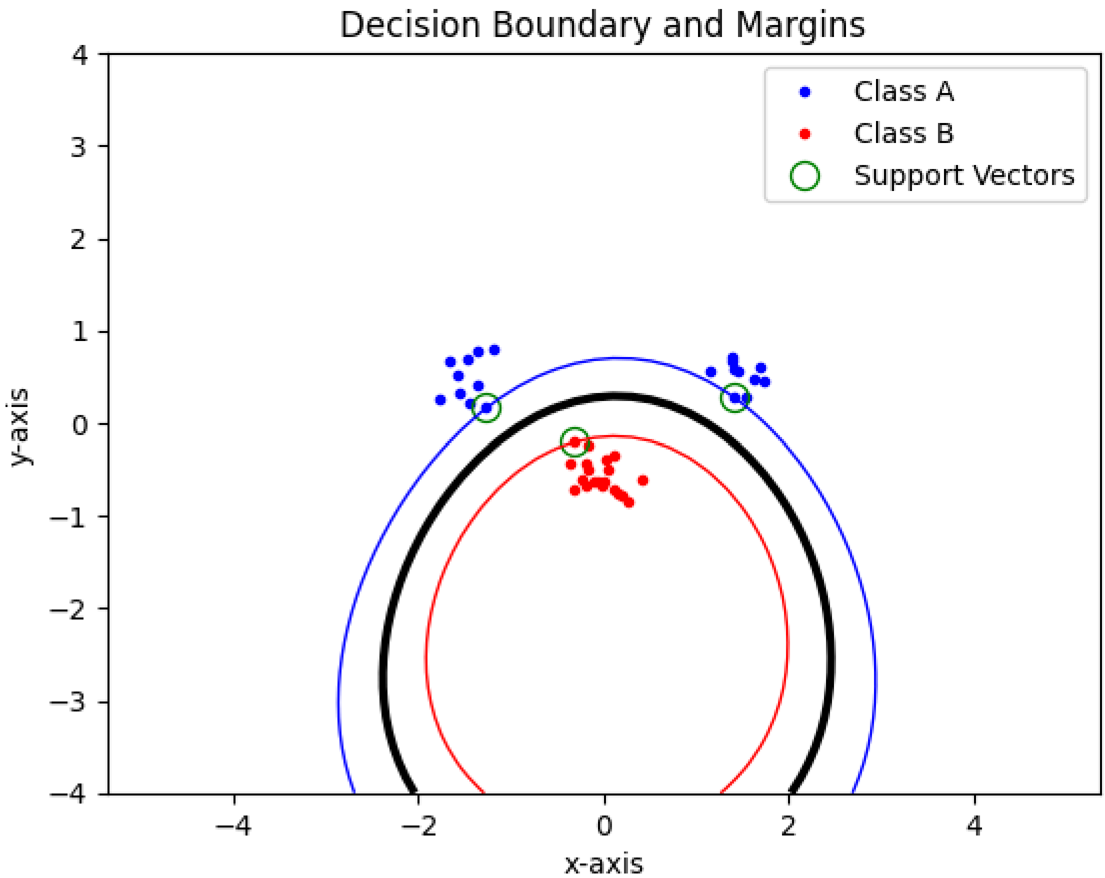
7.3.2 RBF kernel

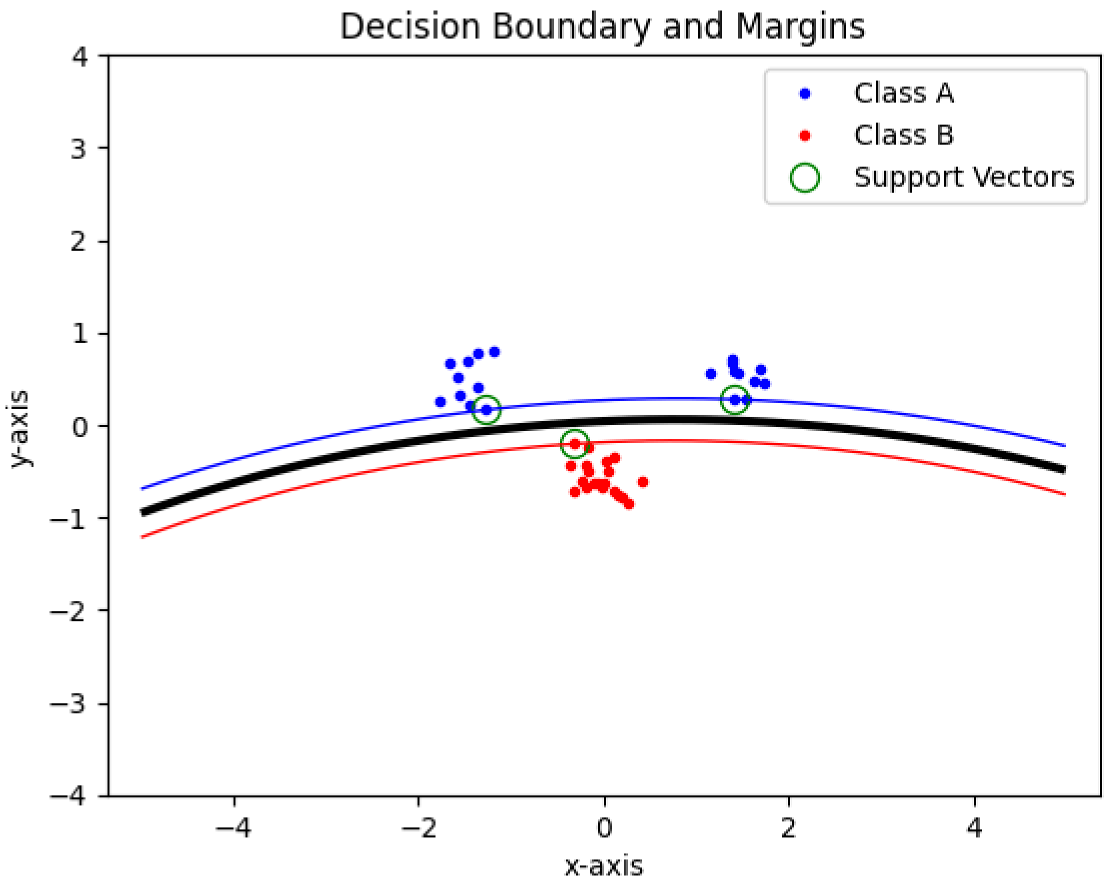
- Low Sigma: complex decision boundary -> overfitting -> high variance & low bias
- High Sigma: smooth decision boundary -> underfitting -> low variance & high bias

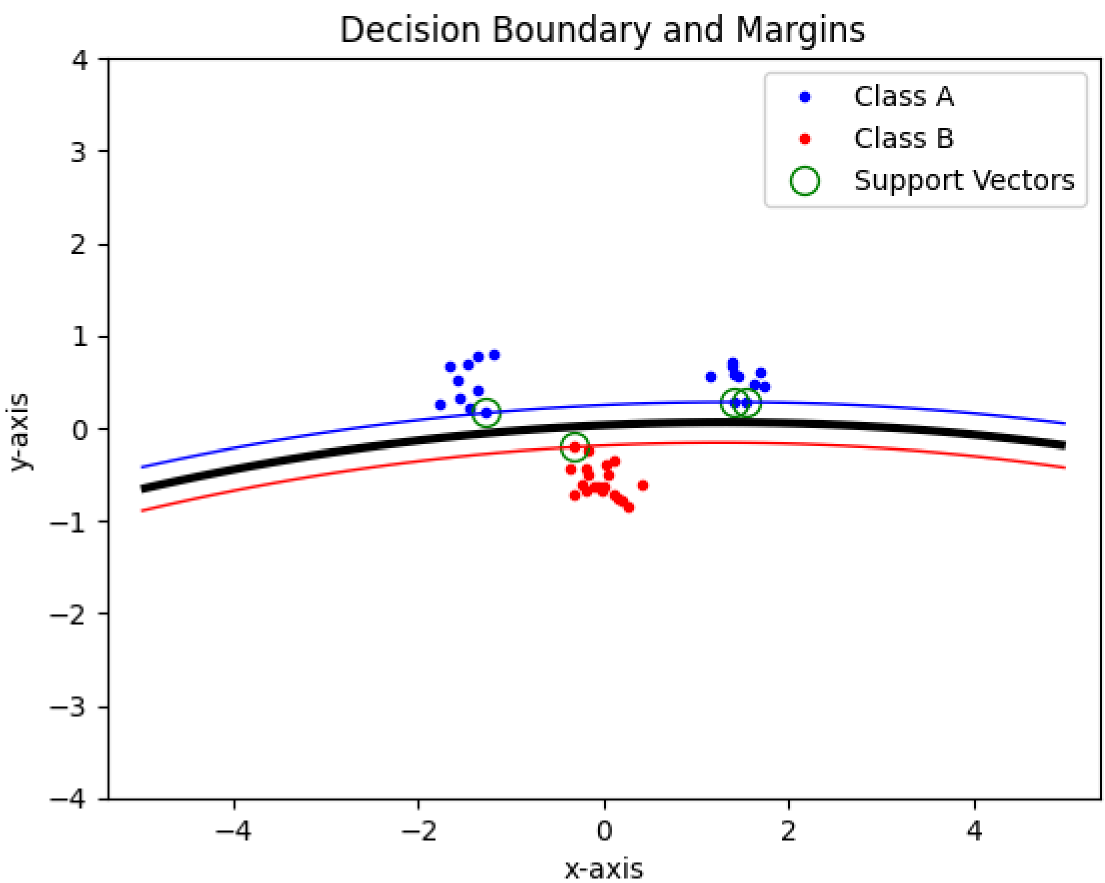
In [15]:
```
display(Image('src/7_3_rbf_0.3.png'))
display(Image('src/7_3_rbf_1.png'))
display(Image('src/7_3_rbf_3.png'))
display(Image('src/7_3_rbf_10.png'))
display(Image('src/7_3_rbf_13.png'))
```


Decision Boundary and Margins
Class A
Class B
Support Vectors
y-axis
x-axis
4
3
2
1
0
−1
−2
−3
−4
−4
−2
0
2
4


Decision Boundary and Margins
Class A
Class B
Support Vectors
y-axis
x-axis
4
3
2
1
0
−1
−2
−3
−4
−4
−2
0
2
4


Decision Boundary and Margins
Class A
Class B
Support Vectors
x-axis
y-axis
-4
-2
0
2
4
-4
-3
-2
-1
0
1
2
3
4


Decision Boundary and Margins
Class A
Class B
Support Vectors
x-axis
y-axis
-4
-2
0
2
4
-4
-3
-2
-1
0
1
2
3
4



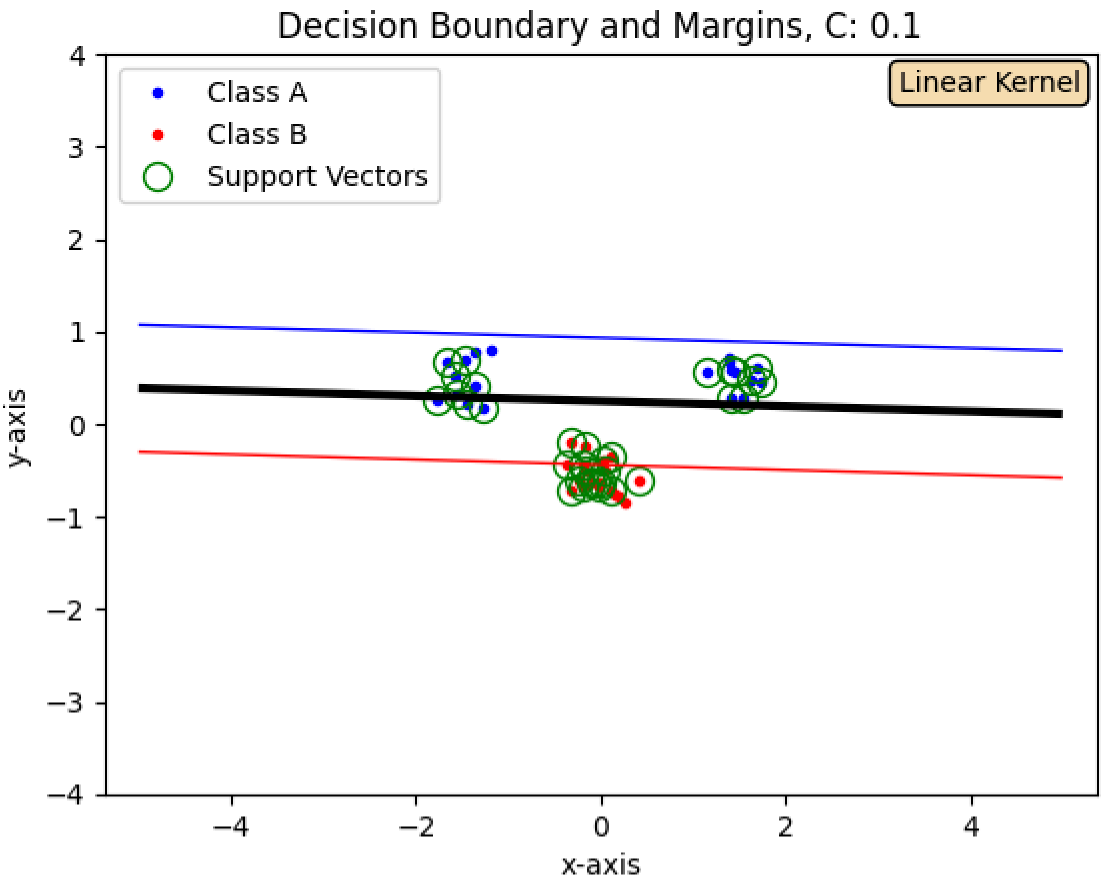
7.4

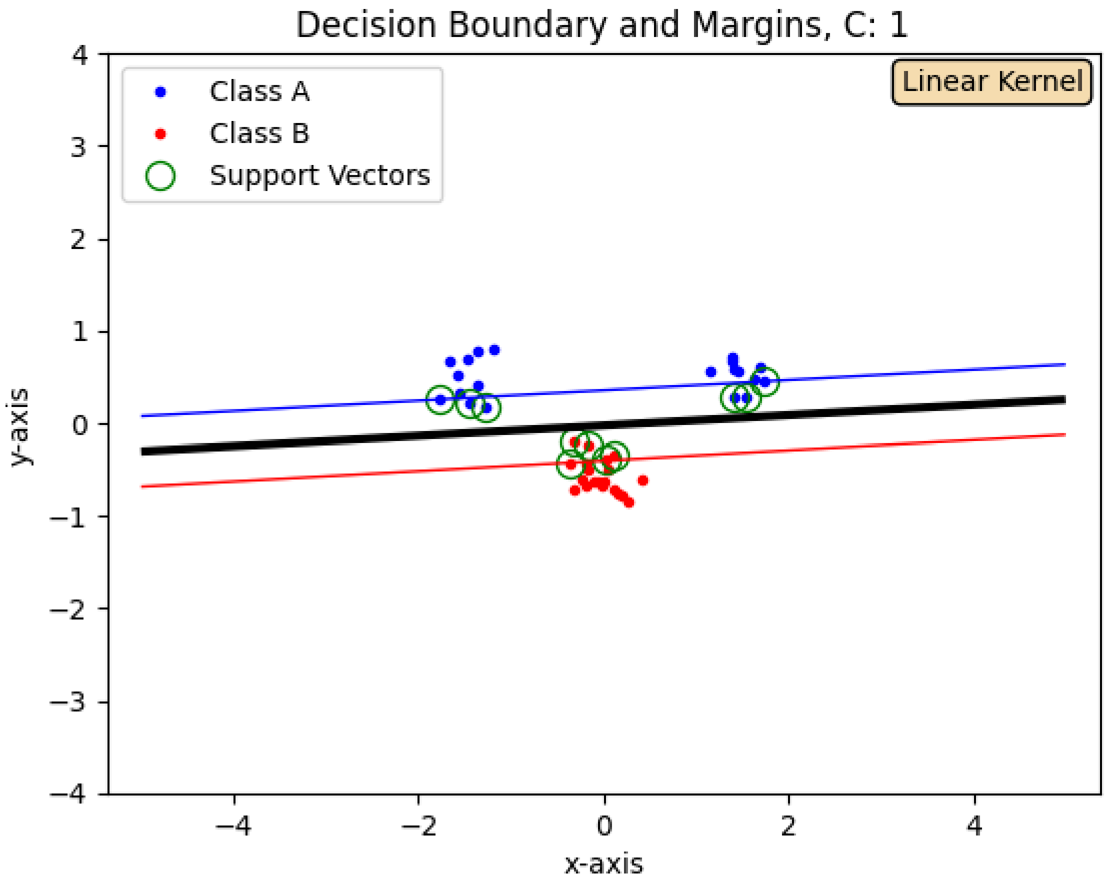
Explore the role of the slack parameter . What happens for very large/small values?

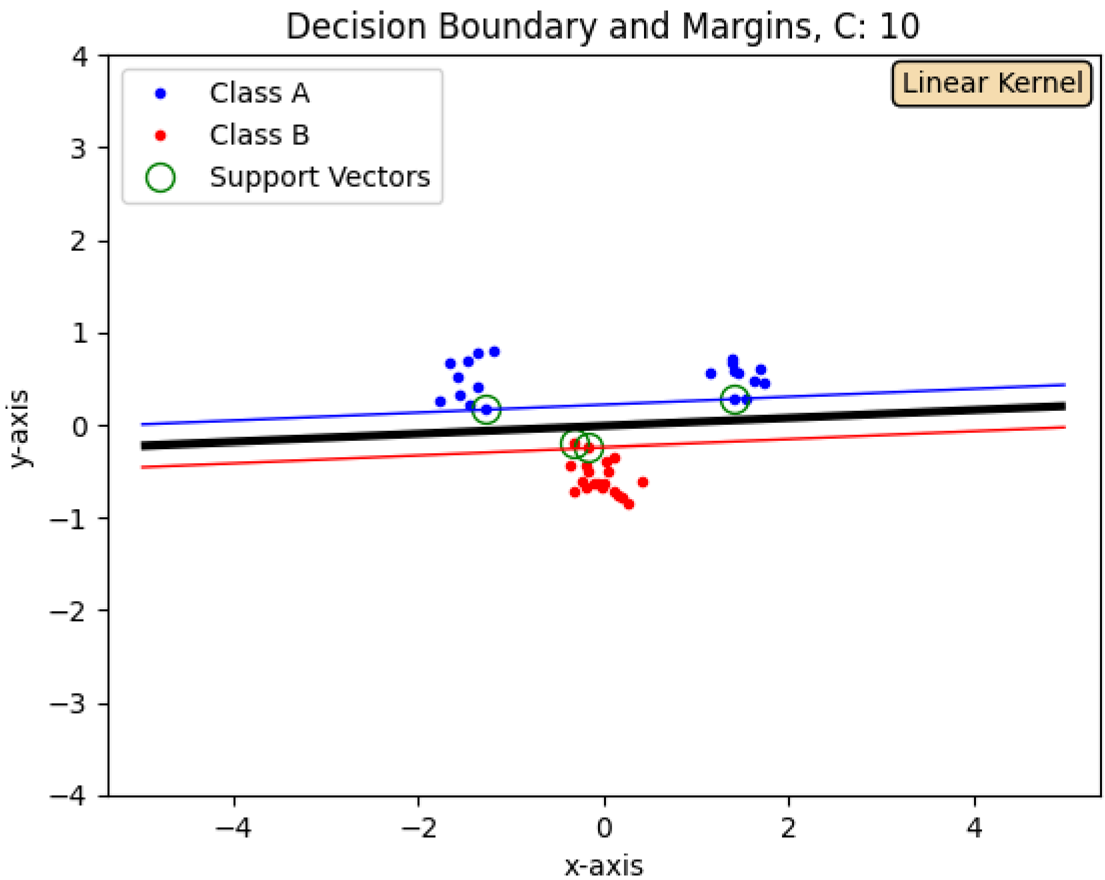
- The slack parameter C in a Support Vector Machine (SVM) controls the tradeoff between **maximizing the margin width** and **minimizing the classification error** on the training data. It is the penalty imposed on margin violations and misclassified points.
- Small C Values: Higher bias, lower variance - underfitting tendency
  - The SVM prioritizes wide margins over correct classification
  - Many training points are allowed to be misclassified or fall within the margin
  - The decision boundary becomes very smooth and simple
  - Observations:
    - Wider margins with many points inside or on the wrong side
    - Fewer support vectors (only those defining the margin)
    - Simple, generalized decision boundaries
- Large C Values: Lower bias, higher variance - overfitting tendency
  - The SVM heavily penalizes any misclassification
  - Tries to classify every training point correctly
  - Creates narrow margins with complex boundaries
  - Observations:
    - Very narrow margins
    - More support vectors (including misclassified points with $\alpha = C$)

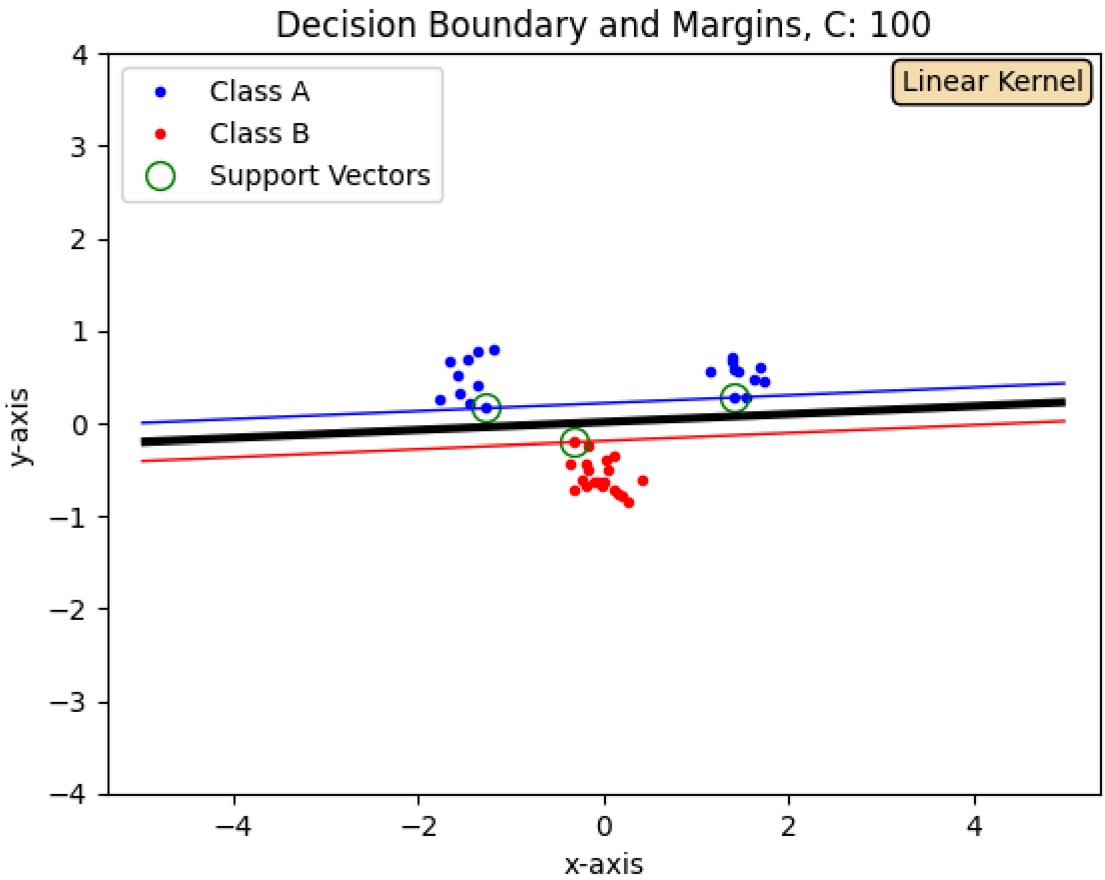
- Complex, wiggly decision boundaries that closely follow training data

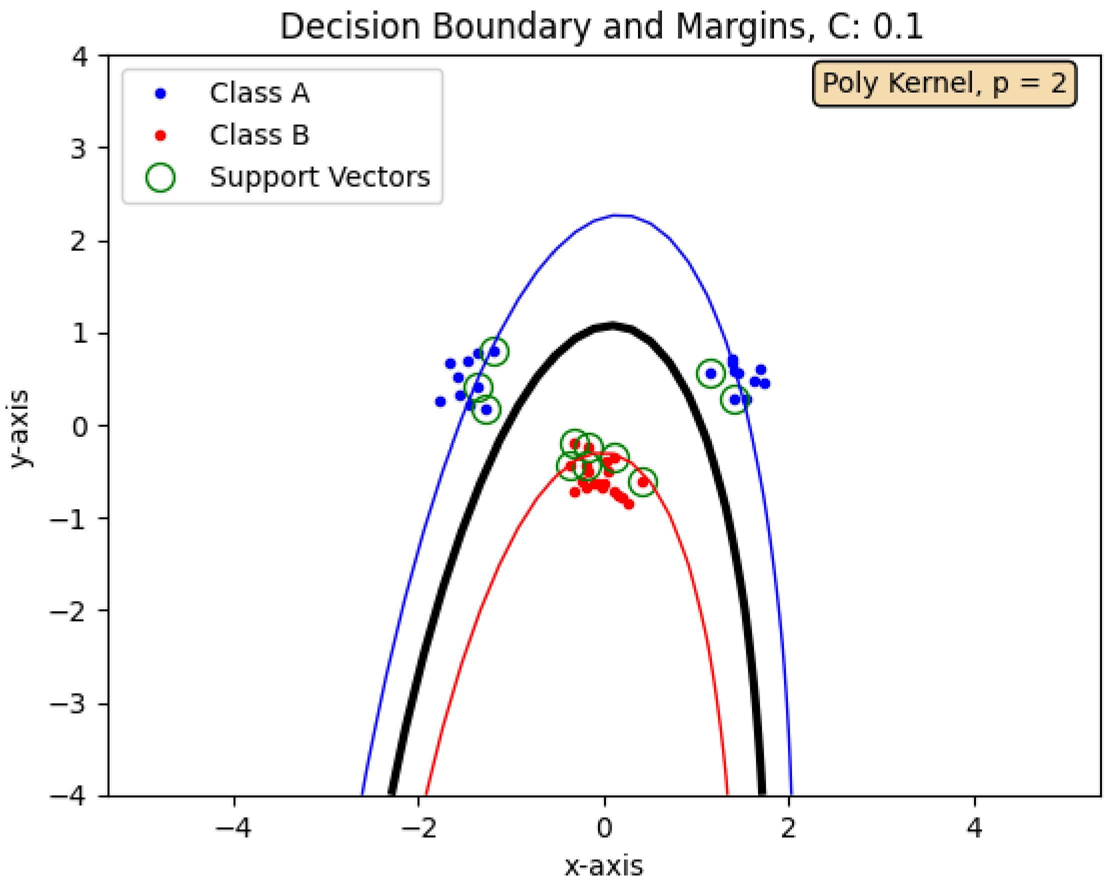
In [16]:
```
display(Image('src/7_4_linear_C_0.1.png'))
display(Image('src/7_4_linear_C_1.png'))
display(Image('src/7_4_linear_C_10.png'))
display(Image('src/7_4_linear_C_100.png'))

display(Image('src/7_4_poly_2_C_0.1.png'))
display(Image('src/7_4_poly_2_C_1.png'))
display(Image('src/7_4_poly_2_C_10.png'))
# display(Image('src/7_4_poly_2_C_100.png'))

# display(Image('src/7_4_rbf_3_C_0.1.png'))
display(Image('src/7_4_rbf_3_C_1.png'))
display(Image('src/7_4_rbf_3_C_10.png'))
display(Image('src/7_4_rbf_3_C_100.png'))
```

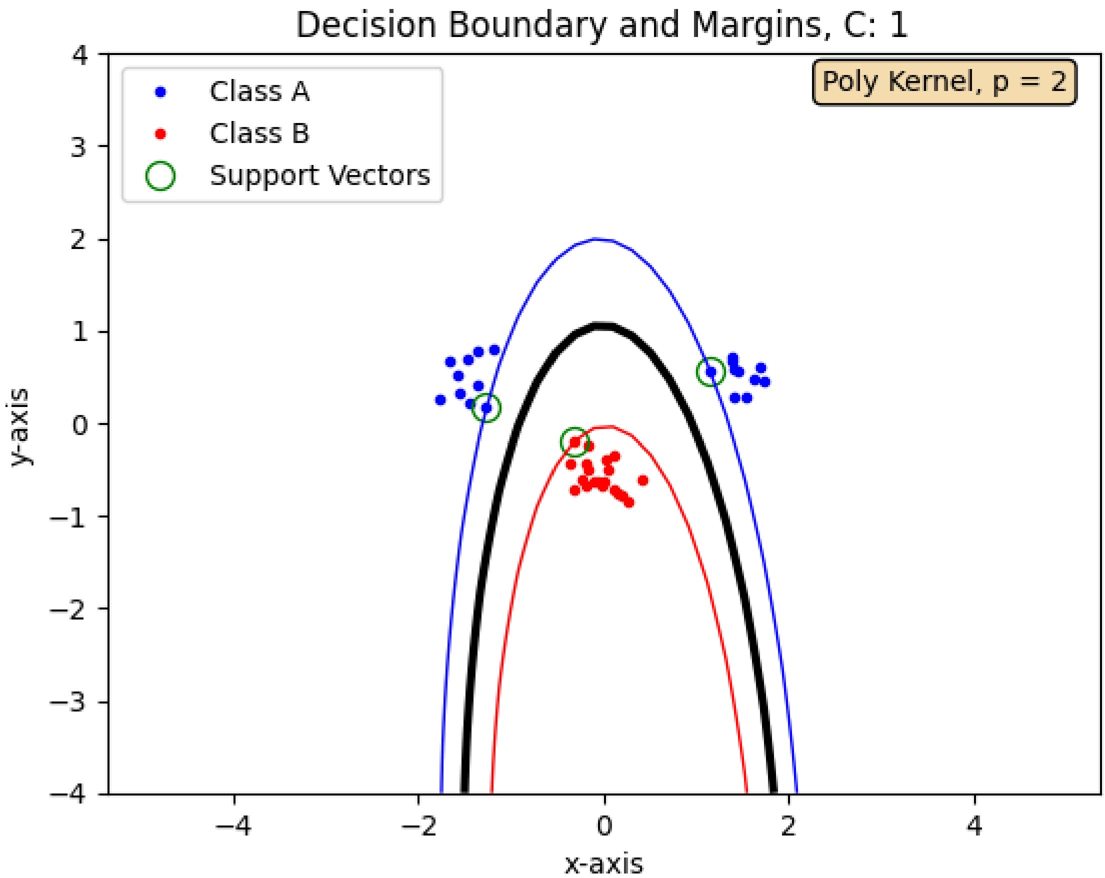


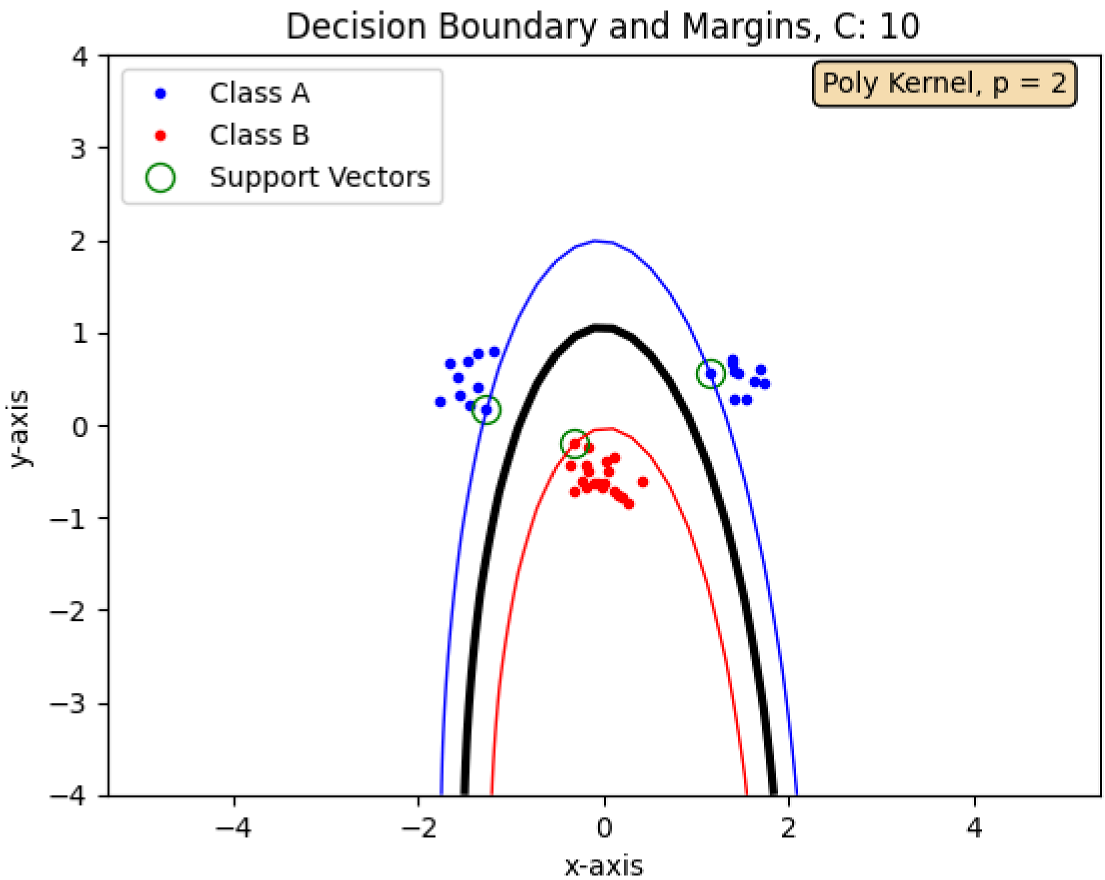

Decision Boundary and Margins, C: 1
Class A
Class B
Support Vectors
Linear Kernel
y-axis
x-axis
4
3
2
1
0
−1
−2
−3
−4
−4
−2
0
2
4

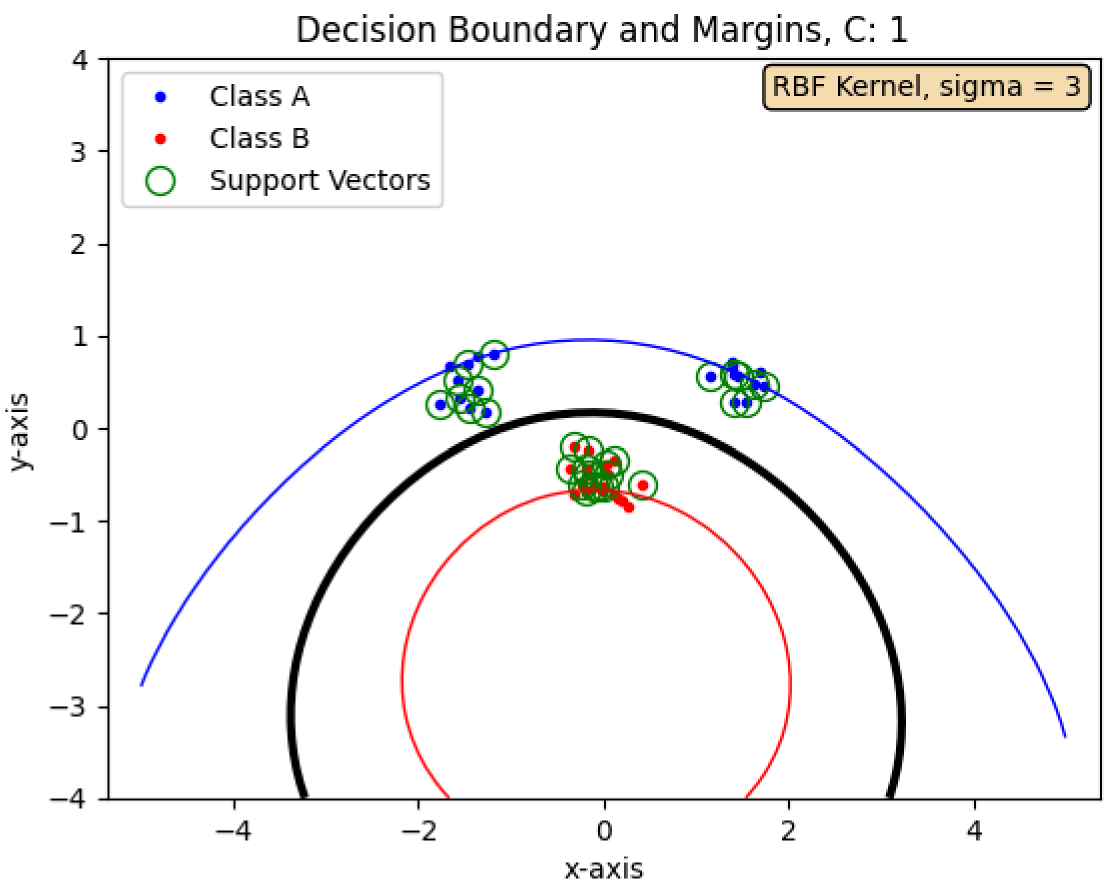

Decision Boundary and Margins, C: 10
Class A
Class B
Support Vectors
Linear Kernel
y-axis
x-axis
4
3
2
1
0
−1
−2
−3
−4
−4
−2
0
2
4

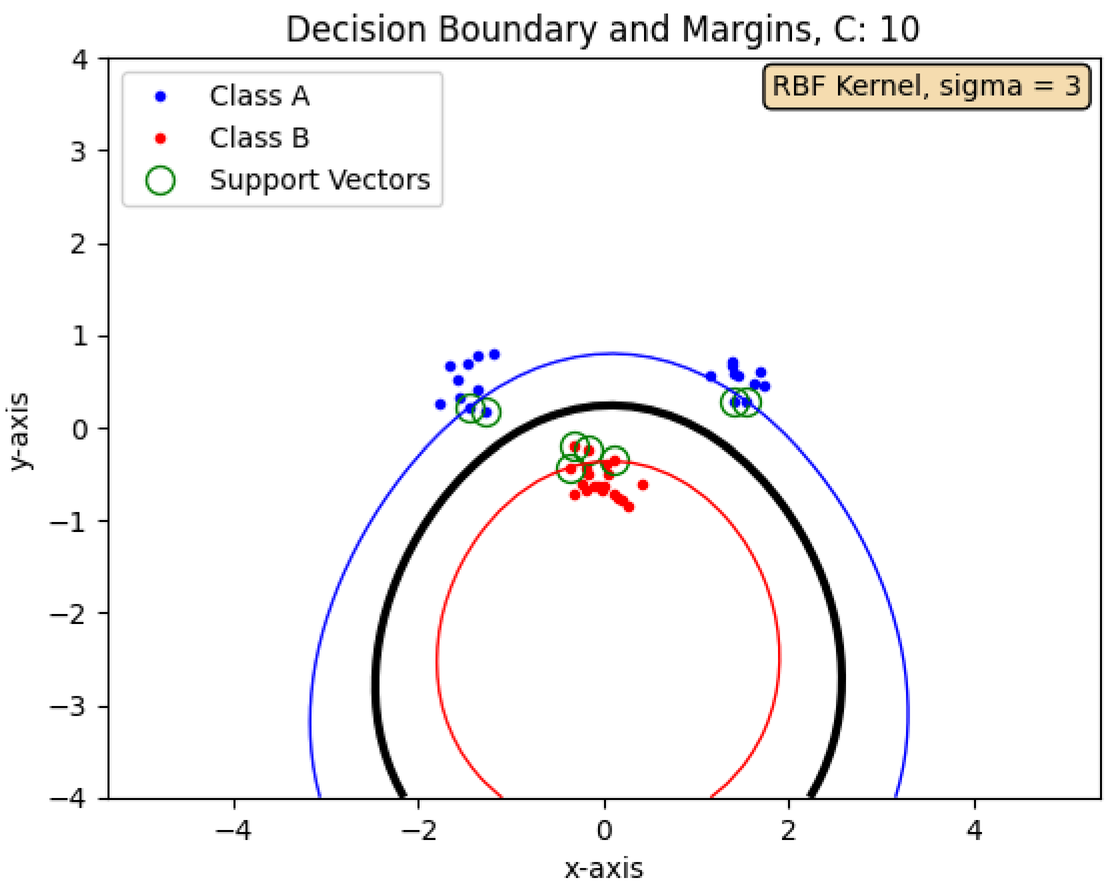

Decision Boundary and Margins, C: 100
Class A
Class B
Support Vectors
Linear Kernel
y-axis
x-axis

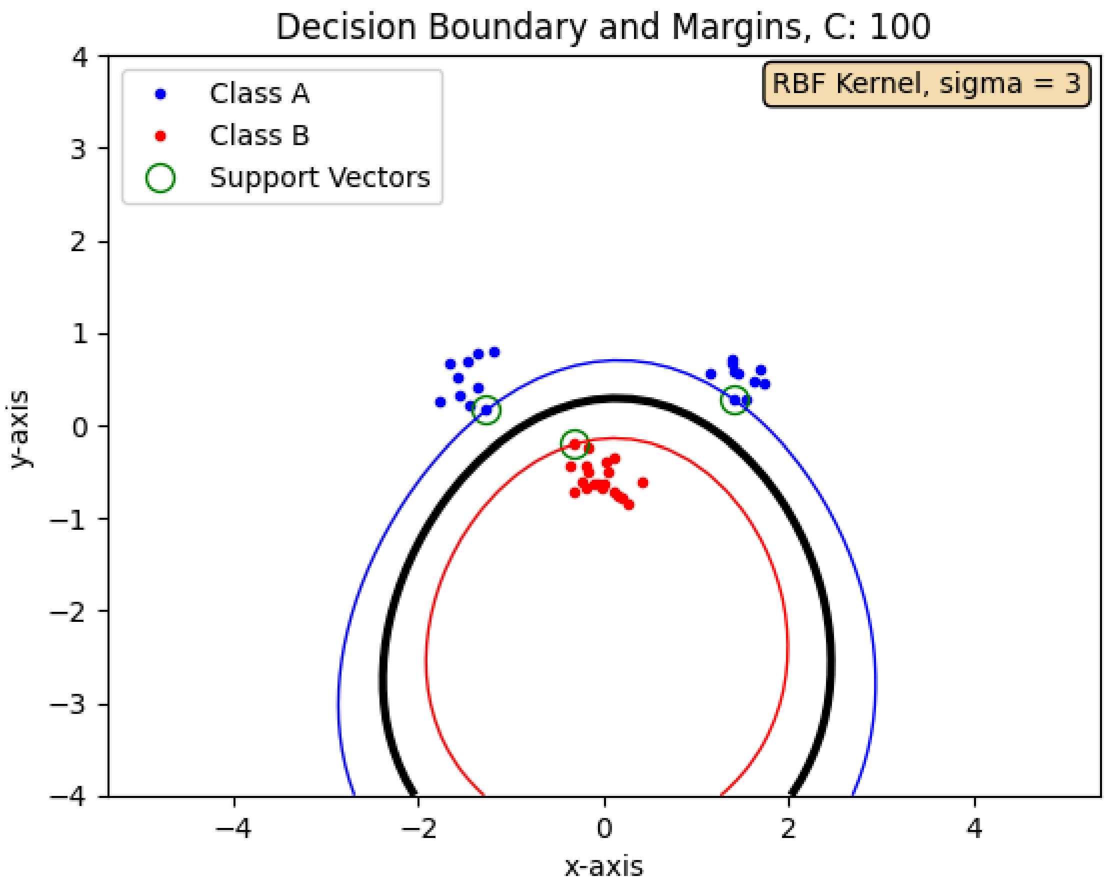

Decision Boundary and Margins, C: 0.1
Class A
Class B
Support Vectors
Poly Kernel, p = 2
y-axis
x-axis


Decision Boundary and Margins, C: 1
Class A
Class B
Support Vectors
Poly Kernel, p = 2
y-axis
x-axis
4
3
2
1
0
−1
−2
−3
−4
−4
−2
0
2
4


Decision Boundary and Margins, C: 10
Class A
Class B
Support Vectors
Poly Kernel, p = 2
y-axis
x-axis
4
3
2
1
0
−1
−2
−3
−4
−4
−2
0
2
4


Decision Boundary and Margins, C: 1
Class A
Class B
Support Vectors
RBF Kernel, sigma = 3
y-axis
x-axis


Decision Boundary and Margins, C: 10
Class A
Class B
Support Vectors
RBF Kernel, sigma = 3
y-axis
x-axis



7.5

Imagine that you are given data that is not easily separable. When should you opt for more slack rather than going for a more complex model (kernel) and vice versa?

- If the data contains noise, outliers, or measurement errors that create a few problematic points in otherwise linearly separable classes, then increasing slack (decreasing C) is the better approach.
  - These outliers shouldn't dictate the entire decision boundary, and allowing the SVM to misclassify a few noisy points will result in a more generalizable model.
  - A complex kernel in this scenario would try to wrap around every outlier, resulting overfitting.
- In contrast, when the non-separability stems from true structural patterns in the data (such as circular, spiral, or XOR-like relationships), a more complex kernel function is required.
  - Slack **can not** help a linear model correctly classify data that follows a ring pattern or other intrinsically non-linear structure.
  - In these cases, the classification difficulty isn't due to noise but rather to the fundamental geometry of the problem, making RBF or polynomial kernels essential for capturing the true decision boundary.