一.

$$\begin{bmatrix} 1 & 4 & 0 & -1 & -1 \\ 1 & -2 & 2 & 3 & 3 \\ 3 & 6 & 2 & 2 & 3 \\ 1 & -8 & 4 & 7 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 0 & -1 & -1 \\ 0 & 6 & -2 & -4 & -4 \\ 0 & -6 & 2 & 5 & 6 \\ 0 & 12 & -4 & -8 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 0 & -1 & -1 \\ 0 & 6 & -2 & -4 & -4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$\sim \begin{bmatrix} 1 & 4 & 0 & -1 & -1 \\ 0 & 3 & -1 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \qquad \therefore \begin{cases} x_1 + 4x_2 - x_4 \cancel{-x_5} = \cancel{0} -1 \\ 3x_2 - x_3 - 2x_4 \cancel{-2x_5} = -2 \\ x_4 = 2. \end{cases}$$

$$\therefore \begin{cases} x_1 = -4x_2 + 1 = -\frac{4}{3}x_3 - \frac{5}{3} \\ x_2 = \frac{1}{3}(x_3 + 2) \end{cases} \qquad \therefore x = x_3 \begin{bmatrix} -\frac{4}{3} \\ \frac{1}{3} \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{5}{3} \\ \frac{2}{3} \\ 0 \\ 2 \end{bmatrix}$$

二.

$$\begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & \frac{7}{2} & \frac{3}{2} & \frac{1}{2} \end{bmatrix} \qquad \therefore A^{-1} = \begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ \frac{7}{2} & \frac{3}{2} & \frac{1}{2} \end{bmatrix}.$$

三.

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 9 & 16 \end{vmatrix} = 2 \qquad D_1 = \begin{vmatrix} 1 & 1 & 1 \\ -1 & 3 & 4 \\ 1 & 9 & 16 \end{vmatrix} = 20 \qquad D_2 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 4 \\ 4 & 1 & 16 \end{vmatrix} = -30.$$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ 4 & 9 & 1 \end{vmatrix} = 12 \qquad \therefore x_1 = 10,\ x_2 = -15\quad x_3 = 6.$$

四.

(1) B （列变换即矩阵右乘）.

(2) $$A^n - 2A^{n-1} = \begin{bmatrix} 2^{n-1} & 0 & 2^{n-1} \\ 0 & 2^n & 0 \\ 2^{n-1} & 0 & 2^{n-1} \end{bmatrix} - \begin{bmatrix} 2^{n-1} & 0 & 2^{n-1} \\ 0 & 2^n & 0 \\ 2^{n-1} & 0 & 2^{n-1} \end{bmatrix} = 0.$$

五. (1)

$(kA)(\frac{1}{k}A^{-1}) = E$ ∴ kA 的逆矩阵为 $(\frac{1}{k}A^{-1})$ ∴ $(kA)^{-1} = \frac{1}{k}A^{-1}$.

(2) $AB - A - B = 0$

~~$B(A$~~ ~~$A(B-E)-B=0$~~

$(A-I)B - A = 0$

$(A-I)B - (A-I) = I$

$(A-I)(B-I) = I$

∴ $(B-I)(A-I) = I$

$BA - A - B = 0$

∴ $BA = AB$

六.

$\beta = c_1\alpha_1 + c_2\alpha_2 + \cdots c_r\alpha_r$

$\alpha_r = \frac{\beta}{c_r} - \frac{c_1}{c_r}\alpha_1 - \frac{c_2}{c_r}\alpha_2 \cdots \frac{c_{r-1}}{c_r}\alpha_{r-1}$

即 $\alpha_r = a_1\beta + a_2\alpha_1 + a_3\alpha_2 + \cdots + a_r\alpha_{r-1}$.

∴ 可以线性表示.

七.

$$A = \begin{vmatrix} a+(n-1)b & b & \cdots & b \\ a+(n-1)b & a & & b \\ \vdots & \vdots & \ddots & \vdots \\ a+(n-1)b & b & \cdots & a \end{vmatrix} = [a+(n-1)b]\begin{vmatrix} 1 & b & \cdots & b \\ 1 & a & & b \\ \vdots & \vdots & \ddots & \vdots \\ 1 & b & \cdots & a \end{vmatrix}$$

$$= [a+(n-1)b]\begin{vmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 1 & a-b & & \vdots \\ \vdots & & \ddots & \vdots \\ 1 & 0 & \cdots & a-b \end{vmatrix} = [a+(n-1)b]\begin{vmatrix} a-b & & \\ & \ddots & \\ & & a-b \end{vmatrix} = [a+(n-1)b]\cdot(a-b)^{n-1}.$$

八.

$C = \begin{vmatrix} c_1 & c_2 \\ c_3 & c_4 \end{vmatrix}$ $AC - CA = \begin{vmatrix} ac_3 - c_2 & c_2 + ac_4 - ac_1 \\ c_1 - c_3 - c_4 & c_2 - ac_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1 & b \end{vmatrix}$.

∴ $\begin{cases} ac_3 = c_2 \\ c_2 + ac_4 - ac_1 = 1 \\ c_1 - c_3 - c_4 = 1 \\ c_2 - ac_3 = b \end{cases}$

$\Rightarrow \begin{bmatrix} 1 & 0 & -1 & -1 & 1 \\ 0 & 1 & -a & 0 & 0 \\ -a & 1 & 0 & a & 1 \\ 0 & 1 & -a & 0 & b \end{bmatrix} \sim$

∴ $ac_3 = c_2 \Rightarrow b = 0$

$\begin{cases} -ac_1 + ac_3 + ac_4 = 1 \\ c_1 - c_3 - c_4 = 1 \end{cases}$

∴ $a = -1$

$b = 0$.