1. (1) $AB=\begin{bmatrix}9&9\\-2&9\\-1&11\end{bmatrix}$ (2) $AB=\begin{bmatrix}1&2&0&-1\\1&-3&2&3\\-1&3&1&4\end{bmatrix}$

2. $AB=\begin{bmatrix}0&14&-3\\17&13&10\end{bmatrix}$ $\cancel{求(AB)^T}$

$(AB)^T=\begin{bmatrix}0&17\\14&13\\-3&10\end{bmatrix}$

$\cancel{\therefore (AB)^T=\begin{bmatrix}\frac{179}{238}&-\frac{13}{238}&\frac{1}{17}\\-\frac{3}{14}&\frac{1}{14}&0\end{bmatrix}}$

3. (a) $Ab_1=\begin{bmatrix}0\\-3\\13\end{bmatrix}$ $Ab_2=\begin{bmatrix}14\\-9\\4\end{bmatrix}$

(b) $AB=\begin{bmatrix}4\times1+(-2)\times2&4\times3+(-2)\times(-1)\\(-3)\times1+0\times2&(-3)\times3+0\times(-1)\\1\times3+2\times5&3\times3+(-1)\times5\end{bmatrix}=\begin{bmatrix}0&14\\-3&-9\\13&4\end{bmatrix}$.

5. $AB=\begin{bmatrix}23&-10+5k\\-9&15+k\end{bmatrix}$ $BA=\begin{bmatrix}23&\cancel{15+k}\ \cancel{-10+5k}\ 15\\6-3k&15+k\end{bmatrix}$ $\therefore\begin{cases}6-3k=-9\\-10+5k=15\end{cases}$ $\therefore k=5$.

4. $\cancel{AX+\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}2&0&2\\0&4&0\\2&0&2\end{bmatrix}+X}$. $\because E^2=E$, $EX=X$ $\therefore AX+E=A^2+X$

$\therefore AX+E^2=A^2+EX$ $\therefore (A-E)X=A^2-E^2$ $\therefore X=A+E=\begin{bmatrix}2&0&1\\0&3&0\\1&0&2\end{bmatrix}$.

6. (1) $A=\begin{bmatrix}0&1\\0&0\end{bmatrix}$ (2) $A=\begin{bmatrix}0&0\\0&1\end{bmatrix}$ (3) $A=\begin{bmatrix}0&0\\0&1\end{bmatrix}$ $X=\begin{bmatrix}0&1\\0&0\end{bmatrix}$ $Y=\begin{bmatrix}1&0\\0&0\end{bmatrix}$

7. (1) $AB=\begin{bmatrix}3&4\\4&6\end{bmatrix}$ $BA=\begin{bmatrix}1&2\\3&8\end{bmatrix}$ $\therefore AB\neq BA$.

(2) $(A+B)^2=\begin{bmatrix}8&14\\14&29\end{bmatrix}$ $A^2+2AB+B^2=\begin{bmatrix}10&16\\15&27\end{bmatrix}$ $\therefore (A+B)^2\neq A^2+2AB+B^2$.

(3) $(A+B)(A-B)=\begin{bmatrix}0&6\\0&9\end{bmatrix}$ $A^2-B^2=\begin{bmatrix}2&8\\1&7\end{bmatrix}$ $\therefore (A+B)(A-B)\neq A^2-B^2$.

8. (1) 有. AB是一个方阵，说明A有3行，B有3列，但是BA不一定是方阵
因为A的列数和B的行数不确定.

(2) 不是. $A=\begin{bmatrix}0&1\\0&0\end{bmatrix}$ $B=\begin{bmatrix}1&0\\0&0\end{bmatrix}$ $BA=\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}0&1\\0&0\end{bmatrix}=\begin{bmatrix}0&1\\0&0\end{bmatrix}$.

$\begin{bmatrix}3&2\\7&4\end{bmatrix}^{-1}=\frac{1}{ad-b\cancel{d}c}\begin{bmatrix}d&-b\\-c&a\end{bmatrix}=\frac{1}{12-14}\begin{bmatrix}4&-2\\-7&3\end{bmatrix}=\begin{bmatrix}-2&1\\\frac{7}{2}&-\frac{3}{2}\end{bmatrix}$.

2. $\begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 0 \\ 3 & 4 & -2 & 0 & 1 & 0 \\ 5 & -4 & 1 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 3 & -1 & 0 \\ 0 & 6 & -11 & 10 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 3 & -1 & 0 \\ 0 & 0 & 7 & -1 & -3 & 1 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 2 & -1 & 3 & -1 & 0 \\ 0 & 0 & 7 & -1 & -3 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 14 & 0 & 20 & -10 & 1 \\ 0 & 0 & 7 & -1 & -3 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 1 & 0 & \frac{10}{7} & -\frac{5}{7} & \frac{1}{14} \\ 0 & 0 & 1 & -\frac{1}{7} & -\frac{3}{7} & \frac{1}{7} \end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ \frac{10}{7} & -\frac{5}{7} & \frac{1}{14} \\ -\frac{1}{7} & -\frac{3}{7} & \frac{1}{7} \end{bmatrix}.$

3. $(A^T - B)^T + C(B^{-1}C)^{-1} = A - B^T + C(B C^{-1}) = \begin{bmatrix} 0 & 2 & 2 \\ 4 & 2 & 6 \\ 6 & 8 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 0 \\ \frac{5}{2} & 1 & 1 \\ 5 & 2 & 0 \end{bmatrix}$

$= \begin{bmatrix} 3 & 3 & 2 \\ \frac{13}{2} & 3 & 7 \\ 11 & 10 & 4 \end{bmatrix}. = \begin{bmatrix} 3 & 3 & 2 \\ \frac{7}{2} & 0 & 5 \\ 11 & 10 & 4 \end{bmatrix}.$

4. $B = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 3 & -3 \\ 6 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 2 & 0 \end{bmatrix}$

$\therefore \cancel{B^{-1} =}$ $\because \frac{1}{ad-bc} \neq 0$ $\therefore B^{-1} = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} \\ -1 & -1 \end{bmatrix}.$

5. (1) $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 5 & -4 & 1 \end{bmatrix}$

(2) $\cancel{A^{-1}}$ $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ -2 & 1 & 0 & 0 & 1 & 0 \\ 5 & -4 & 1 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 5 & -4 & 1 & 0 & 0 & 1 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & 4 & -1 & 5 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 4 & 1 \end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}.$

(3) $x = A^{-1}B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 20 \end{bmatrix}$ $\therefore \begin{cases} x_1 = 2 \\ x_2 = 7 \\ x_3 = 20. \end{cases}$

6. (1) [illegible].

$B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A.$ (2) $B = A\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$

(3) $AB = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$.

不同

7. 证明：可逆矩阵 A 的逆矩阵为 X. Y.

$\begin{cases} AX = I \\ AY = I \end{cases} \Rightarrow X = X(AY) = (XA)Y = IY = Y$ ∴ 逆矩阵唯一.

1. (1) 可逆. $Ax=0$ 仅有平凡解.

(2) 不可逆. 化简 $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

(3) 可逆. 有 4 个主元位置 $\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(4) 该矩阵有 4 个主元位置.

2. $A^2 = A + 2E \Rightarrow A(A-1) = 2E$ ∴ $A^{-1} = \frac{A-1}{2}$

$A^2 - A = 2E$ $A \cdot \frac{A-1}{2} = E$ ∴ A 可逆.

$A^2 - A - 2E = 0$

$(A+2E)(A-3E) = -4E$

∴ $A+2E$ 可逆.

1. (1) $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ $B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$ $AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$

$A_{11}B_{11} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ $A_{11} = I_2$ $A_{12} = 0$ $A_{22} = I_2$

$AB = \begin{bmatrix} B_{11} & B_{12} \\ A_{21}B_{11} + B_{21} & A_{21} + B_{22} \end{bmatrix}$

$A_{12}B_{21} = 0$ $A_{21}B_{11} = \begin{bmatrix} -3 & 4 \\ 0 & 2 \end{bmatrix}$.

∴ $AB = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 2 & 0 & 1 \\ -2 & 4 & 3 & 3 \\ -1 & 1 & 3 & 1 \end{bmatrix}$

(2) $AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{11}B_{12} + B_{22} \\ 0 & A_{22}B_{22} \end{bmatrix}$

$A_{11}B_{12} = \begin{bmatrix} 7 & -1 \\ 2 & -1 \end{bmatrix}$ $A_{22}B_{22} = \begin{bmatrix} -4 & 3 \\ 0 & -9 \end{bmatrix}$

∴ $AB = \begin{bmatrix} 1 & 2 & 5 & 2 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & -4 & 3 \\ 0 & 0 & 0 & -9 \end{bmatrix}$.

2. (1) $A = \left[\begin{array}{c|cc} 5 & 0 & 0 \\ \hline 0 & 3 & 1 \\ 0 & 2 & 1 \end{array}\right]$ $\therefore A^{-1} = \begin{bmatrix} A_{11}^{-1} & -A_{11}^{-1} A_{12} A_{22}^{-1} \\ 0 & A_{22}^{-1} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -2 & 3 \end{bmatrix}$

(2) $A = \left[\begin{array}{cc|cc} 5 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ \hline 0 & 0 & 8 & 3 \\ 0 & 0 & 5 & 2 \end{array}\right]$ $\therefore A^{-1} = \begin{bmatrix} 1 & -2 & 0 & 0 \\ -2 & 5 & 0 & 0 \\ 0 & 0 & 2 & -3 \\ 0 & 0 & -5 & 8 \end{bmatrix}$

A