

$$1. (1) AB = \begin{bmatrix} 9 & 9 \\ -2 & 9 \\ -1 & 11 \end{bmatrix}$$

$$(2) AB = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 1 & -3 & 2 & 3 \\ -1 & 3 & 1 & 4 \end{bmatrix}$$

$$2. AB = \begin{bmatrix} 0 & 14 & -3 \\ 17 & 13 & 10 \end{bmatrix}$$

$$\text{求 } (AB)^T = \begin{bmatrix} 0 & 14 & -3 & 0 \\ 17 & 13 & 10 & 0 \\ 1 & 14 & -39 & 14 \end{bmatrix} \sim \begin{bmatrix} 17 \times 14 & 13 \times 14 & 140 & 0 & 14 \\ 0 & 14 \times 13 & -39 & 13 & 0 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 0 & 17 \\ 14 & 13 \\ -3 & 10 \end{bmatrix}$$

$$\sim \begin{bmatrix} 17 \times 14 & 0 & 179 & -13 & 14 \\ 0 & 14 \times 13 & -39 & 13 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \frac{179}{238} & -\frac{13}{238} & \frac{1}{17} \\ 0 & 1 & -\frac{3}{14} & \frac{1}{14} & 0 \end{bmatrix}$$

$$\therefore (AB)^T = \begin{bmatrix} \frac{179}{238} & -\frac{13}{238} & \frac{1}{17} \\ -\frac{3}{14} & \frac{1}{14} & 0 \end{bmatrix}$$

$$3. (a) Ab_1 = \begin{bmatrix} 0 \\ -3 \\ 13 \end{bmatrix} \quad Ab_2 = \begin{bmatrix} 14 \\ -9 \\ 4 \end{bmatrix}$$

$$(b) AB = \begin{bmatrix} 4 \times 1 + (-2) \times 2 & 4 \times 3 + (-2) \times (-1) \\ (-3) \times 1 + 0 \times 2 & (-3) \times 3 + 0 \times (-1) \\ 1 \times 3 + 2 \times 5 & 3 \times 3 + (-1) \times 5 \end{bmatrix} = \begin{bmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{bmatrix}$$

$$5. AB = \begin{bmatrix} 23 & -10+5k \\ -9 & 15+k \end{bmatrix} \quad B/A = \begin{bmatrix} 23 & \frac{15+k}{6-3k} \\ 6-3k & 15+k \end{bmatrix} \quad \therefore \begin{cases} 6-3k = -9 \\ -10+5k = 15 \end{cases} \quad \therefore k = 5$$

$$4. AX + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 4 & 0 \\ 2 & 0 & 2 \end{bmatrix} \quad \therefore E^2 = E, \quad EX = X \quad \therefore AX + E = A^2 + X$$

$$\therefore AX + E^2 = A^2 + EX \quad \therefore (A - E)X = A^2 - E^2 \quad \therefore X = A + E = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$$6. (1) A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad (2) A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad (3) A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad Y = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$7. (1) AB = \begin{bmatrix} 3 & 4 \\ 4 & 6 \end{bmatrix} \quad BA = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix} \quad \therefore AB \neq BA$$

$$(2) (A+B)^2 = \begin{bmatrix} 8 & 14 \\ 14 & 29 \end{bmatrix} \quad A^2 + 2AB + B^2 = \begin{bmatrix} 10 & 16 \\ 15 & 27 \end{bmatrix} \quad \therefore (A+B)^2 \neq A^2 + 2AB + B^2$$

$$(3) (A+B)(A-B) = \begin{bmatrix} 0 & 6 \\ 0 & 9 \end{bmatrix} \quad A^2 - B^2 = \begin{bmatrix} 2 & 8 \\ 1 & 7 \end{bmatrix} \quad \therefore (A+B)(A-B) \neq A^2 - B^2$$

8. (1) 有. AB 是一个方阵. 说明 A 有 3 行, B 有 3 列, 但是 B/A 不一定是方阵
因为 A 的列数和 B 的行数不确定.

$$(2) \text{不是. } A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad BA = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ 7 & 4 \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{12-14} \begin{bmatrix} 4 & -2 \\ -7 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{7}{2} & -\frac{3}{2} \end{bmatrix}$$

$$\begin{aligned}
 2. \quad & \begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 0 \\ 3 & 4 & -2 & 0 & 1 & 0 \\ 5 & -4 & 1 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 3 & -1 & 0 \\ 0 & 6 & -11 & 10 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 3 & -1 & 0 \\ 0 & 0 & 7 & -1 & -3 & 1 \end{bmatrix} \\
 & \sim \begin{bmatrix} 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 2 & -1 & 3 & -1 & 0 \\ 0 & 0 & 7 & -1 & -3 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 14 & 0 & 20 & -10 & 1 \\ 0 & 0 & 7 & -1 & -3 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 1 & 0 & \frac{10}{7} & -\frac{5}{7} & \frac{1}{14} \\ 0 & 0 & 1 & -\frac{1}{7} & -\frac{3}{7} & \frac{1}{7} \end{bmatrix} \\
 & \therefore A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ \frac{10}{7} & -\frac{5}{7} & \frac{1}{14} \\ -\frac{1}{7} & -\frac{3}{7} & \frac{1}{7} \end{bmatrix}.
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & (A^T - B)^T + C(B^T C)^{-1} = A - B^T + C(B^T C)^{-1} = \begin{bmatrix} 0 & 2 & 2 \\ 4 & 2 & 6 \\ 6 & 8 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 0 \\ \frac{5}{2} & 1 & 1 \\ 5 & 2 & 0 \end{bmatrix} \\
 & = \begin{bmatrix} 3 & 3 & 2 \\ \frac{13}{2} & 3 & 7 \\ 11 & 10 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 2 \\ \frac{7}{2} & 0 & 5 \\ 11 & 10 & 4 \end{bmatrix}.
 \end{aligned}$$

$$4. \quad B = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 3 & -3 \\ 6 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 2 & 0 \end{bmatrix}$$

$$\therefore B^{-1} \text{ exists} \quad \because \frac{1}{ad-bc} \neq 0 \quad \therefore B^{-1} = \frac{1}{2} \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} \\ -1 & -1 \end{bmatrix}.$$

$$\begin{aligned}
 5. \quad (1) \quad & \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 5 & -4 & 1 \end{bmatrix} \quad (2) \quad A^{-1} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ -2 & 1 & 0 & 0 & 1 & 0 \\ 5 & -4 & 1 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 5 & -4 & 1 & 0 & 0 & 1 \end{bmatrix} \\
 & \sim \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & 4 & -1 & 5 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 4 & 1 \end{bmatrix} \\
 & \therefore A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}.
 \end{aligned}$$

$$(3) \quad X = A^{-1}B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 3 & & \\ 2 & & \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 20 \end{bmatrix} \quad \therefore \begin{cases} x_1 = 2 \\ x_2 = 7 \\ x_3 = 20. \end{cases}$$

6. (1) $B \in \mathbb{Q}$ 及逆线性变换.

$$B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A.$$

$$(2) \quad B = A \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

$$(3) AB = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}.$$

不同.

7. 证明: 可逆矩阵 A 的逆矩阵为 X, Y .

$$\begin{cases} AX = I \\ AY = I \end{cases} \Rightarrow X = X(AY) = (XA)Y = IY = Y \quad \therefore \text{逆矩阵唯一.}$$

1. (1) 可逆. $AX=0$ 仅有平凡解.

(2) 不可逆. 化简 $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

(3) 可逆. 有 4 个主元位置 $\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(4) 该矩阵有 4 个主元位置.

$$\begin{aligned} 2. \quad A^2 = A + 2E &\Rightarrow A(A-1) = 2E & \therefore A^{-1} = \frac{A-1}{2} \\ A^2 - A = 2E & \quad A \cdot \frac{A-1}{2} = E & \therefore A \text{ 可逆.} \end{aligned}$$

$$A^2 - A - 2E = 0$$

$$(A+2E)(A-3E) = -4E$$

$$\therefore A+2E \text{ 可逆.}$$

$$\begin{aligned} 1. \quad (1) \quad A &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \quad AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix} \\ A_{11}B_{11} &= \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \quad A_{11} = I_2 \quad A_{12} = 0 \quad A_{22} = I_2 \\ A_{12}B_{21} &= 0 \quad A_{21}B_{11} = \begin{bmatrix} -3 & 4 \\ 0 & 2 \end{bmatrix} \end{aligned}$$

$$AB = \begin{bmatrix} B_{11} & B_{12} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 2 & 0 & 1 \\ -2 & 4 & 3 & 3 \\ -1 & 1 & 3 & 1 \end{bmatrix}$$

$$(2) \quad AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{11}B_{12} + B_{22} \\ 0 & A_{22}B_{22} \end{bmatrix}$$

$$A_{11}B_{12} = \begin{bmatrix} 7 & -1 \\ 2 & -1 \end{bmatrix} \quad A_{22}B_{22} = \begin{bmatrix} -4 & 3 \\ 0 & -9 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 1 & 2 & 5 & 2 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & -4 & 3 \\ 0 & 0 & 0 & -9 \end{bmatrix}$$

$$2. (1) A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 2 & 1 \end{bmatrix} \quad \therefore A^{-1} = \begin{bmatrix} A_{11}^{-1} & -A_{11}^{-1}A_{12}A_{22}^{-1} \\ 0 & A_{22}^{-1} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -2 & 3 \end{bmatrix}$$

$$(2) A = \begin{bmatrix} 5 & 2 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 8 & 3 \\ 0 & 0 & 1 & 5 & 2 \end{bmatrix} \quad \therefore A^{-1} = \begin{bmatrix} 1 & -2 & 0 & 0 \\ -2 & 5 & 0 & 0 \\ 0 & 0 & 2 & -3 \\ 0 & 0 & -5 & 8 \end{bmatrix}.$$