1.

$$D = (-1)^{1+4} \begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (-1)^{1+4} \cdot (2 \times 3 \times 1) = -6.$$

2.

(1)
$$D = 3 \cdot (-1)^{1+1} \begin{vmatrix} 3 & 2 \\ 5 & 1 \end{vmatrix} + 4 \cdot (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = 3 \cdot (3-10) + 4(10-0)$$
$$= -21 + 40 = 19.$$

(2)
$$D = 3 \cdot (-1)^{1+2} \begin{vmatrix} 4 & 3 \\ 6 & 5 \end{vmatrix} + 1 \cdot (-1)^{2+3} \begin{vmatrix} 2 & -3 \\ 4 & 3 \end{vmatrix} = -3 \cdot (20-18) - (6+12)$$
$$= -6 - 18 = -24.$$

3.

(1)
$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 2^2 & 3^2 & 4^2 \end{vmatrix} = (3-2)(4-2)(4-3) = 1 \times 2 \times 1 = 2$$

(2)
$$D = (a_2 - a_1)(a_3 - a_2)(a_3 - a_1)$$

4.

$$D_n = \prod_{1 \le j < i \le n} (x_i - x_j)$$

5. (1)
$$D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & 0 \\ 1 & 0 & 4 \end{vmatrix} = \begin{vmatrix} 2-\frac{1}{3} & 1 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 4 \end{vmatrix} = \begin{vmatrix} 2-\frac{1}{3}-\frac{1}{4} & 1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{vmatrix} = 12 \times \frac{17}{12} = 17.$$

(2)
$$D = \begin{vmatrix} a_1 - \frac{1}{a_2} - \frac{1}{a_3} - \cdots - \frac{1}{a_n} & 1 & 1 & \cdots & 1 \\ 0 & a_2 & 0 & \cdots & 0 \\ 0 & 0 & a_3 & & \\ \vdots & \vdots & & \ddots & \\ 0 & 0 & & & a_n \end{vmatrix} = \left(a_1 - \frac{1}{a_2} - \cdots - \frac{1}{a_n}\right) a_2 a_3 \cdots a_n.$$

6.

(1)
$$\begin{vmatrix} 1+a_1+\cdots a_n & 1+a_1+\cdots a_n & \cdots & 1+a_1\cdots a_n \\ a_2 & 1+a_2 & & \\ \vdots & a_3 & \ddots & \\ \vdots & \vdots & & \\ a_n & a_n & & 1+a_n \end{vmatrix} = (1+a_1+\cdots a_n) \begin{vmatrix} 1 & 1 & \cdots & 1 \\ a_2 & 1+a_2 & a_2 \cdots & a_2 \\ \vdots & \vdots & \ddots & \\ \vdots & \vdots & \cdots & \ddots \\ a_n & a_n & & 1+a_n \end{vmatrix}$$

$$= (1+a_1+\cdots a_n)\begin{vmatrix} 1 & 0 & 0 & \cdots & 0 \\ a_2 & 1 & 0 & & \\ \vdots & 0 & 1 & & \\ \vdots & \vdots & & \ddots & \\ a_n & 0 & & & 1 \end{vmatrix} = 1+a_1+\cdots a_n.$$

(2)

$$D = \begin{vmatrix} x+4y & y & y & y & y \\ x+4y & x & y & y & y \\ x+4y & y & x & y & y \\ x+4y & y & y & x & y \\ x+4y & y & y & y & x \end{vmatrix} = (x+4y)\begin{vmatrix} 1 & y & y & y & y \\ 1 & x & & & \\ 1 & & x & & \\ 1 & & & x & \\ 1 & & & & x \end{vmatrix} = (x+4y)\begin{vmatrix} 1 & 0 & 0 & \cdots & \\ 1 & x-y & & & \\ 1 & & x-y & & \\ 1 & & & \ddots & \\ 1 & & & & x-y \end{vmatrix}$$

$$= (x+4y)(x-y)^4.$$

(3)

$$D = x\cdot(-1)^{1+1}\begin{vmatrix} x & y & & \\ 0 & x & y & \\ 0 & 0 & x & y \\ 0 & 0 & 0 & x \end{vmatrix} + y\cdot(-1)^{5+1}\begin{vmatrix} y & & & \\ x & y & & \\ & x & y & \\ & & x & y \end{vmatrix}$$

$$= x^5 + y^5.$$

7.

$$D = a_1x^{n-1} + (-1)(-1)^{1+2}\begin{vmatrix} a_2 & -1 & & & \\ a_3 & x & -1 & & \\ \vdots & & x & \ddots & \\ \vdots & & & \ddots & -1 \\ a_n & & & & x \end{vmatrix} = a_1x^{n-1} + a_2x^{n-2} + \cdots$$

以此类推，$D = a_1x^{n-1} + a_2x^{n-2} + \cdots a_nx.$

1.

$$D=\begin{vmatrix}1&2&1\\2&3&3\\1&1&-2\end{vmatrix}=-6+6+2-3+8-3=4 \qquad D_1=\begin{vmatrix}1&2&1\\3&3&3\\1&1&-2\end{vmatrix}=-6+6+3-3+8-3=5.$$

$$D_2=\begin{vmatrix}1&1&1\\2&3&3\\1&1&-2\end{vmatrix}=\cancel{8}-6+2+3-3+4-3=-3$$

$$D_3=\begin{vmatrix}1&2&1\\2&3&3\\1&1&1\end{vmatrix}=3+2+6-3-3-4=1.$$

$$\therefore x_1=\frac{5}{4}\quad x_2=-\frac{3}{4}\quad x_3=\frac{1}{4}.$$

2. $$C_{11}=\begin{vmatrix}1&0\\-1&1\end{vmatrix}=1\qquad C_{12}=-\begin{vmatrix}1&0\\2&1\end{vmatrix}=-1\qquad C_{13}=\begin{vmatrix}1&1\\2&-1\end{vmatrix}=-3.$$

$$C_{21}=-\begin{vmatrix}1&2\\-1&1\end{vmatrix}=-3\qquad C_{22}=\begin{vmatrix}0&2\\2&1\end{vmatrix}=4\qquad C_{23}=-\begin{vmatrix}0&1\\2&-1\end{vmatrix}=-2.$$

$$C_{31}=\begin{vmatrix}1&2\\1&0\end{vmatrix}=-2\qquad C_{32}=-\begin{vmatrix}0&2\\1&0\end{vmatrix}=-2\qquad C_{33}=\begin{vmatrix}0&1\\1&1\end{vmatrix}=-1.$$

$$\therefore A^{-1}=\frac{1}{-1\cancel{+}-6}\cdot\begin{bmatrix}1&-3&-2\\-1&4&-2\\-3&-2&-1\end{bmatrix}=\begin{bmatrix}-\frac{1}{7}&\frac{3}{7}&\frac{2}{7}\\\frac{1}{7}&-\frac{4}{7}&\frac{2}{7}\\\frac{3}{7}&\frac{2}{7}&\frac{1}{7}\end{bmatrix}.$$

3.

$$D=\begin{vmatrix}\lambda&1&1\\1&\lambda&1\\1&1&\lambda\end{vmatrix}=0.\qquad \therefore \lambda^3+\lambda+\lambda-\lambda-\lambda-\lambda=\lambda^3-\lambda=0.$$

$\therefore \lambda=0$ 或 1 或 -1.

第4题范德蒙德证明.

(按第一列展开).

$$\begin{vmatrix}1&1&1&\cdots&1\\a_1&a_2&a_3&\cdots&a_n\\a_1^2&a_2^2&a_3^2&\cdots&a_n^2\\\vdots&&&&\\a_1^{n-1}&a_2^{n-1}&a_3^{n-1}&\cdots&a_n^{n-1}\end{vmatrix}=\begin{vmatrix}1&1&\cdots&1\\0&a_2-a_1&\cdots&a_n-a_1\\0&a_2^2-a_2a_1&\cdots&a_n^2-a_na_1\\\vdots&\vdots&\ddots&\vdots\\0&a_2^{n-1}-a_2^{n-2}a_1&\cdots&a_n^{n-1}-a_n^{n-2}a_1\end{vmatrix}=\begin{vmatrix}a_2-a_1&\cdots&a_n-a_1\\\vdots&\ddots&\\a_2^{n-1}-a_2^{n-2}a_1&&a_n^{n-1}-a_n^{n-2}a_1\end{vmatrix}$$

$$=(a_2-a_1)(a_3-a_1)(a_4-a_1)\cdots(a_n-a_1)\begin{vmatrix}1&1&\cdots&1\\a_2&\ddots&&\vdots\\\vdots&&\ddots&\vdots\\a_2^{n-2}&&&a_n^{n-2}\end{vmatrix}$$

重复以上过程. 可得. $D_n=\prod_{1\le j<i\le n}(x_i-x_j)$.