

Proof of Critical Complex Equilibrium

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Formal Proof

Step 1: Eigenvalue Definition

$$\mu = -\eta + i\lambda, \quad |\mu|^2 = \eta^2 + \lambda^2 = 1 \quad (1)$$

Step 2: Balance Condition

$$|\operatorname{Re}(\mu)| = |\operatorname{Im}(\mu)| \Rightarrow \eta = \lambda \quad (2)$$

Step 3: Solve for the Critical Constants

$$2\lambda^2 = 1 \Rightarrow \boxed{\lambda = \eta = \frac{1}{\sqrt{2}} \approx 0.7071} \quad (3)$$

At this equilibrium point, the real (damping) and imaginary (coupling) components are equal in magnitude, representing perfect balance in the complex domain. This yields the universal critical constant:

$$\lambda = \eta = \frac{1}{\sqrt{2}}. \quad (4)$$