1, 2, 3

For all three of the problems below, view $\mathbb{R}$ as a metric space with the usual metric.

(1) Prove or disprove the following: There is only one open set in $\mathbb{R}$ that contains all of $\mathbb{Q}$, and it is the set $\mathbb{R}$.

∎

(2) How many subsets of $\mathbb{R}$ are both open and closed?

(3) Rudin uses Theorem 1.20 to say that "$\mathbb{Q}$ is dense in $\mathbb{R}$." But is $\mathbb{Q}$ still dense in $\mathbb{R}$ if we instead use the definition for "dense" given in Definition 2.18? What is the relationship between these two definitions of "dense" when applied to $\mathbb{Q}$ in $\mathbb{R}$? Does each one imply the other?

∎