Power Systems Economics: Homework #12

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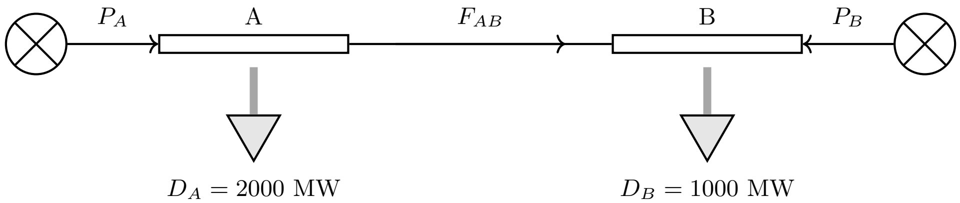
November 18, 2025

6.10

Given: *Consider the two-bus power system of Problem 6.2. Given that $K = R/V^2 = 0.0001\ MW^{-1}$ for the line connecting buses A and B. Calculate the value of the flow that minimizes the total variable cost of production.*



First, we define the problem mathematically. We want to minimize the total cost $C_{Total} = C_A(P_A)+C_B(P_B)$. The marginal cost functions are:

$$MC_A = 20 + 0.03P_A$$

$$MC_B = 15 + 0.02P_B$$

We define F as the power flow received at Bus A from Bus B. Since Generator B is cheaper (15$ base vs 20$), power will flow from B to A. At Bus A, the load is met by local generation plus the received flow:

$$P_A + F = D_A \implies P_A = 2000 - F$$

At Bus B, the generation must supply the local load, plus the flow sent to A, plus the transmission losses.

$$P_B = D_B + F + P_{Loss}$$

We are given $P_{Loss} = K \cdot F^2 = 0.0001F^2$.

$$P_B = 1000 + F + 0.0001F^2$$

To find the minimum cost, we look for the point where the marginal benefit of importing one more MW at Bus A equals the marginal cost of producing and sending it from Bus B. Mathematically, this is the chain rule derivative of the cost function with respect to flow F:

$$\frac{dC}{dF} = \frac{dC_A}{dP_A}\frac{dP_A}{dF} + \frac{dC_B}{dP_B}\frac{dP_B}{dF} = 0$$

Substituting our derivatives: 1. From $P_A = 2000-F$, we get $\frac{dP_A}{dF} = -1$. 2. From $P_B = 1000+F+0.0001F^2$, we get $\frac{dP_B}{dF} = 1 + 0.0002F$.
Substituting these into the optimality condition:

$$MC_A \cdot (-1) + MC_B \cdot (1 + 0.0002F) = 0$$

$$MC_A = MC_B(1 + 0.0002F)$$

This equation tells us that at the optimal point, the price at A must equal the price at B scaled by the penalty factor due to incremental losses.
Now, we solve for F. Let's test the value $F = 730$ MW to see if it satisfies this condition.

- **Calculate P_A:** $P_A = 2000 - 730 = 1270$ MW.
- **Calculate MC_A:** $MC_A = 20 + 0.03(1270) = 20 + 38.1 = 58.10$ \$/MWh.
- **Calculate P_B:** $P_B = 1000 + 730 + 0.0001(730^2) = 1730 + 53.29 = 1783.29$ MW.

- **Calculate MC_B:** $MC_B = 15 + 0.02(1783.29) = 15 + 35.666 = 50.666$ \$/MWh.
- **Check Optimality:**

$$\text{RHS} = MC_B(1 + 0.0002 \times 730) = 50.666(1 + 0.146) = 50.666(1.146)$$

$$\text{RHS} = 58.06 \approx 58.10$$

The values match very closely (within rounding error). Therefore, the optimal flow is indeed **730 MW**.

Results:

- Flow $F_{BA} = 730$ MW.
- Generation $P_A = 1270$ MW.
- Generation $P_B = 1783$ MW.
- Losses = 53 MW.
- Nodal Price $\pi_A = MC_A = \$58.10/\text{MWh}$.
- Nodal Price $\pi_B = MC_B = \$50.67/\text{MWh}$.

Merchandising Surplus Calculation:

$$\text{Consumer Pay} = (2000 \times 58.10) + (1000 \times 50.67) = 116,200 + 50,670 = \$166,870$$

$$\text{Gen Revenue} = (1270 \times 58.10) + (1783 \times 50.67) = 73,787 + 90,344.61 = \$164,131.61$$

$$\text{Surplus} = 166,870 - 164,131.61 = \$2,738.39$$

6.15

Given: *Show that purchasing 100 MW of point-to-point financial rights between buses 3 and 1 provides a perfect hedge to Generator D.*
First, we recall the nodal prices from Problem 6.8: $\pi_1 = \$13.33$ (Consumer) and $\pi_3 = \$10.00$ (Generator).
Generator D wants to sell 100 MW at a strike price of $K = \$11.00$.
We analyze the cash flows:
1. Market Revenue: Generator D sells 100 MW at its local price π_3.

$$\text{Rev}_{Mkt} = 100 \times 10.00 = +\$1000$$

2. CfD Payment: Generator pays the difference between the sink price π_1 and the strike price K.

$$\text{Pay}_{CfD} = 100 \times (\pi_1 - K) = 100 \times (13.33 - 11.00) = 100 \times 2.33 = -\$233$$

3. FTR Payout: The FTR pays the difference between sink and source prices.

$$\text{Pay}_{FTR} = 100 \times (\pi_1 - \pi_3) = 100 \times (13.33 - 10.00) = 100 \times 3.33 = +\$333$$

Total Net Revenue:

$$\text{Total} = 1000 - 233 + 333 = \$1100$$

Target:

$$100 \text{ MW} \times \$11.00 = \$1100$$

Since Total Revenue equals Target Revenue, the hedge is perfect.