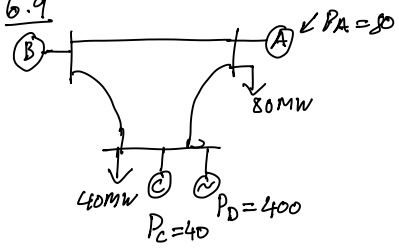
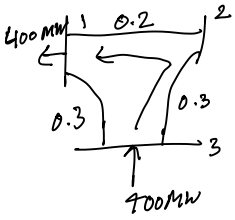
HW11

Monday, November 17, 2025 6:30 PM

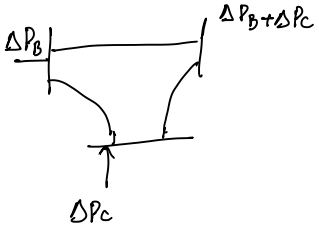
6.9

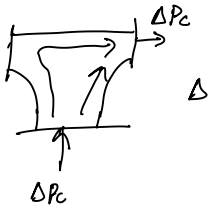




$F_{21} = 400 \times \frac{0.3}{0.8} = 150\ MW$

$F_{31} = 400 \times \frac{0.5}{0.8} = 250\ MW$

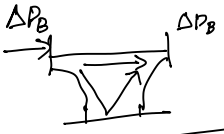




$\Delta P_C \cdot \frac{0.3}{0.8} = F_{12}^{A}$

① First condition

Create counter flow of 10MW on line (1-2)

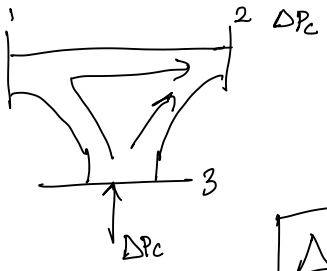


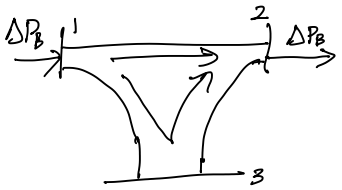
$\Delta P_B \cdot \frac{0.6}{0.8} = F_{12}^{B}$

$$\Delta P_C \cdot \frac{0.3}{0.8} + \Delta P_B \frac{0.6}{0.8} = 10$$

② Second Condition

No flow added on line 3-1.





$$\Delta P_C \cdot \frac{0.3}{0.8} - \Delta P_B \frac{0.2}{0.8} = 0$$

$P_C = 6.6 + 40 = 46.6\ MW$

$P_B = 10\ MW$

$P_D = 400\ MW$

$$P_A = 520 - (46.6 + 10 + 400)$$
$$= 63.3 \text{ MW}$$

$\pi_1 = 15$ \$/MWh $\pi_2 = 12$ \$/MWh $\pi_3 = 10$ \$/MWh

6.12

For each bus, we have

$$\sum_j \frac{1}{x_{ij}} (\pi_i - \pi_j + \mu_{ij} - \mu_{ji}) = 0$$

For bus 1, i=1;

$$\frac{1}{x_{12}} = (\pi_1 - \pi_2 + \mu_{12} - \mu_{21}) + \frac{1}{x_{13}} (\pi_1 - \pi_3 + \mu_{13} - \mu_{31}) = 0 \quad (1)$$

For bus 2, i=2

$$\frac{1}{x_{21}} (\pi_2 - \pi_1 + \mu_{21} - \mu_{12}) + \frac{1}{x_{23}} (\pi_2 - \pi_3 + \mu_{23} - \mu_{32}) = 0 \quad (2)$$

Plug numbers:

$x_{12} = 0.2$, $x_{13} = 0.3$, $\mu_{12} = \mu_{21} = 0$, $\mu_{13} = 0$, $\pi_2 = 12$, $\pi_3 = 10$

$$5(\pi_1 - 12) + \frac{1}{0.3} (\pi_1 - 10 - \mu_{31}) = 0 \quad \text{—— (3)}$$

plug numbers into 2

$x_{21} = 0.2$, $x_{23} = 0.3$, $\pi_3 = 10$, $\pi_2 = 12$, $\mu_{23} = \mu_{32} = 0$, $\mu_{12} = \mu_{21} = 0$

$$5(12 - \pi_1) + \frac{1}{0.3} (12 - b) = 0 \quad \text{—— (4)}$$

$$\Rightarrow \pi_1 = 13.33$$

plug $\pi_1 = 13.33$ into

$$\mu_{31} = 5.33$$

6.14

Conditions from 6.9:

optimal dispatch

$P_A = 63.33$ MW

$P_B = 10$ MW

$P_C = 46.67$ MW

$P_D = 400$ MW

BUS 1 SLACK

$i=2$ and $i=3$

$Y_{21} = \frac{1}{0.2} = 5 \quad Y_{31} = \frac{1}{0.3} = 3.33, \quad Y_{23} = \frac{1}{0.3} = 3.33$

$Y_{22} = 5 + 3.33 = 8.33 \quad Y_{33} = Y_{31} + Y_{32} = 3.33 + 3.33 = 6.66$

Finding μ_{21} when $i=2$

$$-Y_{21}(\pi_2 - \pi_1 + \mu_{21} - \mu_{12}) - Y_{23}(\pi_2 - \pi_3 + \mu_{23} - \mu_{32}) = 0$$

all $\mu = 0$

$$Y_{21}\mu_{21} = -Y_{22}\pi_2 + Y_{21}\pi_1 + Y_{23}\pi_3$$

$$5 \cdot \mu_{21} = -(8.33 \times 12) + (5 \times 15) + (3.33 \times 10)$$

$$\mu_{21} = \$1.67 \text{ /MWh}$$

$i=3$ for μ_{31}

$$-Y_{31}(\pi_3 - \pi_1 + \mu_{31} - \mu_{13}) - Y_{32}(\pi_3 - \pi_2 + \mu_{32} - \mu_{23}) = 0$$

$$3.33 \cdot \mu_{31} = -(6.66 \times 10) + (3.33 \times 15) + (3.33 \times 12)$$

$$\mu_{31} = \$7.00 \text{/MWh}$$