Power Systems Economics: Homework #7

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4.10

Given: Consider a market for electrical energy that is supplied by two generating companies whose cost functions are:

$$C_A = 36 * P_A \frac{\$}{h}$$

$$C_B = 31 * P_B \frac{\$}{h}$$

The inverse demand curve for this market is estimated to be:

$$\pi = 120 - D \frac{\$}{\text{MWh}}$$

Assuming a Cournot model of competition, use a table similar to the one used in Example 4.8 to calculate the equilibrium point of this market (price, quantity, production, and profit of each firm). (Hint: Use a spreasheet. A resolution of 5 MW is acceptable)
First we have to find the Profit Maximimation of Firm A, or Ω_A, is simply the revenue minus its cost. However, the price, π, depends on the output of both P_A and P_B. Therefore we shall get the following equations:

$$\Omega_A = \pi * P_A - C_A * P_A = (120 - (P_A + P_B)) * P_A - 36 * P_A = 120 * P_A - P_A^2 - P_A * P_B - 36 * P_A$$

$$\Omega_A = 84 * P_A - P_A^2 - P_A * P_B$$

Now, to maximize profit (and find Firm A's reaction curve), we simply find when the derivative (with respect to P_A) is zero:

$$\frac{d\Omega_A}{dP_A} = 0 \rightarrow 84 - 2 * P_A - P_B = 0 \rightarrow 2 * P_A = 84 - P_B \rightarrow P_A = 42 - \frac{P_B}{2}$$

Now, to find Firm B's profit maximization equation, and respective reaction curve we go through the exact same steps. First we must find the Ω_B:

$$\Omega_B = \pi * P_B - C_B * P_B = (120 - (P_A + P_B)) * P_B - 31 * P_B = 120 * P_B - P_A * P_B - P_B^2 - 31 * P_B$$

$$\Omega_B = 89 * P_B - P_B^2 - P_A * P_B$$

And after, we now find the derivative (with respect to P_B) and set it to zero:

$$\frac{d\Omega_B}{dP_B} = 0 \rightarrow 0 = 89 - 2 * P_B - P_A \rightarrow 2 * P_B = 89 - P_A \rightarrow P_B = \frac{89 - P_B}{2}$$

Now, to find equilibrium we simply take these two equations, and substitute one of them into the other. In this case I will be substituting the P_B that we found above from Firm B into Firm A's reaction curve:

$$P_A = 42 - \frac{\frac{89 - P_A}{2}}{2} \rightarrow P_A = 42 - \frac{89 - P_A}{4} \rightarrow \frac{3}{4} * P_A = \frac{79}{4} \rightarrow P_A = \frac{79}{4} * \frac{4}{3} = \frac{79}{3} = 26.33... \text{ MWh} \approx \textbf{25 MWh}$$

Now, to find P_B, we simply substitute the exact P_A we found into Firm B's reaction curve:

$$P_B = \frac{89 - \frac{79}{3}}{2} = \frac{\frac{188}{3}}{2} = \frac{188}{6} = \frac{94}{3} = 31.33... \text{ MWh} \approx \textbf{30 MWh}$$

(The rounding/approximation was done due to the *hint* stating that it is recommended).

Now, to find the Equilibrium results we simply will be using the *rounded* values of P_A and P_B:

- Total Demand (D) $= P_A + P_B = 25 + 30 = \mathbf{55}$ MW
- Market Price $(\pi) = 120 - D = 120 - 55 = \mathbf{65}\frac{\$}{\text{MWh}}$
- Firm A Profit $(\Omega_A) = (\pi * P_A) - (C_A * P_A) = 65 * 25 - 36 * 25 = 1625 - 900 = \$\mathbf{725}$
- Firm B Profit $(\Omega_B) = (\pi * P_B) - (C_B * P_B) = 65 * 30 - 31 * 30 = 1950 - 930 = \$\mathbf{1020}$

And now, putting this all into a Cournot Iteration Table, it will be the following:

Step	**Assumed** P_B	**Optimal** P_A	**Assumed** P_A	**Optimal** P_B	**Market Price** π
1	0	40 (42)	40	25 (24.5)	55
2	25	30 (29.5)	30	30 (29.5)	60
3	30	**25** (27)	**25**	**30** (32)	**65**

Now, what in the world does the table mean? The table shows that if Firm B produces at the 30 MW, then Firm A's best response to that is to produce 25 MW. And if, say Firm A starts producing 25 MW, then Firm B's response should be *logically* to produce 30 MW. Meaning, that it is a stable Nash Equilibrium.