Power Systems Economics: Homework #5

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4.1

Given: Cheapo Electrons is an electricity retailer. The table below shows theload that it forecast its consumers would use over a 6-hour period. Cheapo Electrons purchased in the forward market nd the power exchange exactly enough energy to cover this forecast. The table shows the average price that it paid for this energy for each hour. As one might expect, the actual consumption of its customers did not exactly match the load forecast and it had to purchase or sell the difference on the spot market and the prices nidicated. Assuming that Cheapo Electrons sells energy to its customers at a flat rate of 24.00$/MWh, calculate the profit or loss that it made during this 6-hour period. What would the rate that it should have charged its customers to break even?

Period	1	2	3	4	5	6
Load Forecast (MWh)	120	230	310	240	135	110
Average Cost ($/MWh)	22.50	24.50	29.30	25.20	23.10	21.90
Actual Load (MWh)	110	225	330	250	125	105
Spot Price ($/MWh)	21.60	25.10	32.00	25.90	22.50	21.50

A) Calculate the profit or loss that Cheapo Electrons made during this 6-hour period.

To calculate the total revenue we simply need to multiply the actual load amounts per hour by the rate of 24.00$/MWh and then sum them all up as shown below:

$$110 * 24 + 225 * 24 + 330 * 24 + 250 * 24 + 125 * 24 + 105 * 24 = \$27,480.00$$

After this, we simply need to calculate the total cost, of which is just difference between the actual load and forecasted load multiplied by that hours spot price (shown in the table below) and then add that to the multiplication of the forward cost and load forecasted produced amount:

Period	1	2	3	4	5	6
Load Forecast (MWh)	120	230	310	240	135	110
Actual Load (MWh)	110	225	330	250	125	105
Difference	**-10**	**-5**	**20**	**10**	**-10**	**-5**
Spot Price ($/MWh)	21.60	25.10	32.00	25.90	22.50	21.50
Spot Transaction ($)	**-216.00**	**-125.50**	**640.00**	**259.00**	**-225.00**	**-107.50**
Load Forecast (MWh)	120	230	310	240	135	110
Average Cost ($/MWh)	22.50	24.50	29.30	25.20	23.10	21.90
Forward Expense ($)	**2700.00**	**5635.00**	**9083.00**	**6048.00**	**3118.50**	**2409.00**

And now, when you add together all the expenses/transaction that had to occur (not the revenue, yet!), you get a total cost incurred of $29,218.50. This therefore means that the total profit/loss will be the following:

$$Profit = Revenue - Cost = 27,480.00 - 29,218.50 = \$ - 1,738.50$$

Since the profit is negative this means that **Cheapo Electrons made a loss of $1,738.50.**

B) What is the rate that it should have charged to its customers to break even?

To calculate the break even rate that Cheapo Electrons should have charged we simply divide the total cost by the total actual load over the 6-hour period, as shown below:

$$\frac{29,218.50\$}{(110 + 225 + 330 + 250 + 125 + 105)\text{MWh}} = \frac{29,218.50\$}{1145\text{MWh}} = \mathbf{25.52\$/MWh}$$

4.2

Given: The input-output curve of a gas-fired generating unit is approximated by the following function:

$$H(P) = 120 + 9.3 * P + 0.0025 * P^2 \frac{\text{MJ}}{\text{h}}$$

This unit has a minimum stable generation of 200MW and a maximum output of 500MW. The cost of has is 1.20$/MJ. Over a 6-hour period, the output of this unit is sold in a market for electrical energy at the prices shown in the table below.

Period	1	2	3	4	5	6
Price ($/MWh)	12.5	10	13	13.5	15	11

A) Assuming that this unit is optimally dispatched, is initial on-line and cannot be shut down, calculate its operational profit or loss for this period.

First we must create a Cost Function, C(P) via simply just using the input-output curve H(P) and multiplying it by the cost of gas:

$$C(P) = H(P) * (\text{Gas Cost}) = (120 + 9.3 * P + 0.0025 * P^2) * (1.20) = 144 + 11.16 * P + 0.003 * P^2$$

After this, we can find the marginal cost by simply taking the derivative of the Cost Function with respect to the input power needed:

$$MC(P) = \frac{dC(P)}{dP} = 11.16 + 0.006 * P$$

After this, we also know that the optimal dispatch is simply where $MC(P) = MarketPrice$, therefore the production amount is simply $P = \frac{MarketPrice - 11.16}{0.006}$. And this function is, as always, subject to the upper and lower bounds of the unit's stable generation minimum and maximum. Those minimums and maximums of power generation therefore mean that the marginal cost at below 200 and above 500 are fixed like so:

- Marginal Cost at Mimimum (P = 200): $MC(200) = 11.16 + 0.006 * (200) = 12.36\$/MWh$.
- Marginal Cost at Maximum (P = 500): $MC(500) = 11.16 + 0.006 * (500) = 14.16\$/MWh$.

NOW, with all of this information, we can collate it to the following table to show the final profit/loss that the unit will generate:

Period	Price ($/MWh)	Output P (MW)	Revenue ($)	Cost ($)	Profit ($)
1	12.50	223.33	2791.67	2786.03	5.63
2	10.00	200.00	2000.00	2496.00	-496.00
3	13.00	306.67	3986.67	3848.53	138.13
4	13.50	390.00	5265.00	4952.70	312.30
5	15.00	500.00	7500.00	6474.00	1026.00
6	11.00	200.00	2200.00	2496.00	-296.00

This therefore means, after summing up all the profit values, that **the profit for the 6-hour period of the unit running is $690.07.**

Python Script Output Verification

```
[nix-shell:~/Documents/School/Classes/Fall25/EEL4298/HW/hw5]$ python hw5.py
Total Cost Function C(P) = 0.003*P**2 + 11.16*P + 144.0
Marginal Cost Function MC(P) = 0.006*P + 11.16
--------------------------------------------------
Profit/Loss Calculation per Period:
 Period  Price ($/MWh)    Output P (MW)      Revenue ($)         Cost ($)       Profit ($)
      1          12.50 223.333333333333 2791.66666666667 2786.03333333333 5.63333333333367
      2          10.00              200         2,000.00         2,496.00          -496.00
      3          13.00 306.666666666667 3986.66666666667 3848.53333333333 138.133333333334
      4          13.50 390.000000000000 5265.00000000000 4952.70000000000 312.300000000000
      5          15.00              500         7,500.00         6,474.00         1,026.00
      6          11.00              200         2,200.00         2,496.00          -296.00
--------------------------------------------------
Total Profit for the 6-hour period: $690.07

[nix-shell:~/Documents/School/Classes/Fall25/EEL4298/HW/hw5]$
```