HW12

Tuesday, November 18, 2025 5:41 PM

6.10, 6.15

6.10

minimize total cost $C_T = C_A(P_A) + C_B(P_B)$

$$MC_A = 20 + 0.03 P_A$$

$$MC_B = 15 + 0.02 P_B$$

F = Bus B → Bus A – power flow received

Gen B = \$15 base

Gen A = \$20 base

$$P_A + F = D_A \longrightarrow P_A = 2000 - F$$

$$P_B = D_B + F + P_{Loss}$$

$P_{Loss} = K \cdot F^2 = 0.0001 F^2$

$$P_B = 1000 + F + 0.0001 F^2$$

For min cost,

$$\frac{dC}{dF} = \frac{dC_A}{dP_A} \cdot \frac{dP_A}{dF} + \frac{dC_B}{dP_B} \cdot \frac{dP_B}{dF} = 0$$

substitute derivates;

- $P_A = 2000 - F$

  $\frac{dP_A}{dF} = -1$

- $P_B = 1000 + F + 0.0001 F^2$

  $\frac{dP_B}{dF} = 1 + 0.0002F$

$$MC_A \cdot (-1) + MC_B \cdot (1 + 0.0002F) = 0$$

$$MC_A = MC_B(1 + 0.0002F)$$

Test 730 MW:

P_A : $P_A = 2000 - 730 = 1270$ MW

MC_A : $20 + 0.03(1270) = 20 + 38.1 = 58.10$ \$/MWh = (Nodal price π_A)

P_B : $1000 + 730 + 0.0001(730)^2 = 1730 + 53.29 = 1783.29$ MW

MC_B : $15 + 0.02(1783.29) = 15 + 35.66 = 50.66$ \$/MWh = (Nodal price π_B)

Merchandising Surplus

Consumer pays:

$(2000 \times 58.10) + (1000 \times 50.67) = 116,200 + 50,670 = \$166,870$

Gen Revenue:

$(1270 \times 58.10) + (1783 \times 50.67) = 73,787 + 90,344 = \$164,131$

SURPLUS: $166,870 - 164,131 = \$2738$

K factor (R/V2):	0.0001									
Flow from B to A	**Losses (MW)**	**PA (MW)**	**PB (MW)**	**Cost at A ($)**	**Cost at B ($)**	**Total Cost ($)**	**MC at A ($/MWh)**	**MC at B ($/MWh)**	**Payments ($)**	**Revenu ($)**
0	0	2000	1000	100000	25000	125000.00	80.00	35.00	195000.00	195000.
100	1	1900	1101	92150	28637.01	120787.01	77.00	37.02	191020.00	187059.
200	4	1800	1204	84600	32556.16	117156.16	74.00	39.08	187080.00	180252.
300	9	1700	1309	77350	36769.81	114119.81	71.00	41.18	183180.00	174604.
400	16	1600	1416	70400	41290.56	111690.56	68.00	43.32	179320.00	170141.
500	25	1500	1525	63750	46131.25	109881.25	65.00	45.50	175500.00	166887.
600	36	1400	1636	57400	51304.96	108704.96	62.00	47.72	171720.00	164869.
700	49	1300	1749	51350	56825.01	108175.01	59.00	49.98	167980.00	164115.
730	53.29	1270	1783	49594	58550.58	108144.08	58.10	50.67	166865.80	164138.
800	64	1200	1864	45600	62704.96	108304.96	56.00	52.28	164280.00	164649.
900	81	1100	1981	40150	68958.61	109108.61	53.00	54.62	160620.00	166502.
1000	100	1000	2100	35000	75600	110600.00	50.00	57.00	157000.00	169700.

6.15

Nodal prices from 6.8

$\pi_1 = \$13.33$ — consumer

$\pi_3 = \$10.00$ — generator

Gen D wants to sell 100MW at a strike price K = $11.00

$P_A = 80$ MW

$P_B = 0$ MW

$P_C = 40$ MW

$P_D = 400$ MW

Dispatch produced:

$F_{12} = -150$ MW

$F_{13} = -250$ MW

$F_{23} = -150$ MW

contract will be settled:

- Consumer pays → $100 \times 13.33 =$ \$1333.33
  (for extracting 100 MWh at bus 1)

- Gen Receives → $100 \times 10 =$ \$1000
  (for injecting 100 MWh at bus 3)

- Consumer Pays → $100 \times (11 - 10) =$ \$100
  (to gen to settle CFD)

- Consumer who owns the point-to-point financial rights of 100 MWh between 3 and 1 collects
  $100 \times (13.33 - 10) =$ \$333.33

- Consumer pays a total of
  $1333.33 + 100 - 333.33$
  → \$1100 for 100 MWh

− + ...

(price of $11.00 MWh)

Total Rev = Target Rev.

→ hedge is perfect.