4.10

Market for electrical energy that is supplied by two generating companies which cost functions are:

$$CA = 36 * P_A \ \$/h$$
$$CB = 31 * P_B \$/h$$

Inverse demand function can be estimated as

$$\pi = 120 - D \ \$/MWh$$

Cournot model of competition calculate the equilibrium point of this market.

Find the Profit Maximization of Firm A -- the revenue minus its cost.

Price, π depends on the output of both PA and PB. Therefore, we shall get the following equations:

$$\Omega A = \pi * PA - CA * PA$$
$$= \big(120 - (PA + PB)\big) * PA - 36 * PA$$
$$120 * PA - P^2A - PA * PB - 26 * PA$$
$$\Omega A = 84 * PA - P^2A - PA * PB$$

To maximize profit, find derivative w.r.t to PA = 0

$$\frac{d\Omega A}{d\Omega PA} = 0$$

$$84 - 2 * PA - PB = 0$$
$$2 * PA = 84 - PB$$
$$PA = 42 - PB/2$$

For Firm B profit maximization equation, we do the same steps. Find the ΩB:

$$\Omega B = \pi * PB - CB * PB$$
$$\big(120 - (PA + PB)\big) * PB - 32 * PB$$
$$= 120 * PB - PA * PB * P^2B - 32 * PB$$
$$\Omega B = 89 * PB - P^2B - PA * PB$$

Find derivative and set to zero

$$\frac{d\Omega B}{dPB} = 0$$
$$0 = 89 - 2 * PB - PA$$
$$2 * PB = 89 - PA$$
$$PB = (89 - PA)/2$$

Using two equations, substitute to find equilibrium

$$PA = 42 - \frac{\frac{89 - PA}{2}}{2}$$
$$PA = 26.33..$$
$$\mathbf{PA \approx 25 \; MWh}$$

Find PB substitute PA we found

$$PB = \frac{89 - \left(\frac{79}{3}\right)}{2}$$
$$PB = 31.33..$$
$$\mathbf{PB \approx 30MWh}$$

Using rounded values of PA and PB:

$$\text{Total Demand (D)} = PA + PB = 25 + 30 = \mathbf{55MW}$$
$$Market\ Price\ (\pi) = 120 - D = 120 - 55 = \mathbf{65\ \$/MWh}$$
$$Firm\ A\ Profit = (\pi * PA) - (CA * PA) = \mathbf{\$725}$$
$$Firm\ B\ Profit = (\pi * PB) - (CB * PB) = \mathbf{\$1020}$$

Step	Assumed PB	Optimal PA	Assumed PA	Optimal PB	Market Price π
1	0	40(42)	40	25(24.5)	55
2	25	30(29.5)	30	30(29.5)	60
3	30	25(27)	25	30(32)	65

The table shows if Firm B produces 30 MW, Firm As best option would be to produce 25 MW. If Firm A produces 25MW, Firm Bs best option would be two produce 30 MW.

This would be a stable **Nash Equilibrium**