COMP4418
Assignment 2

Z5045582
Yunhe Zhang

Q1.

(a)

% instance
v(1). v(2). v(3). v(4). v(5). v(6).
e(1, 2). e(1, 3). e(1, 4).
e(2, 1). e(2, 4). e(2, 5). e(2, 6).
e(3, 1). e(3, 4). e(3, 5). e(3, 6).
e(4, 1). e(4, 2). e(4, 3). e(4, 5).
e(5, 2). e(5, 3). e(5, 4). e(5, 6).
e(6, 2). e(6, 3). e(6, 5).

% encoding
{c(X) : v(X)} = k.
:- c(X), c(Y), v(X), v(Y),
X!=Y, not e(X,Y).

(b)

the number of 3-cliques is 6.
the number of 4-cliques is 0.
the number of 5-cliques is 0.
the number of 6-cliques is 0.

Q2.

	Reduct P^S	Stable model?
{a,b,c,d}	d:-a. d:-b. d:-c.	no
{a, b, c}	d:-a. d:-b. d:-c.	no
{a, b, d}	d:-a. d:-b. d:-c.	no
{a, c, d}	d:-a. d:-b. d:-c.	no
{b, c, d}	d:-a. d:-b. d:-c.	no
{a, b}	d:-a. d:-b. d:-c.	no

{a, c}	d:-a. d:-b. d:-c.	no
{a, d}	a. d:-a. d:-b. d:-c.	yes
{b, c}	d:-a. d:-b. d:-c.	no
{b, d}	b. d:-a. d:-b. d:-c.	yes
{c, d}	c. d:-a. d:-b. d:-c.	yes
{a}	a. d:-a. d:-b. d:-c.	no
{b}	b. d:-a. d:-b. d:-c.	no
{c}	c. d:-a. d:-b. d:-c.	no
{d}	a. b. c. d:-a. d:-b. d:-c.	no
{}	a. b. c. d:-a. d:-b. d:-c.	no

Q3.

(a)

$KB = \forall x(P(x) \Leftrightarrow x = \#1 \lor x = \#2) \land \forall x(P(x) \Leftrightarrow \neg Q(x))$

$OKB \models K(P(n1) \land Q(n2))$

$\Leftrightarrow \models \|K(P(n1) \land Q(n2))\|_{KB}$

$\Leftrightarrow \models RES[KB, \|P(n1) \land Q(n2)\|_{KB}]$

$\Leftrightarrow \models RES[KB, P(n1) \land Q(n2)]$

$= (n1 = \#1 \land n2 = \#1 \land RES[KB, P(\#1) \land Q(\#1)]) \lor$
$(n1 = \#1 \land n2 = \#2 \land RES[KB, P(\#1) \land Q(\#2)]) \lor$
$(n1 = \#1 \land n2 = \#3 \land RES[KB, P(\#1) \land Q(\#3)]) \lor$
$\ldots\ldots$
$(n1 = \#2 \land n2 = \#1 \land RES[KB, P(\#2) \land Q(\#1)]) \lor$
$(n1 = \#2 \land n2 = \#2 \land RES[KB, P(\#2) \land Q(\#2)]) \lor$
$(n1 = \#2 \land n2 = \#3 \land RES[KB, P(\#2) \land Q(\#3)]) \lor$
$\ldots\ldots$
$(n1 = \#3 \land n2 = \#1 \land RES[KB, P(\#3) \land Q(\#1)]) \lor$
$(n1 = \#3 \land n2 = \#2 \land RES[KB, P(\#3) \land Q(\#2)]) \lor$
$(n1 = \#3 \land n2 = \#3 \land RES[KB, P(\#2) \land Q(\#2)]) \lor$
$\ldots\ldots$
$(n1 \neq \#1 \land n1 \neq \#2 \land \ldots \land n2 \neq \#1 \land n2 \neq \#2 \land \ldots \land RES[KB, P(n1') \land Q(n2')])$

$= (n1 = \#1 \land n2 = \#1 \land FALSE) \lor$
$(n1 = \#1 \land n2 = \#2 \land FALSE) \lor$
$(n1 = \#1 \land n2 = \#3 \land TRUE) \lor$

$\quad\quad\quad$ ......(all TRUE in this part)

$(n1 = \#2 \wedge n2 = \#1 \wedge FALSE) \vee$
$(n1 = \#2 \wedge n2 = \#2 \wedge FALSE) \vee$
$(n1 = \#2 \wedge n2 = \#3 \wedge TRUE) \vee$

$\quad\quad\quad$ ......(all TRUE in this part)

$(n1 = \#3 \wedge n2 = \#1 \wedge FALSE) \vee$
$(n1 = \#3 \wedge n2 = \#2 \wedge FALSE) \vee$
$(n1 = \#3 \wedge n2 = \#3 \wedge FALSE) \vee$

$\quad\quad\quad$ ......(all FALSE in this part)

$(n1 \neq \#1 \wedge n1 \neq \#2 \wedge \ldots \wedge n2 \neq \#1 \wedge n2 \neq \#2 \wedge \ldots \wedge FALSE)$

$= (n1 = \#1 \wedge n2 = \#3) \vee (n1 = \#1 \wedge n2 = \#4) \vee \ldots (n1 = \#2 \wedge n2 = \#3) \vee (n1 = \#2 \wedge n2 = \#4) \vee \ldots$

$\models (n1 , n2) \in \{\# 1, \# 2\} \times \{\# 3, \#4 . . .\}$

thus known instances is $(n1 , n2) \in \{\# 1, \# 2\} \times \{\# 3, \#4 . . .\}$

(b)

$KB = \forall x(P(x) \Leftrightarrow x = \#1 \vee x = \#2) \wedge \forall x(P(x) \Leftrightarrow \neg Q(x))$

$RES[KB, P(x) \wedge \neg Q(y)]$

$= (x = \#1 \wedge y = \#1 \wedge RES[KB, P(\#1) \wedge \neg Q(\#1)]) \vee$
$(x = \#1 \wedge y = \#2 \wedge RES[KB, P(\#1) \wedge \neg Q(\#2)]) \vee$
$(x = \#2 \wedge y = \#1 \wedge RES[KB, P(\#2) \wedge \neg Q(\#1)]) \vee$
$(x = \#2 \wedge y = \#2 \wedge RES[KB, P(\#2) \wedge \neg Q(\#2)]) \vee$
$(x \neq \#1 \wedge x \neq \#2 \wedge y \neq \#1 \wedge y \neq \#2 \wedge RES[KB, P(x') \wedge \neg Q(y')])$
$= (x = \#1 \wedge y = \#1 \wedge TRUE) \vee$
$(x = \#1 \wedge y = \#2 \wedge TRUE) \vee$
$(x = \#2 \wedge y = \#1 \wedge TRUE) \vee$
$(x = \#2 \wedge y = \#2 \wedge TRUE) \vee$
$(x \neq \#1 \wedge x \neq \#2 \wedge y \neq \#1 \wedge y \neq \#2 \wedge FALSE)$
$= (x = \#1 \wedge y = \#1) \vee (x = \#1 \wedge y = \#2) \vee (x = \#2 \wedge y = \#1) \vee (x = \#2 \wedge y = \#2)$

Q4.

(a)

$(F \rightarrow C) \wedge (C \leftrightarrow \neg U)$

$\Leftrightarrow (\neg F \vee C) \wedge ((C \rightarrow \neg U) \wedge (\neg U \rightarrow C))$

$\Leftrightarrow (\neg F \vee C) \wedge (\neg C \vee \neg U) \wedge (U \vee C)$

(b)

let $s = OKB = O(\neg F \vee C) \wedge (\neg C \vee \neg U) \wedge (U \vee C)$

$UP^+(s) = \{(\neg F \vee C), (\neg C \vee \neg U), (U \vee C)\}$

If $k = 0$

$s \mid\approx K_0 (F \vee U \vee C)$

$\Leftrightarrow$ s is obviously inconsistent or $s \mid\approx (F \vee U \vee C)$

$\Leftrightarrow$ s is obviously inconsistent or $(F \vee U \vee C) \in UP^+(s)$

$U \vee C$ subsumes $F \vee U \vee C$, so get $(F \vee U \vee C) \in UP^+(s)$

Therefore $s \mid\approx K_0 (F \vee U \vee C)$ comes out true.

so minimal k is 0

(c)

let $s = OKB = O(\neg F \vee C) \wedge (\neg C \vee \neg U) \wedge (U \vee C)$

$s \mid\approx \neg K_k \neg(\neg F \wedge \neg U) \Leftrightarrow s \mid\approx M_k(\neg F \wedge \neg U)$

If $k = 0$

$s \mid\approx M_0(\neg F \wedge \neg U)$

$\Leftrightarrow$ s is obviously consistent and $s \mid\approx (\neg F \wedge \neg U)$

$\Leftrightarrow$ s is obviously consistent and $s \mid\approx \neg F$ and $s \mid\approx \neg U$

$\Leftrightarrow$ s is obviously consistent and $\neg F \in UP^+(s)$ and $\neg U \in UP^+(s)$

$UP^+(s) = \{(\neg F \vee C), (\neg C \vee \neg U), (U \vee C)\}$

$UP^-(s) = \{(\neg F \vee C), (\neg C \vee \neg U), (U \vee C)\}$

since s is not obviously consistent (C and U occurs pos and neg in $UP^-(s)$) and

also $\neg F \notin UP^+(s)$ and $\neg U \notin UP^+(s)$

so cannot get $s \mid\approx M_0(\neg F \wedge \neg U)$ is true

If $k = 1$

$s \mid\approx M_1(\neg F \wedge \neg U)$ for some literal L, $s \cup \{L\} \mid\approx M_0(\neg F \wedge \neg U)$

let L be $\neg F$

$s \mid\approx M_1(\neg F \wedge \neg U)$

$\Leftrightarrow s \cup \{\neg F\} \mid\approx M_0(\neg F \wedge \neg U)$

$\Leftrightarrow s \cup \{\neg F\}$ is obviously consistent and $s \cup \{\neg F\} \mid\approx (\neg F \wedge \neg U)$

$\Leftrightarrow s \cup \{\neg F\}$ is obviously consistent and $s \cup \{\neg F\} \mid\approx \neg F$ and $s \cup \{\neg F\} \mid\approx \neg U$

$\Leftrightarrow s \cup \{\neg F\}$ is obviously consistent and $\neg F \in UP^+(s \cup \{\neg F\})$ and $\neg U \in UP^+(s \cup \{\neg F\})$

$UP^+(s \cup \{\neg F\}) = \{(\neg F \vee C), (\neg C \vee \neg U), (U \vee C), \neg F\} \cup \{c \mid c \supseteq \neg F\}$

$UP^-(s \cup \{\neg F\}) = \{\neg F\}$

$s \cup \{\neg F\}$ is obviously consistent and $\neg F \in UP^+(s \cup \{\neg F\})$

but $\neg U \notin UP^+(s \cup \{\neg F\})$

so cannot get $s \cup \{\neg F\} \mid\approx (\neg F \wedge \neg U)$ is true

similar to other possible literals like $\neg U$, U, C, $\neg C$, F, all of them cannot get true

so cannot get $s \mid\approx M_1(\neg F \wedge \neg U)$ is true

If $k = 2$

$s \mid\approx M_2(\neg F \wedge \neg U)$

$\Leftrightarrow s \cup \{\neg F\} \mid\approx M_1(\neg F \wedge \neg U)$

$\Leftrightarrow s \cup \{\neg F\} \cup \{\neg U\} \mid\approx M_0(\neg F \wedge \neg U)$

$\Leftrightarrow s \cup \{\neg F\} \cup \{\neg U\}$ is obviously consistent and $s \cup \{\neg F\} \cup \{\neg U\} \mid\approx (\neg F \wedge \neg U)$

$\Leftrightarrow s \cup \{\neg F\} \cup \{\neg U\}$ is obviously consistent

and $s \cup \{\neg F\} \cup \{\neg U\} \mid\approx \neg F$

and $s \cup \{\neg F\} \cup \{\neg U\} \mid\approx \neg U$

⇔ s∪{¬F}∪{¬U} is obviously consistent

and ¬F ∈ UP^+(s∪{¬F}∪{¬U})

and ¬U ∈ UP^+(s∪{¬F}∪{¬U})

UP^+(s∪{¬F}∪{¬U}) = {(¬F ∨ C), (¬ C ∨ ¬U), (U ∨ C), ¬F, ¬U, C}

∪ {c| c ⊇ ¬F or c ⊇ ¬U or c ⊇ C}

UP^-(s∪{¬F}∪{¬U}) = {¬F, ¬U, C}

s∪{¬F} is obviously consistent

and ¬F ∈ UP^+(s∪{¬F}∪{¬U})

and ¬U∈ UP^+(s∪{¬F}∪{¬U})

Therefore s |≈ M_2(¬F ∧ ¬U) comes out true which means s|≈ ¬K_2¬(¬F ∧ ¬U) is true

so minimal k is 2

Q5.

(a)

$\sum_0 \land \sum_{dyn} \models \forall x[p(x)]I(x)$

$\Leftrightarrow \sum_0 \models \forall x R[\langle\rangle, [p(x)]I(x)]$

$\Leftrightarrow \sum_0 \models \forall x R[p(x), I(x)]$

$\Leftrightarrow \sum_0 \models \forall x R[\langle\rangle, \gamma_{I}{}_{p(x)}^{a}{}_{x}^{x}]$

$\Leftrightarrow \sum_0 \models \forall x(p(x) = p(x) \lor I(x))$

$\forall x(p(x) = p(x))$ is valid

$\Leftrightarrow \sum_0 \models \forall x(TRUE \lor I(x))$

$\Leftrightarrow \sum_0 \models TRUE$

Prove

(b)

$\sum_0 \land \sum_{dyn} \models \forall y[m(y)]\ \forall x(I(x) \rightarrow I(x) = y)$

$\Leftrightarrow \sum_0 \models \forall y R[\langle\rangle, [m(y)]\ \forall x(I(x) \rightarrow I(x) = y)]$

⇔ $\sum_0$ |= ∀yR[m(y), ∀x(I(x) → l(x) = y)]

⇔ $\sum_0$ |= ∀yR[m(y), ∀x(¬I(x) ∨ l(x) = y)]

⇔ $\sum_0$ |= ∀y∀xR[m(y), ¬I(x)] ∨ ∀y∀xR[m(y), *l*(x) = y]

⇔ $\sum_0$ |= ∀y∀x¬R[m(y), I(x)] ∨ ∀y∀xR[m(y), *l*(x) = y]

⇔ $\sum_0$ |= ∀y∀x¬R[⟨⟩, $\gamma_{I}{}^{a}_{m(y)}{}^{x}_{x}$] ∨ ∀y∀xR[⟨⟩, $\gamma_{l}{}^{a}_{m(y)}{}^{x}_{x}$]

⇔ $\sum_0$ |= ∀y∀x¬(m(y) = p(x) ∨ I(x)) ∨ ∀y∀xR[⟨⟩,$\gamma_{l}{}^{a}_{m(y)}{}^{x}_{x}$]

⇔$\sum_0$ |= ∀y∀x¬(m(y) = p(x) ∨ I(x)) ∨ ∀y∀xR[⟨⟩,$\gamma_{l}{}^{a}_{m(y)}{}^{x}_{x}$]

∀y∀x(m(y) = p(y)) is invalid / ∀x(I(x)) is invalid ($\sum_0$ = {∃xI(x)})

⇔$\sum_0$ |= ∀y∀x¬(FALSE ∨ FALSE) ∨ ∀y∀xR[⟨⟩,$\gamma_{l}{}^{a}_{m(y)}{}^{x}_{x}$]

⇔$\sum_0$ |= ∀y∀x¬FALSE ∨ ∀y∀xR[⟨⟩,$\gamma_{l}{}^{a}_{m(y)}{}^{x}_{x}$]

⇔$\sum_0$ |= TRUE ∨ ∀y∀xR[⟨⟩,$\gamma_{l}{}^{a}_{m(y)}{}^{x}_{x}$]

⇔$\sum_0$ |= TRUE

Prove

(c)

$\sum_0$ ∧ $\sum_{dyn}$ ∧ **O**$\sum_{dyn}$ |= [s]**K**∃xI(x)

⇔ $\sum_0$ |= R[⟨⟩, [s]**K**∃xI(x)]

⇔ $\sum_0$ |= R[s, **K**∃xI(x)]

⇔ $\sum_0$ |= R[s, (SF(s) → **K**∃x(SF(s) → I(x)))] ∧ R[s, (¬SF(s) → **K**∃x(¬SF(s) → I(x)))]

⇔ $\sum_0$ |= R[s, ¬SF(s) ∨ **K**∃x(¬SF(s) ∨ I(x))]] ∧ R[s, (¬SF(s) → **K**∃x(¬SF(s) → I(x)))]

⇔ $\sum_0$ |= (¬R[s, SF(s)] ∨ **K**∃x(¬R[s, SF(s)] ∨ R[s, I(x)]])) ∧

(R[s, SF(s)] ∨ **K**∃x(R[s, SF(s)] ∨ R[s, I(x)]]))

⇔ $\sum_0$ |= (¬R[⟨⟩, ϕ^{a}_{s}] ∨ **K**∃x(¬R[⟨⟩, ϕ^{a}_{s}] ∨ R[⟨⟩, $\gamma_{I}{}^{a}_{s}{}^{x}_{x}$]])) ∧

(R[⟨⟩, ϕ^{a}_{s}] ∨ **K**∃x(R[⟨⟩, ϕ^{a}_{s}] ∨ R[⟨⟩,$\gamma_{I}{}^{a}_{s}{}^{x}_{x}$]]))

⇔ $\sum_0$ |= (¬(s = s → ∃xI(x)) ∨ **K**∃x(¬(s = s → ∃xI(x))∨ (s = p(x) ∨ I(x)))) ∧

((s = s → ∃xI(x)) ∨ **K**∃x((s = s → ∃xI(x)) ∨ (s = p(x) ∨ I(x))))

$\Leftrightarrow \sum_0 \models (\neg(s = s \rightarrow \exists x I(x)) \vee \mathbf{K}\exists x(\neg(s = s \rightarrow \exists x I(x)) \vee (s = p(x) \vee I(x)))) \wedge$

$((s = s \rightarrow \exists x I(x)) \vee \mathbf{K}\exists x((s = s \rightarrow \exists x I(x)) \vee (s = p(x) \vee I(x))))$

$(s = s \rightarrow \exists x I(x))$ is valid ($\sum_0 = \{\exists x I(x)\}$)

$\Leftrightarrow \sum_0 \models (\neg TRUE \vee \mathbf{K}\exists x(\neg TRUE \vee (s = p(x) \vee I(x)))) \wedge$

$(TRUE \vee \mathbf{K}\exists x(TRUE \vee (s = p(x) \vee I(x))))$

$\Leftrightarrow \sum_0 \models \mathbf{K}\exists x(s = p(x) \vee I(x)) \wedge TRUE$

$\Leftrightarrow \sum_0 \models \mathbf{K}\exists x(s = p(x) \vee I(x))$

$\exists x(s = p(x))$ is invalid

$\Leftrightarrow \sum_0 \models \mathbf{K}\exists x I(x)$

it means I know that some x is in the box, so it's valid

$\Leftrightarrow \sum_0 \models TRUE$

Prove

(d)

$\sum_0 \wedge \sum_{dyn} \wedge \mathbf{O}\sum_{dyn} \models [s]\exists x\mathbf{K}I(x)$

$\Leftrightarrow \sum_0 \models R[\langle\rangle, [s]\exists x\mathbf{K}I(x)]$

$\Leftrightarrow \sum_0 \models R[s, \exists x\mathbf{K}I(x)]$

$\Leftrightarrow \sum_0 \models R[s, (SF(s) \rightarrow \exists x\mathbf{K}(SF(s) \rightarrow I(x)))] \wedge R[s, (\neg SF(s) \rightarrow \exists x\mathbf{K}(\neg SF(s) \rightarrow I(x)))]$

$\Leftrightarrow \sum_0 \models R[s, \neg SF(s) \vee \exists x\mathbf{K}(\neg SF(s) \vee I(x))]] \wedge R[s, (\neg SF(s) \rightarrow \exists x\mathbf{K}(\neg SF(s) \rightarrow I(x)))]$

$\Leftrightarrow \sum_0 \models (\neg R[s, SF(s)] \vee \exists x\mathbf{K}(\neg R[s, SF(s)] \vee R[s, I(x)])) \wedge$

$(R[s, SF(s)] \vee \exists x\mathbf{K}(R[s, SF(s)] \vee R[s, I(x)]))$

$\Leftrightarrow \sum_0 \models (\neg R[\langle\rangle, \phi^a_s] \vee \exists x\mathbf{K}(\neg R[\langle\rangle, \phi^a_s] \vee R[\langle\rangle, \gamma_I{}^{a\,x}_{s\,x}])) \wedge$

$(R[\langle\rangle, \phi^a_s] \vee \exists x\mathbf{K}(R[\langle\rangle, \phi^a_s] \vee R[\langle\rangle, \gamma_I{}^{a\,x}_{s\,x}]))$

$\Leftrightarrow \sum_0 \models (\neg(s = s \rightarrow \exists x I(x)) \vee \exists x\mathbf{K}(\neg(s = s \rightarrow \exists x I(x)) \vee (s = p(x) \vee I(x)))) \wedge$

$((s = s \rightarrow \exists x I(x)) \vee \exists x\mathbf{K}((s = s \rightarrow \exists x I(x)) \vee (s = p(x) \vee I(x))))$

$\Leftrightarrow \sum_0 \models (\neg(s = s \rightarrow \exists x I(x)) \vee \exists x\mathbf{K}(\neg(s = s \rightarrow \exists x I(x)) \vee (s = p(x) \vee I(x)))) \wedge$

$((s = s \rightarrow \exists x I(x)) \vee \exists x \mathbf{K}((s = s \rightarrow \exists x I(x)) \vee (s = p(x) \vee I(x))))$

$(s = s \rightarrow \exists x I(x))$ is valid ($\sum_0 = \{\exists x I(x)\}$)

$\Leftrightarrow \sum_0 \models (\neg TRUE \vee \exists x \mathbf{K}(\neg TRUE \vee (s = p(x) \vee I(x)))) \wedge$

$(TRUE \vee \exists x \mathbf{K}(TRUE \vee (s = p(x) \vee I(x))))$

$\Leftrightarrow \sum_0 \models \exists x \mathbf{K}(s = p(x) \vee I(x)) \wedge TRUE$

$\Leftrightarrow \sum_0 \models \exists x \mathbf{K}(s = p(x) \vee I(x))$

$s = p(x)$ is invalid

$\Leftrightarrow \sum_0 \models \exists x \mathbf{K} I(x)$

$\exists x \mathbf{K} I(x)$ means I know which x is in the box, so it's invalid

$\Leftrightarrow \sum_0 \models FALSE$

Disprove