COMP4418, 2017–Assignment 1

Yunhe Zhang
Z5045582

1.

(a) $p \lor (q \land r) \models (p \lor q) \land (p \lor r)$

p	q	r	$q \land r$	$p \lor (q \land r)$	$p \lor q$	$p \lor r$	$(p \lor q) \land (p \lor r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

In all rows where $p \lor (q \land r)$ is true, $(p \lor q) \land (p \lor r)$ is also true.
Therefore, inference is valid

(b) $\models p \rightarrow (q \rightarrow p)$

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

In all rows $p \rightarrow (q \rightarrow p)$ is true.
Therefore, inference is valid

(c) $p \rightarrow q \models \neg p \rightarrow \neg q$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

In 3^{rd} row, when the $p \rightarrow q$ is true, $\neg p \rightarrow \neg q$ is false.
Therefore, inference is not valid.

(d) $p \rightarrow q, \neg p \rightarrow \neg q \models \neg p \leftrightarrow \neg q$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$	$\neg p \leftrightarrow \neg q$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

In all rows, where both $p \rightarrow q$ and $\neg p \rightarrow \neg q$ are true, $\neg p \leftrightarrow \neg q$ is also true.

Therefore, inference is valid.

(e) $\neg q \rightarrow \neg p, \neg r \rightarrow \neg q \models p \rightarrow r$

q	p	r	$\neg q$	$\neg p$	$\neg r$	$\neg q \rightarrow \neg p$	$\neg r \rightarrow \neg q$	$p \rightarrow r$
T	T	T	F	F	F	T	T	T
T	T	F	F	F	T	T	F	F
T	F	T	F	T	F	T	T	T
T	F	F	F	T	T	T	F	T
F	T	T	T	F	F	F	T	T
F	T	F	T	F	T	F	T	F
F	F	T	T	T	F	T	T	T
F	F	F	T	T	T	T	T	T

In all rows, where both $\neg q \rightarrow \neg p$ and $\neg r \rightarrow \neg q$ are true, $p \rightarrow r$ is also true.
Therefore, inference is valid.

(f) $p \wedge (q \vee r) \vdash (p \wedge q) \vee (p \wedge r)$

Convert premises into CNF:
$p \wedge (q \vee r)$

Convert negated conclusion into CNF:
$\neg((p \wedge q) \vee (p \wedge r)) \equiv (\neg p \vee \neg q) \wedge (\neg p \vee \neg r)$

Proof:

1. p [Premises]
2. $q \vee r$ [Premises]
3. $\neg p \vee \neg q$ [¬ Conclusion]
4. $\neg p \vee \neg r$ [¬ Conclusion]
5. $\neg p \vee r$ [2, 3. Resolution]
6. $\neg p$ [4, 5. Resolution]
7. [] [1, 6. Resolution]

Therefore, inferences hold in propositional logic.

(g) $p \vdash p \rightarrow q$

Convert premises into CNF:
p

Convert negated conclusion into CNF:
$\neg(p \rightarrow q) \equiv \neg(\neg p \vee q)$
$\equiv p \wedge \neg q$

Proof:

1. p [Premises]
2. p [¬ Conclusion]
3. $\neg q$ [¬ Conclusion]

Cannot obtain empty clause using resolution, therefore inferences not hold in propositional logic.

(h) $p \leftrightarrow q \vdash (q \leftrightarrow r) \rightarrow (p \leftrightarrow r)$

Convert premises into CNF:

$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (p \vee \neg q)$

Convert negated conclusion into CNF:

$$
\begin{aligned}
\neg((q \leftrightarrow r) \rightarrow (p \leftrightarrow r)) &\equiv \neg(\neg(q \leftrightarrow r) \vee (p \leftrightarrow r)) \\
&\equiv (q \leftrightarrow r) \wedge \neg(p \leftrightarrow r) \\
&\equiv ((\neg q \vee r) \wedge (q \vee \neg r)) \wedge \neg((\neg p \vee r) \wedge (p \vee \neg r)) \\
&\equiv ((\neg q \vee r) \wedge (q \vee \neg r)) \wedge ((p \wedge \neg r) \vee (\neg p \wedge r)) \\
&\equiv ((\neg q \vee r) \wedge (q \vee \neg r)) \wedge (((p \wedge \neg r) \vee \neg p) \wedge ((p \wedge \neg r) \vee r))) \\
&\equiv ((\neg q \vee r) \wedge (q \vee \neg r)) \wedge (((p \vee \neg p) \wedge (\neg r \vee \neg p)) \wedge ((p \vee r) \wedge (\neg r \vee r))) \\
&\equiv ((\neg q \vee r) \wedge (q \vee \neg r)) \wedge ((\neg r \vee \neg p) \wedge (p \vee r)) \\
&\equiv (\neg q \vee r) \wedge (q \vee \neg r) \wedge (\neg r \vee \neg p) \wedge (p \vee r)
\end{aligned}
$$

Proof:

1.	$\neg p \vee q$	[Premises]
2.	$p \vee \neg q$	[Premises]
3.	$\neg q \vee r$	[¬ Conclusion]
4.	$q \vee \neg r$	[¬ Conclusion]
5.	$\neg r \vee \neg p$	[¬ Conclusion]
6.	$p \vee r$	[¬ Conclusion]
7.	$\neg p \vee r$	[1, 3. Resolution]
8.	$\neg p$	[5, 7. Resolution]
9.	$p \vee \neg r$	[2, 4. Resolution]
10.	p	[6, 9. Resolution]
11.	[]	[8, 10. Resolution]

Therefore, inferences hold in propositional logic.

(i) $\neg p \wedge \neg q \vdash p \leftrightarrow q$

Convert premises into CNF:

$\neg p \wedge \neg q$

Convert negated conclusion into CNF:

$$
\begin{aligned}
\neg(p \leftrightarrow q) &\equiv \neg((\neg p \vee q) \wedge (p \vee \neg q)) \\
&\equiv ((p \wedge \neg q) \vee (\neg p \wedge q)) \\
&\equiv ((p \wedge \neg q) \vee \neg p) \wedge ((p \wedge \neg q) \vee q) \\
&\equiv ((p \vee \neg p) \wedge (\neg q \vee \neg p)) \wedge ((p \vee q) \wedge (\neg q \vee q)) \\
&\equiv (\neg q \vee \neg p) \wedge (p \vee q)
\end{aligned}
$$

Proof:

1.	$\neg p$	[Premises]
2.	$\neg q$	[Premises]

3. $\neg q \lor \neg p$ [¬ Conclusion]
4. $p \lor q$ [¬ Conclusion]
5. q [1, 4. Resolution]
6. [] [2, 5. Resolution]

Therefore, inferences hold in propositional logic.

(j) $\neg q \rightarrow \neg p, \neg r \rightarrow \neg q \vdash p \rightarrow r$

Convert premises into CNF:

$\neg q \rightarrow \neg p \equiv q \lor \neg p$

$\neg r \rightarrow \neg q \equiv r \lor \neg q$

Convert negated conclusion into CNF:

$\neg(p \rightarrow r) \equiv \neg(\neg p \lor r)$

$\equiv p \land \neg r$

Proof:

1. $q \lor \neg p$ [Premises]
2. $r \lor \neg q$ [Premises]
3. p [¬ Conclusion]
4. $\neg r$ [¬ Conclusion]
5. q [1, 3. Resolution]
6. r [2, 5. Resolution]
7. [] [4, 6. Resolution]

Therefore, inferences hold in propositional logic.

2.

(a)

'$<(x, y)$' relation means 'x smaller than y'

1. $\forall x[((age(x, age2) \land colour(x, green)) \rightarrow \exists y(age(y, age1) \land (y=huey) \land <(age1, age2))]$

2. $\forall x[age(x, 5) \rightarrow design(x, camel)]$

3. $\exists x[(x=dewey) \land colour(x, yellow)]$

4. $\exists x[(x=louie) \land design(x, giraffe)]$

5. $\forall x[design(x, panda) \rightarrow \neg colour(x, white)]$

(b)

KB = { $\forall x[((age(x, age2) \land colour(x, green)) \rightarrow \exists y(age(y, age1) \land (y=huey) \land <(age1, age2)))$
$\forall x[age(x, 5) \rightarrow design(x, camel)]$
$\exists x[(x=dewey) \land colour(x, yellow)]$
$\exists x[(x=louie) \land design(x, giraffe)]$
$\forall x[design(x, panda) \rightarrow \neg colour(x, white)]$
$\exists x \exists y \exists z[age(x, 4) \land age(y, 5) \land age(z, 6)]$
$\exists x \exists y \exists z[colour(x, green) \land colour(y, yellow) \land colour(z, white)]$
$\exists x \exists y \exists z[design(x, panda) \land design(y, giraffe) \land design(z, camel)]$
}

1.Let ***I*** **|= KB**

2.Then ***I*** |= $\forall x[((age(x, age2) \land colour(x, green)) \rightarrow \exists y(age(y, age1) \land (y=huey) \land <(age1, age2)))$
(From KB, Huey is younger than the boy in the green T-shirt)

3.So ***I*** |= $\exists x[(x=huey) \land \neg colour(x, green)]$
(Conclude Huey's tee-shirt was not green)

4.Also ***I*** |= $\exists x[(x=dewey) \land colour(x, yellow)]$
(From KB, Dewey's T-shirt was yellow)

5.And ***I*** |= $\exists x \exists y \exists z[colour(x, green) \land colour(y, yellow) \land colour (z, white)]$
(From KB, 3 boys wear T-shirt of different colour)

6.So ***I*** |= $\exists x[(x=huey) \land colour(x, white)]$, $\exists x[(x=louie) \land colour(x, green)]$
(3 boys wear T-shirt of different colour and Dewey's T-shirt was yellow, also Huey's T-shirt is not green, thus, Huey's T-shirt can only be white, and the rest boy Louie wear green T-shirt.
Conclude Huey's T-shirt was white; Louie's T-shirt was green)

7.Also $\boldsymbol{I} \models \forall x[design(x, panda) \rightarrow \neg colour(x, white)]$
(From KB, the panda design was not featured on the white T-shirt)

8.And $\boldsymbol{I} \models \exists x[(x=louie) \land design(x, giraffe)]$
(From KB, Louie's tee-shirt bore the giraffe design)

9.And $\boldsymbol{I} \models \exists x \exists y \exists z[design(x, panda) \land design(y, giraffe) \land design(z, camel)]$
(From KB, 3 boys wear T-shirt with a different design)

10.So $\boldsymbol{I} \models \exists x[(x=dewey) \land design(x, panda)], \exists x[(x=huey) \land design(x, camel)]$
(3 boys wear T-shirt of different colour, panda design was not in white colour, so panda design T-shirt can only in green or yellow.
Louie's T-shirt is green and with giraffe design, also 3 boys wear T-shirt with a different design, thus, panda design was in yellow which is Dewey's T-shirt's colour, and the rest boy Huey, his T-shirt is camel design.
Conclude Dewey's T-shirt was panda design; Huey's T-shirt was camel design.)

11.Also $\boldsymbol{I} \models \forall x[age(x, 5) \rightarrow design(x, camel)]$
(From KB, the five-year-old wore the tee-shirt with the camel design)

12.And $\boldsymbol{I} \models \exists x \exists y \exists z[age(x, 4) \land age(y, 5) \land age(z, 6)]$
(From KB, 3 boys aged 4, 5 and 6)

13.So $\boldsymbol{I} \models \forall x[design(x, camel) \rightarrow age(x, 5)]$
(Since 3 boys aged 4, 5 and 6 and their T-shirts are with different design, from the KB 'five-year-old wore the tee-shirt with the camel design', we can conclude that the boy who wore the T-shirt with camel design is five-year-old)

14.So $\boldsymbol{I} \models \exists x[(x=huey) \land age(x, 5)]$
(Since the boy who wear the T-shirt with camel design is five-year-old and Huey's T-shirt design was camel, thus Huey is five-year-old.
Conclude Huey's age is 5)

15.So $\boldsymbol{I} \models \exists x[(x=louie) \land age(x, age1) \land <(5, age1)]$
(Since Huey is younger than the boy in the green tee-shirt and Huey's age is 5 and Louie's T-shirt is green, Louie's age is greater than 5.
Conclude Louie's age is greater than 5)

16.So $\boldsymbol{I} \models \exists x[(x=louie) \land age(x, 6)], \exists x[(x=dewey) \land age(x, 4)]$
(Since Louie's age is greater than 5 and 3 boys aged 4, 5 and 6, Louie can only be 6 years old, thus the rest boy Dewey is 4 years old.
Conclude Louie's age is 5; Dewey's age is 4)

Now we have a conclusion:

17. $\boldsymbol{I} \models \exists x \exists y \exists z[(x=dewey) \land age(x, 4) \land colour(x, yellow) \land design(x, panda)$

$\wedge$(y=huey)$\wedge$age(y, 5)$\wedge$colour(y, white)$\wedge$design(y,camel)
$\wedge$(z=louie)$\wedge$age(z, 6)$\wedge$colour(z, green)$\wedge$design(z, giraffe)]

So, my answer is yes, it is possible to conclude the age of each boy together with the colour and design of the tee-shirt they're wearing, above is the proof and get conclusion that
Dewey is *4* years old, his T-shirt was *yellow* and *panda* design;
Huey is *5* years old, his T-shirt was *white* and *camel* design;
Louie is *6* years old, his T-shirt was *green* and *giraffe* design.

(c)

My answer in (b) is yes