COMP4418, 2017–Assignment 1

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1.

(a) $p \vee (q \wedge r) \models (p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

In all rows where $p \vee (q \wedge r)$ is true, $(p \vee q) \wedge (p \vee r)$ is also true.
Therefore, inference is valid

(b) $\models p \rightarrow (q \rightarrow p)$

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

In all rows $p \rightarrow (q \rightarrow p)$ is true.
Therefore, inference is valid

(c) $p \rightarrow q \models \neg p \rightarrow \neg q$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

In 3rd row, when the $p \rightarrow q$ is true, $\neg p \rightarrow \neg q$ is false.
Therefore, inference is not valid.

(d) $p \rightarrow q, \neg p \rightarrow \neg q \models \neg p \leftrightarrow \neg q$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$	$\neg p \leftrightarrow \neg q$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

In all rows, where both $p \rightarrow q$ and $\neg p \rightarrow \neg q$ are true, $\neg p \leftrightarrow \neg q$ is also true.

Therefore, inference is valid.

(e) $\neg q \rightarrow \neg p, \neg r \rightarrow \neg q \models p \rightarrow r$

q	p	r	¬q	¬p	¬r	¬q → ¬p	¬r → ¬q	p → r
T	T	T	F	F	F	T	T	T
T	T	F	F	F	T	T	F	F
T	F	T	F	T	F	T	T	T
T	F	F	F	T	T	T	F	T
F	T	T	T	F	F	F	T	T
F	T	F	T	F	T	F	T	F
F	F	T	T	T	F	T	T	T
F	F	F	T	T	T	T	T	T

In all rows, where both $\neg q \rightarrow \neg p$ and $\neg r \rightarrow \neg q$ are true, $p \rightarrow r$ is also true.
Therefore, inference is valid.

(f) $p \wedge (q \vee r) \vdash (p \wedge q) \vee (p \wedge r)$

Convert premises into CNF:
$p \wedge (q \vee r)$

Convert negated conclusion into CNF:
$\neg((p \wedge q) \vee (p \wedge r)) \equiv (\neg p \vee \neg q) \wedge (\neg p \vee \neg r)$

Proof:

1. p [Premises]
2. $q \vee r$ [Premises]
3. $\neg p \vee \neg q$ [¬ Conclusion]
4. $\neg p \vee \neg r$ [¬ Conclusion]
5. $\neg p \vee r$ [2, 3. Resolution]
6. $\neg p$ [4, 5. Resolution]
7. [] [1, 6. Resolution]

Therefore, inferences hold in propositional logic.

(g) $p \vdash p \rightarrow q$

Convert premises into CNF:
p

Convert negated conclusion into CNF:
$\neg(p \rightarrow q) \equiv \neg(\neg p \vee q)$
$\equiv p \wedge \neg q$

Proof:

1. p [Premises]
2. p [¬ Conclusion]
3. $\neg q$ [¬ Conclusion]

Cannot obtain empty clause using resolution, therefore inferences not hold in propositional logic.

(h) $p \leftrightarrow q \vdash (q \leftrightarrow r) \rightarrow (p \leftrightarrow r)$

Convert premises into CNF:

$p \leftrightarrow q \equiv (\neg p \lor q) \land (p \lor \neg q)$

Convert negated conclusion into CNF:

$$
\begin{aligned}
\neg((q \leftrightarrow r) \rightarrow (p \leftrightarrow r)) &\equiv \neg(\neg(q \leftrightarrow r) \lor (p \leftrightarrow r)) \\
&\equiv (q \leftrightarrow r) \land \neg(p \leftrightarrow r) \\
&\equiv ((\neg q \lor r) \land (q \lor \neg r)) \land \neg((\neg p \lor r) \land (p \lor \neg r)) \\
&\equiv ((\neg q \lor r) \land (q \lor \neg r)) \land ((p \land \neg r) \lor (\neg p \land r)) \\
&\equiv ((\neg q \lor r) \land (q \lor \neg r)) \land (((p \land \neg r) \lor \neg p) \land ((p \land \neg r) \lor r))) \\
&\equiv ((\neg q \lor r) \land (q \lor \neg r)) \land (((p \lor \neg p) \land (\neg r \lor \neg p)) \land ((p \lor r) \land (\neg r \lor r))) \\
&\equiv ((\neg q \lor r) \land (q \lor \neg r)) \land ((\neg r \lor \neg p) \land (p \lor r)) \\
&\equiv (\neg q \lor r) \land (q \lor \neg r) \land (\neg r \lor \neg p) \land (p \lor r)
\end{aligned}
$$

Proof:

1.	$\neg p \lor q$	[Premises]
2.	$p \lor \neg q$	[Premises]
3.	$\neg q \lor r$	[¬ Conclusion]
4.	$q \lor \neg r$	[¬ Conclusion]
5.	$\neg r \lor \neg p$	[¬ Conclusion]
6.	$p \lor r$	[¬ Conclusion]
7.	$\neg p \lor r$	[1, 3. Resolution]
8.	$\neg p$	[5, 7. Resolution]
9.	$p \lor \neg r$	[2, 4. Resolution]
10.	p	[6, 9. Resolution]
11.	[]	[8, 10. Resolution]

Therefore, inferences hold in propositional logic.

(i) $\neg p \land \neg q \vdash p \leftrightarrow q$

Convert premises into CNF:

$\neg p \land \neg q$

Convert negated conclusion into CNF:

$$
\begin{aligned}
\neg(p \leftrightarrow q) &\equiv \neg((\neg p \lor q) \land (p \lor \neg q)) \\
&\equiv ((p \land \neg q) \lor (\neg p \land q)) \\
&\equiv ((p \land \neg q) \lor \neg p) \land ((p \land \neg q) \lor q) \\
&\equiv ((p \lor \neg p) \land (\neg q \lor \neg p)) \land ((p \lor q) \land (\neg q \lor q)) \\
&\equiv (\neg q \lor \neg p) \land (p \lor q)
\end{aligned}
$$

Proof:

1.	$\neg p$	[Premises]
2.	$\neg q$	[Premises]

3. $\neg q \lor \neg p$ [¬ Conclusion]
4. $p \lor q$ [¬ Conclusion]
5. q [1, 4. Resolution]
6. [] [2, 5. Resolution]

Therefore, inferences hold in propositional logic.

(j) $\neg q \rightarrow \neg p, \neg r \rightarrow \neg q \vdash p \rightarrow r$

Convert premises into CNF:
$\neg q \rightarrow \neg p \equiv q \lor \neg p$
$\neg r \rightarrow \neg q \equiv r \lor \neg q$

Convert negated conclusion into CNF:
$\neg(p \rightarrow r) \equiv \neg(\neg p \lor r)$
$\equiv p \land \neg r$

Proof:

1. $q \lor \neg p$ [Premises]
2. $r \lor \neg q$ [Premises]
3. p [¬ Conclusion]
4. $\neg r$ [¬ Conclusion]
5. q [1, 3. Resolution]
6. r [2, 5. Resolution]
7. [] [4, 6. Resolution]

Therefore, inferences hold in propositional logic.

2.

(a)

'$<(x, y)$' relation means 'x smaller than y'

1. $\forall x[((age(x, age2) \wedge colour(x, green)) \rightarrow \exists y(age(y, age1) \wedge (y=huey) \wedge <(age1, age2))]$

2. $\forall x[age(x, 5) \rightarrow design(x, camel)]$

3. $\exists x[(x=dewey) \wedge colour(x, yellow)]$

4. $\exists x[(x=louie) \wedge design(x, giraffe)]$

5. $\forall x[design(x, panda) \rightarrow \neg colour(x, white)]$

(b)

KB = {$\forall x[((age(x, age2) \wedge colour(x, green)) \rightarrow \exists y(age(y, age1) \wedge (y=huey) \wedge <(age1, age2)))$
$\forall x[age(x, 5) \rightarrow design(x, camel)]$
$\exists x[(x=dewey) \wedge colour(x, yellow)]$
$\exists x[(x=louie) \wedge design(x, giraffe)]$
$\forall x[design(x, panda) \rightarrow \neg colour(x, white)]$
$\exists x \exists y \exists z[age(x, 4) \wedge age(y, 5) \wedge age(z, 6)]$
$\exists x \exists y \exists z[colour(x, green) \wedge colour(y, yellow) \wedge colour(z, white)]$
$\exists x \exists y \exists z[design(x, panda) \wedge design(y, giraffe) \wedge design(z, camel)]$
}

1.Let $I \models$ **KB**

2.Then $I \models \forall x[((age(x, age2) \wedge colour(x, green)) \rightarrow \exists y(age(y, age1) \wedge (y=huey) \wedge <(age1, age2)))$
(From KB, Huey is younger than the boy in the green T-shirt)

3.So $I \models \exists x[(x=huey) \wedge \neg colour(x, green)]$
(Conclude Huey's tee-shirt was not green)

4.Also $I \models \exists x[(x=dewey) \wedge colour(x, yellow)]$
(From KB, Dewey's T-shirt was yellow)

5.And $I \models \exists x \exists y \exists z[colour(x, green) \wedge colour(y, yellow) \wedge colour (z, white)]$
(From KB, 3 boys wear T-shirt of different colour)

6.So $I \models \exists x[(x=huey) \wedge colour(x, white)], \exists x[(x=louie) \wedge colour(x, green)]$
(3 boys wear T-shirt of different colour and Dewey's T-shirt was yellow, also Huey's T-shirt is not green, thus, Huey's T-shirt can only be white, and the rest boy Louie wear green T-shirt.
Conclude Huey's T-shirt was white; Louie's T-shirt was green)

7.Also $I \models \forall x[design(x, panda) \rightarrow \neg colour(x, white)]$
(From KB, the panda design was not featured on the white T-shirt)

8.And $I \models \exists x[(x=louie) \wedge design(x, giraffe)]$
(From KB, Louie’s tee-shirt bore the giraffe design)

9.And $I \models \exists x \exists y \exists z[design(x, panda) \wedge design(y, giraffe) \wedge design(z, camel)]$
(From KB, 3 boys wear T-shirt with a different design)

10.So $I \models \exists x[(x=dewey) \wedge design(x, panda)], \exists x[(x=huey) \wedge design(x, camel)]$
(3 boys wear T-shirt of different colour, panda design was not in white colour, so panda design T-shirt can only in green or yellow.
Louie’s T-shirt is green and with giraffe design, also 3 boys wear T-shirt with a different design, thus, panda design was in yellow which is Dewey’s T-shirt’s colour, and the rest boy Huey, his T-shirt is camel design.
Conclude Dewey’s T-shirt was panda design; Huey’s T-shirt was camel design.)

11.Also $I \models \forall x[age(x, 5) \rightarrow design(x, camel)]$
(From KB, the five-year-old wore the tee-shirt with the camel design)

12.And $I \models \exists x \exists y \exists z[age(x, 4) \wedge age(y, 5) \wedge age(z, 6)]$
(From KB, 3 boys aged 4, 5 and 6)

13.So $I \models \forall x[design(x, camel) \rightarrow age(x, 5)]$
(Since 3 boys aged 4, 5 and 6 and their T-shirts are with different design, from the KB ‘five-year-old wore the tee-shirt with the camel design’, we can conclude that the boy who wore the T-shirt with camel design is five-year-old)

14.So $I \models \exists x[(x=huey) \wedge age(x, 5)]$
(Since the boy who wear the T-shirt with camel design is five-year-old and Huey’s T-shirt design was camel, thus Huey is five-year-old.
Conclude Huey’s age is 5)

15.So $I \models \exists x[(x=louie) \wedge age(x, age1) \wedge <(5, age1)]$
(Since Huey is younger than the boy in the green tee-shirt and Huey’s age is 5 and Louie’s T-shirt is green, Louie’s age is greater than 5.
Conclude Louie’s age is greater than 5)

16.So $I \models \exists x[(x=louie) \wedge age(x, 6)], \exists x[(x=dewey) \wedge age(x, 4)]$
(Since Louie’s age is greater than 5 and 3 boys aged 4, 5 and 6, Louie can only be 6 years old, thus the rest boy Dewey is 4 years old.
Conclude Louie’s age is 5; Dewey’s age is 4)

Now we have a conclusion:

17. $I \models \exists x \exists y \exists z[(x=dewey) \wedge age(x, 4) \wedge colour(x, yellow) \wedge design(x, panda)$

$\wedge$(y=huey)$\wedge$age(y, 5)$\wedge$colour(y, white)$\wedge$design(y,camel)
$\wedge$(z=louie)$\wedge$age(z, 6)$\wedge$colour(z, green)$\wedge$design(z, giraffe)]

So, my answer is yes, it is possible to conclude the age of each boy together with the colour and design of the tee-shirt they're wearing, above is the proof and get conclusion that
Dewey is *4* years old, his T-shirt was *yellow* and *panda* design;
Huey is *5* years old, his T-shirt was *white* and *camel* design;
Louie is *6* years old, his T-shirt was *green* and *giraffe* design.

(c)
My answer in (b) is yes

3.

Two files in total:
'assn1q3_prolog.pl' and 'assn1q3'
where 'assn1q3_prolog.pl' is the prolog file, 'assn1q3' is the script written in python3 when run command like:

./assn1q3 '[p imp q, (neg r) imp (neg q)] seq [p imp r]'

the python file 'assn1q3' will transform '[p imp q, (neg r) imp (neg q)] seq [p imp r]' into first order logic format, like:

seq([imp(p, q), imp(neg(r), neg(q))], [imp(p, r)])

then call prolog file 'assn1q3_prolog.pl' to run the command like:

rule_hw(seq([imp(p, q), imp(neg(r), neg(q))], [imp(p, r)])).

The 'rule_hw()' is the function I write in prolog file 'assn1q3_prolog.pl', it will return the result and the proof process like:

seq([imp(p, q), imp(neg(r), neg(q))], [imp(p, r)]) P5a

Finally, python file print get the return, then print result 'true' or 'false' and if it is 'true', transform the proof process from first order logic format back to original format, then print it out, here is part of proof :

12.[p imp q, (neg r) imp (neg q)] seq [p imp r]   P5a