

COMP4418, 2017–Assignment 1

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1.

(a) $p \vee (q \wedge r) \models (p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

In all rows where $p \vee (q \wedge r)$ is true, $(p \vee q) \wedge (p \vee r)$ is also true.
Therefore, inference is valid

(b) $\models p \rightarrow (q \rightarrow p)$

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

In all rows $p \rightarrow (q \rightarrow p)$ is true.
Therefore, inference is valid

(c) $p \rightarrow q \models \neg p \rightarrow \neg q$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

In 3rd row, when the $p \rightarrow q$ is true, $\neg p \rightarrow \neg q$ is false.
Therefore, inference is not valid.

(d) $p \rightarrow q, \neg p \rightarrow \neg q \models \neg p \leftrightarrow \neg q$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$	$\neg p \leftrightarrow \neg q$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

In all rows, where both $p \rightarrow q$ and $\neg p \rightarrow \neg q$ are true, $\neg p \leftrightarrow \neg q$ is also true.

Therefore, inference is valid.

(e) $\neg q \rightarrow \neg p, \neg r \rightarrow \neg q \models p \rightarrow r$

q	p	r	$\neg q$	$\neg p$	$\neg r$	$\neg q \rightarrow \neg p$	$\neg r \rightarrow \neg q$	$p \rightarrow r$
T	T	T	F	F	F	T	T	T
T	T	F	F	F	T	T	F	F
T	F	T	F	T	F	T	T	T
T	F	F	F	T	T	T	F	T
F	T	T	T	F	F	F	T	T
F	T	F	T	F	T	F	T	F
F	F	T	T	T	F	T	T	T
F	F	F	T	T	T	T	T	T

In all rows, where both $\neg q \rightarrow \neg p$ and $\neg r \rightarrow \neg q$ are true, $p \rightarrow r$ is also true.
Therefore, inference is valid.

(f) $p \wedge (q \vee r) \vdash (p \wedge q) \vee (p \wedge r)$

Convert premises into CNF:

$$p \wedge (q \vee r)$$

Convert negated conclusion into CNF:

$$\neg((p \wedge q) \vee (p \wedge r)) \equiv (\neg p \vee \neg q) \wedge (\neg p \vee \neg r)$$

Proof:

1. p [Premises]
2. $q \vee r$ [Premises]
3. $\neg p \vee \neg q$ [\neg Conclusion]
4. $\neg p \vee \neg r$ [\neg Conclusion]
5. $\neg p \vee r$ [2, 3. Resolution]
6. $\neg p$ [4, 5. Resolution]
7. [] [1, 6. Resolution]

Therefore, inferences hold in propositional logic.

(g) $p \vdash p \rightarrow q$

Convert premises into CNF:

$$p$$

Convert negated conclusion into CNF:

$$\begin{aligned} \neg(p \rightarrow q) &\equiv \neg(\neg p \vee q) \\ &\equiv p \wedge \neg q \end{aligned}$$

Proof:

1. p [Premises]
2. p [\neg Conclusion]
3. $\neg q$ [\neg Conclusion]

Cannot obtain empty clause using resolution, therefore inferences not hold in propositional logic.

(h) $p \leftrightarrow q \vdash (q \leftrightarrow r) \rightarrow (p \leftrightarrow r)$

Convert premises into CNF:

$$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (p \vee \neg q)$$

Convert negated conclusion into CNF:

$$\begin{aligned} \neg((q \leftrightarrow r) \rightarrow (p \leftrightarrow r)) &\equiv \neg(\neg(q \leftrightarrow r) \vee (p \leftrightarrow r)) \\ &\equiv (q \leftrightarrow r) \wedge \neg(p \leftrightarrow r) \\ &\equiv ((\neg q \vee r) \wedge (q \vee \neg r)) \wedge \neg((\neg p \vee r) \wedge (p \vee \neg r)) \\ &\equiv ((\neg q \vee r) \wedge (q \vee \neg r)) \wedge ((p \wedge \neg r) \vee (\neg p \wedge r)) \\ &\equiv ((\neg q \vee r) \wedge (q \vee \neg r)) \wedge (((p \wedge \neg r) \vee \neg p) \wedge ((p \wedge \neg r) \vee r)) \\ &\equiv ((\neg q \vee r) \wedge (q \vee \neg r)) \wedge (((p \vee \neg p) \wedge (\neg r \vee \neg p)) \wedge ((p \vee r) \wedge (\neg r \vee r))) \\ &\equiv ((\neg q \vee r) \wedge (q \vee \neg r)) \wedge ((\neg r \vee \neg p) \wedge (p \vee r)) \\ &\equiv (\neg q \vee r) \wedge (q \vee \neg r) \wedge (\neg r \vee \neg p) \wedge (p \vee r) \end{aligned}$$

Proof:

- | | |
|-------------------------|----------------------|
| 1. $\neg p \vee q$ | [Premises] |
| 2. $p \vee \neg q$ | [Premises] |
| 3. $\neg q \vee r$ | [\neg Conclusion] |
| 4. $q \vee \neg r$ | [\neg Conclusion] |
| 5. $\neg r \vee \neg p$ | [\neg Conclusion] |
| 6. $p \vee r$ | [\neg Conclusion] |
| 7. $\neg p \vee r$ | [1, 3. Resolution] |
| 8. $\neg p$ | [5, 7. Resolution] |
| 9. $p \vee \neg r$ | [2, 4. Resolution] |
| 10. p | [6, 9. Resolution] |
| 11. [] | [8, 10. Resolution] |

Therefore, inferences hold in propositional logic.

(i) $\neg p \wedge \neg q \vdash p \leftrightarrow q$

Convert premises into CNF:

$$\neg p \wedge \neg q$$

Convert negated conclusion into CNF:

$$\begin{aligned} \neg(p \leftrightarrow q) &\equiv \neg((\neg p \vee q) \wedge (p \vee \neg q)) \\ &\equiv ((p \wedge \neg q) \vee (\neg p \wedge q)) \\ &\equiv ((p \wedge \neg q) \vee \neg p) \wedge ((p \wedge \neg q) \vee q) \\ &\equiv ((p \vee \neg p) \wedge (\neg q \vee \neg p)) \wedge ((p \vee q) \wedge (\neg q \vee q)) \\ &\equiv (\neg q \vee \neg p) \wedge (p \vee q) \end{aligned}$$

Proof:

- | | |
|-------------|------------|
| 1. $\neg p$ | [Premises] |
| 2. $\neg q$ | [Premises] |

3. $\neg q \vee \neg p$ [¬ Conclusion]
4. $p \vee q$ [¬ Conclusion]
5. q [1, 4. Resolution]
6. $[]$ [2, 5. Resolution]

Therefore, inferences hold in propositional logic.

(j) $\neg q \rightarrow \neg p, \neg r \rightarrow \neg q \vdash p \rightarrow r$

Convert premises into CNF:

$$\neg q \rightarrow \neg p \equiv q \vee \neg p$$

$$\neg r \rightarrow \neg q \equiv r \vee \neg q$$

Convert negated conclusion into CNF:

$$\begin{aligned}\neg(p \rightarrow r) &\equiv \neg(\neg p \vee r) \\ &\equiv p \wedge \neg r\end{aligned}$$

Proof:

1. $q \vee \neg p$ [Premises]
2. $r \vee \neg q$ [Premises]
3. p [¬ Conclusion]
4. $\neg r$ [¬ Conclusion]
5. q [1, 3. Resolution]
6. r [2, 5. Resolution]
7. $[]$ [4, 6. Resolution]

Therefore, inferences hold in propositional logic.

2.

(a)

1. $\forall x[(\text{age}(x, \text{age2}) \wedge \text{colour}(x, \text{green})) \rightarrow \exists y(\text{age}(y, \text{age1}) \wedge (y=\text{huey}) \wedge (\text{age1} < \text{age2}))]$
2. $\forall x[\text{age}(x, 5) \rightarrow \text{design}(x, \text{camel})]$
3. $\exists x[(x=\text{dewey}) \wedge \text{colour}(x, \text{yellow})]$
4. $\exists x[(x=\text{louie}) \wedge \text{design}(x, \text{giraffe})]$
5. $\forall x[\text{design}(x, \text{panda}) \rightarrow \neg \text{colour}(x, \text{white})]$

(b)

$\text{KB} = \{ \forall x[(\text{age}(x, \text{age2}) \wedge \text{colour}(x, \text{green})) \rightarrow \exists y(\text{age}(y, \text{age1}) \wedge (y=\text{huey}) \wedge (\text{age1} < \text{age2}))]$
 $\forall x[\text{age}(x, 5) \rightarrow \text{design}(x, \text{camel})]$
 $\exists x[(x=\text{dewey}) \wedge \text{colour}(x, \text{yellow})]$
 $\exists x[(x=\text{louie}) \wedge \text{design}(x, \text{giraffe})]$
 $\forall x[\text{design}(x, \text{panda}) \rightarrow \neg \text{colour}(x, \text{white})]$
 $\exists x \exists y \exists z[\text{age}(x, 4) \wedge \text{age}(y, 5) \wedge \text{age}(z, 6)]$
 $\exists x \exists y \exists z[\text{colour}(x, \text{green}) \wedge \text{colour}(y, \text{yellow}) \wedge \text{colour}(z, \text{white})]$
 $\exists x \exists y \exists z[\text{design}(x, \text{panda}) \wedge \text{design}(y, \text{giraffe}) \wedge \text{design}(z, \text{camel})]$
 $\}$

1. Let $I \models \text{KB}$

2. Then $I \models \forall x[(\text{age}(x, \text{age2}) \wedge \text{colour}(x, \text{green})) \rightarrow \exists y(\text{age}(y, \text{age1}) \wedge (y=\text{huey}) \wedge (\text{age1} < \text{age2}))]$

(From KB, Huey is younger than the boy in the green tee-shirt)

3. So $I \models \exists x[(x=\text{huey}) \wedge \neg \text{colour}(x, \text{green})]$

(Conclude Huey's T-shirt is not green)

4. Also $I \models \exists x[(x=\text{dewey}) \wedge \text{colour}(x, \text{yellow})]$

(From KB, Dewey's tee-shirt was yellow)

5. And $I \models \exists x \exists y \exists z[\text{colour}(x, \text{green}) \wedge \text{colour}(y, \text{yellow}) \wedge \text{colour}(z, \text{white})]$

(From KB, 3 boys wear T-shirt of different colour)

6. So $I \models \exists x[(x=\text{huey}) \wedge \text{colour}(x, \text{white})], \exists x[(x=\text{louie}) \wedge \text{colour}(x, \text{green})]$

(3 boys wear T-shirt of different colour and Dewey's T-shirt is yellow. Also Huey's T-shirt is not green, thus, his T-shirt can only be white, and the rest boy Louie wear green T-shirt.

Conclude Huey's T-shirt is white; Louie's T-shirt is green)

7. Also $I \models \forall x[\text{design}(x, \text{panda}) \rightarrow \neg \text{colour}(x, \text{white})]$

(From KB, the panda design was not featured on the white tee-shirt)

8. And $I \models \exists x[(x=\text{louie}) \wedge \text{design}(x, \text{giraffe})]$
(From KB, Louie's tee-shirt bore the giraffe design)

9. And $I \models \exists x \exists y \exists z [\text{design}(x, \text{panda}) \wedge \text{design}(y, \text{giraffe}) \wedge \text{design}(z, \text{camel})]$
(From KB, 3 boys wear T-shirt with a different design)

10. So $I \models \exists x[(x=\text{dewey}) \wedge \text{design}(x, \text{panda})], \exists x[(x=\text{huey}) \wedge \text{design}(x, \text{camel})]$
(3 boys wear T-shirt of different colour, panda design is not in white colour, so it can only in green or yellow. Louie's T-shirt is green and with giraffe design and 3 boys wear T-shirt with a different design, thus, panda design is in yellow which is Dewey's T-shirt's colour, and the rest boy Huey, his T-shirt is camel design.
Conclude Dewey's T-shirt is panda design; Huey's T-shirt is camel design.)

11. Also $I \models \forall x[\text{age}(x, 5) \rightarrow \text{design}(x, \text{camel})]$
(From KB, the five-year-old wore the tee-shirt with the camel design)

12. And $I \models \exists x \exists y \exists z [\text{age}(x, 4) \wedge \text{age}(y, 5) \wedge \text{age}(z, 6)]$
(From KB, 3 boys aged 4, 5 and 6)

13. So $I \models \forall x[\text{design}(x, \text{camel}) \rightarrow \text{age}(x, 5)]$
(Since 3 boys aged 4, 5 and 6 and their T-shirts are with different design, from the KB 'five-year-old wore the tee-shirt with the camel design', we can conclude that the boy who wear the T-shirt with camel design is five-year-old)

14. So $I \models \exists x[(x=\text{huey}) \wedge \text{age}(x, 5)]$
(Since the boy who wear the T-shirt with camel design is five-year-old and Huey's T-shirt design is camel, thus Huey is five-year-old.
Conclude Huey's age is 5)

15. So $I \models \exists x[(x=\text{louie}) \wedge \text{age}(x, \text{age1}) \wedge <(5, \text{age1})]$
(Since Huey is younger than the boy in the green tee-shirt and Huey's age is 5 and Louie's T-shirt is green, Louie's age is greater than 5.
Conclude Louie's age is greater than 5)

16. So $I \models \exists x[(x=\text{louie}) \wedge \text{age}(x, 6)], \exists x[(x=\text{dewey}) \wedge \text{age}(x, 4)]$
(Since Louie's age is greater than 5 and 3 boys aged 4, 5 and 6, Louie can only be 6 years old, thus the rest boy Dewey is 4 years old.
Conclude Louie's age is 6; Dewey's age is 4)

Now we have a conclusion:

17. $I \models \exists x \exists y \exists z [(x=\text{dewey}) \wedge \text{age}(x, 4) \wedge \text{colour}(x, \text{yellow}) \wedge \text{design}(x, \text{panda})$
 $\wedge (y=\text{huey}) \wedge \text{age}(y, 5) \wedge \text{colour}(y, \text{white}) \wedge \text{design}(y, \text{camel})$
 $\wedge (z=\text{louie}) \wedge \text{age}(z, 6) \wedge \text{colour}(z, \text{green}) \wedge \text{design}(z, \text{giraffe})]$

So, my answer is yes, it is possible to conclude the age of each boy together with the colour and design of the tee-shirt they're wearing, above is the proof and get conclusion that
Dewey is 4 years old, his T-shirt is *yellow* and *panda* design;
Huey is 5 years old, his T-shirt is *white* and *camel* design;
Louie is 6 years old, his T-shirt is *green* and *giraffe* design.

(c)

My answer in (b) is yes