

# COMP4418, 2017 – Assignment 3

Z5045582

Yunhe Zhang

## 1. Social Choice and Game Theory

(a)

the domination set of each alternative

a {b, d, f, g}

b {c, d, e, g}

c {a, d, e, f, g}

d {e, f, g}

e {a, f, g}

f {b, g}

g {}

- **the uncovered set:**

Alternative in two steps:

a {b, c, d, e, f, g} ✓

b {a, c, d, e, f, g} ✓

c {a, b, d, e, f, g} ✓

d {a, b, e, f, g} ✗

e {a, b, d, f, g} ✗

f {b, c, d, e, g} ✗

g {} ✗

The uncovered set is {a, b, c}

- **the top cycle:**

The top cycle is {a, b, c, d, e, f}

- **the set of Copeland winners:**

the set of Copeland winners is {c}

- **the set of Banks winners:**

maximal acyclic subgraph length is 5

$b \Rightarrow c \Rightarrow d \Rightarrow e \Rightarrow g$

$c \Rightarrow a \Rightarrow d \Rightarrow f \Rightarrow g$

the set of Banks winners is  $\{b, c\}$

- **the set of Condorcet winners:**

no alternative is satisfied, so the set of Condorcet winners is  $\{\}$

**(b)**

Pure Nash equilibria

(A, E)

(B, D)

Mixed Nash equilibria

Player 1 plays A with  $p$ , plays B with  $1 - p$

$$4p + 6(1 - p) = 5p + 4(1 - p)$$

$$4p + 6 - 6p = 5p + 4 - 4p$$

$$3p = 2$$

$$p = 2 / 3$$

Player 2 plays D with  $q$ , plays E with  $1 - q$

$$2q + 8(1 - q) = 6q + 4(1 - q)$$

$$2q + 8 - 8q = 6q + 4 - 4q$$

$$8q = 4$$

$$q = 1 / 2$$

Mixed Nash equilibria

$$(A, B) = (2 / 3, 1 / 3)$$

$$(D, E) = (1 / 2, 1 / 2)$$

## 2. Decision Making

**(a)**

1. Blackjack: (D) POMDP
2. Candy Crush: (B) MDP
3. Chess: (E) None/Other
4. Minesweeper: (D) POMDP
5. Snakes and Ladders: (A) MP
6. Texas Hold 'em Poker: (E) None/Other

(b)

$\pi(S1) = \text{Stay}$

$\pi(S2) = \text{Stay}$

$\pi(S3) = \text{Stay or Leave}$

If the discount factor is very high, the value of state is hard to converge, so consider the value of state with state transition, I determine the S1 stay is positive, leave is negative or 0; S2 stay is 0, leave is negative; S3 both are negative.

(c)

$\pi(S1) = \text{Stay}$

$\pi(S2) = \text{Leave}$

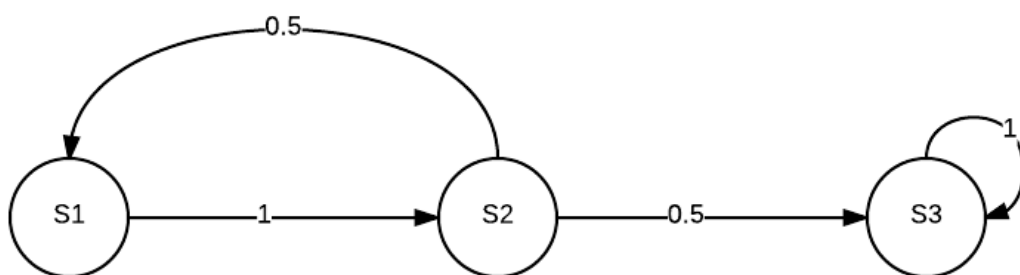
$\pi(S3) = \text{Stay or Leave}$

Since the discount factor is very low, the future part could be ignored, thus just chose the action whose associated reward is higher.

(d)

	$V_0(s)$	$V_0(s, S)$	$V_0(s, L)$	$V_1(s)$	$V_1(s, S)$	$V_1(s, L)$	$V_2(s)$	$V_2(s, S)$	$V_2(s, L)$	$V_3(s)$
s1	0	1	0	1	1.6	3	3	2.8	2.82	2.82
s2	0	0	5	5	3	4.7	4.7	7.82	4.94	7.82
s3	0	-2	-2	-2	-3.2	-3.2	-3.2	-3.92	-3.92	-3.92

(e)



(f)

Since agent uses  $\pi$  this could see as Markov process problem

when  $i = 0$

$$v_0(s1) = v_0(s2) = v_0(s3) = 0$$

when  $i > 0$

$$v_i(s1)$$

$$= u(s1, L) + \delta P(s1, L, s2)v_i(s2)$$

$$= \delta v_i(s2)$$

$$v_i(s2)$$

$$= u(s2, L) + \delta(P(s2, L, s1)*v_{i-1}(s1) + P(s2, L, s3)*v_{i-1}(s3))$$

$$= 5 + \delta(0.5*v_i(s1) + 0.5*v_i(s3))$$

$$v_i(s3)$$

$$= u(s3, S) + \delta P(s3, S, s3)v_i(s3)$$

$$= -2 + \delta v_i(s3)$$

According to

$$(1) v_i(s1) = \delta v_i(s2)$$

$$(2) v_i(s2) = 5 + \delta(0.5*v_i(s1) + 0.5*v_i(s3))$$

$$(3) v_i(s3) = -2 + \delta v_i(s3)$$

Get:

$$(1) v_i(s1) = \frac{\delta(10-12\delta)}{(1-\delta)*(2-\delta^2)}$$

$$(2) v_i(s2) = \frac{10-12\delta}{(1-\delta)*(2-\delta^2)}$$

$$(3) v_i(s3) = \frac{-2}{1-\delta}$$

When  $\delta$  is very high like  $\delta = 0.999$

$V_i$  of all 3 states will be negative. This could part support my intuition.

To fully support my intuition, need to consider more actions.

Consider about other possible policy like

$\pi_2(S1) = \text{Stay}$ ,  $\pi_2(S2) = \text{Stay}$ ,  $\pi_2(S3) = \text{Stay or Leave}$  (since two action result in same value)

Then

$$v_i(s1) = \frac{1}{1-\delta}$$

which is positive if  $\delta$  is very high

$$v_i(s2) = 0$$

always 0

$$v_i(s3) = \frac{-2}{1-\delta}$$

which is negative if  $\delta$  is very high

$\pi_3(S1) = \text{Stay}, \pi_3(S2) = \text{Leave}, \pi_3(S3) = \text{Stay or Leave}$

$v_i(s1) = \frac{1}{1-\delta}$  which is positive if  $\delta$  is very high

$v_i(s2) = \frac{10-11\delta}{2-2\delta}$  which is negative if  $\delta$  is very high

$v_i(s3) = \frac{-2}{1-\delta}$  which is negative if  $\delta$  is very high

$\pi_4(S1) = \text{Leave}, \pi_4(S2) = \text{Stay}, \pi_4(S3) = \text{Stay or Leave}$

$v_i(s1) = 0$  always 0

$v_i(s2) = 0$  always 0

$v_i(s3) = \frac{-2}{1-\delta}$  which is negative if  $\delta$  is very high

Thus  $\pi_2(S1) = \text{Stay}, \pi_2(S2) = \text{Stay}, \pi_2(S3) = \text{Stay or Leave}$  is the best.  
After considering more actions, my intuition in 2b is supported.

When  $\delta$  is very low like  $\delta = 0.001$

$V_i$  of all 3 states will be close to their reward of action in  $\pi$

$v_i(s1)$  close to 0

$v_i(s2)$  close to 5

$v_i(s3)$  close to -2

It supports my intuition of question 2c since  $v_i$  is close to action associated reward, so the higher associated reward action will be the optimal policy.