Eigenvalues of the Adjoint Representation

This program attempts to provide a unique visualization of the root space of a finite-dimensional semi-simple Lie algebra.

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Overview

The images this program produces is achieved in the following 9 steps:

1. The user provides two values: n and k.
2. Generate a basis for the Lie algebra $\mathfrak{sl}_n(\mathbb{C})$.
3. Find a basis, $\mathcal{B}$, for a Cartan subalgebra of $\mathfrak{sl}_n(\mathbb{C})$.

4. For each $X_i \in \mathcal{B}$, calculate the adjoint representation ad_{X_i}.
5. Calculate a subset $C \subset \mathbb{C}$, such that $|C| = k$.
6. Calculate a finite spanning set, $S = \{\sum_{i=1}^{k} c_i \mathrm{ad}_{X_i} \mid c_i \in C\}$.
7. Letting $d = \dim(\mathfrak{sl}_n(\mathbb{C}))$, then for each $s \in S$, calculate the set of all Eigenvectors $E_s = \{v \in \mathbb{C}^d \mid \exists \lambda : sv = \lambda v\}$.
8. Select a fixed vector $w \in \mathbb{C}^d$.
9. For each Eigenvector $e \in \bigcup_{s \in S} E_s$, consider its Eigenvalue, $\lambda \in \mathbb{C}$ and the angle $0 \leq \theta < 2\pi$, between e and w.
10. For each of these (λ, θ) pairs, plot λ as a pixel in $\mathbb{R}^2$, and give it an RGB value as a function of θ.

Definitions