STAT 215A Test #2-Take Home Portion, Spring 2022

Instructions. This test consists of submission of your answers to two problems among the four problems given below (one option from each pair). You are allowed to collaborate for this assignment. You can work in pairs or groups of size three. It is sufficient for one group member to submit the completed solutions (if all group members have contributed to the solutions considerably) but you are also allowed to submit your solutions individually by stating a list of your collaborators. I will be available via Zoom at 12:30-1:30 pm on Wednesday for any issues or questions. Show your work and submit the solution file via Canvas by 1 pm (Pacific time) on Friday, April 29th.

1. For the first problem, you have two choices: Option A and Option B. Select one of them, and complete all of the parts in that problem.

Option A. Let $X_1, X_2, ..., X_n$ be n i.i.d random variables, each with mean μ and standard deviation σ. Let $\bar{X} = \frac{X_1+X_2+...+X_n}{n}$ represent the variable for the sample mean of $X_1, X_2, ..., X_n$.

(a) (1 pt) Determine the expected value and variance of $\bar{X}$.

(b) (2 pts) Let $Y_k = X_k - \bar{X}$ for each $k = 1, 2, ..., n$. Determine the expected value and variance of Y_k for each $k = 1, 2, ..., n$.

(c) (1 pt) Compute $Cov(Y_1, Y_2)$.

Option B. Consider a sequence of i.i.d random variables $X_1, X_2, ...,$ each with the following pmf: For $p + q + r$,

$$P(X_k = x) = \begin{cases} p, & \text{for } x = 1 \\ q, & \text{for } x = 0 \\ r, & \text{for } x = -1 \end{cases}.$$

(a) (1 pt) Determine the mean and variance of each X_k, $k \in \mathbb{N}$.

(b) (1 pt) Define $S_n = \sum_{k=1}^{n} X_k$ for $n \in \mathbb{N}$. Compute the mean and variance of S_n.

(c) (1 pt) Compute $Cov(S_2, S_4)$.

(d) (1 pt) Determine the pmf of S_2.

2. You have two options for this problem, too. Again, select one of them for submission and complete all of the parts in that question.

Option A. Let A be the upper half of the unit disk in $\mathbb{R}^2$: $A = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \le 1 \text{ and } 0 \le y \le 1\}$. Assume that the joint pdf of two jointly continuous r.v. X and Y is given by

$$f_{X,Y}(x, y) = \begin{cases} C(1 + xy), & \text{if } (x, y) \in A \\ 0, & \text{otherwise.} \end{cases}$$

(a) (1.5 pts) Determine $C > 0$ so that $f_{X,Y}$ is a valid (joint) pdf.

(b) (1.5 pts) Compute $P(X + Y > 1)$.

(c) (1 pt) Determine the marginal pdf of X.

Option B. Let T be the set of pairs of nonnegative integers such that the sum of the numbers in each pair is at most 4: $T = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : 0 \le x + y \le 4,\ x \ge 0 \text{ and } y \ge 0\}$. One can check that there are 15 such pairs in T.

Define a joint pmf of the following form:

$$p_{X,Y}(x, y) = P(X = x, Y = y) = \begin{cases} C(1 + xy), & \text{if } (x, y) \in T \\ 0, & \text{otherwise.} \end{cases}$$

(a) (1.5 pts) Determine $C > 0$ so that $p_{X,Y}$ is a valid (joint) pmf.

(b) (1.5 pts) Compute $P(X + Y > 1)$.

(c) (1 pt) Determine the marginal pmf of Y.