

Visualizing S_n

Quin Darcy

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Below is a demonstration of the various images produced by the program `group.py`. This program starts prompting the user to enter a value n . From here the program recursively generates the set of all permutations of the numbers $1, \dots, n$. To do this *Heap's Algorithm* is used. The number of permutations generated is $n!$. At this point, each permutation is assigned a unique color in the form (R, G, B) , where $0 \leq R \leq 255$, $0 \leq G \leq 255$, $0 \leq B \leq 255$. This assignment is accomplished by dividing $2\pi/n!$, and to the i th permutation p_i , we assign $i(2\pi/n!)$ for each i . Thus there is a one-to-one correspondance between the $n!$ permutations and $n!$ angles $0 \leq \theta_i < 2\pi$. From here the set of angles is sent to a function which maps the given angle θ_i to a point (R_i, G_i, B_i) .

Equipped with a set of $n!$ (R, G, B) tuples, we then compute the Cayley table for S_n . After this, the program then computes the set of all cyclic subgroups, it computes the alternating group A_n , and prompts the user to choose an element from the group and computes the orbit and stabilizer of this element. After all this sets are computed, an image is constructed, pixel by pixel, referencing the set of (R, G, B) tuples.

Before showing the images, it is worth noting how the permutations are ordered. If $n = 3$, we should expect the top row of the Cayley table to ordered as

$$[1, 2, 3], [1, 3, 2], [2, 1, 3], [2, 3, 1], [3, 1, 2], [3, 2, 1].$$

One thing to notice is that the program keeps the permuations as is instead of converting them to cycle notation.

Below are the Cayley tables for $n = 3, 4, 5, 6, 7$, and various other of the above mentioned sets. $n = 7$ sadly the limit for my current CPU.

A 9x9 grid of colored squares. The colors are arranged in a repeating pattern of three columns and three rows. The colors are: Black, Red, Yellow-green, Green, Cyan, Blue, Magenta, Light green, and Light red. The pattern repeats every three rows and every three columns.

Figure 1: Cayley Table for S_3

The image displays a 16x16 grid of colored squares, each square being one-eighth of a larger 8x8 block. The colors follow a repeating pattern across the grid. Starting from the top-left corner, the colors transition through red, orange, yellow, green, cyan, blue, magenta, and purple. This pattern repeats every two columns and every two rows. The first column contains black, red, orange, yellow, green, cyan, blue, and magenta squares. The second column contains red, orange, yellow, green, cyan, blue, magenta, and purple squares. The third column contains orange, yellow, green, cyan, blue, magenta, purple, and pink squares. The fourth column contains yellow, green, cyan, blue, magenta, purple, pink, and light blue squares. The fifth column contains green, cyan, blue, magenta, purple, pink, light blue, and lime green squares. The sixth column contains cyan, blue, magenta, purple, pink, light blue, lime green, and lime yellow squares. The seventh column contains blue, magenta, purple, pink, light blue, lime green, lime yellow, and lime orange squares. The eighth column contains magenta, purple, pink, light blue, lime green, lime yellow, lime orange, and lime red squares. The ninth column contains purple, pink, light blue, lime green, lime yellow, lime orange, lime red, and lime black squares. The tenth column contains pink, light blue, lime green, lime yellow, lime orange, lime red, lime black, and lime magenta squares. The eleventh column contains light blue, lime green, lime yellow, lime orange, lime red, lime black, lime magenta, and lime cyan squares. The twelfth column contains lime green, lime yellow, lime orange, lime red, lime black, lime magenta, lime cyan, and lime blue squares. The thirteenth column contains lime yellow, lime orange, lime red, lime black, lime magenta, lime cyan, lime blue, and lime green squares. The fourteenth column contains lime orange, lime red, lime black, lime magenta, lime cyan, lime blue, lime green, and lime yellow squares. The fifteenth column contains lime red, lime black, lime magenta, lime cyan, lime blue, lime green, lime yellow, and lime orange squares. The sixteenth column contains lime black, lime magenta, lime cyan, lime blue, lime green, lime yellow, lime orange, and lime red squares.

Figure 2: Cayley Table for S_4

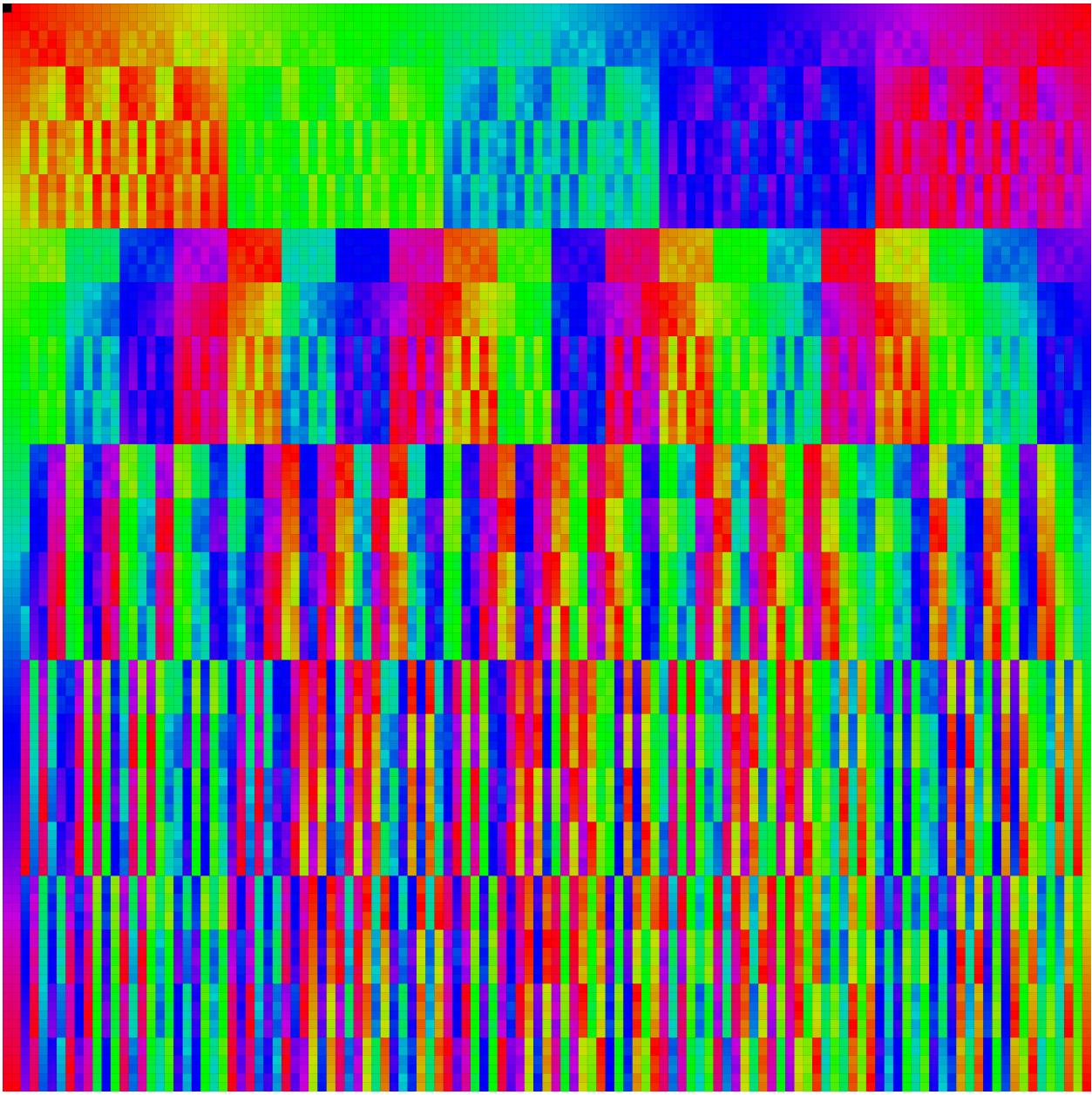


Figure 3: Cayley Table for S_5

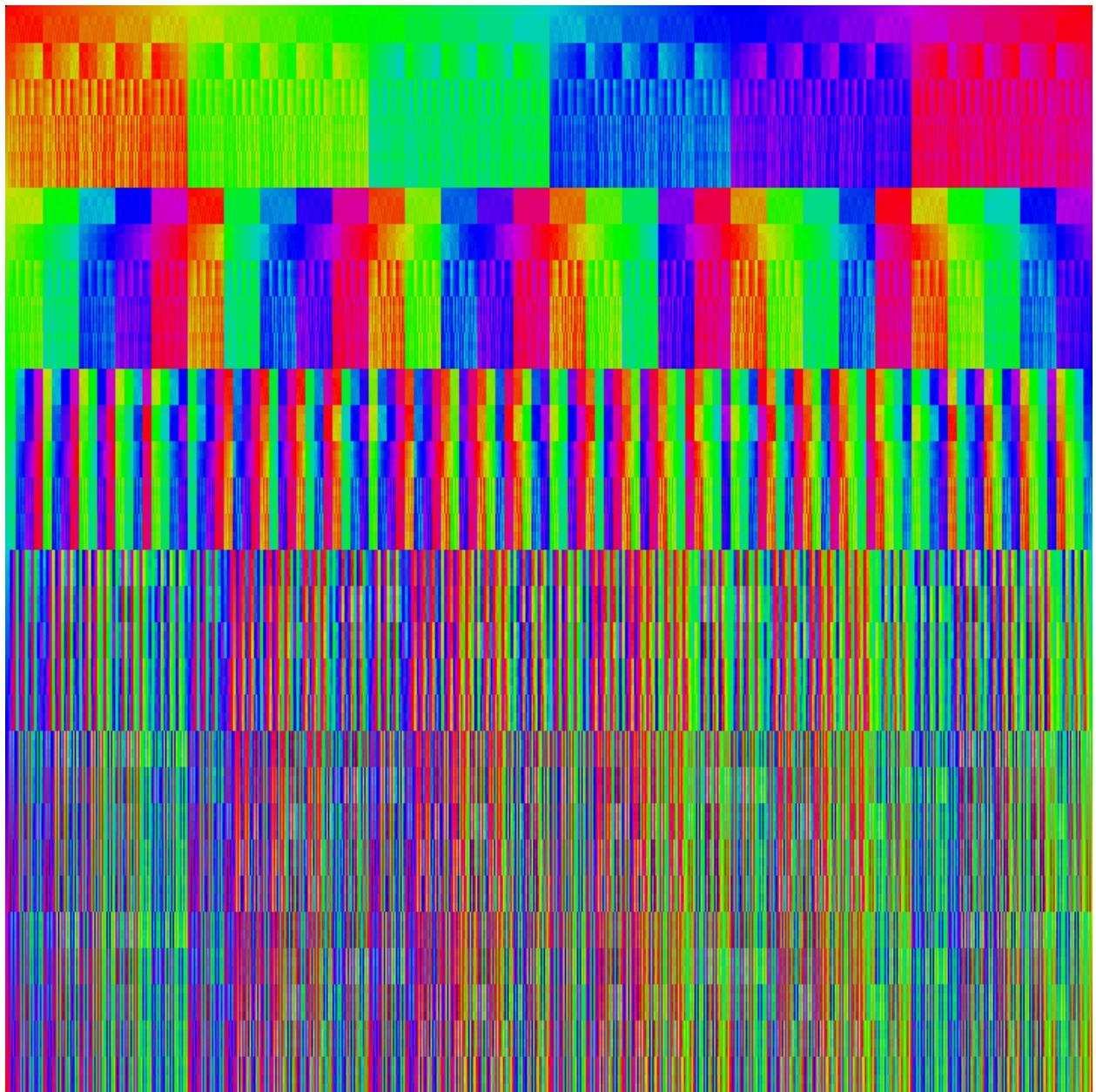


Figure 4: Cayley Table for S_6

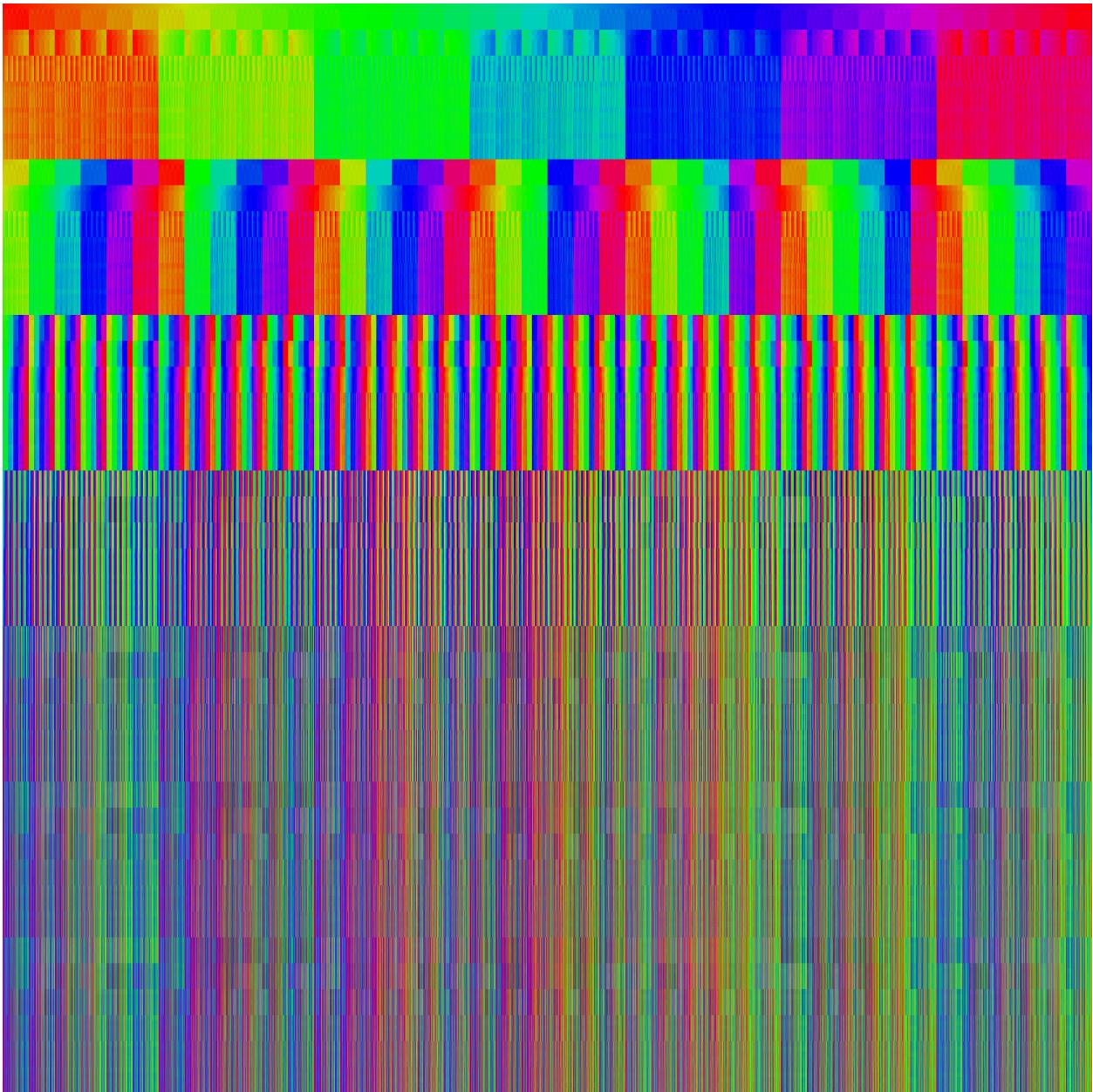


Figure 5: Cayley Table for S_7

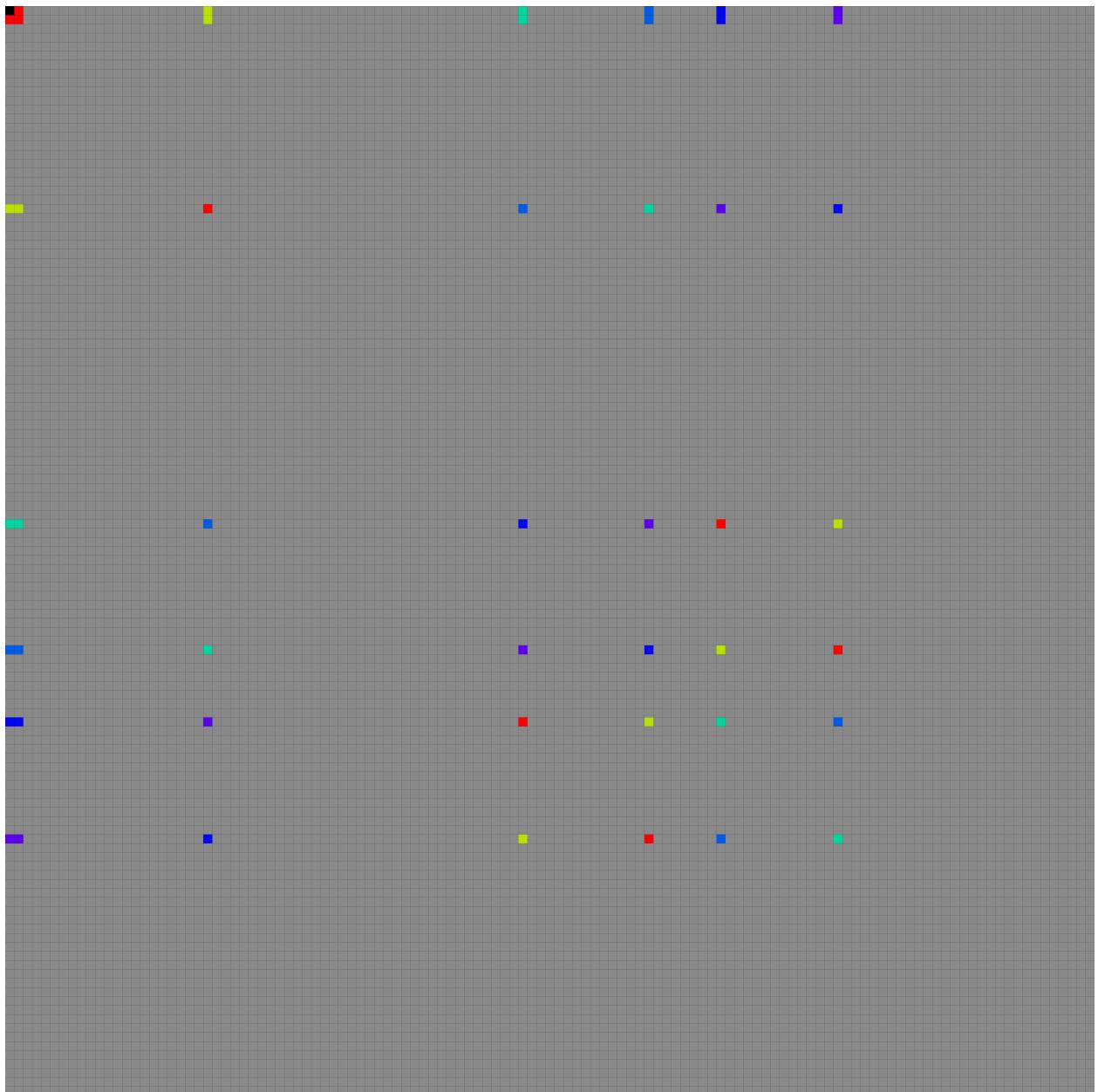


Figure 6: Cyclic Subgroup of (2453) in S_5

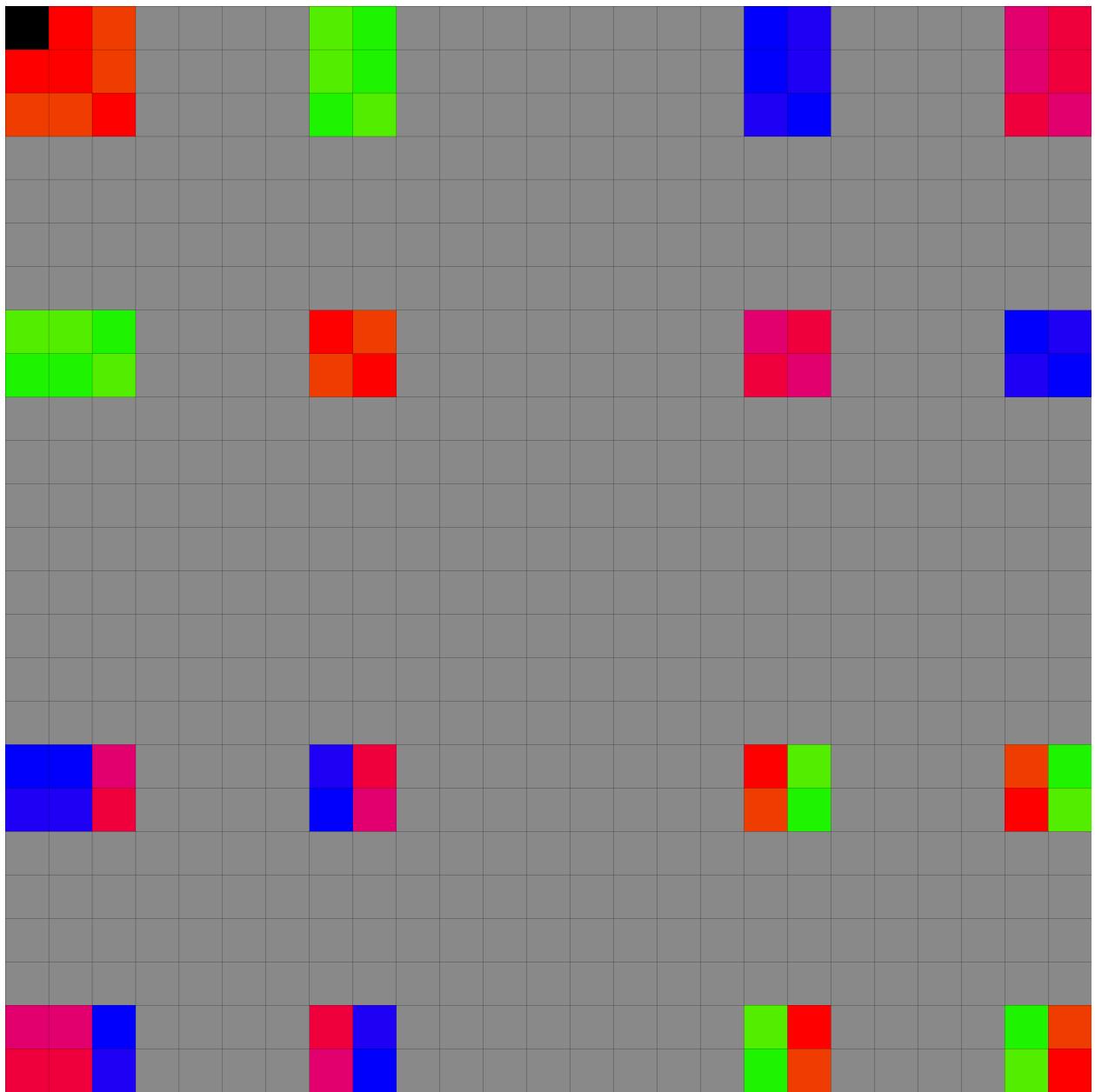


Figure 7: Stabilizer of $(12)(34)$ in S_4

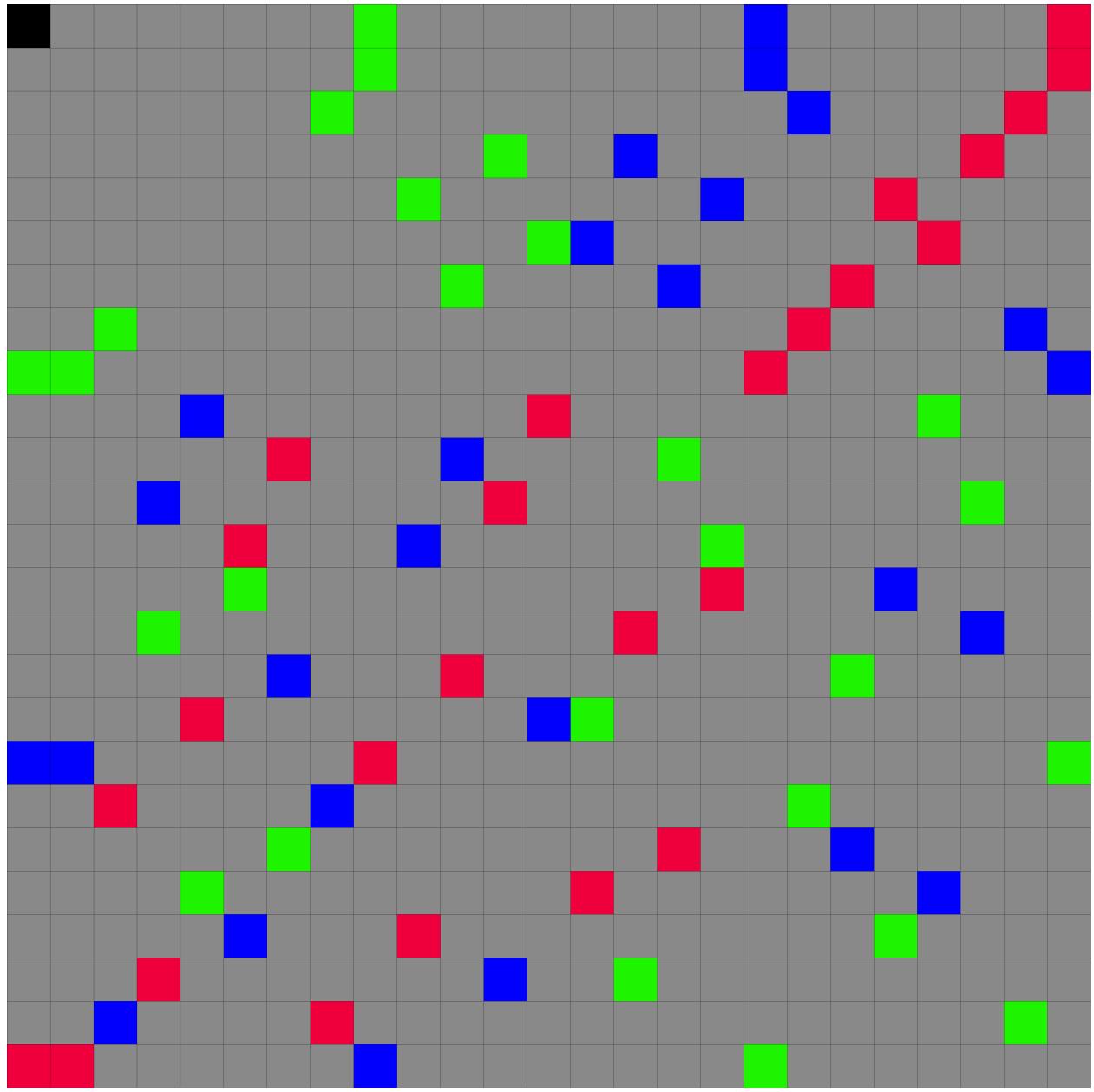


Figure 8: Orbit of $(12)(34)$ in S_4

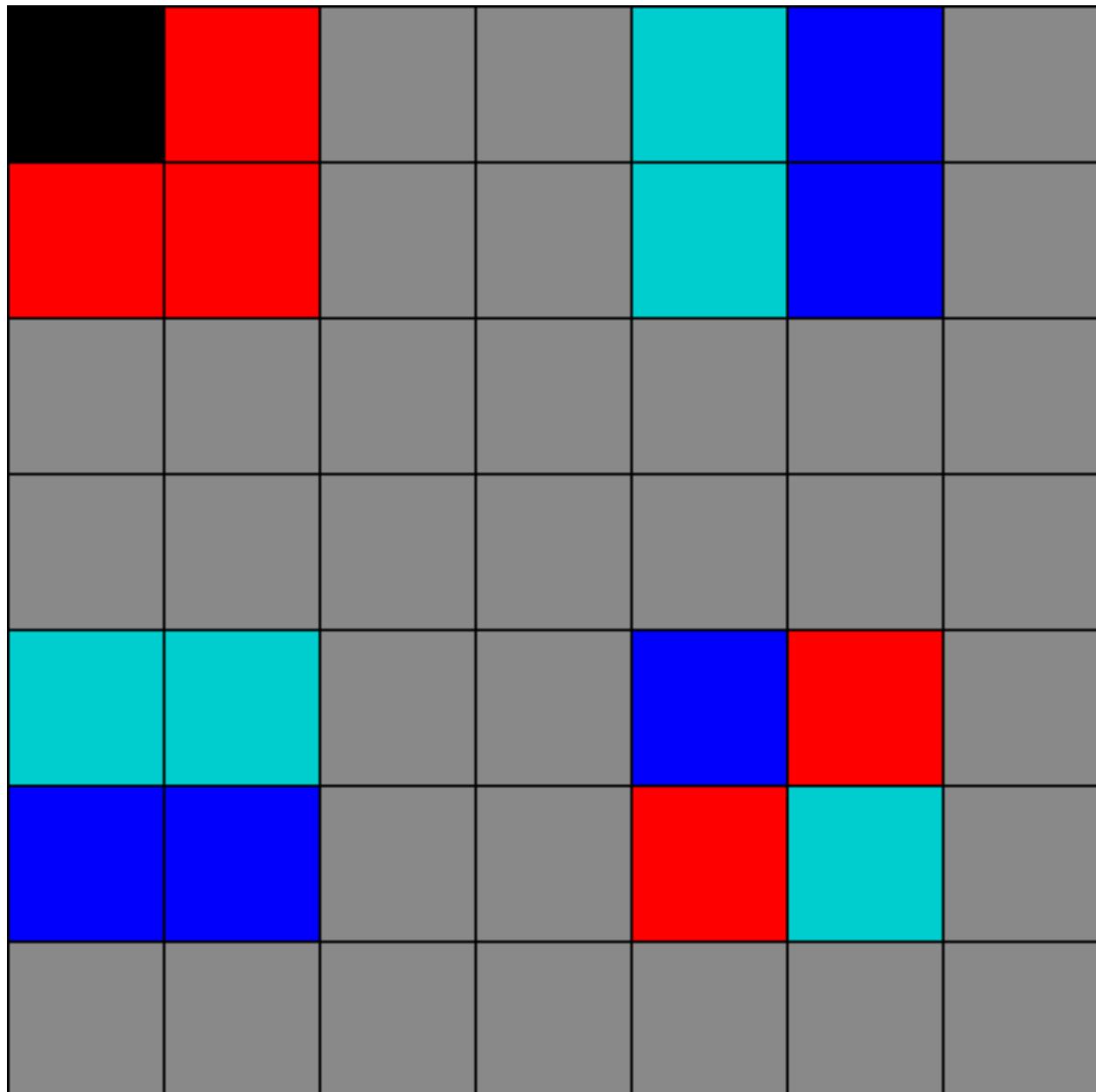


Figure 9: Alternating group of S_3

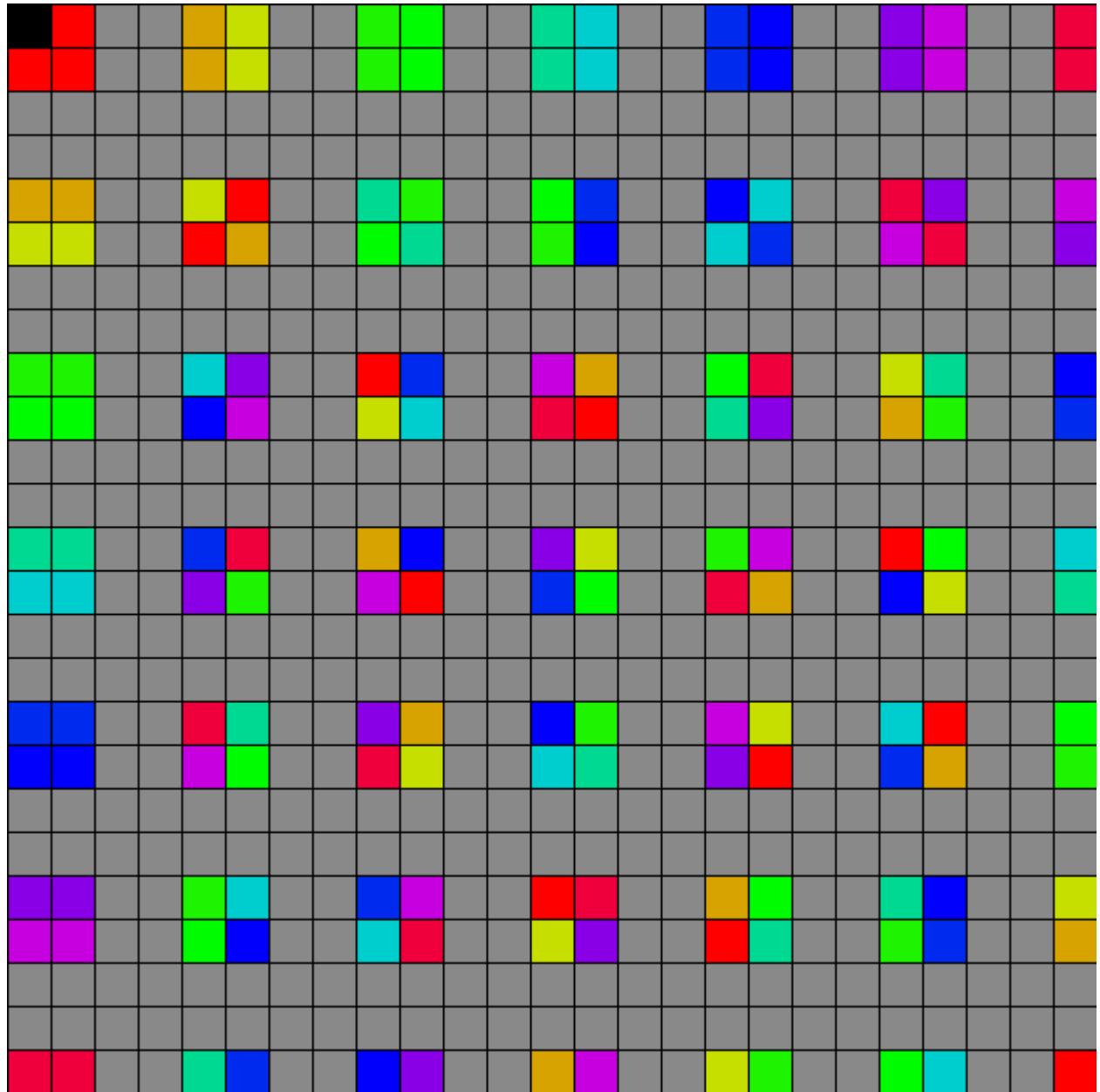


Figure 10: Alternating group of S_4

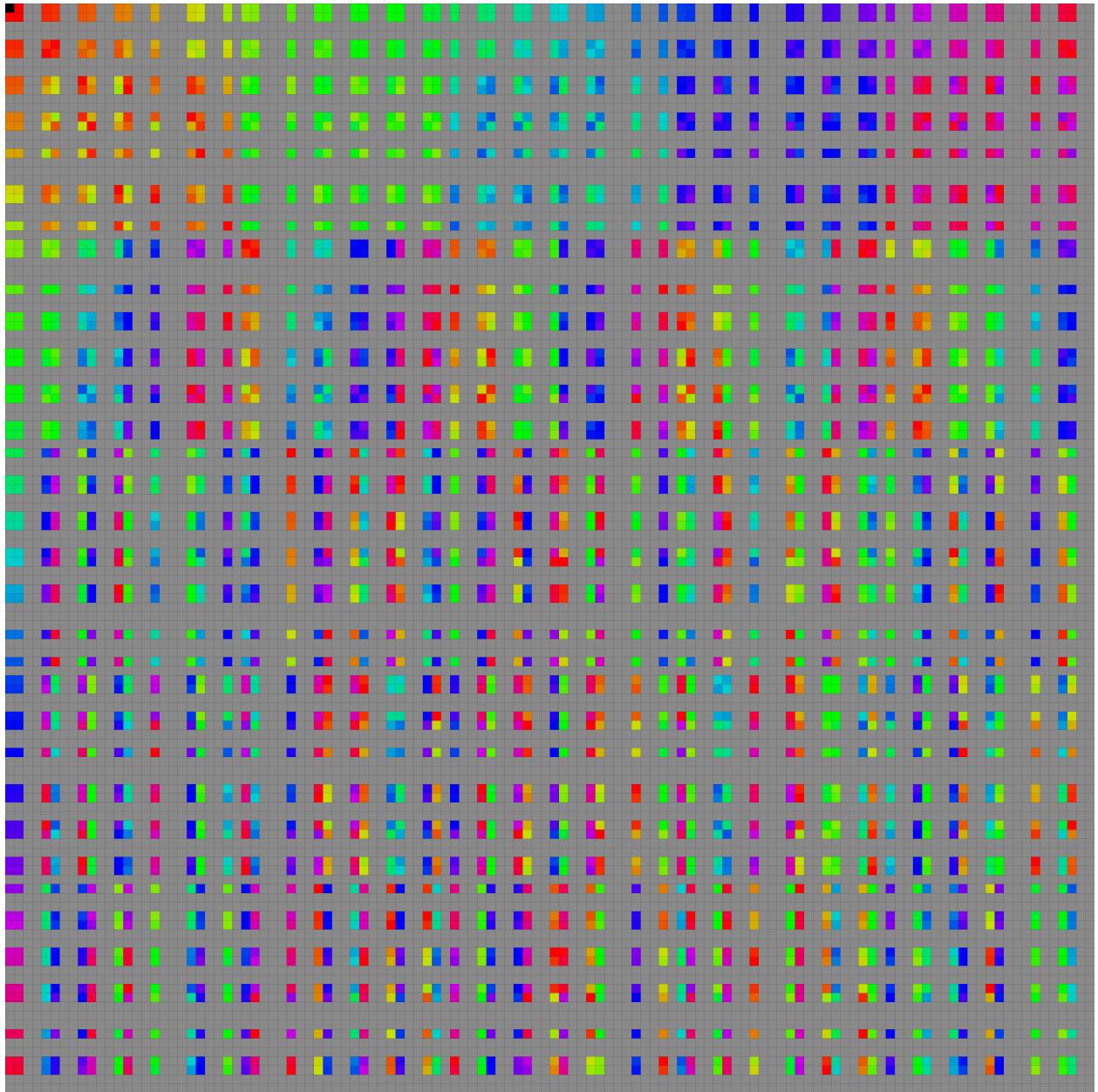


Figure 11: Alternating group od S_5