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STAT 215A
4/28/22

# TEST 2

1.

(a) $f_Y(y) = \frac{1}{35}\sum_{x=0}^{2} 2^x(1+y) = \frac{1+y}{35}\sum_{x=0}^{2} 2^x = \frac{(1+y)}{35}(1+2+4)$

$= \frac{(1+y)7}{35} = \frac{1+y}{5}$

(b) $f_X(x) = \frac{2^x}{35}\sum_{y=1}^{2} 1+y = \frac{2^x}{35}(2+3) = \frac{2^x 5}{35} = \frac{2^x}{7}, \ X=0,1,2$

(c) Letting $f(x) = \frac{1}{35}2^x$ and $h(y) = 1+y$, then $p_{x,y}(x,y) = f(x)h(y)$ and is thus seperable which implies $X$ and $Y$ are independent. Thus $E[XY] = E[X]E[Y]$.

- $E[X] = \frac{1}{7}\sum_{x=0}^{2} x\,2^x = \frac{1}{7}(2+8) = \cancel{8/35}\ 8/7$
- $E[Y] = \frac{1}{5}\sum_{y=1}^{2} y+y^2 = \frac{1}{5}(2+6) = 8/5$
- $E[XY] = E[X]E[Y] = \frac{8}{7}\cdot\frac{8}{5} = \frac{64}{35}$

(d) Since $X$ and $Y$ are independent $Cov(X,Y) = 0$.

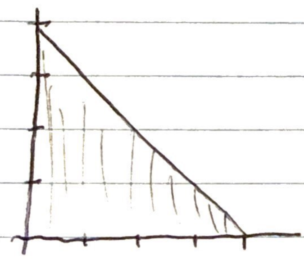
(e) $P(X=Y) = \frac{1}{35}\sum_{y=1}^{2}\sum_{x=1}^{2} 2^x(1+y) = \left((4+8)+(6+12)\right)\frac{1}{35}$

$\approx 0.857$

2.

(a) We have $f_X(x) = \frac{1}{5}$, $f_Y(y) = \frac{1}{2}e^{-y/2}$
and
$$f_{X,Y}(x,y) = \frac{1}{10}e^{-y/2}$$

The values $(x,y) \in \mathbb{R}^2$ such that $x+y<4$ given that $x \in (0,5)$ and $y \geq 0$ implies ~~$0 \leq x \leq 4$ and $0 \leq y \leq 4$~~ for each $0 \leq x \leq 4$ we need $y < 4-x$



$$\int_0^4 \int_0^{4-x} \frac{1}{10} e^{-y/2}\, dy\, dx$$

$$= \frac{1}{10}\int_0^4 \left( -2e^{-y/2} \Big|_0^{4-x} \right) dx$$

$$= \frac{1}{10}\int_0^4 \left(-2\left(e^{(x-4)/2} - 1\right)\right) dx$$

$$= \frac{-1}{5}\left(2e^{x/2-2} - x\right)\Big|_0^4$$

$$= -\frac{1}{5}\left((2e^0 - 4) - (2e^{-2})\right)$$

$$= -\frac{1}{5}\left(-2 - 2e^{-2}\right) \approx \boxed{0.454}$$

(b) By independence $E[X^2Y + 2Y] = E[X^2]E[Y] + 2E[Y]$

$$E[X^2] = \frac{1}{5}\int_0^5 x^2\, dx = \frac{1}{15}x^3 \Big|_0^5 = \frac{5^3}{15} = \frac{25}{3}$$

$$E[Y] = \frac{1}{2}\int_0^\infty y e^{-y/2}\, dy = \frac{1}{2}\left(-2e^{-x/2}(x+2)\right)\Big|_0^\infty = 2$$

$E[X^2]E[Y] + 2E[Y]$

$= \left(\frac{25}{3}\right)(2) + 2(2)$

$= \boxed{\frac{62}{3}}$

3

(a) $f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$

$$F_X(x) = P(X \le x) = P\left(\frac{1}{1+e^Z} \le x\right)$$

$$= P\left(\frac{1}{x} \le 1+e^Z\right)$$

$$= P\left(\ln\left(\frac{1}{x}-1\right) \le Z\right)$$

$$= F_Z\left(-\ln\left(\frac{1}{x}-1\right)\right)$$

$$f_X(x) = \frac{d}{dx}\int_{-\infty}^{-\ln(\frac{1}{x}-1)} \frac{1}{1+e^z}\,dz \;\; ? = \frac{d}{dx}\left(-\ln\left(\frac{1}{x}-1\right) - \ln\left(\frac{1}{2-2x}\right)\right)$$

$$= \boxed{\frac{1}{x}}$$

If the above is true, then $0 < x$.

(b) $\int_0^x \frac{1}{t}\,dt = \ln x = 0.5 \rightarrow x = \boxed{e^{1/2}}$