STAT 215A Quiz #2, Spring 2022

Instructions. This is a take-home assignment that consists of two problems. Attempt both of the problems and submit your solution document via Canvas Assignments by 1 pm on Thursday, April 7. This is a collaborative quiz, and so you are allowed to discuss solution ideas with your classmates, but without directly copying from each other. If you are closely working with one or two classmates, then you can submit your assignment jointly. So joint/ group submissions (with a group size of two or three) are also allowed if all the parties involved contribute to the assignment significantly. In that case, one group member can submit the resulting solution document with the names of the collaborators on the first page (or in the comments section on Canvas). You may also choose to upload your document individually by stating names of your collaborators, if any. If you notice any mistakes or if any part of a question is not clear to you, please let me know.

1. Let $X_1, X_2, ..., X_{10}$ be 10 independent random variables on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, each with a continuous uniform distribution on the interval $(0, a)$ where $a > 0$.

   (a) (1.5 pts) Let $Y = \max\{X_i : i = 1, 2, ..., 10\}$. More explicitly, $Y(\omega) = \max\{X_i(\omega) : i = 1, 2, ..., 10\}$ for $\omega \in \Omega$. Show that Y is a random variable on $(\Omega, \mathcal{F}, \mathbb{P})$. In other words, prove that $\{\omega \in \Omega : Y(w) \leq y\} \in \mathcal{F}$ for all $y \in \mathbb{R}$.

   (b) (1.5 pts) In fact, Y is a continuous random variable. Justify this by writing the cdf of Y, $F_Y(y)$, as an integral of an explicit nonnegative function $f_Y(.)$ such that $F_Y(y) = \int\limits_{-\infty}^{y} f_Y(t)dt$, for all $y \in \mathbb{R}$. Recall that $f_Y(.)$ is a pdf for Y.

   (c) (1 pt) Compute $P(Y > 2a/3)$.

2. An urn contains 5 balls numbered $1, 2, 3, 4$ and 5. We randomly select two balls, one after another and without replacement, and record the number on each ball. Let X_1 denote the number on the first ball selected, and X_2 denote the number on the second ball selected. So, clearly X_1 and X_2 are two discrete random variables on $(\Omega, \mathcal{F})$ where $\Omega = \{1, 2, 3, 4, 5\}$ and $\mathcal{F}$ is the power set of Ω. Moreover, define $Z = \min\{X_1, X_2\}$. One can check that Z is also a discrete random variable on $(\Omega, \mathcal{F})$.

   (a) (1.5 pts) Determine the pmf of Z.

   (b) (1.5 pts) Compute $E[Z]$ and $E[Z^2]$.