MATH 230B

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Assignment: Writeup

Homework 1 Summary

The problems in HW1 probed at certain properties of differentiation and continuity. There was one result that was used in almost all of the exercises in this homework, the Mean Value Theorem. This theorem states that a function f which is continuous over an interval $[a, b]$ and differentiable over (a, b) is said to have a derivative at a particular point $c \in (a, b)$ such that $f'(c) = (f(b) - f(a))/(b - a)$. The power of this theorem is that it presents an explicit relationship between the function evaluated at the endpoints and the derivative of the function. In the case where you know more about the function than you do about its derivative, then this theorem allows you to take advantage of that asymmetry, and vice versa.

Several exercises included the condition of a function with a bounded derivative (see problems 2, A1, A3). What this condition gives you is Lipschitz continuity of the function. This type of continuity allows you to directly relate the distance between the images of two points in the domain and the distance between the two points themselves. For example, in problem 2 the functions derivative was strictly bounded 1. This means that for any x, y in our domain, $|f(x) - f(y)| < |x - y|$. How this was used in this exercise was two take x, y to be two terms in a Cauchy sequence whose distance was less than some ε, which in turn allowed us to show that the distance between the two image of the two sequence terms was less than ε, hence showing that the sequence of image terms was Cauchy and therefore convergent.

Something worth noting is that problem A1 is a generalization of problem 2. Both required that the derivative be strictly bounded by 1 which gives rise to a contraction mapping. The same proof structure was used but instead of referring to a specific Cauchy sequence, we used an arbitrary one. The one in problem 2 converged to the left end point (to 0), whereas in problem A1, we worked with a general Cauchy sequence which converged to the right endpoint of the domain.

Another concept present in this homework was that of using the derivative of the function to indicate whether a function is increasing or decreasing (see problems 4 and 5). In problem 4, we were able to show that the given function was increasing by showing that its derivative was greater than 0 over its domain. Specifically, the function was a quotient function and we needed only to show that the numerator in the resultant derivative was greater than or equal to 0.

Two other problems that I found noteworthy were problems 3 and A3. Both of them had to do with showing the existence of unique fixed points, but the methods used to do it were very different for each. In problem 4, the approach was to use contradiction and assume there was no fixed points. From there one exhausts all the implications of the other premises to show that each one leads to a contradiction. In problem A3, we had another bounded derivative and a recursive function definition. In this case, the derivative was bounded by one and with the recursive definition, we were dealing with a contraction mapping. The approach to this problem began by showing that the sequence defined by the difference between two consecutive terms in the sequence went to 0. From there one shows the general case that the difference between arbitrary terms go to zero which is to show that the sequence is Cauchy. To show uniqueness of the fixed point used the Mean Value Theorem to produce a contradiction when assuming the existence of two distinct fixed points.

Lastly, in problem A4 the approach was to start with any nonzero x and show that any derivative of the function can be re-expressed as a rational polynomial multiplied by the original function. This then gives rise to the infinite differentiability that we were set out to prove.