

## STAT 215A Test #2, Spring 2022

**Instructions.** This test consists of three problems. You can utilize notes, formula, technology (e.g. a calculator, Matlab, R etc.) and a normal area table. Show your work and submit the solution file via Canvas before 5:20 pm today. This is an individual assignment (so no collaboration/ discussion or outside assistance is allowed). I will be available via Zoom at 4-5:15 pm for any issues or questions. After uploading your file on Canvas, please make sure to confirm the submission was successful.

1. Let  $X$  and  $Y$  be jointly discrete random variables on a probability space where their joint pmf is given by  $p(x, y) = p_{X,Y}(x, y) = \frac{2^x + 2^x y}{35}$ , for  $x = 0, 1, 2$  and for  $y = 1, 2$ .
  - (a) (1 pt) Show that the marginal pmf of  $Y$  is  $\frac{1+y}{5}$ , for  $y = 1, 2$ .
  - (b) (1 pt) Determine the marginal pmf of  $X$ .
  - (c) (1.5 pts) Compute  $E[XY]$ .
  - (d) (1 pt) Determine the covariance of  $X$  and  $Y$ .
  - (e) (1 pt) Determine  $P(X = Y)$ .
2. Let  $X$  and  $Y$  be two independent continuous random variables on a probability space where  $X$  has continuous uniform distribution on the interval  $(0, 5)$ , and  $Y$  has  $Exp(1/2)$  distribution. Hence, by independence, their joint pdf,  $f(x, y)$ , is given by the product of individual pdf expressions.
  - (a) (2 pts) Write an explicit double integral for the following probability expression, and then compute the resulting integral:  $P(X + Y < 4)$ .
  - (b) (1.5 pts) Compute  $E[X^2Y + 2Y]$ .
3. Let  $Z \sim N(0, 1)$  and define  $X = \frac{1}{1+e^Z}$ .
  - (a) (2 pts) Determine the pdf of  $X$  along with its domain.
  - (b) (1 pt) Determine the median of  $X$ . Show your work.