STAT 215A HW#1, Spring 2022

Instructions. This homework assignment consists of three problems but you only need to submit your answers to two of them. It is due by 3:50 pm on Tuesday, February 8. You are allowed to collaborate in pairs or groups of size three, and submit the assignment (with answers to two of the problems) jointly. In that case, one group member can submit the resulting solution document with the names of the collaborators on the first page (or in the comments section). You may also choose to upload each document individually by stating names of your collaborators, if any. Each question is worth one point.

1. Let $\mathcal{F}_1$ and $\mathcal{F}_2$ be two sigma-algebras on a sample space Ω. We know that $\mathcal{F}_1 \cap \mathcal{F}_2$ is a sigma-algebra on Ω. Show that $\mathcal{F}_1 \cup \mathcal{F}_2$ is not necessarily a sigma-algebra on Ω.

2. Let $\{A_k : k = 1, 2, ..., n\}$ be a finite collection of events in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Prove the following inequality for any such finite collection of n events:

$$\mathbb{P}(\bigcup_{k=1}^{n} A_k) \geq \sum_{k=1}^{n} \mathbb{P}(\mathbb{A}_k) - \sum_{1 \leq j < k \leq n} \mathbb{P}(\mathbb{A}_j \cap \mathbb{A}_k).$$

   Hint: You may try a proof by induction on n.

3. Let A, B and C be three events in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

   (a) Let D be the event that exactly one event in the set $\{A, B\}$ occurs. Show that $\mathbb{P}(D) = \mathbb{P}(A) + \mathbb{P}(B) - 2\mathbb{P}(A \cap B)$.

   (b) Let E be the event that at least one event in $\{A, B, C\}$ occurs. Describe $\mathbb{P}(E)$ in terms of the probabilities of the events in $\{A, B, C\}$ and/or their intersection probabilities.