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STAT 215A
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HW #5
(Quiz #3 Take Home)

Option B

$$P(W=w) = \sum_{x=1}^{4} P_{x,y}(x, w-x)$$

$$= C\left(\frac{1^{w-1}}{(w-1)!} + \frac{2^{w-2}}{(w-2)!} + \frac{3^{w-3}}{(w-3)!} + \frac{4^{w-4}}{(w-4)!}\right)$$

$$= C\left(\frac{1^{w}}{(w-1)!} + \frac{2^{w}}{2(w-2)!} + \frac{3^{w}}{3(w-3)!} + \frac{4^{w}}{4(w-4)!}\right)$$

$$= C\sum_{x=1}^{4} \frac{x^{w}}{x(w-x)!}$$

Since we need $w-x \geq 0$, then this is valid for $w \in \{4,5,6\}$.

Bonus

We need C such that $\sum_{y=0}^{2}\sum_{x=1}^{4} C\frac{x^y}{y!} = 1$.

$$C\sum_{y=0}^{2}\sum_{x=1}^{4}\frac{x^y}{y!} = C\sum_{y=0}^{2}\left(\frac{1}{y!} + \frac{2^y}{y!} + \frac{3^y}{y!} + \frac{4^y}{y!}\right)$$

$$= C\left((1+1+1+1)+(1+2+3+4)+\left(\tfrac{1}{2}+\tfrac{4}{2}+\tfrac{9}{2}+\tfrac{16}{2}\right)\right)$$

$$= C(4+10+15$$

$$= 39C \longrightarrow C = \frac{1}{39}.$$