
STAT 215A

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Due Date: NONE
Assignment: PS03

1. Out of 12 balls in an urn, 5 of them are green. Consider a sample size of $n = 4$ without replacement. Let A_k be the event that a sample of size 4 contains exactly k green balls for $0 \leq k \leq 4$. Moreover, let B_j be the event that the ball selected at the j th step is green for $1 \leq j \leq 4$.

- (a) Determine $P(B_j)$ for each $1 \leq j \leq 4$.

Solution. Using Lemma 4 from section 1.4, we get the following probabilities

$$\begin{aligned} P(B_1) &= \frac{5}{12} \approx 0.42 \\ P(B_2) &= P(B_2 | B_1)P(B_1) + P(B_2 | B_1^c)P(B_1^c) \\ &= \frac{4}{11} \frac{5}{12} + \frac{5}{11} \frac{7}{12} \approx 0.42 \\ P(B_3) &= P(B_3 | B_2 \cap B_1)P(B_2 \cap B_1) + P(B_3 | B_2^c \cap B_1)P(B_2^c \cap B_1) \\ &\quad + P(B_3 | B_2 \cap B_1^c)P(B_2 \cap B_1^c) + P(B_3 | B_2^c \cap B_1^c)P(B_2^c \cap B_1^c) \\ &= \frac{3}{10} \frac{4}{11} \frac{5}{12} + \frac{4}{10} \frac{7}{11} \frac{5}{12} + \frac{4}{10} \frac{5}{11} \frac{7}{12} + \frac{5}{10} \frac{6}{11} \frac{7}{12} \approx 0.42 \end{aligned}$$

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- (b) Compute $P(A_3 | B_2)$.

Solution. We want to know the probability of our sample size containing 3 green balls given that the second ball selected was green. This can happen in the following ways: balls 1, 2, and 3 are green; balls 1, 2, and 4 are green; balls 2, 3, and 4 are green. The probabilities associated with these three events are

$$\frac{5}{12} \frac{4}{11} \frac{3}{10} \frac{7}{9} + \frac{5}{12} \frac{4}{11} \frac{7}{10} \frac{3}{9} + \frac{7}{12} \frac{5}{11} \frac{4}{10} \frac{3}{9} \approx$$

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- (c) Compute $P(B_j | A_3)$ for each $1 \leq j \leq 4$.

Solution.

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- (d) Would your answers to any of the parts (a)-(c) above change if the sampling is done with replacement?

Solution.

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