Visualizing S_n

Quin Darcy

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Below is a demonstration of the various images produced by the program `group.py`. This program starts prompting the user to enter a value `n`. From here the program recursively generates the set of all permutations of the numbers $1, \ldots, n$. To do this *Heap's Algorithm* is used. The number of permutations generated is $n!$. At this point, each permutation is assigned a unique color in the form (R, G, B), where $0 \leq R \leq 255$, $0 \leq G \leq 255$, $0 \leq B \leq 255$. This assignment is accomplished by dividing $2\pi/n!$, and to the ith permutation p_i, we assign $i(2\pi/n!)$ for each i. Thus there is a one-to-one correspondance between the $n!$ permutations and $n!$ angles $0 \leq \theta_i < 2\pi$. From here the set of angles is sent to a function which maps the given angle θ_i to a point (R_i, G_i, B_i).

Equipped with a set of $n!$ (R, G, B) tuples, we then compute the Cayley table for S_n. After this, the program then computes the set of all cyclic subgroups, it computes the alternating group A_n, and promts the user to choose an element from the group and computes the orbit and stabilizer of this element. After all this sets are computed, an image is constructed, pixel by pixel, referencing the set of (R, G, B) tuples.

Before showing the images, it is worth noting how the permutations are ordered. If $n = 3$, we should expect the top row of the Cayley table to ordered as

$$[1, 2, 3], [1, 3, 2], [2, 1, 3], [2, 3, 1], [3, 1, 2], [3, 2, 1].$$

One thing to notice is that the program keeps the permuations as is instead of converting them to cycle notation.

Below are the Cayley tables for $n = 3, 4, 5, 6, 7$, and various other of the above mentioned sets. $n = 7$ sadly the limit for my current CPU.

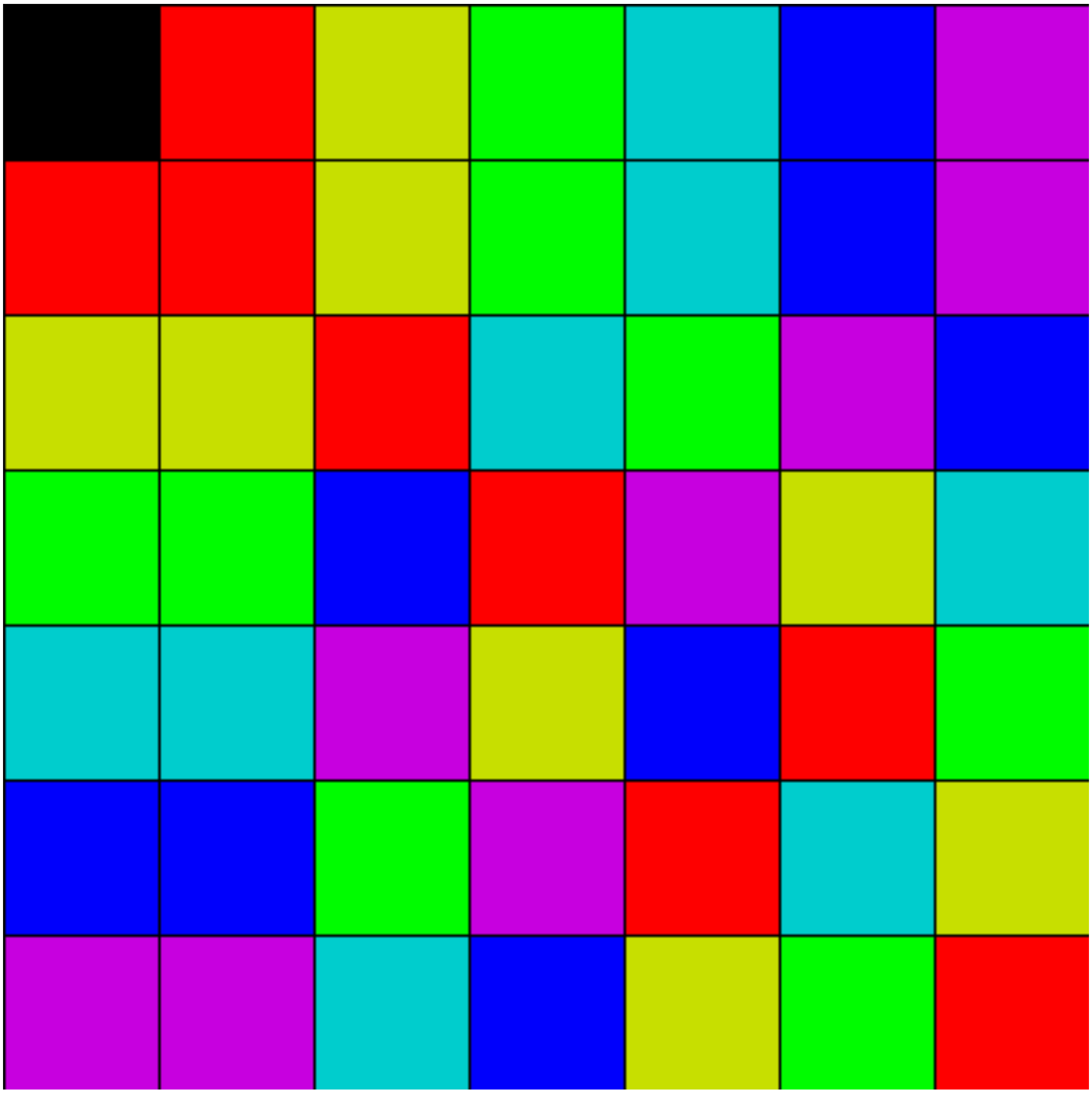

Figure 1: Cayley Table for S_3

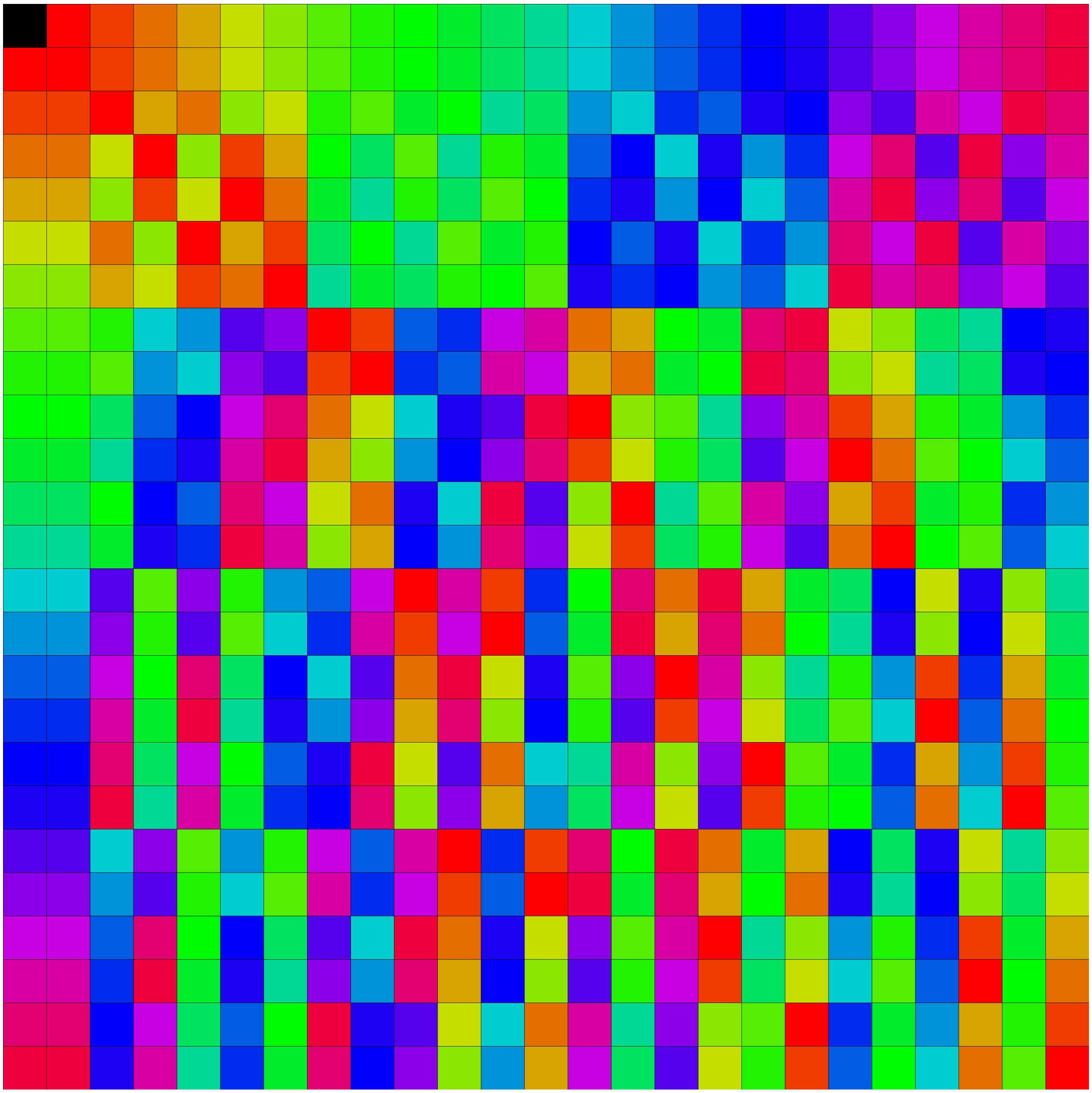

Figure 2: Cayley Table for S_4

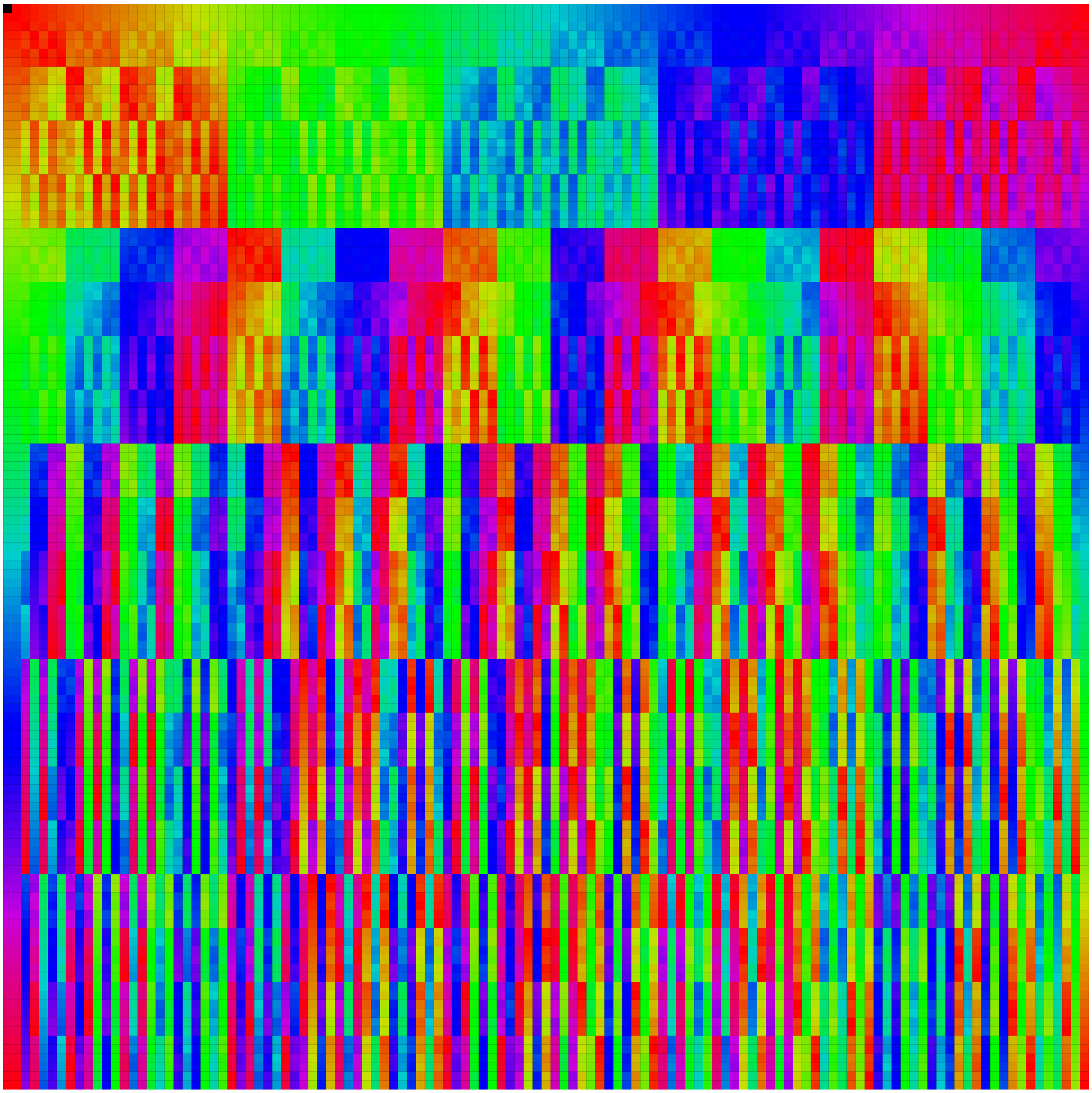

Figure 3: Cayley Table for S_5

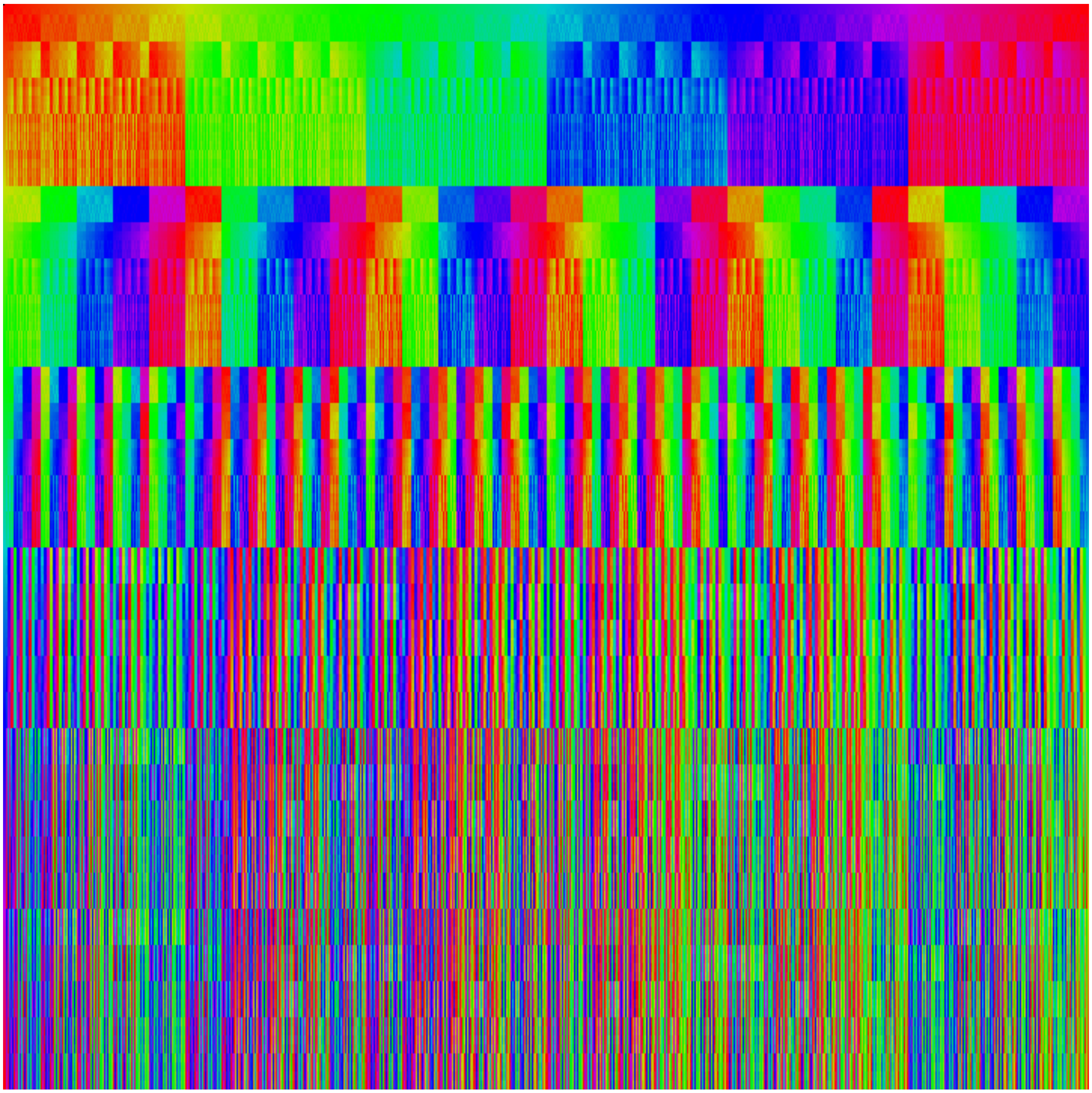

Figure 4: Cayley Table for S_6

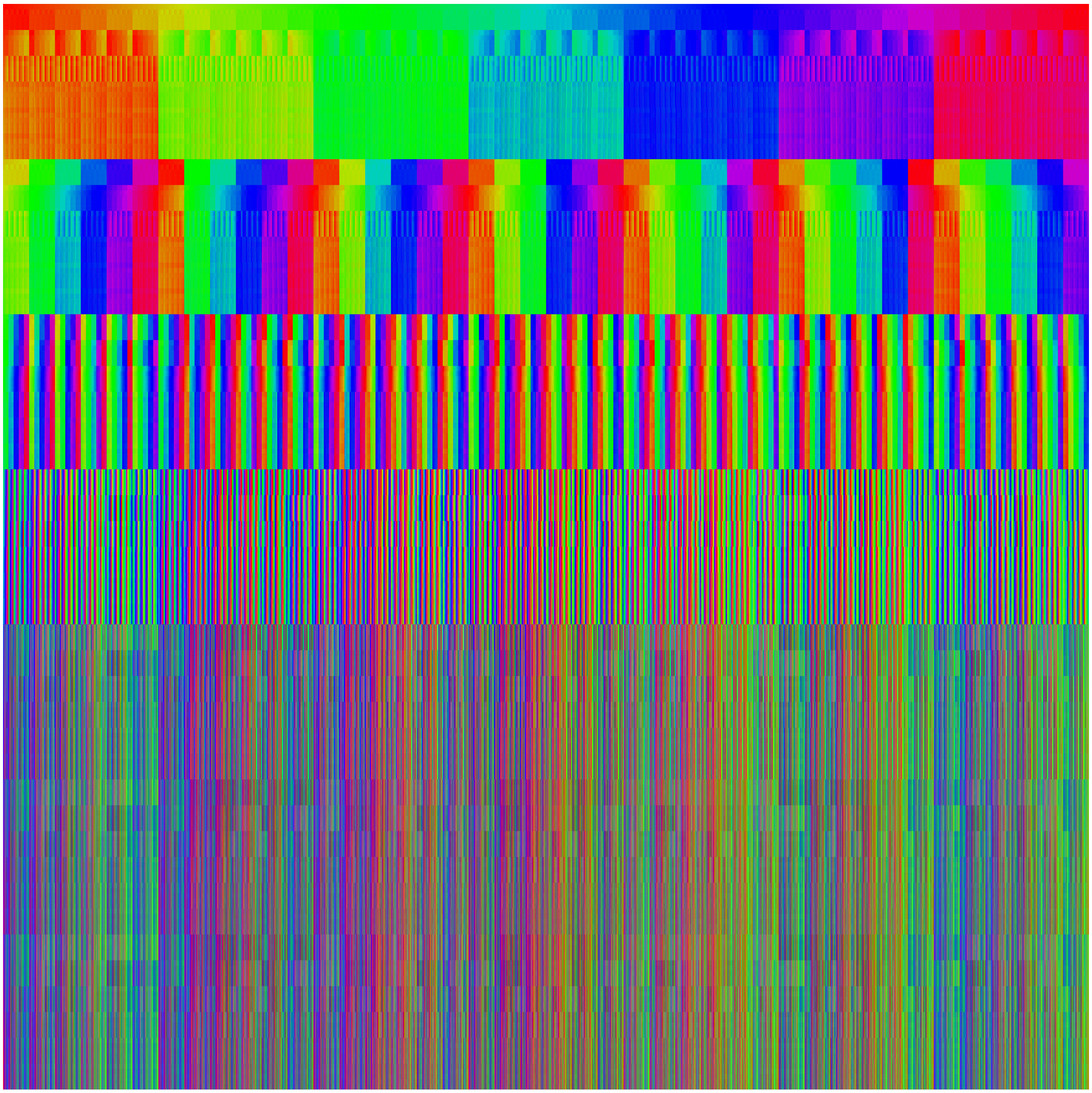

Figure 5: Cayley Table for S_7

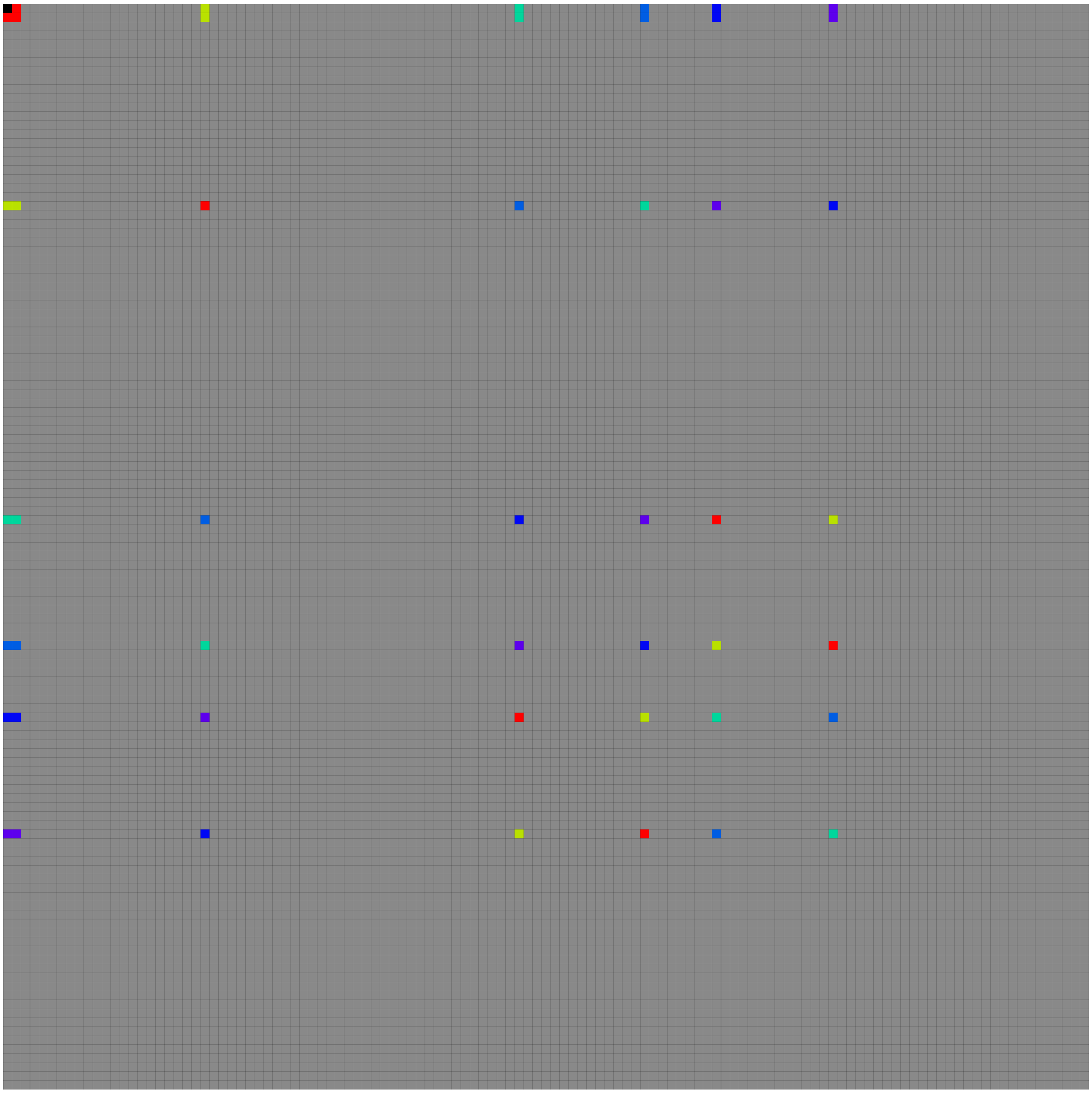

Figure 6: Cyclic Subgroup of (2453) in S_5

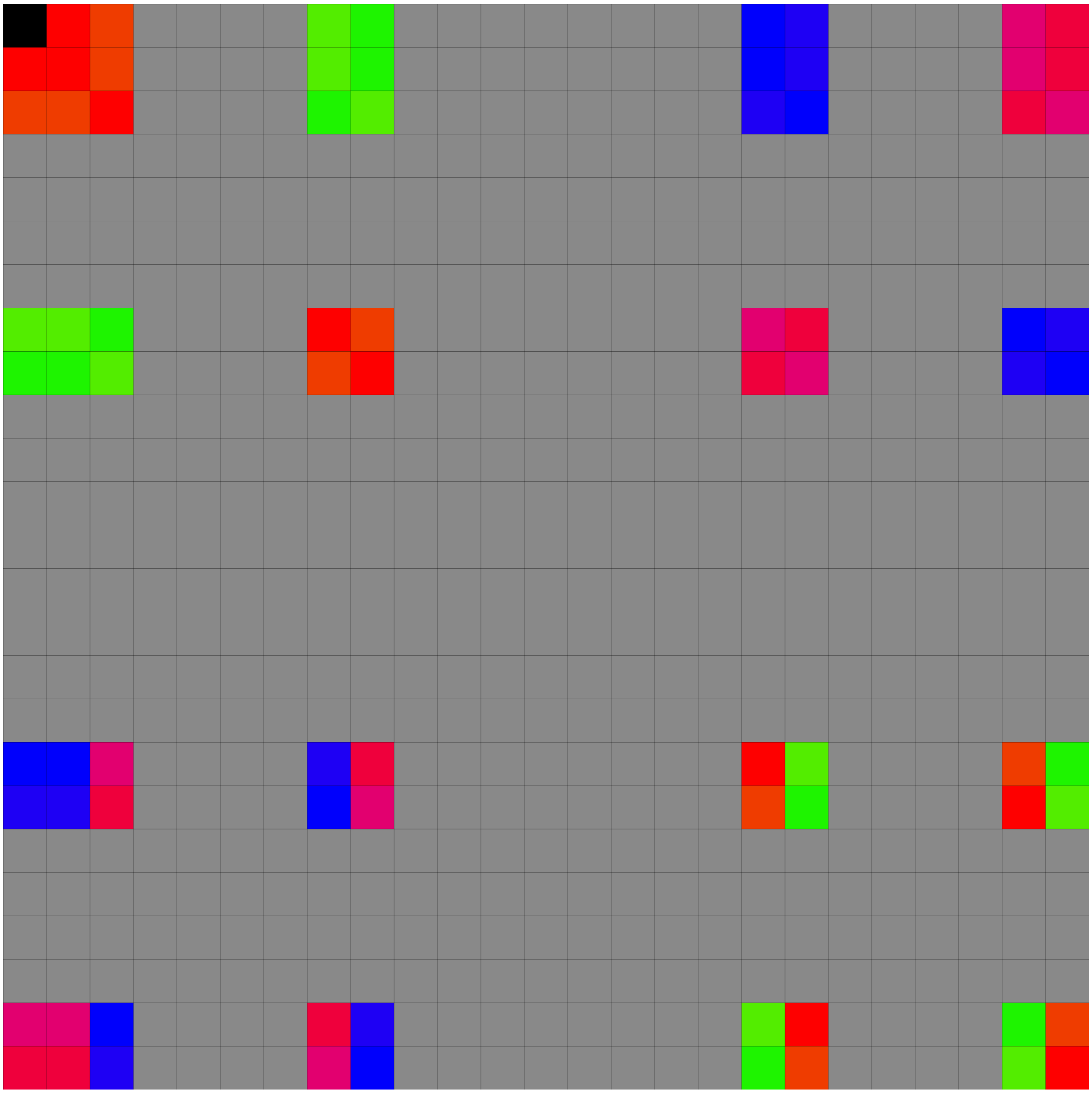

Figure 7: Stabilizer of (12)(34) in S_4

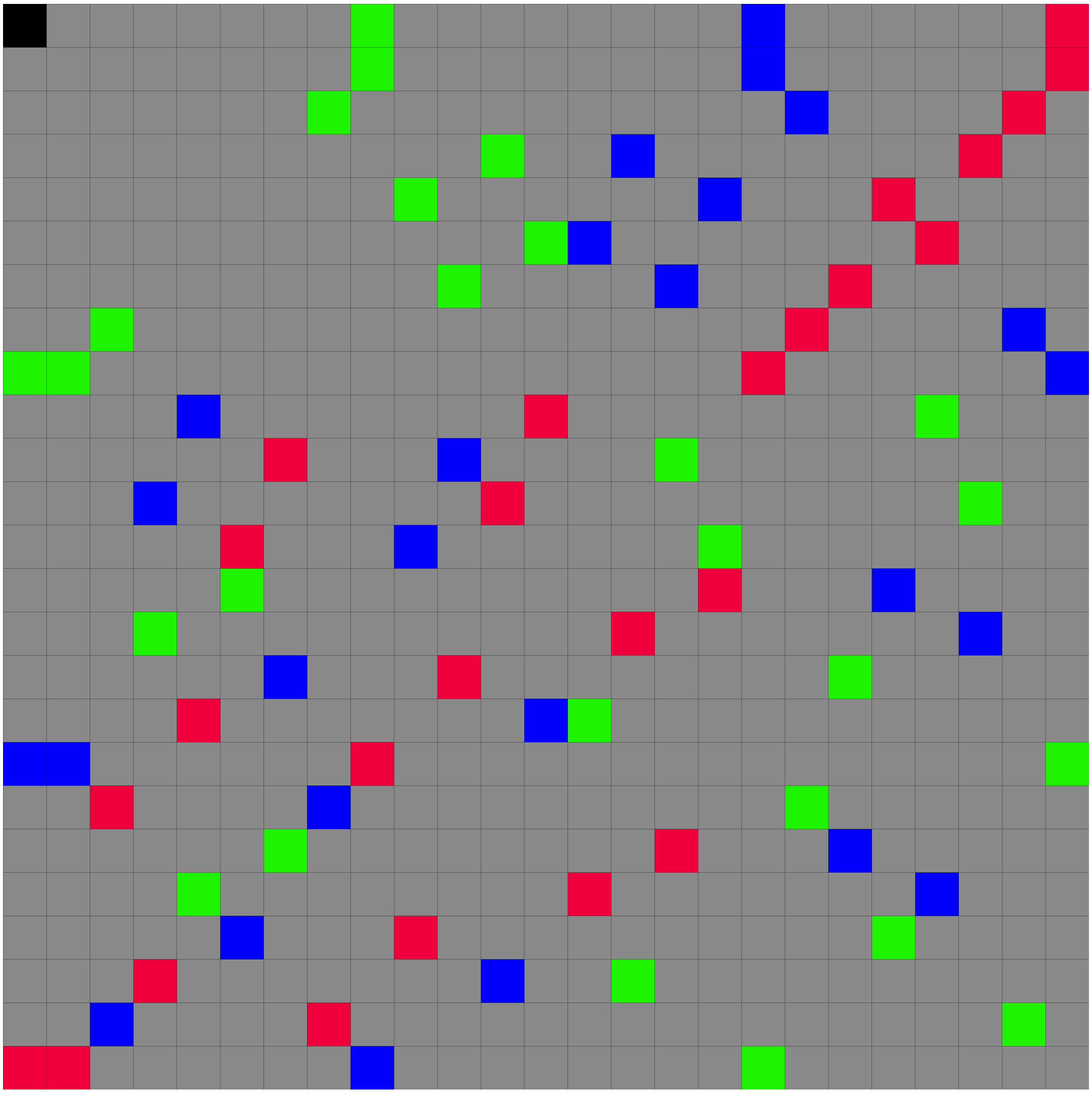

Figure 8: Orbit of (12)(34) in S_4

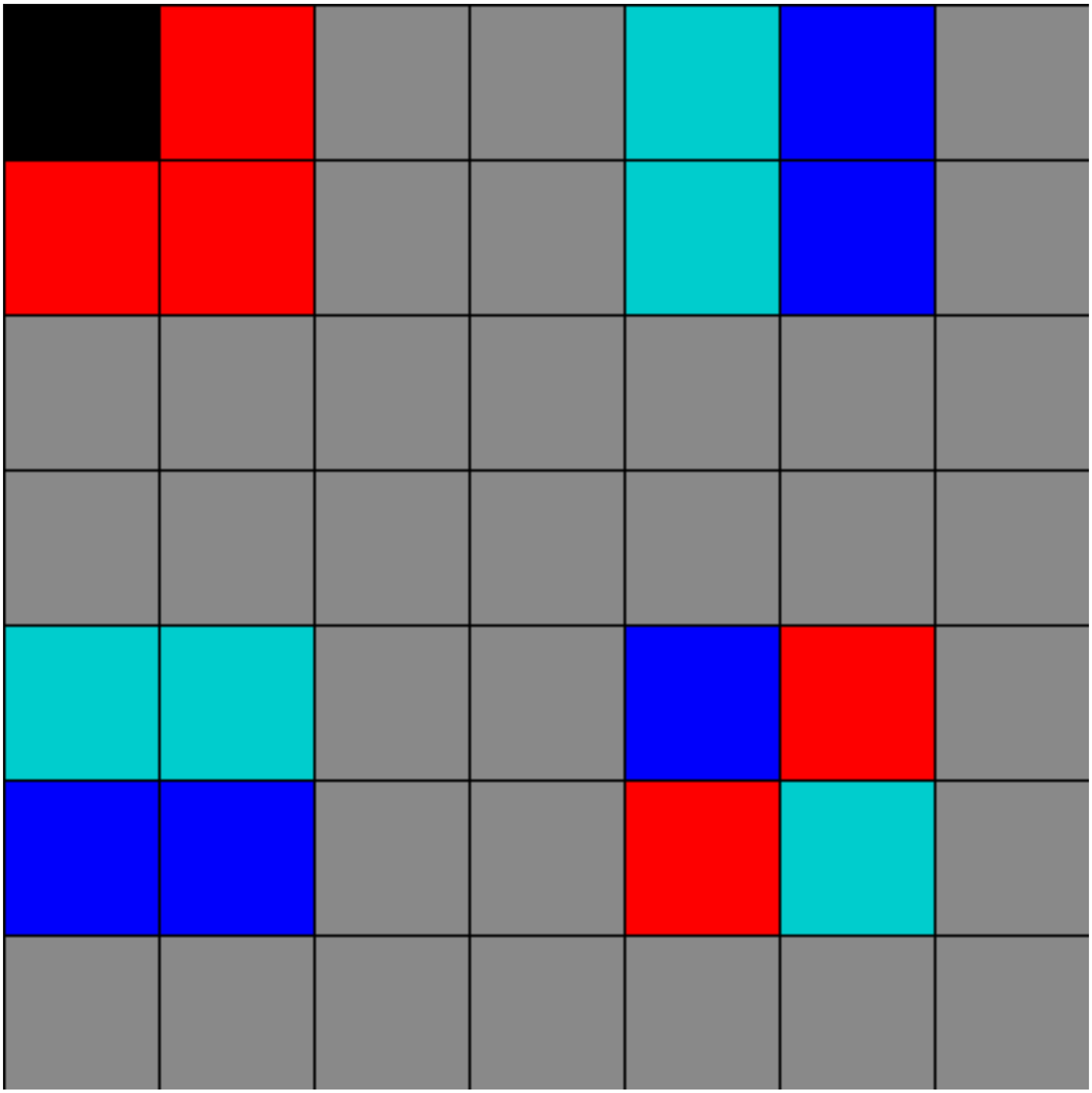

Figure 9: Alternating group of S_3

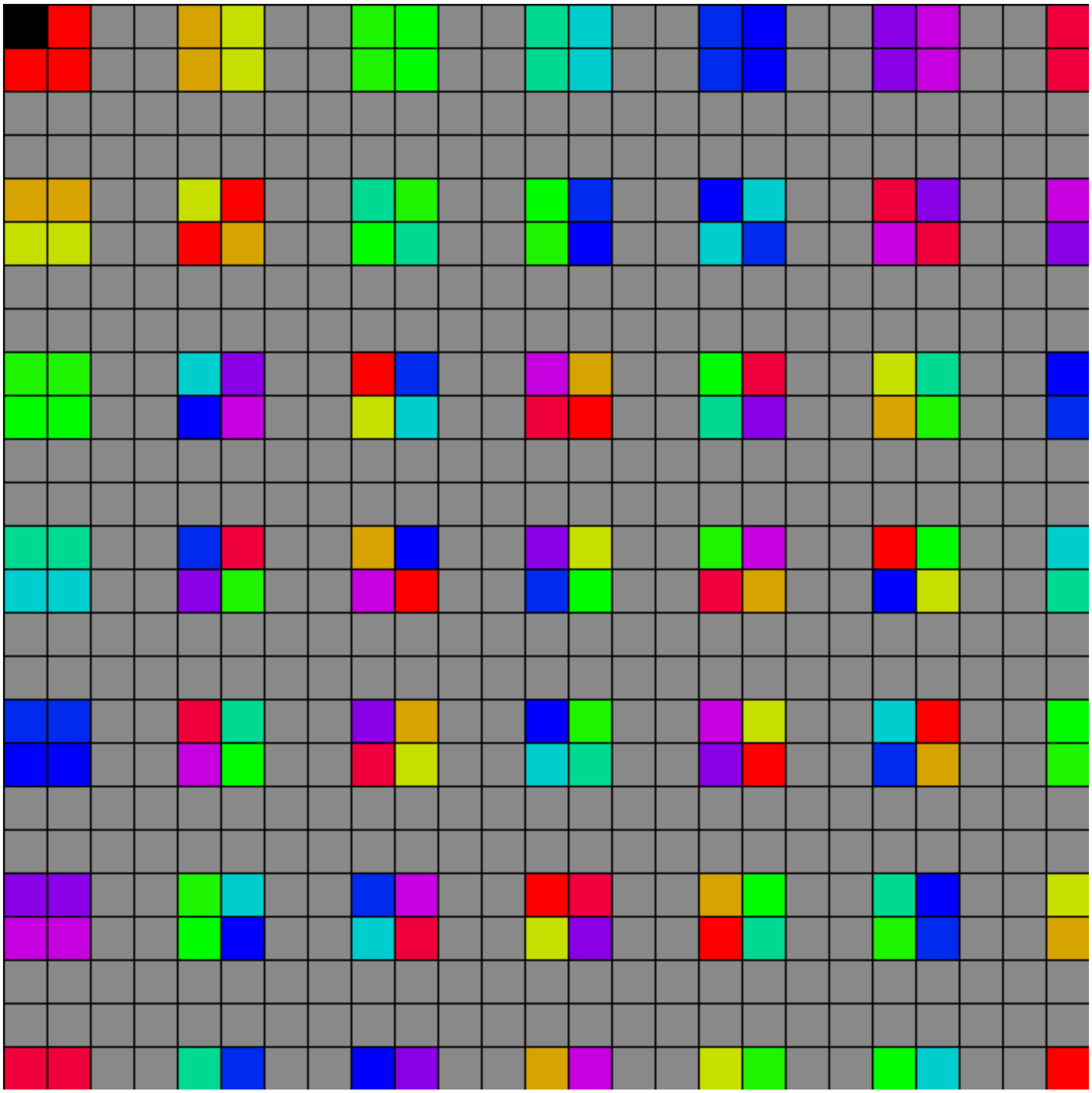

Figure 10: Alternating group of S_4

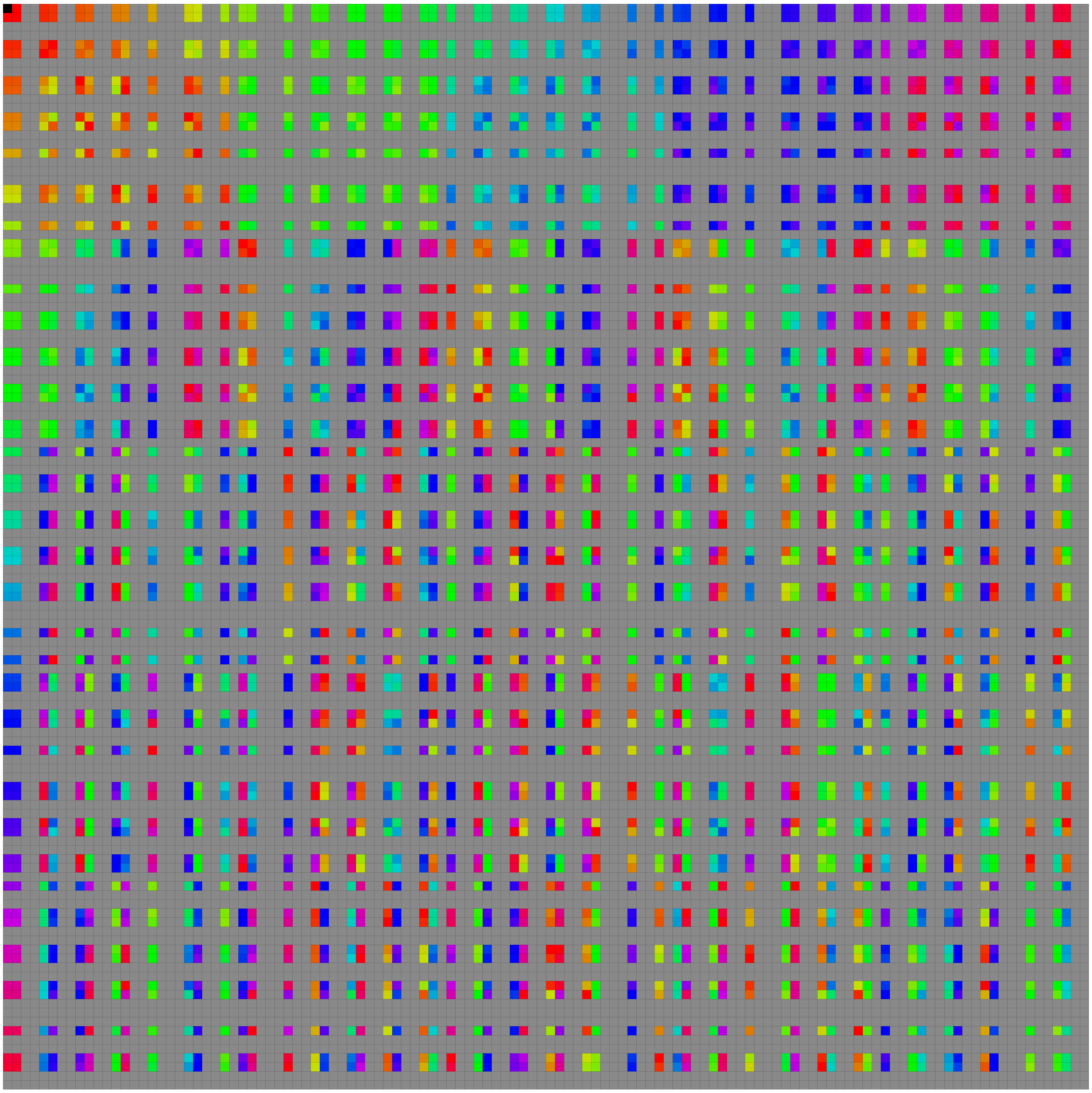

Figure 11: Alternating group od S_5