STAT 215A HW#3, Spring 2022

Instructions. This homework assignment consists of three problems but you only need to submit your solutions to two of them. The assignment is due by 12 noon on Tuesday, March 8. You are allowed to collaborate in pairs or groups of size three, and submit the assignment jointly. In that case, one group member can submit the resulting solution document with the names of the collaborators on the first page (or in the comments section on Canvas). You may also choose to upload your document individually by stating names of your collaborators, if any.

1. (1 pt) Let X be a continuous r.v. with the cdf $F(x) = \int_{-\infty}^{x} f(t)dt$ and pdf $f(.)$. Prove that
   (a) The function kfF^{k-1} is also a pdf for $k \in \mathbb{N}$.
   (b) If G is another cdf and $0 < c < 1$, then $cF + (1-c)G$ is also a cdf.
2. (1 pt) Consider the experiment of rolling a balanced six-sided die repeatedly until a six occurs for the first time. Let Y represent the variable for the number of the rolls before the first six appears. So, the possible values of Y are $0, 1, 2, ...$ (Y is unbounded).
   (a) Determine the pmf of Y.
   (b) Compute $P(Y > 1 + k | Y > 1)$, where $k \in \mathbb{N}$.
3. (1 pt) Let X be a r.v. in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ such that $P(X \geq 0) = 1$. Moreover, suppose that $Y = 1 - e^{-2X}$.
   (a) Explain why Y is also a r.v. on $(\Omega, \mathcal{F}, \mathbb{P})$, and then describe the cdf of Y in terms of the cdf of X.
   (b) If $X \sim Exp(2)$, then determine the pdf of Y.