STAT 215A

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Instructor: Dr. Cetin | Assignment: Homework 02

1. Depending on the risk level of home insurance contacts/policies, an insurance company divided a city into three regions: high risk, medium risk, and low risk.

   - About 10% of its customers live in the high risk region where about one fifth of the customers filed a claim last year.
   - About 40% of its customers live in the medium risk region where about 6% of the customers filed a claim last year.
   - Only about 2% of the customers in the low risk region filed a claim last year.

   One of the customers is selected at random. Determine the probability that

   (a) the customer filed a claim last year

   Solution. We begin by labelling some of the events with the first being C and it will denote the event of filing a claim. The other labels

$$\begin{aligned} A_1 &: \text{Lives in high risk region} \\ A_2 &: \text{Lives in medium risk region} \\ A_3 &: \text{Lives in low risk region} \end{aligned}$$

   Then based on the given information we have that

$$\begin{aligned} \mathbb{P}(A_1) &= 0.1 & \mathbb{P}(C \mid A_1) &= 0.2 \\ \mathbb{P}(A_2) &= 0.4 & \mathbb{P}(C \mid A_2) &= 0.06 \\ \mathbb{P}(A_3) &= 0.5 & \mathbb{P}(C \mid A_3) &= 0.02. \end{aligned}$$

   With this we can calculate $\mathbb{P}(C)$ by using the total probability formula

$$\mathbb{P}(C) = \sum_{i}^{3} \mathbb{P}(A_i)\mathbb{P}(C \mid A_i) = 0.02 + 0.024 + 0.01 = 0.054.$$

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   (b) the customer lived in the high risk region, given that the customer filed a claim last year.

Solution. We want to calculate the probability of A_1 given C. Using the information above we get

$$\mathbb{P}(A_1 \mid C) = \frac{\mathbb{P}(C \mid A_1)\mathbb{P}(A_1)}{\sum_{i=1}^{3} \mathbb{P}(C \mid A_i)\mathbb{P}(A_i)} = \frac{\mathbb{P}(C \mid A_1)\mathbb{P}(A_1)}{\mathbb{P}(C)} = \frac{0.02}{0.054} = 0.37.$$

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2. Assume that three events A, B, and C are pairwise disjoint, each with positive probability. Prove or disprove each of the following statements:

   (a) The events $A \cup B$ and C are independent.

   Solution. Consider an experiment where a coin is tossed twice. The possible outcomes of this experiment are $\Omega = \{HH, HT, TH, TT\}$. Let A denote the event that a heads occurred on the first flip. Then $A = \{HH, HT\}$. Let B denote the event that a heads occurred on the last flip, then $B = \{HH, TH\}$. Finally, let C denote the event that both flips were the same, then $C = \{HH, TT\}$. From this it follows that

   $$(A \cup B) \cap C = HH.$$

   Thus

   $$\mathbb{P}((A \cup B) \cap C) = 1/4$$

   whereas

   $$\mathbb{P}(A \cup B)\mathbb{P}(C) = (3/4)(1/4) = 3/16.$$

   Therefore $\mathbb{P}((A \cup B) \cap C) \neq \mathbb{P}(A \cup B)\mathbb{P}(C)$ and so $A \cup B$ and C are not independent.

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   (b) The events $A \cap B$ and $C \cap B$ are independent.

   Solution. Using the same example as above, we have that $A \cap B = \{HH\}$ and $B \cap C = \{HH\}$ and so $(A \cap B) \cap (B \cap C) = \{HH\}$. Thus

   $$\mathbb{P}((A \cap B) \cap (B \cap C)) = 1/4,$$

   whereas

   $$\mathbb{P}(A \cap B)\mathbb{P}(B \cap C) = (1/4)(1/4) = 1/16.$$

   Thus

   $$\mathbb{P}((A \cap B) \cap (B \cap C)) \neq \mathbb{P}(A \cap B)\mathbb{P}(B \cap C).$$

   Therefore $A \cap B$ and $B \cap C$ are not independent. ■