STAT 215A HW#4, Spring 2022

Instructions. For the submisison of this homework assignment, choose one of the two problems given below. It is due by noon on Tuesday, April 26. You are allowed to collaborate in pairs or groups of size three, and submit the solution on Canvas (either separately or as a joint submission by one member).

1. Let $\{X_k : k \in \mathbb{N}\}$ be a sequence i.i.d. discrete random variables such that each member has the following pmf: $P(X_k = 1) = p$ and $P(X_k = -1) = 1 - p$, where $0 < p < 1$. For $n \in \mathbb{N}$, define $S_n = \sum_{k=1}^{n} X_k$.

   (a) Determine the pmf of $S_3 = \sum_{k=1}^{3} X_k$.

   (b) Find the mean and variance of S_n, for each n.

   (c) Consider the symmetric case where $p = 1/2 = 1 - p$. Determine $Cov(S_n, S_m)$, for all $n, m \in \mathbb{N}$. Hint: First consider $n < m$, and use independence of $\{X_k : k \in \mathbb{N}\}$.

2. Let X be a continuous r.v. with the following pdf: For fixed $C > 0$, let $f(x) = \frac{C}{x^{C+1}}$ for $x > 1$ (and it is zero otherwise).

   (a) Verify that f is a valid pdf for all $C > 0$. Then, determine the cdf of X.

   (b) Compute the expected value of X.

   (c) Determine the median of X.

   (d) Determine the cdf of $Y = 1/X$.