Welcome to Population Ecology!

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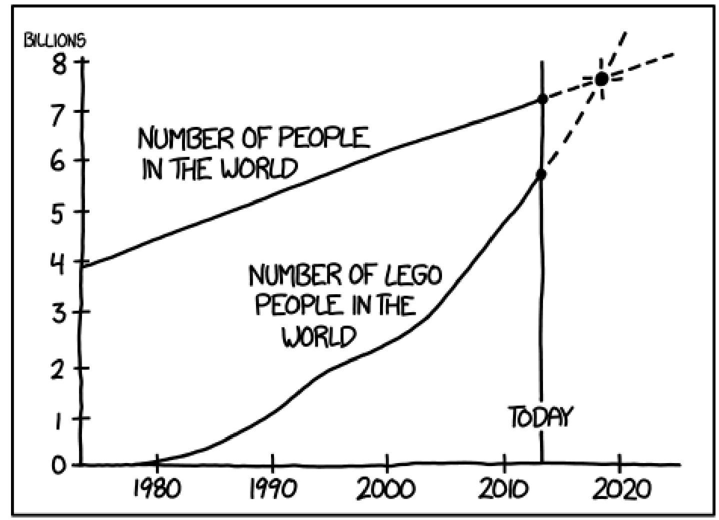
17/04/2019

3 part lecture (with breaks)

- **Part 1** Exponential growth
- **Part 2** Logistic growth - *Density dependent*
- **Part 3** Logistic growth with stochasticity



BY 2019, HUMANS WILL BE OUTNUMBERED.

Part 1: Why be interested in population growth?

- Project future populations
  - Human population expected to be 9.8 billion by 2050
- Conservation of species
- Sustainable use of resources
- Many more

Part 1: Let's start with exponential population growth

Density independent growth

$$\frac{dN}{dt} = rN$$

Exponential growh

What is required for a population to grow?

How many births and how many deaths?

$$N_{t+1} = N_t + B - D + I - E$$

- B = *Births*
- D = *Deaths*
- I = *Immigration*
- E = *Emigration*

If we assume that immigration and emmigration are equal, then the change in population size is:

$$\Delta N = B - D$$

Exponential growth

$$\Delta N = B - D$$

- More births than deaths the population grows

- More deaths than births the population dies

Exponential growth

Change in population (dN) *over a very small interval of time* (dt) can be described as:

$$\frac{dN}{dt} = B - D$$

- Births and deaths are also described in rates:

$$B = bN$$

b = instantaneous birth rate

[births / (individual * time)]

$$D = dN$$

d = instantaneous death rate

[deaths / (individual * time)]

Exponential growth

SO change in population over time can be described as

$$\frac{dN}{dt} = (b - d)N$$

$$\frac{dN}{dt} = (0.55 - 0.50)N$$

$$\frac{dN}{dt} = (0.55 - 0.50) * 100$$

$$\frac{dN}{dt} = 0.05 * 100$$

$$\frac{dN}{dt} = 5$$

Exponential growth

If we let $b - d$ become the constant r, the **intrinsic rate of increase**, we have the continuous exponential growth equation:

$$\frac{dN}{dt} = rN$$

Exponential growth

Change in population is equal to the intrinsic rate of increase (r) multiplied by the population size (N)

$$\frac{dN}{dt} = rN$$

N = population size

r = intrinsic rate or increase

r defines how fast a population is growing or declining

- $r = 0$ no growth
- $r > 0$ positive growth
- $r < 0$ negative growth

The differential equation tells us growth **rate** not population size

Exponential growth

The pooulation of an exponentially growing population at a given time t can be worked out by:

$$N_t = N_0 e^{rt}$$

Look familiar from assignment 1?

The discrete version of the exponential equation tells us the number added to the population per time-step:

$$N_{t+1} = N_t + r_d N_t$$

$$N_{t+1} = 100 + 0.05 * 100$$

$$N_{t+1} = 105$$

N_t = Population size at time t

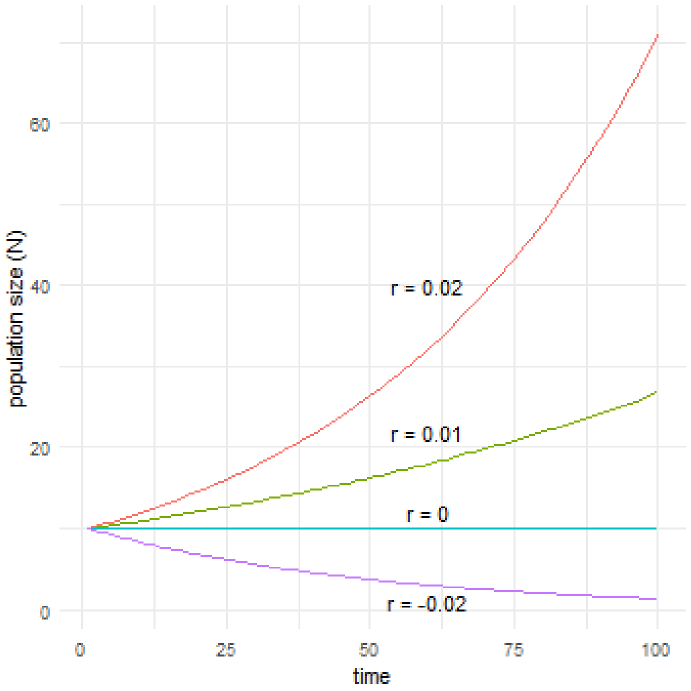
r_d = discrete growth factor

Exponential growth

Theoretical populations of populations growing (and declining) as a result of different values of r

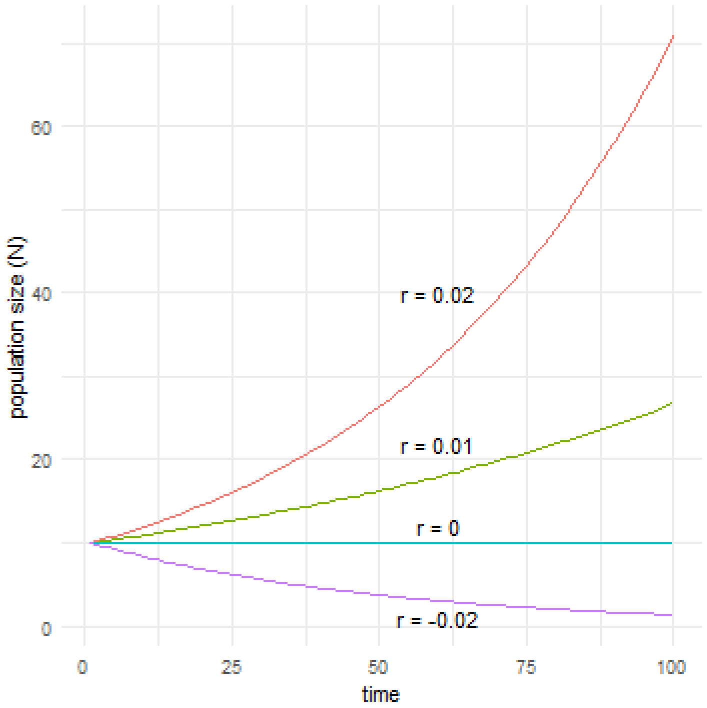
- $r = 0$ no growth
- $r > 0$ positive growth
- $r < 0$ negative growth

Because growth rate is exponential, by taking the natural logarithm of the population size the graphed lines become straight

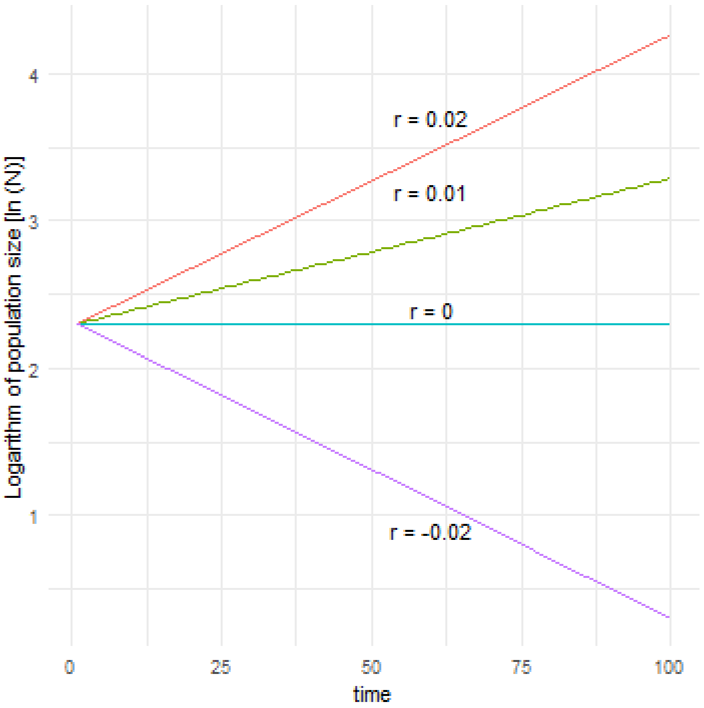


Exponential growth

Exponential growth



Logarithm of exponential growth

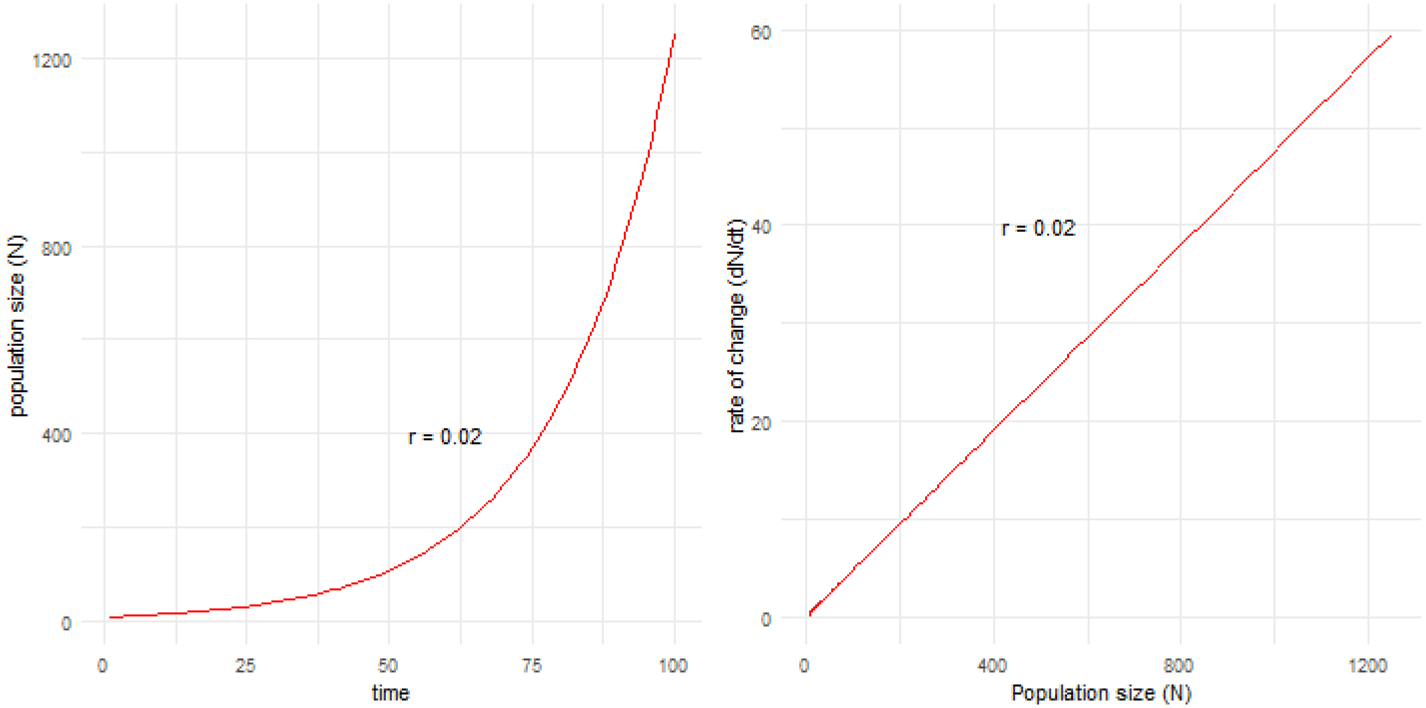


Growth rates

Common name	**r [individuals/(individual*day)]**	**Doubling time**
Virus	300.0	3.3 minutes
Bacterium	58.7	17 minutes
Protozoan	1.59	10.5 hours
Hydra	0.34	2 days
Flour beetle	0.101	6.9 days
Brown rat	0.0148	46.8 days
Domestic cow	0.001	1.9 years
Mangrove	0.00055	3.5 years
Southern beech	0.000075	25.3 years

Growth rates

Population increases exponentially but **Growth rate** over **population size** increases proportionately with the population



Exponential growth

- Populations growing exponentially have a doubling time
- The doubling time depends on the growth rate r and is *not* every year
- Surely no species can grow forever exponentially?!?!?!
    - *Correct!* Welcome to part 2, **density dependence**

Take 5 minutes to discuss the assumptions of the exponential growth model

Part 2: Logistic population growth

Density dependent growth

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

Logistic growth

Now we will look at populations which do not grow forever but reach a **carrying capacity** (K)

K represents the maximum population size that can be supported considering limiting factors such as food, shelter and space

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

N = Population size

r = Intrinsic rate of increase

K = Carrying capacity

Logistic growth

Consider the term $\left(1 - \frac{N}{K}\right)$ as a penalty on the growth of the population depending on the number of individuals in the community

- Very crowded communities have a high penalty compared to ones with plenty of space and resources

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

- At the point the population reaches the carrying capacity: N will = K, and the fraction $\frac{N}{K}$ will = 1

- The term $\left(1 - \frac{N}{K}\right)$ will collapse to 0 and the change in population will = 0. The equation will be multiplied by 0 and equal 0

- So the population will remain at size K

Logistic growth

If the population is very small, N is small relative to K, then the penalty is small

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

However, as we learned from exponential growth, a population grows in proportion to its size.

- A population of 1000 seabirds will produce more eggs than a population of 100.
- In the logistic growth equation the proportion added to the population decreases as the population grows reaching 0 when $N = K$

Logistic growth

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

- If $K = 100$ and $N = 7$ then the unused space is $1 - (7/100) = 0.93$ and the population is growing at 93% of the growth rate of an exponentially growing population
- If $K = 100$ and $N = 98$ then the unused proportion of capacity is $1 - (98/100) = 0.02$ and growth is at 2% of the growth rate of an exponentially growing population

The point at which a population is the largest relative to the penalty for its size is at $K/2$

- What this means is the growth rate of a population is fastest at half its carrying capacity
- As it grows bigger than $K/2$ the penalty becomes stronger but below $K/2$ the population is small and the proportion added to the population is small

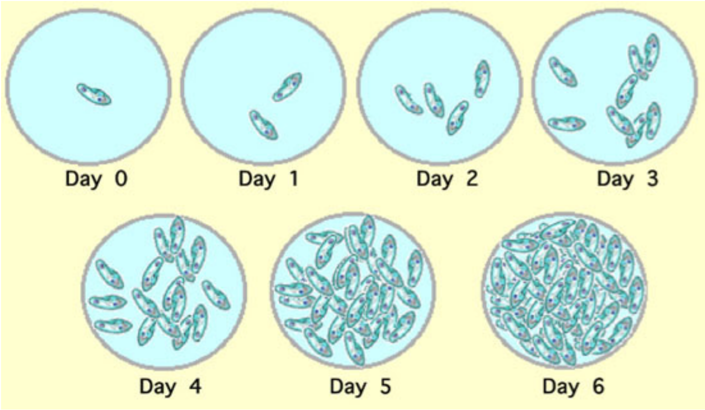
Logistic growth

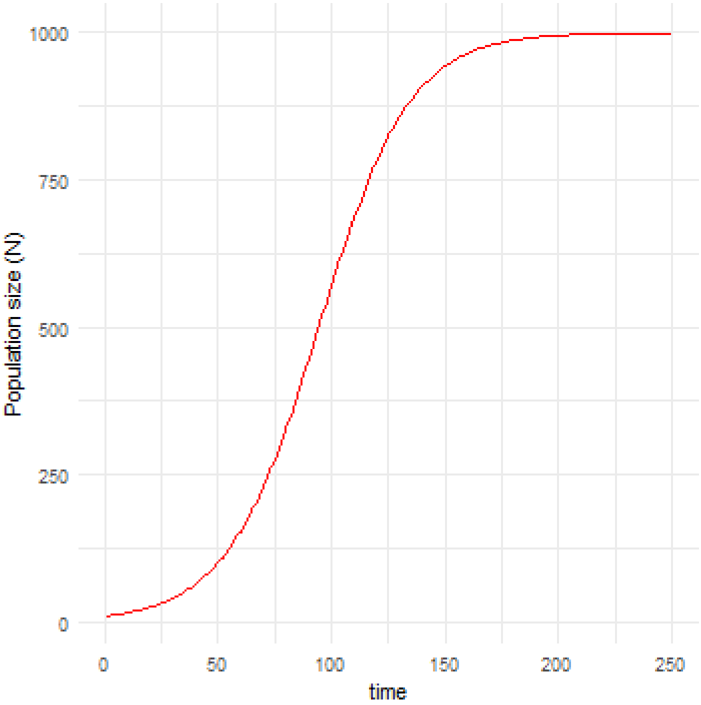
Thanos should have studied population ecology

- If a species is at capacity, its growth rate will increase to maximum if you cut it in half.
- Not all species are equal so cutting all in half will not have an equal effect…

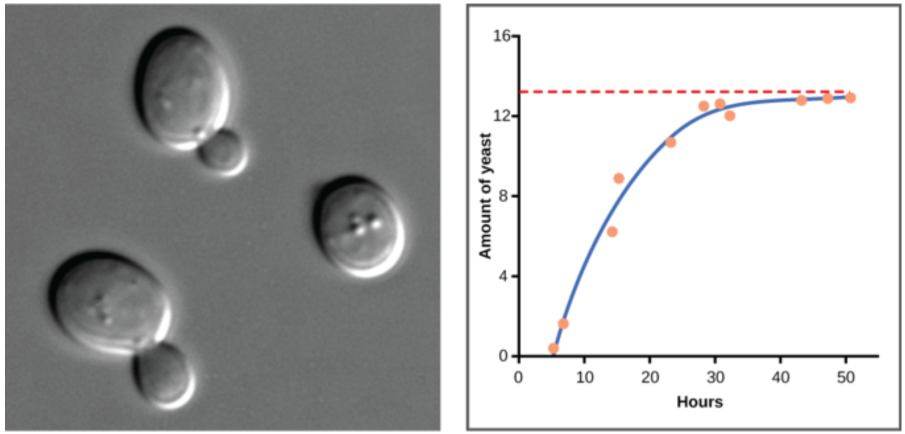
Logistic growth

Paramecium growing to capacity.

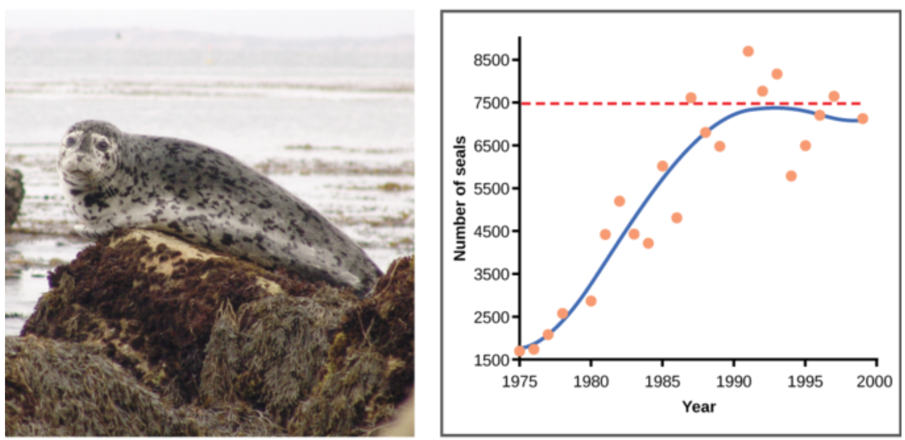




Logistic growth



(a)



(b)

Logistic growth

Now we will look at the discrete form of the equation:

$$N_{t+1} = N_t + r_d N_t \left(1 - \frac{N_t}{K}\right)$$

N_t = Population size at time t

r_d = discrete growth factor

K = Carrying capacity

- Instead of telling us the change in a population at an infinitely small point in time, the discrete equation tells us the size of the population at a given time.

Logistic growth

The size of the population at the next time-step is equal to:

- The size of the current population plus the current population multiplied by the discrete growth rate and the density penalty.

$$N_{t+1} = N_t + r_d N_t \left(1 - \frac{N_t}{K}\right)$$

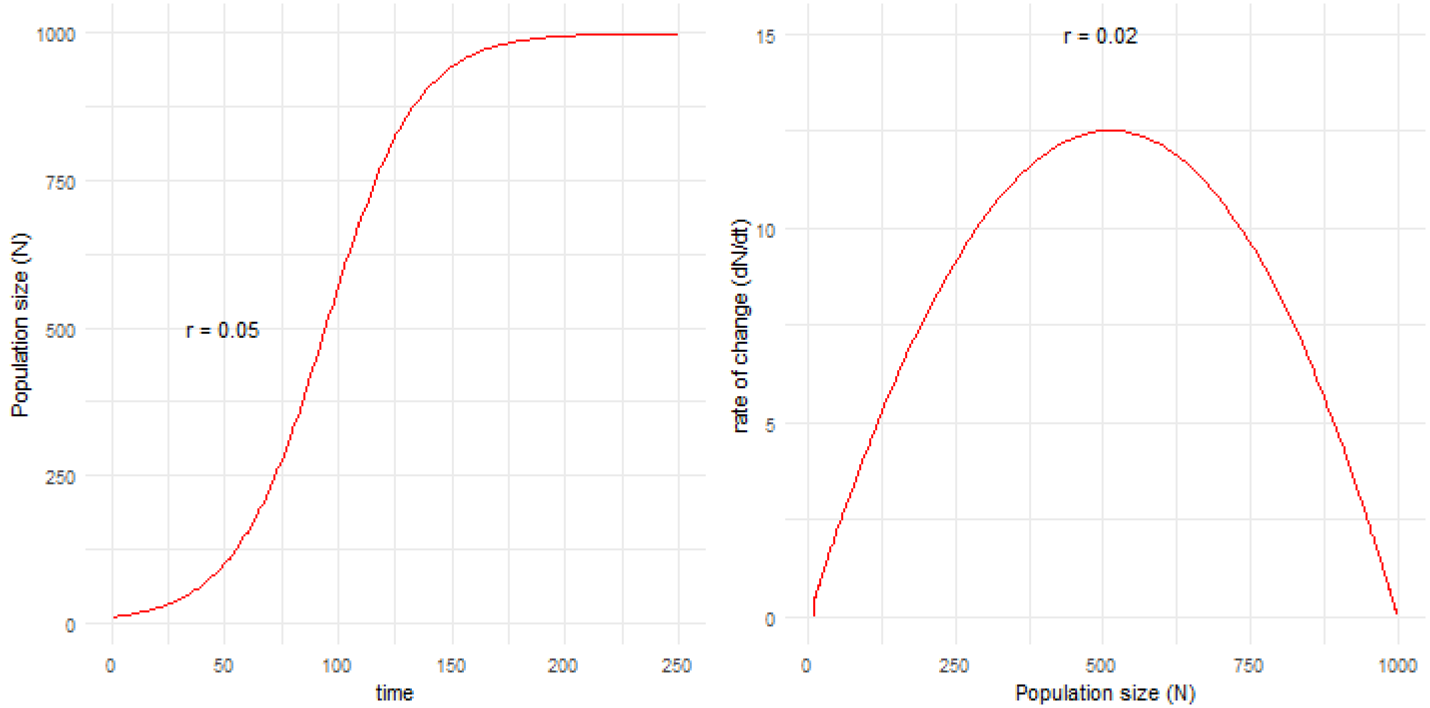
$$N_{t+1} = 100 + 0.05 * 100 \left(1 - \frac{100}{200}\right)$$

$$N_{t+1} = 102.5$$

Remember how in the exponential model $N_{t+1} = 105$?

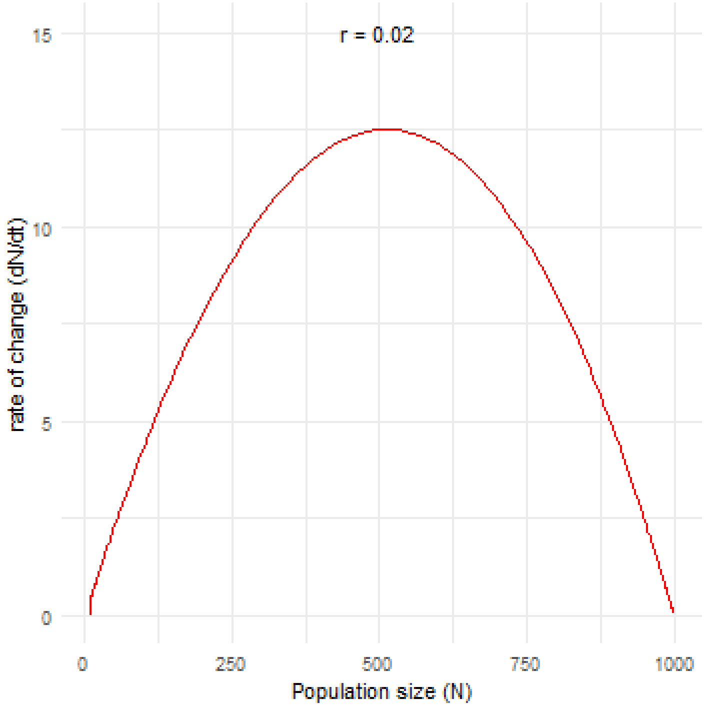
Logistic growth

Unlike the exponential model, the logistic model growth rate is dependent on population size and reaches its peak at half of the carrying capacity

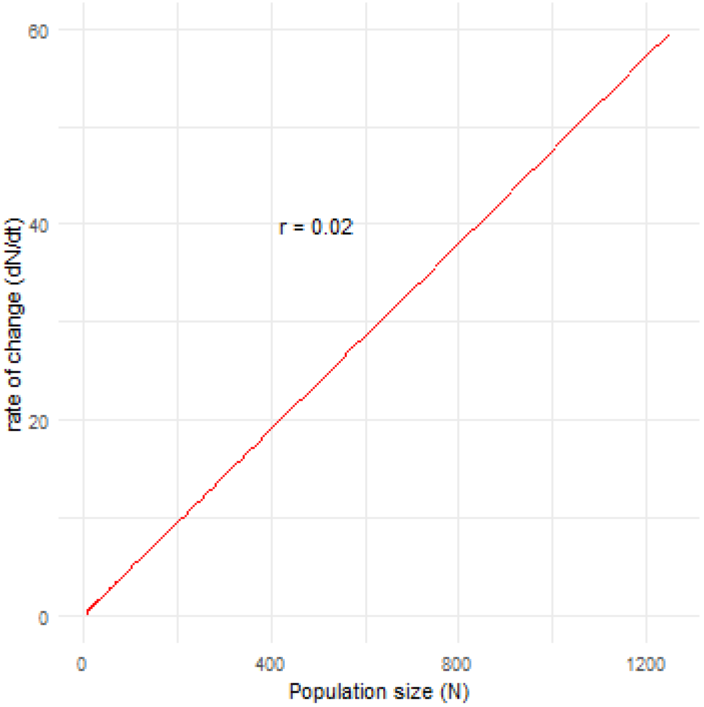


Logistic growth

Logistic growth rate vs population size



Exponential growth rate vs population size



Take 5 minutes to discuss the assumptions of the logistic growth model

Part 3: Introducing stochasticity

Non-deterministic growth

Stochasticity

Until now, everything we have looked at has been *entirely* deterministic but is this true of the real world?

$$N_{t+1} = N_t + r_d N_t \left(1 - \frac{N_t}{K}\right)$$

Environmental stochasticity

- Populations go through good and bad times and are not constant
- We can represent this by adding variance to the growth rate r_d

Demographic stochasticity

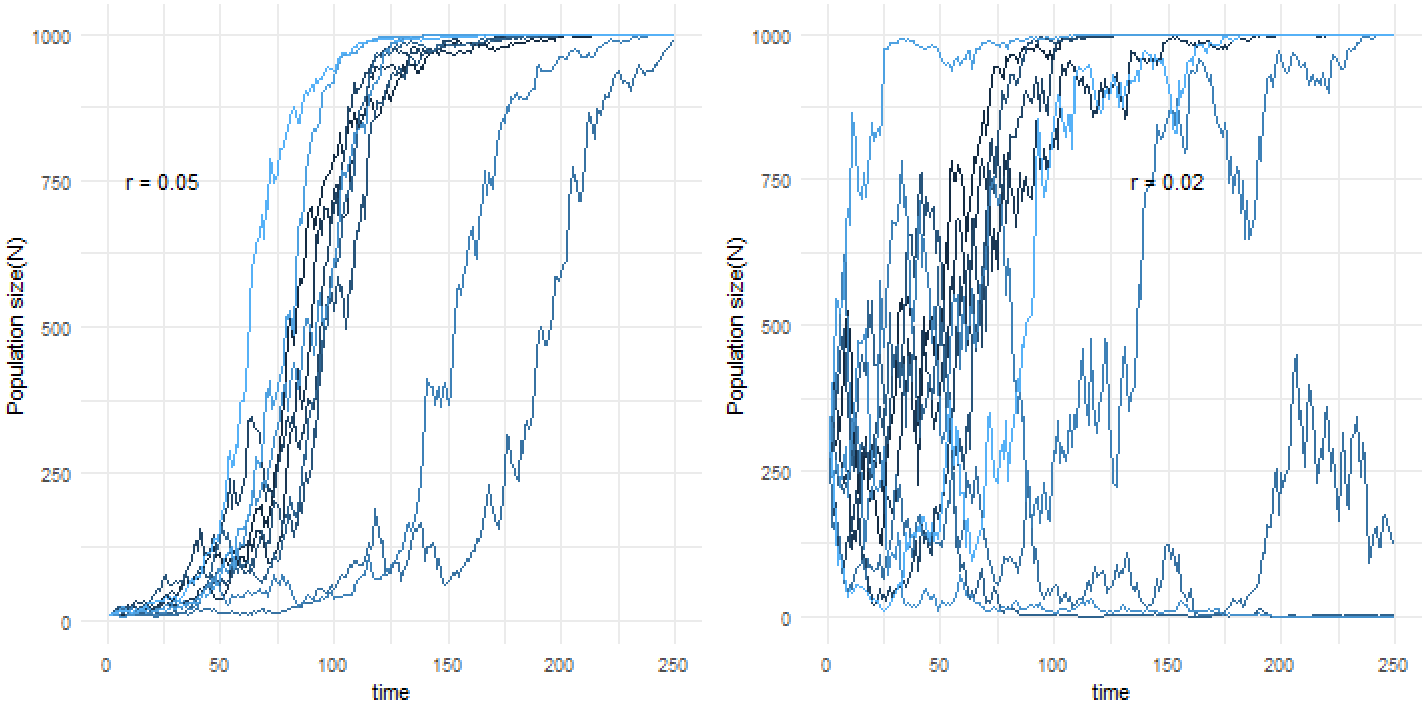
- By chance a population might have a run of births or a run of deaths
- Demographic stochasticity includes the probability of births and deaths in the parmeter r

- Variability can also be included in K, the carrying capacity!

We will focus on *environmental stochasticity*

Stochasticity

Even if **average** growth rate is positive some stochasticity can drive extinction



Recap

We have looked at:

- discrete and continuous equations for **exponential growth**
  - also known as **density independent** growth
- discrete and continuous equations for **logistic growth**
  - also known as **density dependent** growth
- **Stochastic** logistic growth
- We have discussed model **assumptions**

Key points

- **Exponentially** growing populations grow proportional to their size indefinitely
- **Logistically** growing populations experience a penalty that increases with population size
  - Growth rate is greatest at half the carrying capacity when the population size is at its largest relative to the density penalty
- **Stochasticity** can have a serious effect on populations and deterministic models do not account for stochasticity
- **Assumptions are key**, *think about these for the assignment*

Things to be aware of

- Differences in continuous and discrete equation
  - Different notation and descriptions of r
  - Mathematical differences
- Many diferent population growth equations
  - We have not talked about the model $N_{t+1} = \lambda N_t \left(1 - \frac{N_t}{K}\right)$. I recommend reading about it
  - Concepts can be transferable but be aware while reading results may not be the same
- This is just the beginning
  - Population growth models get *much much* more complicated to deal with more complicated species, for example, age structured population and competition

Go forth and model!!!