# Welcome to Population Ecology!

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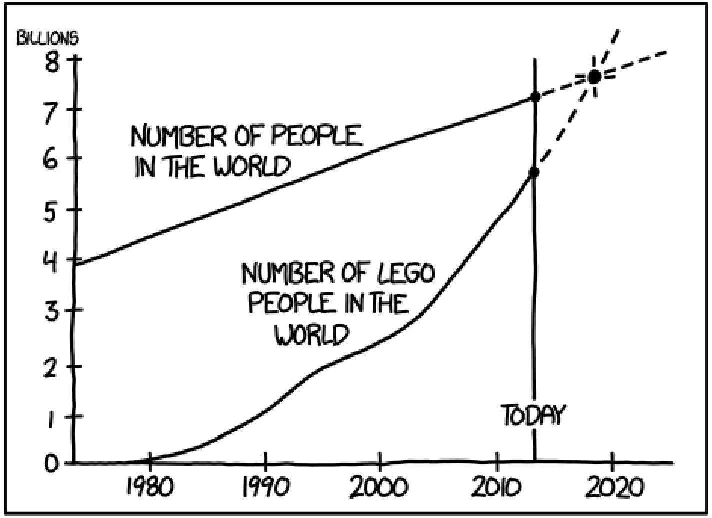
14/08/2019

# 3 part lecture (with breaks)

- **Part 1** Exponential growth
- **Part 2** Logistic growth - *Density dependent*
- **Part 3** Logistic growth with stochasticity



BY 2019, HUMANS WILL BE OUTNUMBERED.

# Part 1: Why be interested in population growth?

- Project future populations
  - Human population expected to be 9.8 billion by 2050
- Conservation of species
- Sustainable use of resources
- Many more

# Part 1: Let's start with exponential population growth

Density independent growth

$$\frac{dN}{dt} = rN$$

# Exponential growth

What is required for a population to grow?

How many births and how many deaths?

$$N_{t+1} = N_t + B - D + I - E$$

- $B$ = *Births*
- $D$ = *Deaths*
- $I$ = *Immigration*
- $E$ = *Emigration*

If we assume that immigration and emigration are equal, then the change in population size is:

$$\Delta N = B - D$$

# Exponential growth

$$\Delta N = B - D$$

- More births than deaths the population grows

- More deaths than births the population dies

# Exponential growth

*Change in population* ( $dN$ ) *over a very small interval of time* ( $dt$ ) can be described as:

$$\frac{dN}{dt} = B - D$$

- Births and deaths are also described in rates:

$$B = bN$$

$b$ = instantaneous birth rate

[births / (individual * time)]

$$D = dN$$

$d$ = instantaneous death rate

[deaths / (individual * time)]

# Exponential growth

*SO* change in population over time can be described as

$$\frac{dN}{dt} = (b - d)N$$

$$\frac{dN}{dt} = (0.55 - 0.50)N$$

$$\frac{dN}{dt} = (0.55 - 0.50) * 100$$

$$\frac{dN}{dt} = 0.05 * 100$$

$$\frac{dN}{dt} = 5$$

# Exponential growth

If we let $b - d$ become the constant $r$, the **intrinsic rate of increase**, we have the continuous exponential growth equation:

$$\frac{dN}{dt} = rN$$

# Exponential growth

Change in population is equal to the intrinsic rate of increase ( $r$ ) multiplied by the population size ( $N$ )

$$\frac{dN}{dt} = rN$$

*$N$ = population size*

*$r$ = intrinsic rate or increase*

$r$ defines how fast a population is growing or declining

- $r = 0$ no growth
- $r > 0$ positive growth
- $r < 0$ negative growth

The differential equation tells us growth **rate** not population size

# Exponential growth

The population of an exponentially growing population at a given time $t$ can be worked out by:

$$N_t = N_0 e^{rt}$$

Look familiar from assignment 1?

The discrete version of the exponential equation tells us the number added to the population per time-step:

$$N_{t+1} = N_t + r_d N_t$$

$$N_{t+1} = 100 + 0.05 * 100$$

$$N_{t+1} = 105$$

$N_t$ = Population size at time t

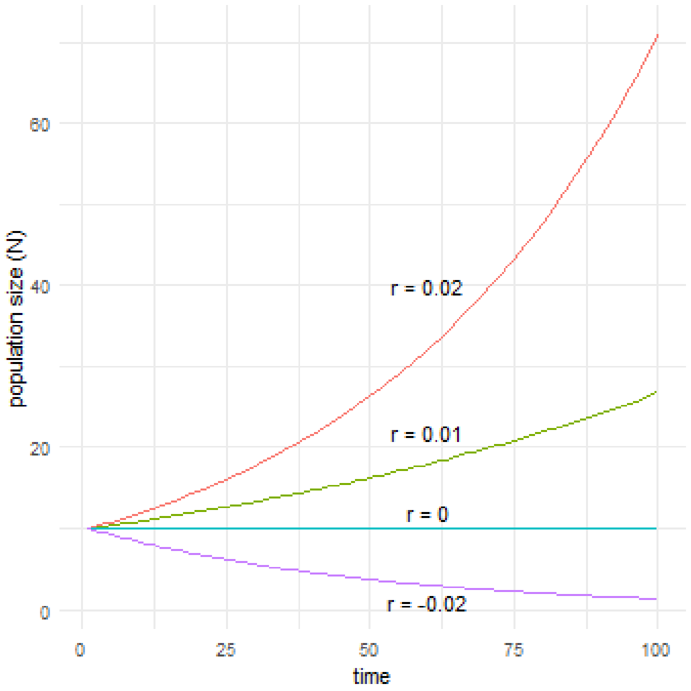
$r_d$ = discrete growth factor

# Exponential growth

Theoretical populations of populations growing (and declining) as a result of different values of $r$

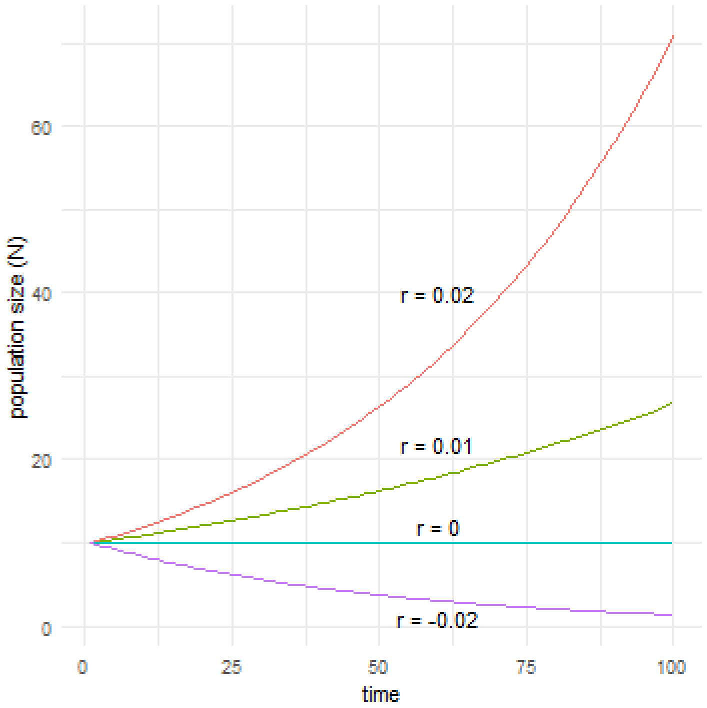
- $r = 0$ no growth
- $r > 0$ positive growth
- $r < 0$ negative growth

Because growth rate is exponential, by taking the natural logarithm of the population size the graphed lines become straight

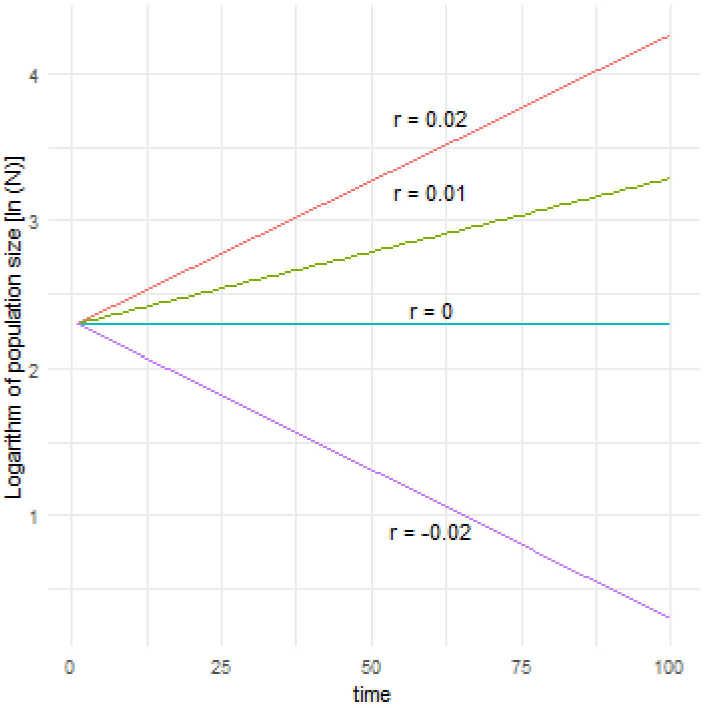


# Exponential growth

Exponential growth




Logarithm of exponential growth

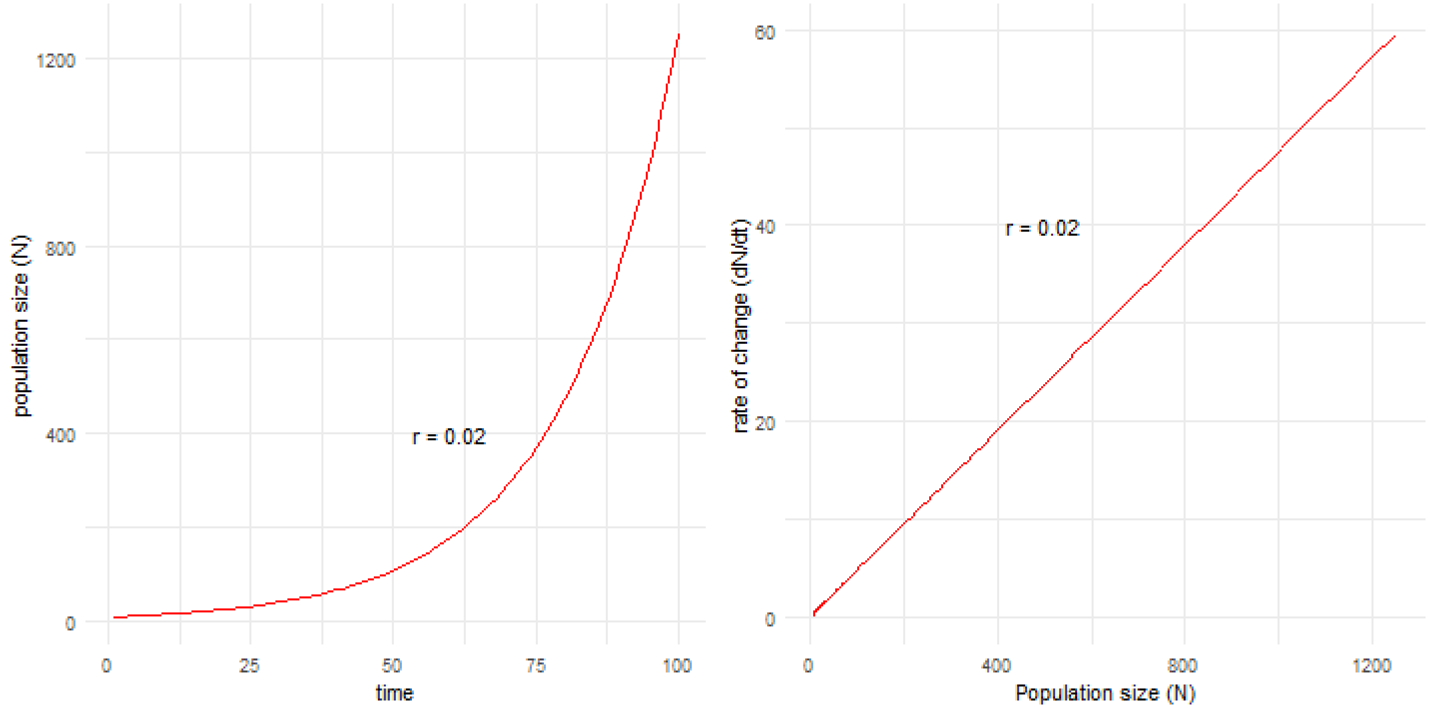


# Growth rates

| Common name | $r$ [individuals/(individual*day)] | Doubling time |
| --- | --- | --- |
| Virus | 300.0 | 3.3 minutes |
| Bacterium | 58.7 | 17 minutes |
| Protozoan | 1.59 | 10.5 hours |
| Hydra | 0.34 | 2 days |
| Flour beetle | 0.101 | 6.9 days |
| Brown rat | 0.0148 | 46.8 days |
| Domestic cow | 0.001 | 1.9 years |
| Mangrove | 0.00055 | 3.5 years |
| Southern beech | 0.000075 | 25.3 years |

# Growth rates

Population increases exponentially but **Growth rate** over **population size** increases proportionately with the population



# Exponential growth

- Populations growing exponentially have a doubling time
- The doubling time depends on the growth rate $r$ and is *not* every year
- Surely no species can grow forever exponentially?!?!?!
    - *Correct!* Welcome to part 2, **density dependence**

Take 5 minutes to discuss the assumptions of the exponential growth model

# Part 2: Logistic population growth

Density dependent growth

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

# Logistic growth

Now we will look at populations which do not grow forever but reach a **carrying capacity** ( $K$ )

$K$ represents the maximum population size that can be supported considering limiting factors such as food, shelter and space

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

$N$ = Population size

$r$ = Intrinsic rate of increase

$K$ = Carrying capacity

# Logistic growth

Consider the term $\left(1 - \frac{N}{K}\right)$ as a penalty on the growth of the population depending on the number of individuals in the community

- Very crowded communities have a high penalty compared to ones with plenty of space and resources

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

- At the point the population reaches the carrying capacity: $N$ will = $K$, and the fraction $\frac{N}{K}$ will = 1
- The term $\left(1 - \frac{N}{K}\right)$ will collapse to 0 and the change in population will = 0. The equation will be multiplied by 0 and equal 0
- So the population will remain at size $K$

# Logistic growth

If the population is very small, $N$ is small relative to $K$, then the penalty is small

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

However, as we learned from exponential growth, a population grows in proportion to its size.

- A population of 1000 seabirds will produce more eggs than a population of 100.
- In the logistic growth equation the proportion added to the population decreases as the population grows reaching 0 when $N = K$

# Logistic growth

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

- If $K = 100$ and $N = 7$ then the unused space is $1 - (7/100) = 0.93$ and the population is growing at 93% of the growth rate of an exponentially growing population
- If $K = 100$ and $N = 98$ then the unused proportion of capacity is $1 - (98/100) = 0.02$ and growth is at 2% of the growth rate of an exponentially growing population

The point at which a population is the largest relative to the penalty for its size is at $K/2$

- What this means is the growth rate of a population is fastest at half its carrying capacity
- As it grows bigger than $K/2$ the penalty becomes stronger but below $K/2$ the population is small and the proportion added to the population is small

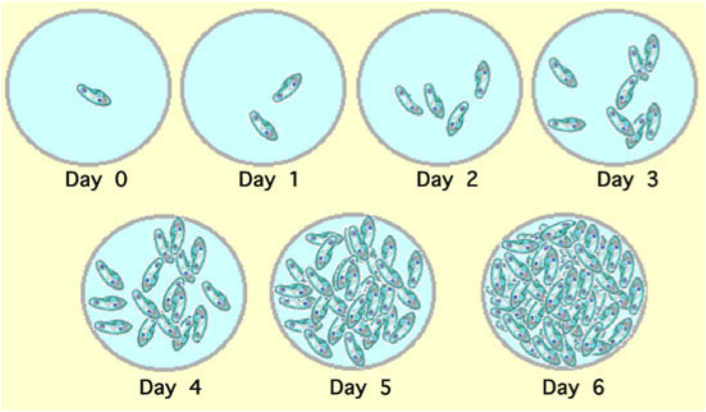
# Logistic growth

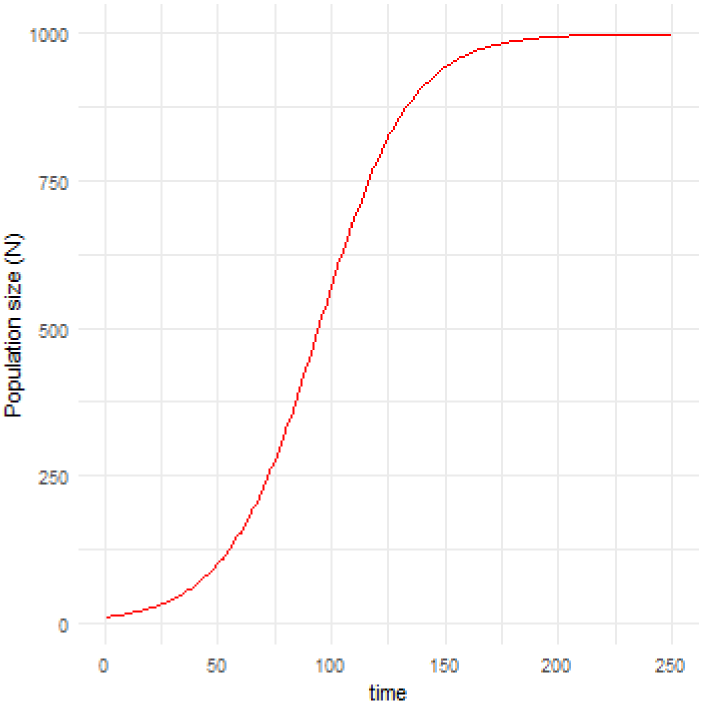
*Thanos should have studied population ecology*

- If a species is at capacity, its growth rate will increase to maximum if you cut it in half.
- Not all species are equal so cutting all in half will not have an equal effect…

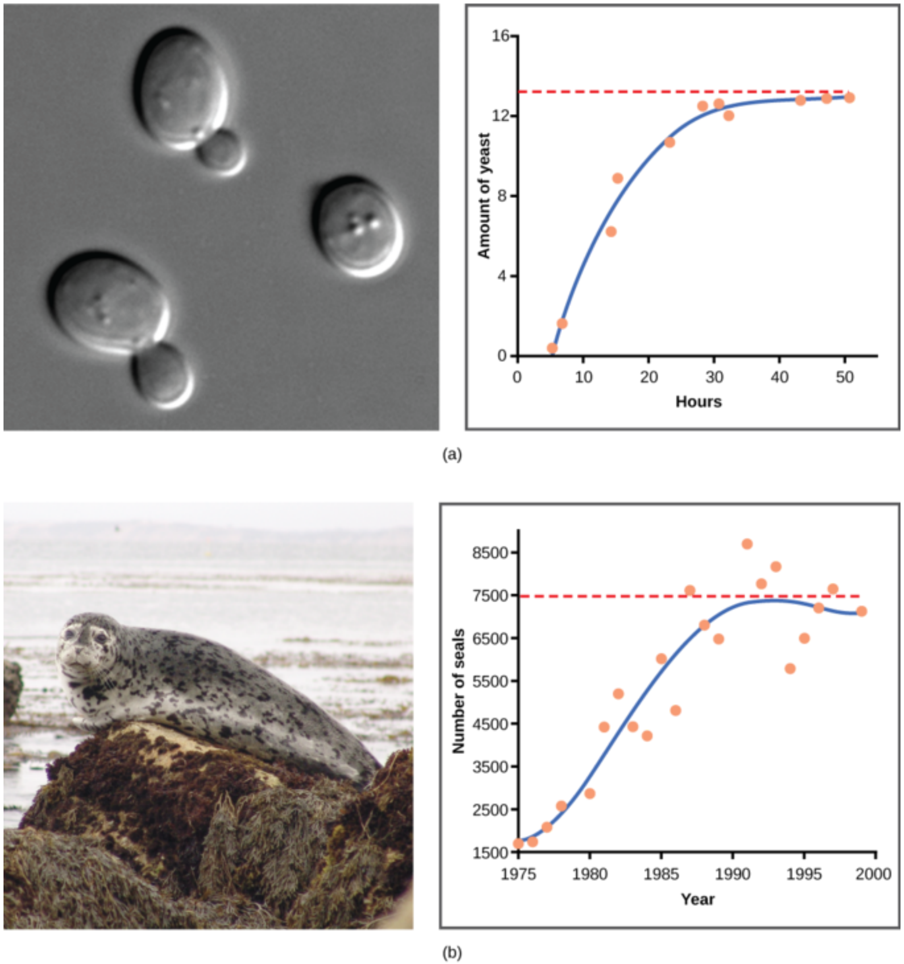
# Logistic growth

*Paramecium* growing to capacity.



# Logistic growth



# Logistic growth

Now we will look at the discrete form of the equation:

$$N_{t+1} = N_t + r_d N_t \left(1 - \frac{N_t}{K}\right)$$

$N_t$ = Population size at time t

$r_d$ = discrete growth factor

$K$ = Carrying capacity

- Instead of telling us the change in a population at an infinitely small point in time, the discrete equation tells us the size of the population at a given time.

# Logistic growth

The size of the population at the next time-step is equal to:

- The size of the current population plus the current population multiplied by the discrete growth rate and the density penalty.

$$N_{t+1} = N_t + r_d N_t \left(1 - \frac{N_t}{K}\right)$$

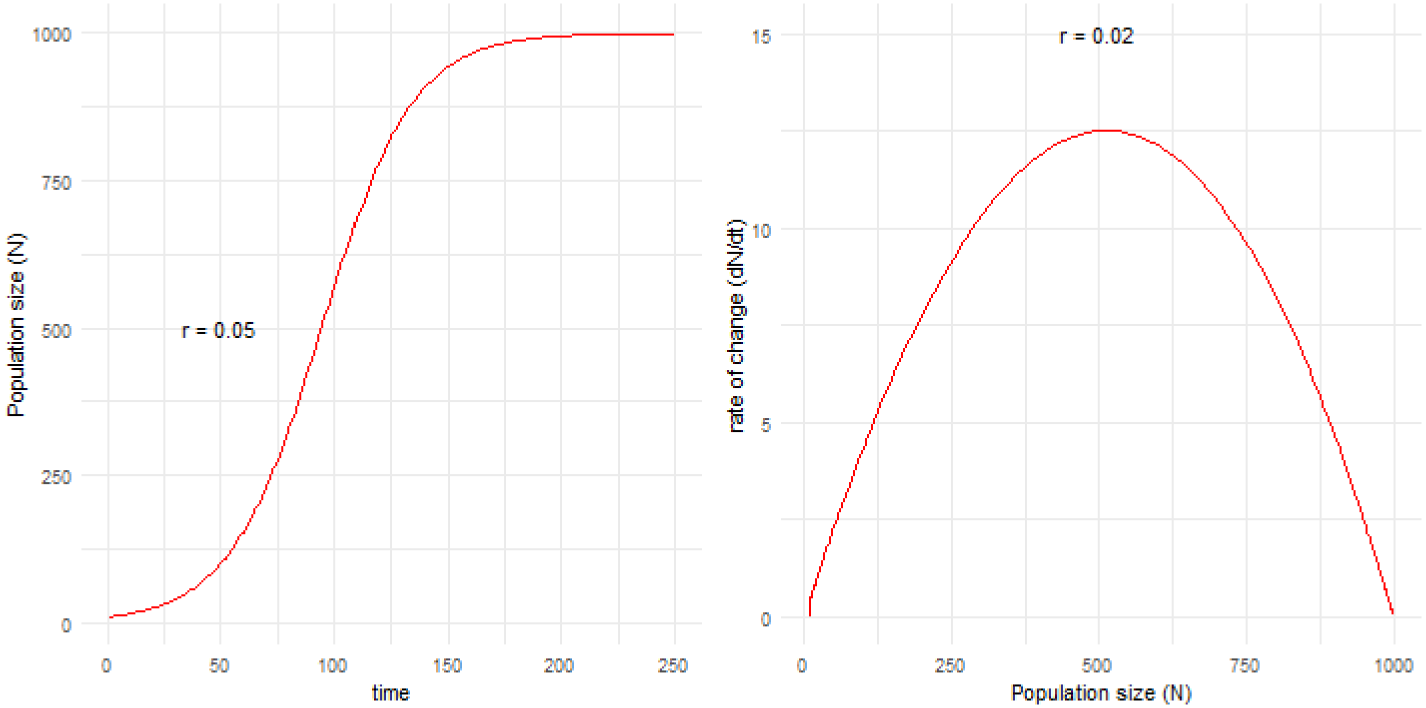
$$N_{t+1} = 100 + 0.05 * 100 \left(1 - \frac{100}{200}\right)$$

$$N_{t+1} = 102.5$$

Remember how in the exponential model $N_{t+1} = 105$ ?

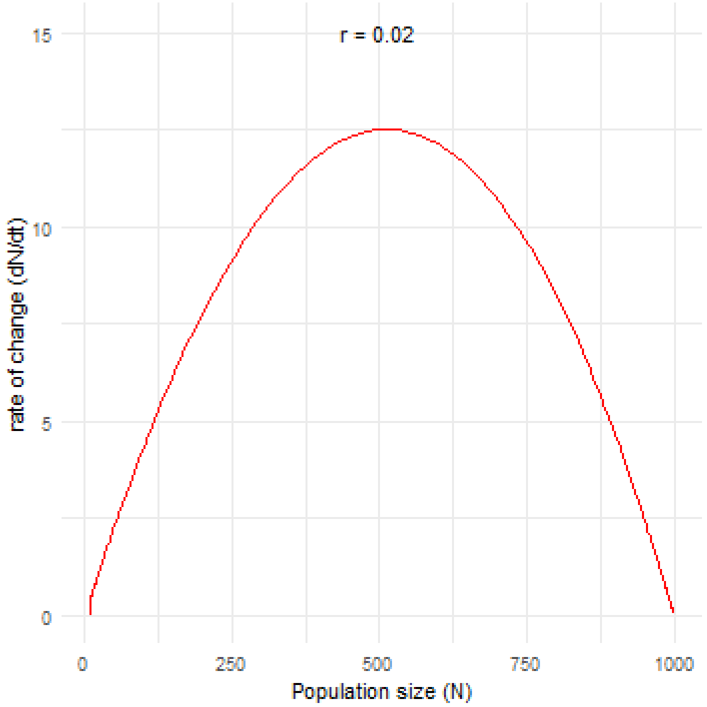
# Logistic growth

**Unlike** the exponential model, the logistic model growth rate is dependent on population size and reaches its peak at half of the carrying capacity

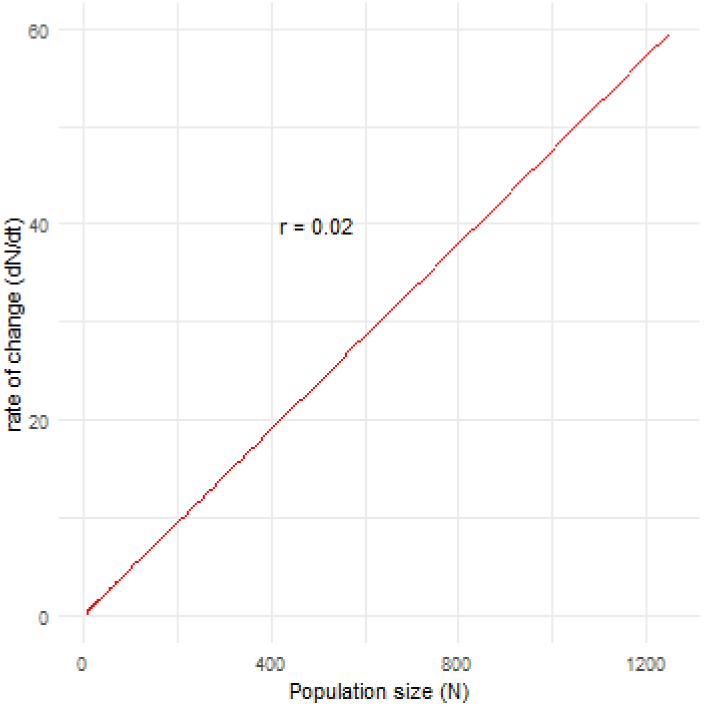


# Logistic growth

Logistic growth rate vs population size




Exponential growth rate vs population size



Take 5 minutes to discuss the assumptions of the logistic growth model

# Part 3: Introducing stochasticity

Non-deterministic growth

# Stochasticity

Until now, everything we have looked at has been *entirely* deterministic but is this true of the real world?

$$N_{t+1} = N_t + r_d N_t \left(1 - \frac{N_t}{K}\right)$$

Environmental stochasticity

- Populations go through good and bad times and are not constant
- We can represent this by adding variance to the growth rate $r_d$

Demographic stochasticity

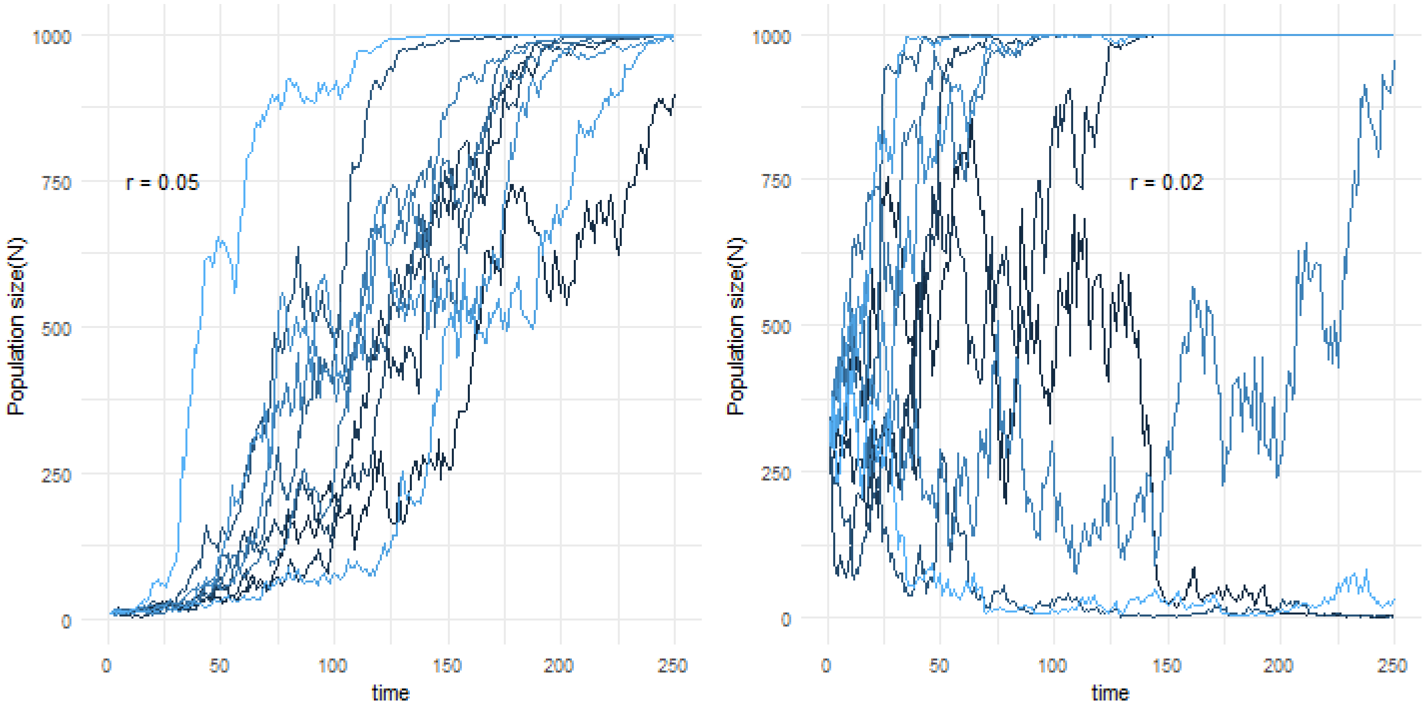
- By chance a population might have a run of births or a run of deaths
- Demographic stochasticity includes the probability of births and deaths in the parameter $r$

- Variability can also be included in $K$, the carrying capacity!

We will focus on *environmental stochasticity*

# Stochasticity

Even if **average** growth rate is positive some stochasticity can drive extinction



# Recap

We have looked at:

- discrete and continuous equations for **exponential growth**
  - also known as **density independent** growth
- discrete and continuous equations for **logistic growth**
  - also known as **density dependent** growth
- **Stochastic** logistic growth
- We have discussed model **assumptions**

# Key points

- **Exponentially** growing populations grow proportional to their size indefinitely
- **Logistically** growing populations experience a penalty that increases with population size
    - Growth rate is greatest at half the carrying capacity when the population size is at its largest relative to the density penalty
- **Stochasticity** can have a serious effect on populations and deterministic models do not account for stochasticity
- **Assumptions are key**, *think about these for the assignment*

# Things to be aware of

- Differences in continuous and discrete equation
    - Different notation and descriptions of $r$
    - Mathematical differences
- Many different population growth equations
    - We have not talked about the model $N_{t+1} = \lambda N_t \left(1 - \frac{N_t}{K}\right)$.
    - Concepts can be transferable but be aware while reading results may not be the same
- This is just the beginning
    - Population growth models get *much much* more complicated to deal with more complicated species, for example, age structured population and competition

Go forth and model!!!