Solution **Final Exam** ISyE 6420 December 8, 2020

1 Vasoconstriction.

(a) The transformation of v and r is shown in the attached code (Appendix A).

(b) We obtain the following results based on a logit model (shown in Figure 1).

	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
beta0	-31.03	9.771	0.2201	-52.22	-30.03	-14.58	1001	100000
beta1	6.32	2.036	0.04622	2.896	6.112	10.8	1001	100000
beta2	5.602	1.864	0.0392	2.441	5.433	9.564	1001	100000
deviance	32.38	2.449	0.04555	29.5	31.77	38.65	1001	100000
p.star	0.7615	0.1038	0.001202	0.5326	0.7724	0.9305	1001	100000

Figure 1: OpenBUGS results for logit model

(c) Based on the results shown in Figure 1, the probability of vasoconstriction is estimated as 0.7615.

(d) We fit the model with `probit` link and show the result of deviance in Figure 2.

	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
deviance	37.87	15.61	0.8763	29.83	32.6	105.5	1001	100000

Figure 2: Deviance for the model with probit link

We see that the logit model has smaller deviance compared with probit model.

2 Magnesium Ammonium Phosphate and Chrysanthemums.

We run the OpenBUGS code (attached in Appendix B) and obtain the results shown in Figure 3.

(a) We let treatment 1, 2, 3, and 4 denote the treatment of concentration of $MgNH_4PO_4$ of 50 g/bu, 100 g/bu, 200 g/bu, and 400 g/bu, respectively. By checking the results of the pairwise comparison, we have the following conclusion.

- Treatment 1 and 2 has mean difference as -1.833 and 95% credible set as $[-5.65, 2.003]$. As the credible set contains 0, we conclude that there is no significant different between those two treatments.
- Treatment 1 and 3 has mean difference as -2.972 and 95% credible set as $[-6.804, 0.865]$. As the credible set contains 0, we conclude that there is no significant different between those two treatments.

	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
beta[1]	-4.785	2.379	0.01804	-9.461	-4.799	-0.07329	1001	100000
beta[2]	-2.951	1.375	0.007809	-5.651	-2.955	-0.2386	1001	100000
beta[3]	-1.813	1.37	0.008277	-4.502	-1.811	0.8828	1001	100000
beta[4]	-0.02028	1.376	0.0076	-2.737	-0.02579	2.687	1001	100000
cb[1,2]	-1.833	1.94	0.01304	-5.65	-1.843	2.003	1001	100000
cb[1,3]	-2.972	1.949	0.01251	-6.804	-2.976	0.865	1001	100000
cb[1,4]	-4.764	1.939	0.01364	-8.593	-4.773	-0.9241	1001	100000
cb[2,3]	-1.138	1.943	0.008544	-4.984	-1.139	2.682	1001	100000
cb[2,4]	-2.931	1.94	0.008999	-6.738	-2.937	0.8864	1001	100000
cb[3,4]	-1.793	1.943	0.009081	-5.636	-1.788	2.024	1001	100000
contrast	-0.04057	2.751	0.0152	-5.474	-0.05157	5.374	1001	100000

Figure 3: OpenBUGS results for problem 2

- Treatment 1 and 4 has mean difference as -4.764 and 95% credible set as $[-8.593, -0.9241]$. As the credible set does not contain 0, we conclude that there exists significant different between those two treatments.
- Treatment 2 and 3 has mean difference as -1.138 and 95% credible set as $[-4.984, 2.682]$. As the credible set contains 0, we conclude that there is no significant different between those two treatments.
- Treatment 2 and 4 has mean difference as -2.931 and 95% credible set as $[-6.738, 0.8864]$. As the credible set contains 0, we conclude that there is no significant different between those two treatments.
- Treatment 3 and 4 has mean difference as -1.793 and 95% credible set as $[-5.636, 2.024]$. As the credible set contains 0, we conclude that there is no significant different between those two treatments.

Therefore, we failed to reject the null hypothesis (there is no significant difference between treatments). Based on the results from the pairwise comparison, we see that treatment 1 and 4 have significant difference.

(b) Based on the results shown in Figure 3, the 95% credible set for the contrast $\mu_1 - \mu_2 - \mu_3 + \mu_4$ is found as $[-5.474, 5.374]$.

3 Hocking–Pendleton Data.

We run the OpenBUGS code (attached in Appendix C) and obtain the results shown in Figure 4.

(a) By fitting the regression model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3,$$

	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
BR2	0.8488	0.05011	4.992E-4	0.7253	0.8583	0.9176	1001	100000
beta0	8.615	9.676	0.4659	-10.59	8.597	26.57	1001	100000
beta1	3.431	0.5529	0.02652	2.404	3.43	4.535	1001	100000
beta2	-1.444	0.239	0.005999	-1.912	-1.445	-0.9688	1001	100000
beta3	0.3409	0.2716	0.009879	-0.1849	0.3391	0.8802	1001	100000
icpo[1]	9.528	4.955	0.06922	5.546	8.233	21.37	1001	100000
icpo[2]	10.55	3.525	0.04015	6.478	9.721	19.52	1001	100000
icpo[3]	6.77	1.099	0.01116	5.028	6.629	9.296	1001	100000
icpo[4]	6.961	1.124	0.01068	5.18	6.822	9.56	1001	100000
icpo[5]	6.787	1.108	0.01085	5.037	6.647	9.354	1001	100000
icpo[6]	9.107	3.353	0.06052	5.657	8.265	17.65	1001	100000
icpo[7]	8.088	1.771	0.01705	5.642	7.767	12.41	1001	100000
icpo[8]	8.359	3.832	0.06257	5.275	7.46	17.1	1001	100000
icpo[9]	8.165	2.441	0.03642	5.413	7.61	14.37	1001	100000
icpo[10]	8.543	2.787	0.04651	5.514	7.881	15.59	1001	100000
icpo[11]	10.81	5.587	0.05589	5.979	9.301	24.59	1001	100000
icpo[12]	6.811	1.117	0.01195	5.052	6.669	9.392	1001	100000
icpo[13]	7.731	2.094	0.02777	5.276	7.295	12.87	1001	100000
icpo[14]	7.799	2.213	0.03025	5.282	7.322	13.3	1001	100000
icpo[15]	9154.0	188400.0	602.1	57.04	732.6	42100.0	1001	100000
icpo[16]	7.864	2.292	0.01474	5.293	7.36	13.65	1001	100000
icpo[17]	31.25	18.37	0.1119	13.9	26.41	77.44	1001	100000
icpo[18]	101.3	207.8	1.074	15.63	55.14	459.2	1001	100000
icpo[19]	8.524	2.851	0.01932	5.492	7.84	15.81	1001	100000
icpo[20]	10.04	3.539	0.05376	6.144	9.174	19.02	1001	100000
icpo[21]	7.083	1.34	0.02349	5.13	6.879	10.28	1001	100000
icpo[22]	7.772	1.414	0.02699	5.621	7.564	11.1	1001	100000
icpo[23]	6.908	1.147	0.01166	5.109	6.762	9.57	1001	100000
icpo[24]	27.12	1062.0	5.612	5.42	8.702	80.24	1001	100000
icpo[25]	7.212	1.505	0.02163	5.14	6.953	10.83	1001	100000
icpo[26]	7.158	1.349	0.01139	5.172	6.95	10.39	1001	100000
y.pred	37.43	3.773	0.1252	29.91	37.44	44.79	1001	100000
y.star	37.41	2.645	0.1243	32.14	37.42	42.33	1001	100000

Figure 4: OpenBUGS results for problem 3

we find the estimated parameters as $\beta_0 = 8.615, \beta_1 = 3.431, \beta_2 = -1.444$ and $\beta_3 = 0.3409$. The Bayesian R^2 is found as 0.8488.

(b) Based on the values of `icpo` shown in Figure 4, we compute the valus of CPO using 1/`icpo`. We show the results for all 26 observations as follows.

i	1	2	3	4	5	6	7
CPO[i]	0.1050	0.0948	0.1477	0.1437	0.1473	0.1098	0.1236

i	8	9	10	11	12	13	14
CPO[i]	0.1196	0.1225	0.1171	0.0925	0.1468	0.1293	0.1282

i	15	16	17	18	19	20	21
CPO[i]	0.00011	0.1272	0.0320	0.0099	0.1173	0.0996	0.1412

i	22	23	24	25	26
CPO[i]	0.1287	0.1448	0.0369	0.1387	0.1397

We that observation 15 and 18 are influential observations/potential outliers (the cumulative shows the same results).

(c) Based on the results shown in Figure 4, the mean response has its mean value as 37.41 and its 95% credible set as $[32.14, 42.33]$. The prediction reponse has its mean value as 37.43 and its 95% credible set as $[29.91, 44.79]$.

A OpenBUGS Code for Problem 1

```
model {

for (i in 1:n) {
x1[i] <- log(10*v[i])
x2[i] <- log(10*r[i])
}

for (i in 1:n) {
y[i] ~ dbern(p[i])

# logit link
#logit(p[i]) <- beta0+beta1*x1[i]+beta2*x2[i]

# probit link
p[i] <- phi(beta0+beta1*x1[i]+beta2*x2[i])

}

beta0 ~ dnorm(0, 0.0001)
beta1 ~ dnorm(0, 0.0001)
beta2 ~ dnorm(0, 0.0001)

# posterior probability when v=r=1.5
x1.star <- log(10*1.5)
x2.star <- log(10*1.5)
logit(p.star) <- beta0+beta1*x1.star+beta2*x2.star

}

# DATA
list(n=39, y=c(1,1,1,1,1,1,0,0,0,0,0,0,0,
1,1,1,1,1,0,1,0,0,0,0,1,0,1,0,
1,0,1,0,0,1,1,1,0,0,1),
v=c(3.7, 3.5, 1.25, 0.75, 0.8, 0.7, 0.6, 1.1, 0.9, 0.9,
0.8, 0.55, 0.6, 1.4, 0.75, 2.3, 3.2, 0.85, 1.7, 1.8,
0.4, 0.95, 1.35, 1.5, 1.6, 0.6, 1.8, 0.95, 1.9, 1.6,
2.7, 2.35, 1.1, 1.1, 1.2, 0.8, 0.95, 0.75, 1.3),
r=c(0.825, 1.09, 2.5, 1.5, 3.2, 3.5, 0.75, 1.7, 0.75,
0.45, 0.57, 2.75, 3, 2.33, 3.75, 1.64, 1.6, 1.415,
1.06, 1.8, 2, 1.36, 1.35, 1.36, 1.78, 1.5, 1.5, 1.9,
```

```
0.95, 0.4, 0.75, 0.3, 1.83, 2.2, 2, 3.33, 1.9, 1.9, 1.625))
```

B OpenBUGS Code for Problem 2

```
model {
for (i in 1:n) {
height[i] ~ dnorm(mu[i], tau)
mu[i] <- mu0+beta[treatment[i]]
}

## STZ (sum-to-zero) constraints
beta[1] <- sum(beta[2:level]);

## priors
mu0 ~ dnorm(0, 0.0001)
for (l in 2:level) {
beta[l] ~ dnorm(0, 0.0001)
}
tau ~ dgamma(0.001,0.001)
sigma2 <- 1/tau

## pairwise comparison
for (i in 1:level-1) {
for (j in i+1:level) {
cb[i,j] <- beta[i]-beta[j]
}
}

contrast <- beta[1]-beta[2]-beta[3]+beta[4]

}

# DATA
list(n=40, level=4)
treatment[] height[]
1 13.2
1 12.4
1 12.8
1 17.2
1 13.0
1 14.0
```

```
1 14.2
1 21.6
1 15.0
1 20.0
2 16.0
2 12.6
2 14.8
2 13.0
2 14.0
2 23.6
2 14.0
2 17.0
2 22.2
2 24.4
3 7.8
3 14.4
3 20.0
3 15.8
3 17.0
3 27.0
3 19.6
3 18.0
3 20.2
3 23.2
4 21.0
4 14.8
4 19.1
4 15.8
4 18.0
4 26.0
4 21.1
4 22.0
4 25.0
4 18.2
END

# INITS
list(mu0=0, beta=c(NA,0,0,0), tau=1)
```

C OpenBUGS Code for Problem 3

```
model {
for (i in 1:n) {
y[i] ~ dnorm(mu[i], tau)
mu[i] <- beta0+beta1*x1[i]+beta2*x2[i]+beta3*x3[i]
r[i] <- y[i]-mu[i]
f[i] <- sqrt(tau/6.2832)*exp(-0.5*tau*r[i]*r[i])
icop[i] <- 1/f[i]
}

beta0 ~ dnorm(0, 0.0001)
beta1 ~ dnorm(0, 0.0001)
beta2 ~ dnorm(0, 0.0001)
beta3 ~ dnorm(0, 0.0001)
tau ~ dgamma(0.001,0.001)
sigma2 <- 1/tau

# Bayesian R2
p <- 4
nminusp <- n-p
sse <- nminusp*sigma2
for (i in 1:n) {
cy[i] <- y[i]-mean(y[])
}
sst <- inprod(cy[], cy[])
BR2 <- 1-sse/sst

# prediction
x1.star <- 10
x2.star <- 5
x3.star <- 5
y.star <- beta0+beta1*x1.star+beta2*x2.star+beta3*x3.star
y.pred ~ dnorm(y.star, tau)

}

# DATA
list(n=26)

x1[] x2[] x3[] y[]
12.98 0.317 9.998 57.702
```

```
14.295 2.028 6.776 59.295
15.531 5.305 2.947 55.166
15.133 4.738 4.201 55.767
15.342 7.038 2.053 51.722
17.149 5.982 -0.055 60.446
15.462 2.737 4.657 60.715
12.801 10.663 3.048 37.447
17.039 5.132 0.257 60.974
13.172 2.039 8.738 55.27
16.125 2.271 2.101 59.289
14.34 4.077 5.545 54.027
12.923 2.643 9.331 53.199
14.231 10.401 1.041 41.896
15.222 1.22 6.149 53.254
15.74 10.612 -1.691 45.798
14.958 4.815 4.111 58.699
14.125 3.153 8.453 50.086
16.391 9.698 -1.714 48.89
16.452 3.912 2.145 62.213
13.535 7.625 3.851 45.625
14.199 4.474 5.112 53.923
15.837 5.753 2.087 55.799
16.565 8.546 8.974 56.741
13.322 8.598 4.011 43.145
15.949 8.29 -0.248 50.706
END

# INIT
list(beta0=0,beta1=0,beta2=0,beta3=0,tau=1)
```