HW1

Rory Michelen

September 5, 2019

Question 1

Part 1

Using the law of total probability, we can express the probability that system, S will be operational as a function of t. First, some definitions:

$S =$ Event that the system will be functional
$E_i =$ Event that component E_i will be functional
$H_1 = E_5(t)$
$H_2 = H_1^c$

$P(S) = P(S \mid H_1)P(H_1) + P(S \mid H_2)P(H_2)$

$P(S \mid H_1) = P(E_1) + P(E_3) - P(E_1)P(E_3) = e^{-t} + e^{-t/2} - e^{-3t/2}$

$P(H_1) = P(E_5) = e^{-t}$

$P(S \mid H_2) = P(E_1)P(E_2) + P(E_3)P(E_4) - P(E_1)P(E_2)P(E_3)P(E_4) = e^{-3t} + e^{-5t/6} - e^{-23t/6}$

$P(H_2) = 1 - P(E_5) = 1 - e^{-t}$

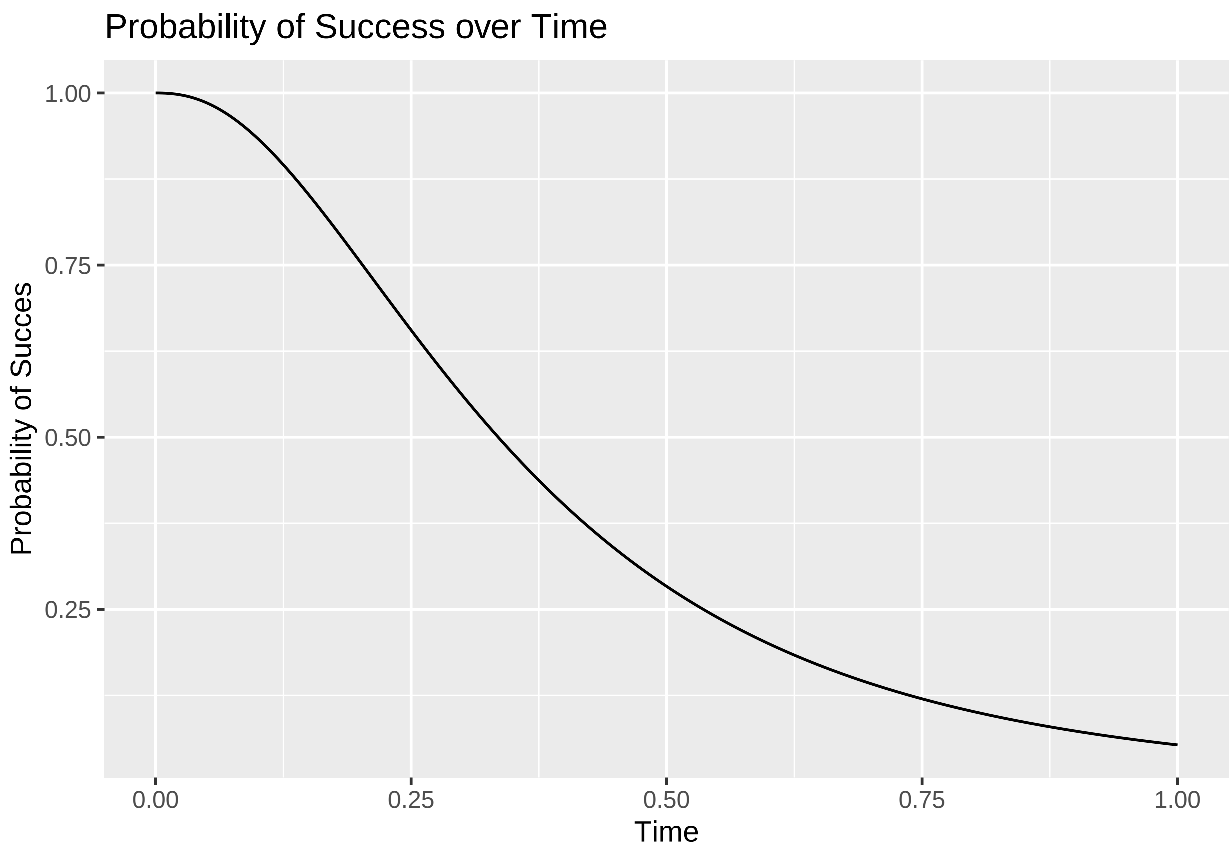
$P(S) = (P(E_1) + P(E_3) - P(E_1)P(E_3)) * P(E_5) + (P(E_1)P(E_2) + P(E_3)P(E_4) - P(E_1)P(E_2)P(E_3)P(E_4)) * (1 - P(E_5))$
$P(s) = (e^{-t} + e^{-t/2} - e^{-3t/2}) * e^{-t} + (e^{-3t} + e^{-5t/6} - e^{-23t/6}) * (1 - e^{-t})$

Well, that was fun to write in LaTex! Let's move on to computation:

```r
# Question 1
t<-seq(0,1,by=0.001)
l<-c(1,2,1/2,1/3,1)

prob_e<-function (t,l){
  return(exp(-1*l*t))
}

probs.at.t<-as.data.frame(t)
probs.at.t$H1<-prob_e(t,5)
probs.at.t$H2<-1-probs.at.t$H1
probs.at.t$s.gv.H1<-prob_e(t,1)+
                      prob_e(t,3)-
                      (prob_e(t,1)*prob_e(t,3))
probs.at.t$s.gv.H2<-(prob_e(t,1)*prob_e(t,2))+
                      (prob_e(t,3)*prob_e(t,4))-
                      (prob_e(t,1)*prob_e(t,2)*prob_e(t,3)*prob_e(t,4))

probs.at.t$s<-with((H1*s.gv.H1)+(H2*s.gv.H2),data=probs.at.t)
```

```r
probs.at.t%>%
  ggplot(aes(x=t,y=s))+
    geom_line(group=1)+
    labs(x="Time",
```

```
        y="Probability of Succes",
        title="Probability of Success over Time")
```



Below is the computation for probability of event S at time t=1/2.

```
# Time =1/2
probs.at.t%>%
  dplyr::filter(t==0.5)%>%
  select(s)
```

```
## Warning: package 'bindrcpp' was built under R version 3.5.1
```

```
##          s
## 1 0.283342
```

Using this code, we get

$P(s) = 0.28$ for t=1/2

Part B

Using the world-famous bayes formula, we can derive the formula for the probability that component 5 was successful at time t=1/2

$P(E_5 \mid S) = \frac{P(S|H_1)}{P(S)} P(H_1)$

Each of these probabilities were expressed as a function of t in part A. Below is the computation for time t=1/2

```
# Part B
probs.at.t$H1.gv.s<-with(s.gv.H1*H1/s,data=probs.at.t)

probs.at.t%>%
  dplyr::filter(t==0.5)%>%
  select(H1.gv.s)
```

```
##     H1.gv.s
## 1 0.2011481
```

And we get the result of

$P(E_5 \mid S) = 0.20$ for t=1/2

Question 2

First, I'll define some important events

Let $B_{i,j}$ = Event that the i^{th} item is selected from batch j
Let C_i = the event that the i^{th} item is conforming

Next, some probabilities that will be useful later:
$P(B_{1,j}) = 0.5$ for $j \in \{1, 2\}$

$P(C_1 \mid B_{1,1}) = 1.0$

$P(C_1 \mid B_{1,2}) = 0.8$

$P(C_1) = P(C_1 \mid B_{1,1})P(B_{1,1}) + P(C_1 \mid B_{1,2})P(B_{1,2}) = (1 * 0.5) + (0.8 * 0.5) = 0.9$

Now, let's formulate the proposed question and plug in the above values. We were asked to find:

$P(C_2^c \mid C_1)$

$P(C_2^c \mid C_1) = 1 - P(C_2 \mid C_1)$

$P(C_2 \mid C_1) = P(C_2 \mid C_1, B_{1,1}) * P(B_{1,1}|C_1) + P(C_2 \mid C_1, B_{1,2}) * P(B_{1,2}|C_1)$

$P(C_2 \mid C_1, B_{1,1}) = P(C_2 \mid B_{1,1}) = 1$
$P(C_2 \mid C_1, B_{1,2}) = P(C_2 \mid B_{1,2}) = 0.8$

$P(B_{1,1}|C_1) = \frac{P(C_1|B_{1,1})}{P(C_1)} P(B_{1,1}) = \frac{1}{0.9} 0.5 = \frac{5}{9}$
$P(B_{1,2}|C_1) = \frac{P(C_1|B_{1,2})}{P(C_1)} P(B_{1,2}) = \frac{0.8}{0.9} 0.5 = \frac{4}{9}$

$P(C_2 \mid C_1) = 1 * \frac{5}{9} + \frac{8}{10} * \frac{4}{9} = \frac{41}{45}$

$P(C_2^c \mid C_1) = \frac{4}{45} = 0.89$

Question 3

Let P_i denote the event that our algorithm predicted class i for $i \in \{0, 1\}$
Let A_i denote the event that an item selected from the population is of class i for $i \in \{0, 1\}$
Let S_i denote the even that an item selected from the sample is of class i for $i \in \{0, 1\}$

It is given that:
$P(A_0) = 0.99$

$P(A_1) = 0.01$

$P(S_0) = \frac{55}{120}$

$P(S_1) = \frac{65}{120}$

$P(P_1) = \frac{70}{120}$

$P(P_1|S_1) = \frac{52}{70}$

We are asked to find $P(A_1 \mid P_1)$

$P(A_1 \mid P_1) = \frac{P(P_1|A_1)}{P(P_1)} P(A_1)$

$P(P_1 \mid A_1) = (P_1 \mid S_1) = \frac{52}{70}$

$P(A_1 \mid P_1) = \frac{\frac{52}{70}}{\frac{70}{120}} * 0.01 = 0.0135$