IYSE 6420 Fall 2020 Homework1

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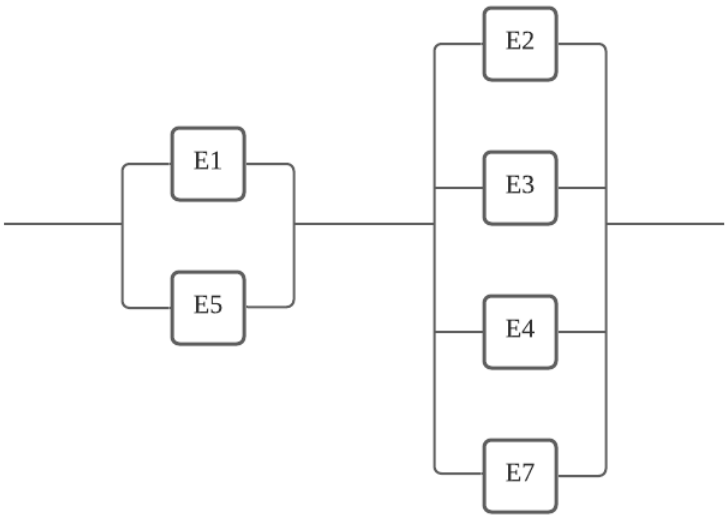
1. Circuit

a) Find the probability that the circuit is operational during time interval T.

S: circuit is operational
H1: E6 is working
H2: E6 is not working

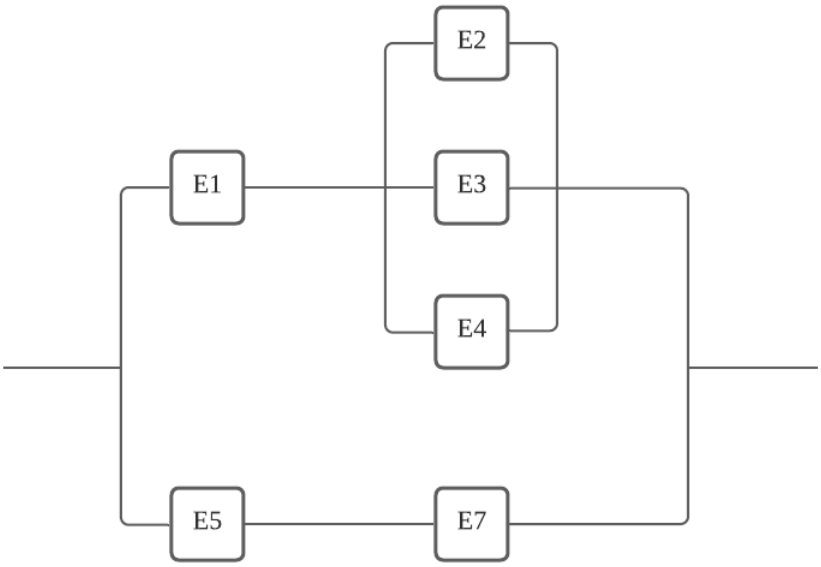
Element	E1	E2	E3	E4	E5	E6	E7
Works (p)	0.5	0.7	0.3	0.4	0.9	0.5	0.7
Fails (q)	0.5	0.3	0.7	0.6	0.1	0.5	0.3

Hypotheses H1:



P(S|H1) = (1 – q1q5) (1 - q2q3q4q7)
= (1 – 0.5 x 0.1) x (1 – 0.3 x 0.7 x 0.6 x 0.3)
= 0.91409

Hypotheses H2:



P(S|H2) = 1 – (1 – p1 x (1 – q2q3q4)) x (1 – p5 x p7)
= 1 – (1 – 0.5 x (1 – 0.3 x 0.7 x 0.6)) x (1 – 0.5 x 0.7)
= 0.63405

By total probability,
P(S) = P(S|H1) x P(H1) + P(S|H2) x P(H2)
= 0.91409 x 0.5 + 0.63405 x 0.5
= 0.77407

b) If the circuit was found operational at the time T, what is the probability that the element E6 was operational.

P(H1|S) = P(S|H1) x P(H1) / P(S)
= 0.91409 x 0.5 / 0.77407
= 0.59

2. Two Batches

What is the probability that the second product, randomly selected from the same batch, is found non-conforming?

H1: product is selected from 1st batch
H2: product is selected from 2nd batch

Batches	Prob. Item Conforming	Prob. Selected
1st batch	1.0	0.5
2nd batch	0.9	0.5

A: product is conforming

P(A) = P(A|H1) P(H1) + P(A|H2) P(H2)
= 1.0 x 0.5 + 0.9 x 0.5
= 0.95

P(H1|A) = P(A|H1) x P(H1) / P(A)
= 1.0 x 0.5 / 0.95
= 0.526

The probability that the product is from 1st batch: 0.526

P(H2|A) = P(A|H2) x P(H2) / P(A)
= 0.9 x 0.5 / 0.95
= 0.474

The probability that the product is from 2nd batch: 0.474

Probability of the next product from same batch is non-conforming:

P(~A) = P(H1|A) x P(A|H1) + P(H2|A) x P(A|H2)
= 0.526 x 1.0 + 0.474 x 0.9
= 0.9526

3. Machine

a) What is the probability that the machine will fail? Evaluate this probability for p = 0.4.

Machine	M1	M2	M3	M4
Work	p	p	p	½
Fail	q = (1-p)	q = (1-p)	q = (1-p)	½

H1: No machine works

P(H1) = q x q x q / 2 = q^3 / 2
= 0.6^3 / 2
= 0.108

H2: Only one machine works

Mi: Only Mi works

P(H2) = P(M1) + P(M2) + P(M3) + P(M4)
= p x q^2 / 2 x 3 + q^3 / 2
= 0.4 x 0.6^2 / 2 x 3 + 0.6^3 / 2
= 0.324

H0: the machine will fail

P(H0) = P(H1) + P(H2) = 0.432

b) If the machine failed, what is the probability that the component which fails with probability 1/2 actually failed.

$P(M4 \text{ failed} | H0) = 1 - P(M4 | H0)$

$= 1 - P(H0 | M4) \times P(M4) / P(H0)$

$= 1 - 1 \times q^3 / 2 / 0.432$

If $q = 1-p = 0.6$, then

$P(M4 \text{ failed} | H0) = 1 - (0.6^3 / 2 / 0.432)$

$= 0.75$