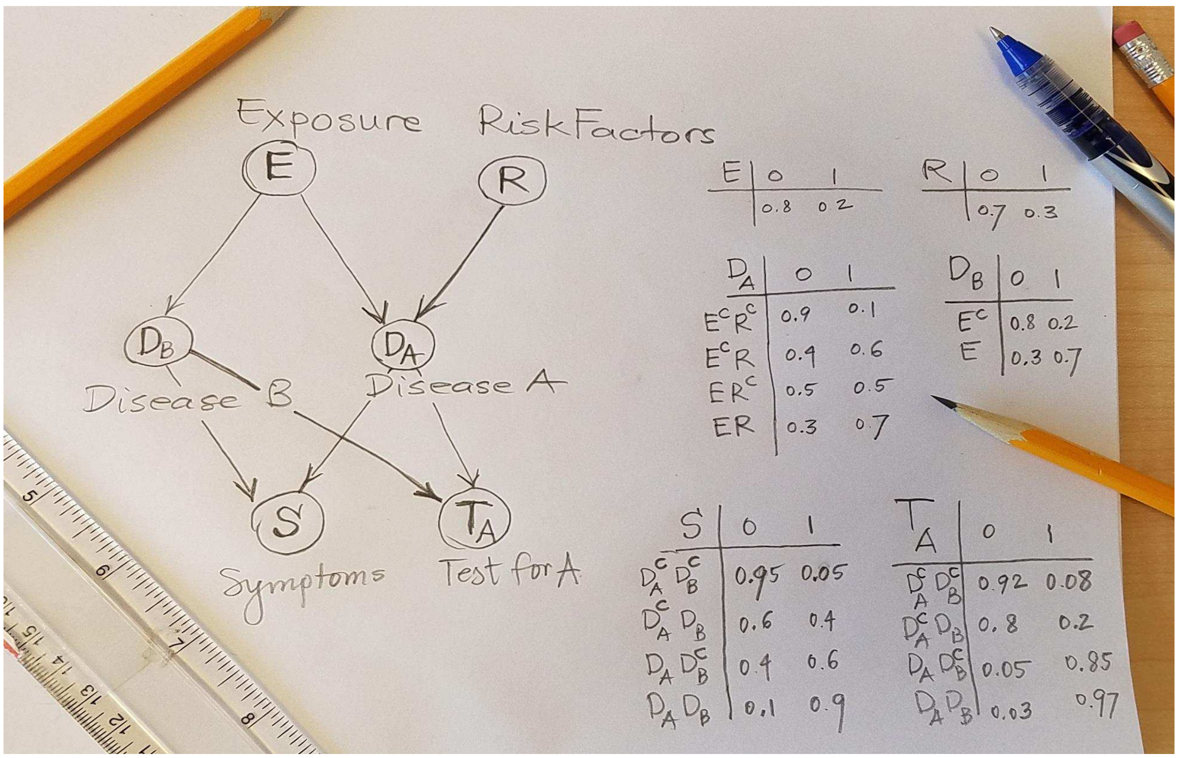
ISyE 6420

HW1 due Tuesday, February 6, 11:55pm

HW 1

1. Bayes Network. Incidences of diseases A and B (D_A, D_B) depend on the exposure (E). Disease A is additionally influenced by risk factors (R). Both diseases lead to symptoms (S). Results of the test for disease A (T_A) are affected also by disease B. Positive test will be denoted as $T_A = 1$, negative as $T_A = 0$. The Bayes Network and needed conditional probabilities are shown in Figure.



(a) What is the probability of disease A ($D_A = 1$), if disease B is not present ($D_B = 0$), but symptoms are present ($S = 1$)

(b) What is the probability of exposure ($E = 1$), if symptoms are present ($S = 1$) and test is positive ($T_A = 1$).

Hint: You can use any of methods or software in solving this problem. Approximate solutions (MATLAB/Octave, R, Python, Win/OpenBUGS) are satisfactory.

2. Jeremy and Variance from Single Observation. Jeremy believes that his IQ test scores have normal distribution with mean 110 and unknown variance σ^2. He takes a test and scores $X = 98$.

(a) Show that inverse gamma prior $\mathcal{IG}(\alpha, \beta)$ is the conjugate for σ^2 if the observation X is normal $\mathcal{N}(\mu, \sigma^2)$ with μ known. What is the posterior?

(b) Find a Bayes estimator of σ^2 and its standard deviation in Jeremy's model if the prior on σ^2 is an inverse gamma $\mathcal{IG}(3, 100)$.

Hint: Random variable Y is said to have an inverse gamma $\mathcal{IG}(\alpha, \beta)$ distribution if its density is given by

$$f(y|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha) y^{\alpha+1}} \exp\left\{-\frac{\beta}{y}\right\}, \ \alpha, \beta > 0.$$

The mean of Y is $EY = \frac{\beta}{\alpha-1}, \ \alpha > 1$ and the variance is $Var(Y) = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}, \ \alpha > 2$.

(c) Use WinBUGS to solve this problem and compare the MCMC approximations with exact values from (b).

Hint: Express the likelihood terms of precision τ with gamma $\mathcal{G}a(\alpha, \beta)$ prior, but then calculate and monitor $\sigma^2 = \frac{1}{\tau}$.