

- (A) Express all problem facts as a propositional logic knowledge base.  
Clearly explain the meaning of your propositional symbols.

A: Caterpillar tells the truth  
B: Bill the Lizard tells the truth  
C: Cheshire Cat tells the truth  
D: Caterpillar ate the salt  
E: Bill the Lizard ate the salt  
F: Cheshire Cat ate the salt

Knowledge Base:

1.  $(D \vee E \vee F)$
2.  $D \Rightarrow (\neg E \wedge \neg F)$
3.  $E \Rightarrow (\neg D \wedge \neg F)$
4.  $F \Rightarrow (\neg D \wedge \neg E)$
5.  $A \Leftrightarrow E$
6.  $B \Leftrightarrow E$
7.  $C \Leftrightarrow \neg F$
8.  $(A \wedge B) \Rightarrow (\neg C)$
9.  $(B \wedge C) \Rightarrow (\neg A)$
10.  $(C \wedge A) \Rightarrow (\neg B)$
11.  $(\neg A \wedge \neg B) \Rightarrow C$
12.  $(\neg B \wedge \neg C) \Rightarrow A$
13.  $(\neg C \wedge \neg A) \Rightarrow B$

- (B) Convert the propositional logic knowledge base to CNF.

KB  $\Rightarrow$  CNF:

$\{(D \vee E \vee F), (D \Rightarrow (\neg E \wedge \neg F)), (E \Rightarrow (\neg D \wedge \neg F)), (F \Rightarrow (\neg D \wedge \neg E)), A \Leftrightarrow E, B \Leftrightarrow E, C \Leftrightarrow \neg F, (A \wedge B) \Rightarrow (\neg C), (B \wedge C) \Rightarrow (\neg A), (C \wedge A) \Rightarrow (\neg B), (\neg A \wedge \neg B) \Rightarrow C, (\neg B \wedge \neg C) \Rightarrow A, (\neg C \wedge \neg A) \Rightarrow B\}$

Eliminate  $\Leftrightarrow$

$\{(D \vee E \vee F), (D \Rightarrow (\neg E \wedge \neg F)), (E \Rightarrow (\neg D \wedge \neg F)), (F \Rightarrow (\neg D \wedge \neg E)), (A \Rightarrow E) \wedge (E \Rightarrow A), (B \Rightarrow E) \wedge (E \Rightarrow B), (C \Rightarrow \neg F) \wedge (\neg F \Rightarrow C), (A \wedge B) \Rightarrow (\neg C), (B \wedge C) \Rightarrow (\neg A), (C \wedge A) \Rightarrow (\neg B), (\neg A \wedge \neg B) \Rightarrow C, (\neg B \wedge \neg C) \Rightarrow A, (\neg C \wedge \neg A) \Rightarrow B\}$

Eliminate  $\Rightarrow$

$\{(D \vee E \vee F), (\neg D \vee (\neg E \wedge \neg F)), (\neg E \vee (\neg D \wedge \neg F)), (\neg F \vee (\neg D \wedge \neg E)), (\neg A \vee E) \wedge (\neg E \vee A), (\neg B \vee E) \wedge (\neg E \vee B), (\neg C \vee \neg F) \wedge (F \vee C), (\neg(A \wedge B) \vee (\neg C)), (\neg(B \wedge C) \vee (\neg A)), (\neg(C \wedge A) \vee (\neg B)), (\neg(\neg A \wedge \neg B) \vee C), (\neg(\neg B \wedge \neg C) \vee A), (\neg(\neg C \wedge \neg A) \vee B)\}$

Move  $\neg$  inward

$\{(D \vee E \vee F), (\neg D \vee (\neg E \wedge \neg F)), (\neg E \vee (\neg D \wedge \neg F)), (\neg F \vee (\neg D \wedge \neg E)), (\neg A \vee E) \wedge (\neg E \vee A), (\neg B \vee E) \wedge (\neg E \vee B), (\neg C \vee \neg F) \wedge (F \vee C), ((\neg A \vee \neg B) \vee (\neg C)), ((\neg B \vee \neg C) \vee (\neg A)), ((\neg C \vee \neg A) \vee (\neg B)), ((A \vee B) \vee C), ((B \vee C) \vee A), ((C \vee A) \vee B)\}$

Distribute  $\vee$  over  $\wedge$

$\{(D \vee E \vee F), ((\neg D \vee \neg E) \wedge (\neg D \vee \neg F)), ((\neg E \vee \neg D) \wedge (\neg E \vee \neg F)), ((\neg F \vee \neg D) \wedge (\neg F \vee \neg E)), (\neg A \vee E) \wedge (\neg E \vee A), (\neg B \vee E) \wedge (\neg E \vee B), (\neg C \vee \neg F) \wedge (F \vee C), (\neg A \vee \neg B \vee \neg C), (A \vee B \vee C)\}$

Rewrite as conjunction of known sentences:

$\{\{D, E, F\},$   
 $\{\neg D, \neg E\}$   
 $\{\neg D, \neg F\}$   
 $\{\neg E, \neg F\}$   
 $\{\neg A, E\},$   
 $\{\neg E, A\}$   
 $\{\neg B, E\},$   
 $\{\neg E, B\},$   
 $\{\neg C, \neg F\},$   
 $\{F, C\}$   
 $\{\neg A, \neg B, \neg C\},$   
 $\{A, B, C\}\}$

(C) Use resolution theorem proving to solve the problem.

1. Proof by contradiction- Attempt to prove that Caterpillar is a liar ( $\neg A$ ). Assume that Caterpillar is a truth teller (A):

(1)	$\{D, E, F\}$		Knowledge Base
(2)	$\{\neg D, \neg E\}$		
(3)	$\{\neg D, \neg F\}$		
(4)	$\{\neg E, \neg F\}$		
(5)	$\{\neg A, E\}$		
(6)	$\{\neg E, A\}$		
(7)	$\{\neg B, E\}$		
(8)	$\{\neg E, B\}$		
(9)	$\{\neg C, \neg F\}$		
(10)	$\{F, C\}$		
(11)	$\{\neg A, \neg B, \neg C\}$		
(12)	$\{A, B, C\}$		
(13)	$\{A\}$		Assumed Negation
(14)	$\{E\}$	(5)(13)	Derived clauses
(15)	$\{B\}$	(14)(8)	
(16)	$\{\neg C\}$	(13)(15)(11)	
(17)	$\{F\}$	(16)(10)	
(18)	$\{\neg E\}$	(17)(4)	
(19)	$\{\}$	(14)(18)	Contradiction
<b><i>Caterpillar is not a truth-teller.</i></b>			

2. Proof by contradiction- Attempt to prove that Bill the Lizard is a liar ( $\neg B$ ). Assume that Bill the Lizard is a truth teller (B):

(1)	$\{D, E, F\}$		Knowledge Base
(2)	$\{\neg D, \neg E\}$		
(3)	$\{\neg D, \neg F\}$		
(4)	$\{\neg E, \neg F\}$		
(5)	$\{\neg A, E\}$		
(6)	$\{\neg E, A\}$		
(7)	$\{\neg B, E\}$		
(8)	$\{\neg E, B\}$		

(9)	$\{\neg C, \neg F\}$		
(10)	$\{F, C\}$		
(11)	$\{\neg A, \neg B, \neg C\}$		
(12)	$\{A, B, C\}$		
(13)	$\{B\}$		Assumed Negation
(14)	$\{E\}$	(7)(13)	Derived clauses
(15)	$\{A\}$	(14)(6)	
(16)	$\{\neg C\}$	(13)(15)(11)	
(17)	$\{F\}$	(16)(10)	
(18)	$\{\neg E\}$	(17)(4)	
(19)	$\{\}$	(13)(18)	Contradiction

***Bill the Lizard is not a truth-teller.***

- Therefore, the Cheshire Cat is telling the truth because both others (Caterpillar and Bill the Lizard) have been proven to be liars and there must be at least one truth-teller.
- Given this new information in the knowledge base, prove who ate the salt:

(1)	$\{D, E, F\}$		Knowledge Base
(2)	$\{\neg D, \neg E\}$		
(3)	$\{\neg D, \neg F\}$		
(4)	$\{\neg E, \neg F\}$		
(5)	$\{\neg A, E\}$		
(6)	$\{\neg E, A\}$		
(7)	$\{\neg B, E\}$		
(8)	$\{\neg E, B\}$		
(9)	$\{\neg C, \neg F\}$		
(10)	$\{F, C\}$		
(11)	$\{\neg A, \neg B, \neg C\}$		
(12)	$\{A, B, C\}$		
(13)	$\{\neg A\}$		
(14)	$\{\neg B\}$		
(15)	$\{C\}$		
(16)	$\{\neg F\}$	(9)(15)	Derived clauses
(17)	$\{\neg E\}$	(8)(14)	
(18)	$\{D\}$	(1)(16)(17)	Resolution

***Caterpillar ate the salt!***