Assignment 1

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60-454 Design and Analysis of Algorithms

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Question 1. (i). We want to prove that when Algorithm Summation terminates its execution,

$$y = \sum_{j=0}^{n} a_j x^j$$

Proof. We shall apply induction to prove that after the mth iteration of the for loop the following invariant holds true:

$$y = \sum_{j=0}^{m-1} a_{n-j} x^{n-i-j}$$

(Induction basis) First, we note that y is initialized to 0 in Line 1. When $m = 1$ and $i = n$ in Line 3,

$$\begin{aligned} y &= a_n + x * y \\ &= a_n + x * 0 \\ &= a_n \\ &= a_n x^0 \\ &= \sum_{j=0}^{0} a_{n-j} x^{n-i-j} \end{aligned}$$

(Induction Hypothesis) Suppose for iteration $m - 1 < n$, $y = \sum_{j=0}^{m-2} a_{n-j} x^{n-i-j-1}$
(Induction Step) When Line 3 is executed for the mth time,

$$
\begin{aligned}
y &= a_{n-i} + x\sum_{j=0}^{m-2} a_{n-j}x^{n-i-j-1} \\
&= a_{n-i} + \sum_{j=0}^{m-2} a_{n-j}x^{n-i-j} \\
&= a_{n-i}x^0 + \sum_{j=0}^{m-2} a_{n-j}x^{n-i-j} && \text{(Induction Hypothesis)} \\
&= \sum_{j=0}^{m-1} a_{n-j}x^{n-i-j}
\end{aligned}
$$

Therefore we can conclude that the invariant $y = \sum_{j=0}^{m-1} a_{n-j}x^{n-i-j}$ holds $\forall m > 0$.

On the n+1th iteration (where $i = 0$) we then have,

$$
\begin{aligned}
y &= \sum_{j=0}^{n} a_{n-j}x^{n-j} \\
&= a_nx^n + a_{n-1}x^{n-1} + ... + a_1x^1 + a_0x^0 \\
&= \sum_{j=0}^{n} a_jx^j && \text{(By reversing the order of the terms)}
\end{aligned}
$$

□

Question 1. (ii).
Key Operation: Multiplication of real numbers.
Input size: n (The size of the input array)

The Summation algorithm must perform the key operation on each of the n elements of the input array.

1. Worst-case Time Complexity: $T(n) = n$
2. Average-case Time Complexity: $T_{ave}(n) = n$