Università
della
Svizzera
italiana

Financial decision making and the law of one price

After this lecture, you will learn

1. Valuing Decisions
   - Analyze costs and benefits
2. The Time Value of Money
   - The value of money today vs. tomorrow
3. Present Value and the NPV Decision Rule
   - Assess the relative merits of two-period projects using net present value.
4. Arbitrage and the Law of One Price
   - Define arbitrage and discuss its role.
5. No-Arbitrage and Security Prices

1. Valuing decisions: what makes financial decision "good"?

- When benefits from the decision exceed the costs.
- How to identify costs and benefits?

Example: Suppose a jewelry manufacturer has the opportunity to trade 400 ounces of silver and receive 10 ounces of gold today

First step?

- To compare the costs and benefits, we first need to convert them to a common unit

1. Valuing decisions: what makes a financial decision "good"?

- When benefits from the decision exceed the costs.
- How to identify costs and benefits?

Example: Suppose a jewelry manufacturer has the opportunity to trade 400 ounces of silver and receive 10 ounces of gold today

- To compare the costs and benefits, we first need to convert them to a common unit, $.
- For that, we need the price of silver and gold.
- Where to find them? Current market price.

1. Valuing decisions: analyzing costs and benefits

Example: Suppose a jewelry manufacturer has the opportunity to trade 400 oz of silver and receive 10 oz of gold today.

- Suppose the current market price for silver is $25 per oz.
  400 oz Silver x $25 = $10,000 today
- Suppose the current market price for gold is $1900 per oz.
  10 oz of gold x $1,900 = $19,000 today

- Benefits – costs = _____, the net value of the project is positive/negative, so the jeweler should accept/reject the trade.

Using market prices to determine value

- Competitive market prices.
  - A market in which goods can be bought and sold at the same price (zero “bid-ask spread”)

- No dispute over whether the price is fair or not.

Using market prices to determine value

Problem (Example 3.1):

You just won a radio contest, and the prize is **four tickets to a lecture about the history of accounting** (face value **€40 each**). As exciting as that sounds, you're not really interested in accounting history. Luckily, you have a second choice: **two tickets to a sold-out comedy show** by your favorite comedian (face value **€45 each**).

While checking eBay, you see that people are selling the accounting **lecture tickets for €30 each**, and the **comedy show tickets are selling for €50 each**.

Which prize should you choose?

Using market prices to determine value

Solution:

Competitive market prices, not your personal preferences (nor the face value of the tickets), are relevant here:

- Four accounting lecture tickets x $30 apiece = $120 market value
- Two comedy show tickets x $50 apiece = $100 market value

- Instead of taking the tickets to the comedy show, you should accept the accounting history lecture tickets, sell them on eBay, and use the proceeds to buy two tickets to the comedy show. You'll even have $20 left over to buy a T-shirt.

Using market prices to determine value

Problem (Example 3.2):

You are the operations manager at your firm. Due to a pre-existing contract, you have the opportunity to acquire **125 barrels of oil and 1,500 pounds of copper for a total of $12,000**. The current competitive market **price of oil is $80 per barrel** and for **copper is $4 per pound**.

You are not sure you need all of the oil and copper, and are concerned that the value of both commodities may fall in the future.

Should you take this opportunity?

Using market prices to determine value

Solution:

First, you need to convert the costs and benefits to a common unit ($) using market prices:

$$(125 \text{ barrels of oil}) \times \$80 \; per \; barrel = \$10{,}000 \text{ today}$$

$$(1500 \text{ pounds of copper}) \times \$4 \; per \; pound = \$6000 \text{ today}$$

The net value of the opportunity is _____. Because the net value is positive/negative, you should ____ it. This value depends only on the **current** market prices for oil and copper. Even if you do not need all the oil or copper, or expect their values to fall, you can sell them at current market prices and obtain their values of $16,000. Thus, the opportunity is a good one for the firm and will increase its value by $4,000.

2. Interest Rates and the Time Value of Money

- Consider an investment opportunity with the following certain cash flows. Should you take it?
  - Cost: $100,000 today
  - Benefit: $105,000 in one year

- The answer depends on the difference in value between money today vs. money in the future.

- Why is the value of money today different from tomorrow?
- Future is discounted

2. The Interest Rate: An Exchange Rate Across Time

- The rate at which we can exchange money today for money in the future is determined by the current **interest rate**
- Suppose the current annual interest rate is 7%.
- Risk-free interest rate (discount rate), r_f

  : The interest rate at which money can be borrowed/lent without risk

2. The Interest Rate: An Exchange Rate Across Time

- Interest rate factor = 1 + r_f
  - To translate today's value into **future value**
  - \$1 today x (1 + r_f) = 1 x 1.07 = \$1.07 (in one year)

- Discount factor = $\frac{1}{1 + r_f}$
  - To translate future value in **present value (PV)**
  - \$1 in one year x $\frac{1}{1 + r_f}$ = $\frac{1}{1.07}$ = \$0.93458 (today)

2. The Interest Rate: An Exchange Rate Across Time

- Value of investment **in one year**
  - Suppose the risk-free interest rate is 7%
  - We can express our future benefit as:

$$\text{Benefit} = (\$100{,}000 \text{ today}) \times 1.07 \text{ \$ in one year}$$

$$= \$107{,}000 \text{ in one year}$$

2. The Interest Rate: An Exchange Rate Across Time

- Consider an investment opportunity with the following certain cash flows. Should you take it?
  - Cost: $100,000 today
  - Benefit: $105,000 in one year

- No, because we can earn $2,000 more in one year by putting our $100,000 in the bank instead of making this investment.

2. The Interest Rate: An Exchange Rate Across Time

- Value of investment **today**
  - Suppose the risk-free interest rate is 7%
  - We can express our future benefit in today's value:

$$(\$105{,}000 \text{ in one year}) \times \frac{1}{1.07} = \$98{,}130.84 \text{ today}$$

- You can invest less than $100,000 to have $105,000 in one year. So, we should reject this investment opportunity.

Comparing Costs at Different Points in Time Problem

Go to Quiz

Comparing Costs at Different Points in Time

Solution

If the project were delayed, it would cost

$$\$3 \text{ billion} \times 1.1 = \$3.3 \text{ billion in } 2005$$

To compare this amount to $3 billion in 2004, we must convert it using the interest rate of 2%:

$$\left(\frac{\$3.3 \text{ billion in } 2005}{\$1.02}\right) = \$3.235 \text{ billion in } 2004$$

Therefore, the cost of a delay of one year was

$$\$3.235 \text{ billion} - \$3 \text{ billion} = \$235 \text{ million in } 2004$$

3. Present Value and the NPV Decision Rule

- The **Net Present Value (NPV)** of a project or investment is the difference between the present value of its benefits and the present value of its costs.

$$\text{NPV} = \text{PV(Benefits)} - \text{PV(Costs)}$$

$$\text{NPV} = \text{PV(All project cash flows)}$$

- Cash flow: the movement of money in and out of a business or a project over time.
- Money flows in when the company earns money (inflows), and money flows out when the company spends money (outflows).

3. The Net Present Value (NPV) Decision Rule

- When making an investment decision, choose the one with the highest NPV (equivalent to receiving its NPV in cash today).
  - Accept those projects with positive NPV because accepting them is equivalent to receiving their NPV in cash today.
  - Reject those projects with negative NPV because accepting them would reduce the wealth of investors.

The NPV Is Equivalent to Cash Today
Problem

Your firm needs to buy a new $9,500 copier. As part of a promotion, the manufacturer has offered to let you pay $10,000 in one year, rather than pay cash today. Suppose the risk-free interest rate is 7% per year.

Is this offer a good deal? Show that its NPV represents cash in your pocket.

[Go to Quiz]

The NPV Is Equivalent to Cash Today Solution

- First, discount the future cost to today's value

$$\text{PV}(Future\ cost\ of\ the\ offer) = \left(\frac{\$10{,}000 \text{ in one year}}{1.07}\right) = \$9345.79 \text{ today}$$

- Then, compute the NPV

The benefit is that you won't have to pay $9,500 today.

PV(Benefit) – PV(Cost) = $9500 - $9345.79 = $152.21 today

The NPV is positive, so take the deal! It is like getting a cash discount of $154.21 today.

3. The Net Present Value (NPV) Decision Rule

- Because NPV is expressed in terms of cash today, it simplifies decision making.
- Choosing Among Alternatives
  - We can also use the NPV decision rule to choose among projects
  - To do so, we must compute the NPV of each alternative, and then select the one with the highest NPV
  - This alternative is the one which will lead to the largest increase in the value of the firm

3. The Net Present Value (NPV) Decision Rule Problem

You have $10,000 to invest and are considering three one-year risk-free investment options.

1. Invest up to $10,000 in a T-Bill paying 2%.
2. Invest in a project that costs $6,000 and returns $6,100 in one year.
3. Invest in a project that costs $4,000 and returns $4,100 in one year.

How should the $10,000 investment be allocated?

3. The Net Present Value (NPV) Decision Rule Problem

Suppose you started a Web site hosting business and then decided to return to school. Now that you are back in school, you are considering selling the business within the next year. An investor has offered to buy the business for $200,000 whenever you are ready. If the interest rate is 10%, which of the following three alternatives is the best choice?

1. Sell the business now.

2. Continue running it while you are in school for one more year, and then sell the business (requiring you to spend $30,000 on expenses now, but generating $50,000 in profit at the end of the year).

3. Hire someone to manage the business while you are in school for one more year, and then sell the business (requiring you to pay $50,000 on expenses now, but generating $100,000 in profit at the end of the year).

NPV and Cash Needs

- Regardless of our preferences for cash today versus cash in the future, we should always maximize NPV first
- We can then borrow or lend to shift cash flows through time and find our most preferred pattern of cash flows

NPV and Cash Needs

- Going back to the example (r = 10%)

-	Today	In One Year	NPV
Sell Now	$200,000	0	$200,000
Scale Back Operations	–$30,000	$50,000	$-\$30{,}000 + \frac{\$250{,}000}{1.10} = \$197{,}273$
Hire a Manager	–$50,000	$100,000 $200,000	$-\$50{,}000 + \frac{\$300{,}000}{1.10} = \$222{,}727$

- #3 has the highest NPV. But, what if you need cash today ($60,000) to pay for school and other expenses? Can #1 be a better choice?

NPV and Cash Needs

- #3 vs. #1 (r = 10%)
- As long as you can borrower and lend at 10%, you can borrow today to have $60,000 you need.

	Today	In One Year
Hire a Manager	−$50,000	$300,000
Borrow	$110,000	−$121,000
Total Cash Flow	$60,000	$179,000
vs.		
Sell Now	$200,000	$0
Invest	−$140,000	$154,000
Total Cash Flow	$60,000	$154,000

4. Arbitrage and the Law of One Price

- So far, we have emphasized the importance of using competitive market prices to compute the NPV.
- But is there always only one such price? What if the same good trades for different prices in different markets?

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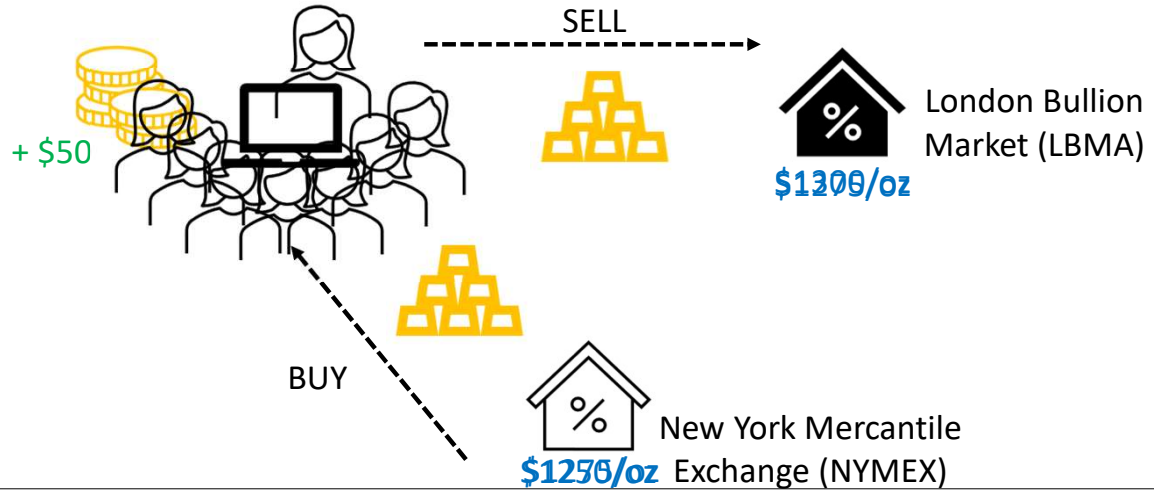


4. Arbitrage and the Law of One Price

- Arbitrage: the practice of buying and selling equivalent goods in different markets to take advantage of a price difference.
- An **arbitrage opportunity** occurs when it is possible to make a profit **without taking any risk** or making any investment.

4. Arbitrage and the Law of One Price
Is this an arbitrage opportunity?

- A trader notices that the same cryptocurrency is trading for $30,000 on one exchange and $30,050 on another exchange. The trader buys the cryptocurrency on the cheaper exchange and sells it on the more expensive one instantly, after accounting for negligible transaction costs.
- A person buys a house in a neighborhood, hoping that the value of the property will increase due to future infrastructure projects planned in the area. The buyer intends to sell the house later at a higher price.
- A market maker quotes a buy price (bid) of $0.90 and a sell price (ask) of $1.10 for a stock. The market maker is hoping to buy the stock at $0.90 and sell it at $1.10, profiting from the $0.20 spread.

4. Arbitrage and the Law of One Price

- Normal Market
  - A competitive market in which there are no arbitrage opportunities
- If the prices in the two markets differ, investors will profit immediately by buying in the market where it is cheap and selling in the market where it is expensive.
- In doing so, they will equalize the prices. As a result, prices will not differ (at least not for long).
- **Law of One Price**: If equivalent investment opportunities trade simultaneously in different competitive markets, then they must trade for the same price in all markets.

3.5 No-Arbitrage and Security Prices

No-Arbitrage Price of a Security

Price(Security) = PV(All cash flows paid by the security)

3.5 No-Arbitrage and Security Prices Problem

Consider a security that pays its owner $100 today and $100 in one year, without any risk. Suppose the risk-free interest rate is 10%.

1. What is the no-arbitrage price of the security today (before the first $100 is paid)?

2. If the security is trading for $195, what arbitrage opportunity is available?

3. At what interest rate would the arbitrage opportunity disappear?

A nerdy arbitrage joke…

"A finance professor and a student are walking down a street. The student notices a $100 bill lying on the pavement and leans down to pick it up. The finance professor immediately intervenes and says, "Don't bother; there is no free lunch. If there was a real $100 bill lying there, somebody would already have picked it up!"

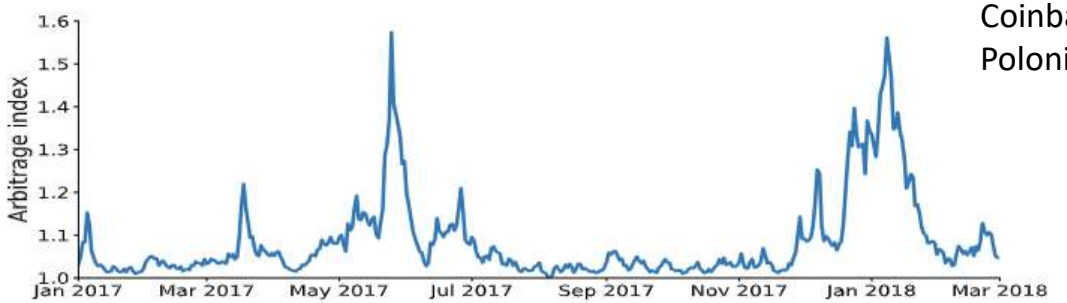
The price of Bitcoin across crypto exchanges

https://coinranking.com/coin/Qwsogvtv82FCd+bitcoin-btc/exchanges

$$Arbitrage\ index = \frac{Max\ (P_1, P_2, P_3, \dots \mathrm{P_n})}{Min\ (P_1, P_2, P_3, \dots \mathrm{P_n})}$$

N = # of exchanges

Binance, Bitfinex, bitFlyer, Bithumb, Bitstamp, Bittrex, Coinbase,Gemini, Kraken, Korbit, Poloniex, Quoine, and Zaif



Makarov and Schoar (2019) "Trading and arbitrage in cryptocurrency markets", Journal of Financial Economics 35, 2 (February 2020): 293-319.

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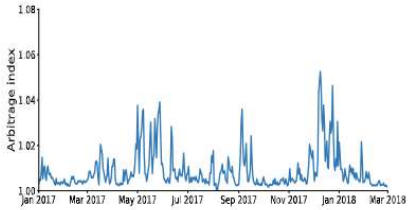
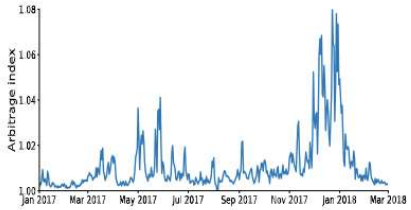
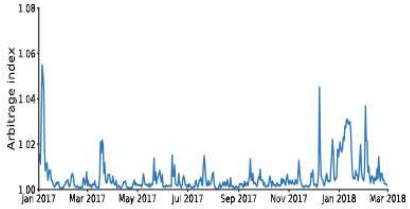


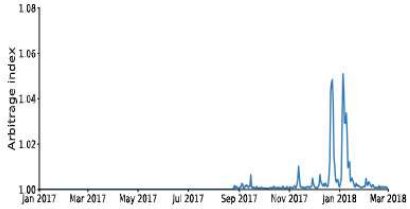
Panel A: USA



Panel B: Europe



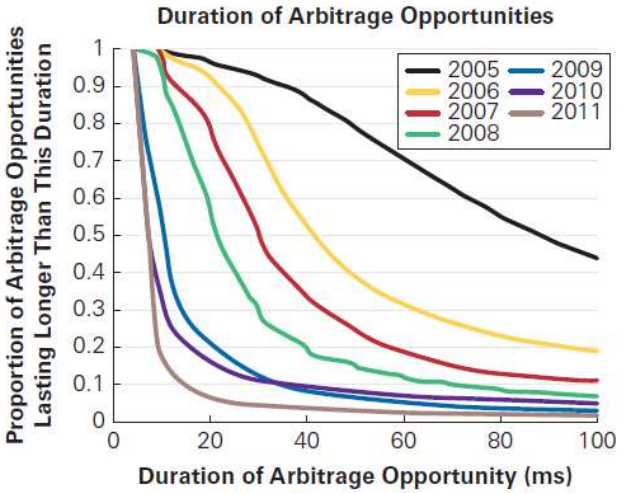
Panel C: Japan

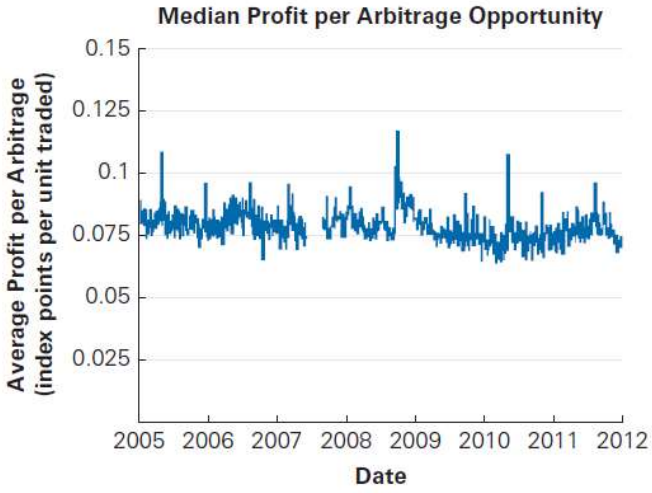


Panel D: Korea

Arbitrage in Markets

- The S&P 500 index is traded in futures exchanges in the Chicago Mercantile Exchange (CME) and as an ETF on the NYSE.
- High-frequency traders (HFTs) aim to execute trades faster than their competitors to exploit mispricings





The High-Frequency Trading Arms Race: Frequent Batch Auctions as a Market Design Response," *Quarterly Journal of Economics* (2015): 1547–1621.

Valuing a portfolio

- So far, no-arbitrage price for individual securities.
- The law of one price also applies to packages of securities (like indexes)
- Consider two securities A and B, and a third security C which has the same cash flows as A and B combined.

Value additivity

Price(C) = Price (A+B) = Price (A) + Price(B)

What if, Price(C) > Price (A+B) = Price (A) + Price(B)

Valuing a portfolio
Problem

Holbrook Holdings is a publicly traded company with only two assets: It owns 60% of Harry's Hotcakes restaurant chain and an ice hockey team. Suppose the market value of Holbrook Holdings is $160 million, and the market value of the entire Harry's Hotcakes chain (which is also publicly traded) is $120 million. What is the market value of the hockey team?

(Market value = # shares * price per share)

Hotcakes is worth $60\% \times \$120 \text{ million} = \$72 \text{ million}$, the hockey team has a value of $\$160 \text{ million} - \$72 \text{ million} = \$88 \text{ million}$.